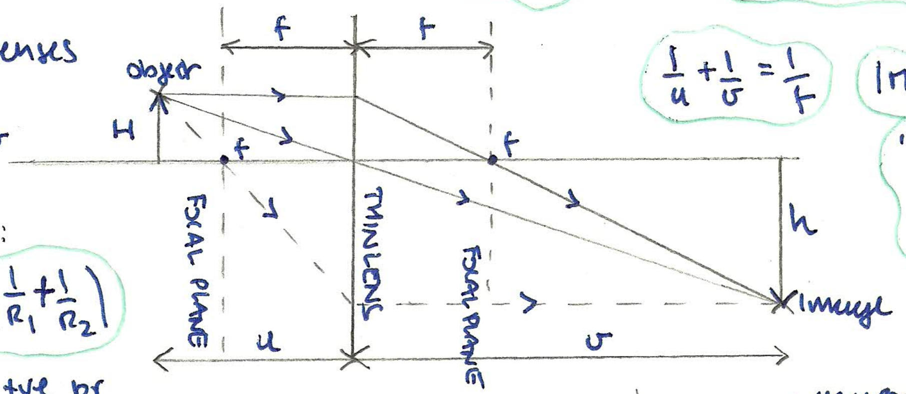
OPTICS – Classical approach

Geometric optics used to describe behaviour of most large scale optical apparatus – based on RAY TRACING (following light ray paths between points in space). No diffraction ($\lambda \to 0$ or large apertures). Fermat's principle gives ray path. "Path taken by ray is s.t. time taken is stationary w.r.t trajectory variations" i.e. $\int_{path} \frac{ds}{v_p} = \int_{path} \frac{n(s)ds}{c}$ is stationary. $n(s)$ = refractive index.

⇒ apply Euler-Lagrange equation to get path: $\int_{path} f(y, \frac{dy}{dx}; x)\,dx$ stationary

⇒ $\frac{\partial f}{\partial y} = \frac{d}{dx}\frac{\partial f}{\partial y'}$ ($y' = \frac{dy}{dx}$) Total derivative ⇒ law of reflection, law of refraction
$\theta_i = \theta_r$, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Imaging and lenses (lens is thin – small angle approx etc).



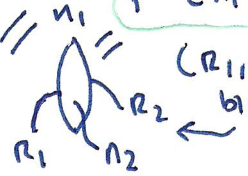
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

transverse magnification $|M_T| = \frac{|h|}{H} = \frac{v}{u}$

'Power' of lens $P = n/f$

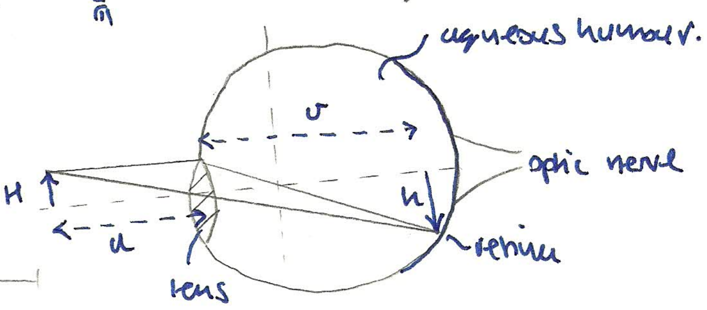
For thin lens:

