PT. IA PHYSICS

REVISION NOTES

BY ANDY FRENCH CHRIST'S COLLEGE

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Errors
Mechanics (classical)
Kinetic Theory
Intermolecular forces
Oscillations
Waves
Fields
Special Relativity
Quantum Mechanics

Measurements, errors, units

The SI set

mass	[M]	-	kilogram	/kg
length	[L]	-	metre	/m
time	[T]	-	second	/s
electric charge		-		
flow - current	[I]	-	Amp	/A
Amount of substance		-	mole	/mol
Thermodynamic temperature		-	kelvin	/K
luminous intensity		-	candela	/cd

Fundamental units used for dimensional analysis (check physical expressions are homogenous)

Error analysis

Sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ Standard deviation $s = \frac{1}{\sqrt{n}}\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

Standard deviation of a population $\sigma = \sqrt{\frac{n}{n-1}}$

Standard deviation of means $\sigma_m = \frac{\sigma}{\sqrt{n}}$

$\Rightarrow$ Quote measurement $\bar{x} \pm \sigma_m$

Error propagation formula (in a product of independent [random] variables)

If $z = z(A, B, C, D\ldots)$ then error in z σ_z is:

$$(\sigma_z)^2 = \left(\frac{\partial z}{\partial A}\sigma_A\right)^2 + \left(\frac{\partial z}{\partial B}\sigma_B\right)^2 + \ldots = \sum_i \left(\frac{\partial z}{\partial i}\sigma_i\right)^2$$

Now if errors are quoted as fractions (or proportions)

If $z = \prod_i i^{k_i} \Rightarrow \left(\frac{\sigma_z}{z}\right)^2 = \sum_i (k_i P_i)^2$

where P_i is the fractional error in i.

For a sum of products i.e. $Z = z_1 + z_2$ we find σ_Z as $\sigma_{z_1} + \sigma_{z_2}$.

Classical Mechanics

Newton's law's of motion

I Every body continues in uniform motion in a straight line unless it is compelled to change by an action of a force

II $\underline{F} = m\underline{a}$ or more generally $\underline{F} = d\underline{p}/dt$

$\underline{F}$ = force $\quad \underline{p}$ = momentum, $m\underline{v}$. (Note directions)

III For every force there is always an equal and opposite reaction on the body which is imposing the said force.

$$\underline{F}_A(B) = -\underline{F}_B(A)$$

Dynamics of a body

(1) Linear dynamics A force $\underline{F}$ will act according to Newton II on the centre of mass of a rigid body. (Reducing a body to a 'particle' with all the mass of the body).

(2) Rotational dynamics A force applied away from the centre of mass of the body will exert a torque as well as a linear acceleration of the C.O.M. Torque is the rate of change of angular momentum.

$$\underline{L} = I\underline{\omega} = \sum_i (\underline{r}_i \wedge \underline{p}_i)$$

($\underline{L}$ = angular momentum, $\underline{\omega}$ = angular velocity, I = moment of inertia, $\underline{p}$ = momentum (linear) $\underline{r}$ = vector ⊥ to axis of rotation through C.O.M to applied force vector*)

* or velocity vector.

$$I = \sum_i m_i |\underline{r}_i|^2 \qquad \underline{\omega} = \frac{d\underline{\theta}}{dt} \qquad \underline{\theta} \parallel \underline{L}$$

$$\text{Torque} = \underline{G} = \underline{r} \wedge \underline{F} = \frac{d}{dt}\underline{L} = \frac{d}{dt}(I\underline{\omega})$$

Centre of mass

For an arbitrary axis and same procedure of $\underline{r}$ as before:

$$\bar{\underline{r}} \wedge M_{tot}\,\underline{g} = \sum_i \underline{r}_i \wedge m_i \underline{g}$$

defines centre of gravity $\bar{\underline{r}}$ of body composed of mass elements m_i. In a uniform $\underline{g}$ field this = centre of mass

Work, energy, impulse

linear work on c.o.m $\quad W_L = \int_r \underline{F}.d\underline{r}$ $\quad d\underline{r}$ = element of path r

rotational work on body $\quad W_R = \int_\theta \underline{G}.d\underline{\theta}$ $\quad d\underline{\theta}$ = angular displacement about axis

Note/ $\int \underline{F}.d\underline{r} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $\quad u$ = initial speed of com, v = final " " "

$\int \underline{G}.d\underline{\theta} = \frac{1}{2}I\omega^2 - \frac{1}{2}I\Omega^2$ $\quad \Omega$ = initial angular speed of body $\quad \omega$ = final " " " "

change of momentum = impulse

linear impulse $\quad \int \underline{F}\,dt = \Delta \underline{p}$

rot. impulse $\quad \int \underline{G}\,dt = \Delta \underline{L}$

Laws of conservation

Mass, energy, charge and momentum are all conserved in an isolated system.