$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$



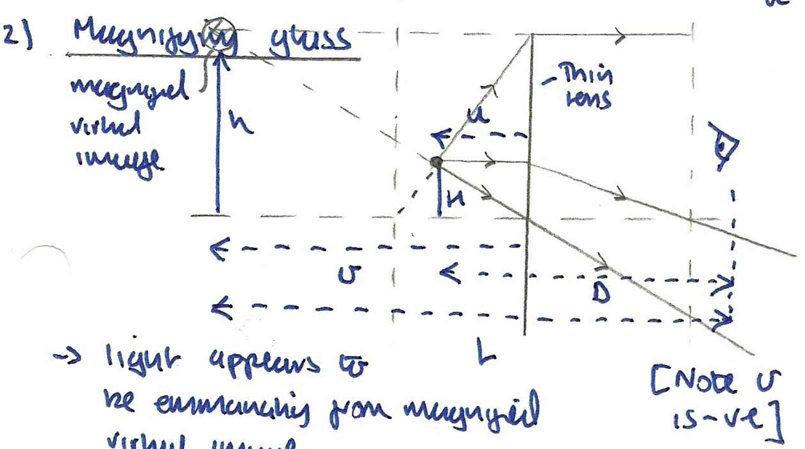
(R_1, R_2 +ve for biconvex, R_1, R_2 -ve for concave lens)(.

Thin lens optical setups: 1) eye



Adjustable focal plane.

2) Magnifying glass



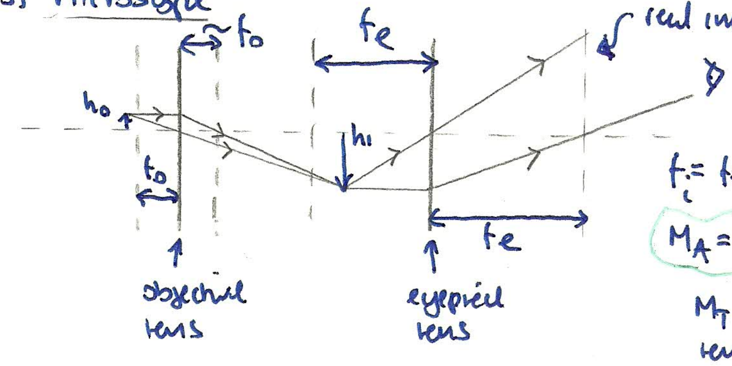
→ light appears to be emanating from magnified virtual image.

← Angular magnification is more useful here.

$M_A = \alpha_i / \alpha_o$ $\quad \alpha_i = \frac{h}{L}$ ← small angle approx

and α_o is $\frac{H}{D}$ where dimensions relate to when original object was in focus.

3) Microscope

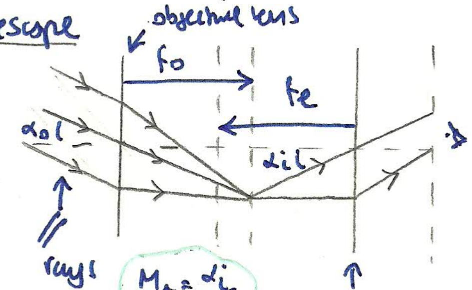


f_i = focal length;

$M_A = M_{To} * M_{Ae}$

M_T for objective lens, M_A for eyepiece.

4) Telescope



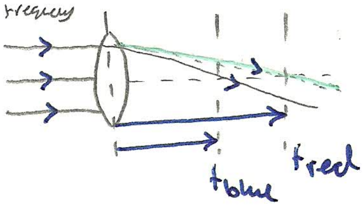
$M_A = \frac{\alpha_i}{\alpha_o} = -\frac{f_o}{f_e}$ inverted

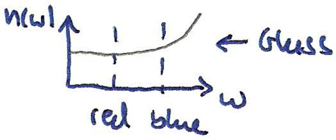
Note: ∥ rays striking thin lens will meet at same focus in the focal plane of the lens. For microscope and telescope treat focussed image from objective lens as object for objective lens.

Aberrations - not perfect images from not perfect lenses!

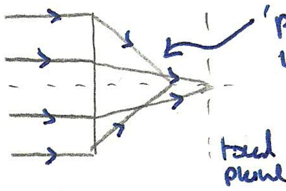
1) Chromatic. n is usually $n(f)$ so different colours will be focused at different distances from the lens.

Corrections: compound lenses with wavelength dependent refractive indices.



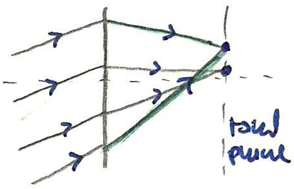


2) Spherical aberration: ← paraxial approximation / small angle approximation breaks down.

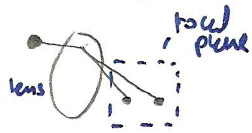


'peripheral' rays are focused before focal length - lens appears to have higher power than it should. Corrections: * aspherical lens * include a diverging lens into system * reduce lens aperture size.

3) Coma - even if spherical aberrations are removed // peripheral rays can still be focused at different points in the focal plane.



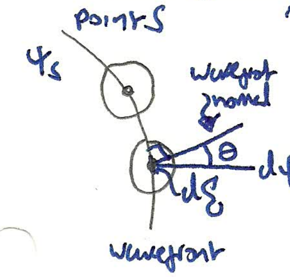
4) Astigmatism - cone of rays leaving a point on the object which is off axis will strike the lens asymmetrically - results in a focus in the vertical and horizontal directions lying in different focal planes.



5) Field curvature 6) Distortion.

[Memory aid: chromatic - splits beams, spherical - shortens f, coma - shifts vertical focus in focal plane. Astigmatism - shifts horizontal and vertical focus in focal plane].

Wave propagation and diffraction - Huygens - Fresnel / Kirchoff Integral



"Each unobstructed element $d\xi$ on a wavefront at a given instant acts as a source of secondary wavelets $d\psi_p$ with the same temporal dependence as the primary wave ψ_s"

spherical wave

$$\psi_p = A \iint_{\xi} k(\theta) \frac{\psi_s}{r} e^{ikr} \, d\xi$$

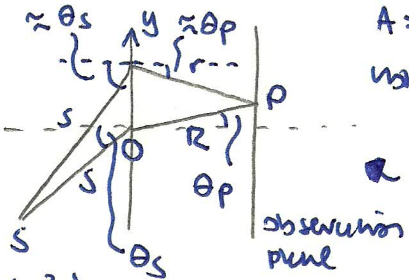
$k(\theta)$ is the obliquity factor which ensures no backward going waves. $k(\theta) = \frac{1}{2}(1+\cos\theta)$.

$A = -\frac{i}{\lambda}$ actually but not often much use.

Now if ψ_s originated from some point source s metres from S at an amplitude ψ_0: $\psi_s = \frac{\psi_0}{s} e^{iks}$.

$$\Rightarrow \psi_p = -\frac{i\psi_0}{\lambda} \iint_{\xi} k(\theta_s, \theta_p) \frac{e^{ik(s+r)}}{sr} \, d\xi$$

$$k(\theta_s, \theta_p) = \frac{1}{2}(\cos\theta_s + \cos\theta_p)$$



If ξ varies only with y dimension.

and $r(y) = R - y\sin\theta_p + \frac{y^2\cos^2\theta_p}{2R} + O\left(\frac{y^3}{R^2}\right)$ $\quad s(y) = S + y\sin\theta_s + \frac{y^2\cos^2\theta_s}{2S} + O\left(\frac{y^3}{S^2}\right)$

Now let $k(\theta_s, \theta_p) \approx 1$ and remove $\frac{1}{sr}$ dependence (assume plane waves).

$\Rightarrow \psi_p \propto \int_y \psi_s e^{ik(s+r)} dy$. Now define ψ_s as $h(y)\psi_0$. $h(y)$ is aperture function (i.e. =1 for 100% transmission, 0 0% transmission, i $\pi/2$ phase change etc).