Note energy can be stored in 'potential wells' → fields.

$$\frac{1}{2}mv_1^2 + \phi_1 = \frac{1}{2}mv_2^2 + \phi_2$$

where $\frac{1}{2}mv^2$ = kinetic energy $\quad \phi$ = potential energy.

Frames of reference — coordinate systems used for quantitatively describing events.

use four vectors. $[\underline{r}, t]$

see "Special Relativity" notes for standard configuration.

In classical mechanics one transforms between inertial frames (where Newton I holds → i.e. no forces (net)) using Galilean relativity.

$$x' = x - vt \qquad z' = z$$
$$y' = y \qquad t' = t$$

Note invariant nature of time.

Note a useful frame of reference for collision problems is the **zero momentum frame**:
→ we add a velocity $\underline{V}$ to all particles such that $\sum_i \underline{p}_i = 0$.

This ⇒ $$\underline{V} = -\sum_i m_i \underline{u}_i \Big/ \sum_i m_i$$

Centre of percussion — For a vertically suspended body: $x = I/mh$

x = distance below c.o.m of c.o. percussion. h = distance / height from c.o.m to pivot.

Gyroscope — Precession rate $\Omega = mgl/I\omega$

Kinetic Theory

Main Assumptions

1. Large numbers of molecules - dynamics* can be treated statistically.

* and kinematics!

2. Molecules are in continuous and rapid motion. (Random). collide elastically

3. Pressure originates from the summation of large numbers of reaction forces as molecules bounce off the body which is feeling the pressure.

For an <u>IDEAL GAS</u> we can make further assumptions

① molecules occupy a negligible fraction of total volume.

② long range forces of attraction between particles are negligible.

⇒ <u>Ideal gas law</u>: $PV = nRT$ or $PV = NkT$

(P = pressure, V = volume, n = no. of moles, R = molar gas constant, N = no. of molecules, k = Boltzmann constant, T = absolute temp.)

<u>Probability of molecules arriving in a direction $\theta \rightarrow \theta + d\theta$</u> (in 2D polars*)

$d\theta \, p(\theta) = \frac{1}{2} \sin\theta \, d\theta$

probability density function.

* Assume spherical symmetry. → applies 3D.

<u>Molecular flux on a surface within an ideal gas</u>

$N = \frac{1}{4} n \langle c \rangle$

(n = number density, $\langle c \rangle$ mean molecular speed).

<u>Microscopic definition of pressure</u>

$P = \frac{1}{3} nm \langle c^2 \rangle$

<u>Equipartition of energy</u> (classical!)

Each quadratic term in the energy expression contributes $\frac{1}{2}kT$ to the average energy $\bar{E}$.

→ must regard statistical importance of quadratic energy term to assess how $\bar{E}$ relates to the $\bar{E}$ averaged over a bulk mass of average temp T.

<u>Mean free path</u> (l)

$l = \frac{1}{\sqrt{2}\pi s^2 n}$ (s = molecular diameter)

<u>Survival probability</u>

$p(x)\,dx = e^{-x/l}$ (x = distance travelled before collision).

probability density

steps = collisions.

Random walks random walk $= l\sqrt{N}$

(N = no. of 'steps'). Note, in time t molecule makes on average $\frac{t\langle c\rangle}{l}$ steps / collisions. So random walk can be written:

$$\text{random walk} = \sqrt{l t \langle c\rangle}$$

Diffusion $$J_z = -\frac{1}{3} l \langle c\rangle \frac{dn}{dz}$$

mean free path.

(J_z = flux of gas in 'z' direction, $\frac{dn}{dz}$ = concentration gradient in z direction).

Thermal conductivity $$\dot{Q}_z = -\frac{1}{3}\rho l \langle c\rangle c_v \frac{dT}{dz}$$

(ρ = density, c_v = heat capacity, Q = heat)

Visisity $$t_x = \frac{1}{3}\rho \langle c\rangle l A \frac{dv_x}{dz}$$

(t_x = ~~[illegible]~~ force on plate of gas as a plate moves at velocity v_x a distance z above it, A = plate Area)

Boltzmann Distribution $n_s = e^{-\varepsilon/kT}$

$$\Rightarrow \quad P(\varepsilon_i) = \frac{\exp(-\varepsilon_i/kT) \leftarrow n_s}{\sum_i \exp(-\varepsilon_i/kT) \leftarrow n_{tot}}$$

(n_s = number of particles in energy state ε (of energy ε)
$P(\varepsilon_i)$ = probability of ~~[illegible]~~ finding a particle of energy ε_i)

→ example: Isothermal Atmosphere: $P = P_0 \exp\left(\frac{-mgh}{kT}\right)$
(use of Boltzmann factor $\exp(-\varepsilon/kT)$.).

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