Now phase at P is $k(s+r)$. ~~___ ___ ___~~ $\Rightarrow$ 3 regimes. 1) Fraunhofer (far field)

$\frac{ky^2}{2}\left(\frac{1}{S}+\frac{1}{R}\right) \ll 2\pi$ $\Rightarrow$ $k(s+r) = \text{const} + ky(\sin\theta_s - \sin\theta_p)$ let $q = -k(\sin\theta_s - \sin\theta_p)$

Now absorbing constants: $\Rightarrow$ $\psi_p \propto \int_y \psi_0 h(y) e^{-iqy}$ i.e. ψ_p = Fourier transform of aperture function $h(y)$ with a constant in front. (in 2D case $h(y) \to h(x,y)$ and slight modifications need to be done on the geometry).

2) Fresnel regime: $\theta_S = \theta_P$ and near field s.t. $\frac{ky^2}{2}\left(\frac{1}{S}+\frac{1}{R}\right) \geq 2\pi$

$\Rightarrow$ $k(s+r) = \text{const} + \frac{k}{2}(y^2/R)$ if SOP is now a straight line. (~~[illegible]~~).
R not same R as before - see 'more on Fresnel'.

3) v. near field regime. Kirchoff integral breaks down - need to solve Maxwell's equations.

More on Fraunhofer: can use all FT theorems like convolution.

i.e. for $h(y)$ = periodic array of top hats, spacing d, width D:

$h(y) = \; | \; | \; | \; | \; | \; * \; \sqcap$ so $FT(h(y)) = FT(||||) \times FT(\sqcap)$
$\leftrightarrow d$ $\leftarrow D \rightarrow$

i.e. a sinc modulated harmonic function ($\leftarrow$ FT of a array of δ functions)

$\downarrow$ for a grating ($h(y)$ is an infinite array of δ functions spacing d)

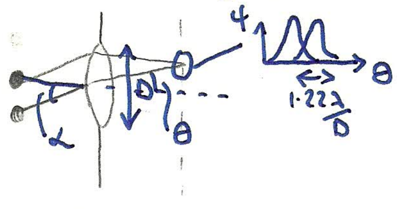
mth order maxima for λ and $\lambda+\delta\lambda$ beams are: $d\sin\theta_\lambda = m\lambda$
$d\sin\theta_{\lambda+d\lambda} = m(\lambda+\delta\lambda)$

Now if grating has ~~[illegible]~~ width W, 1st zero of the modulating sinc function occurs at λ/W. (Note W is $\gg d$ so sinc width $\ll$ periodicity of grating DP - hence sinc function from OVERALL width modulates each peak - slit width has v. large width so modulates entire D.P.).
W fixes 'beam width'

Hence 1st minimum of mth order maxima profile occurs at $\sin\theta_\lambda = \frac{m\lambda}{d} + \frac{\lambda}{W}$

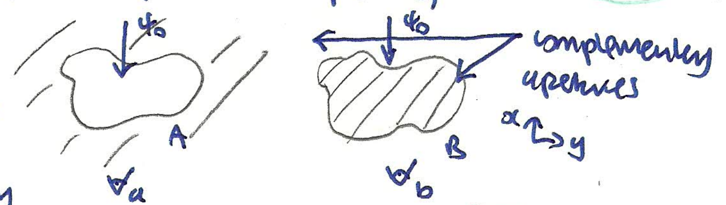
$\rightarrow$ Now ~~minimum~~ maxi minimum resolution requirement is RAYLEIGH CRITERION, i.e. maximum of $\lambda+\delta\lambda$ coincides with λ minimum (1st of mth order peak)

i.e. if $m\lambda + \frac{\lambda d}{W} = m(\lambda+\delta\lambda)$ $\Rightarrow$ $\frac{\lambda}{\delta\lambda} = \frac{mW}{d} = mN$. i.e. chromatic resolving power = highest m $\times$ number of slits in grating. Now can apply Rayleigh criterion to resolution of two objects - i.e. distant stars by a telescope. Slit width / aperture width modulates entire D.P. For a telescope or viewing apparatus - single large slit but 'sinc' modulation with eventually reduce periphery intensity. For two objects of angular separation α - assuming Fraunhofer conditions; objects will be resolved if $\alpha > \beta\lambda/D$ (λ = wavelength, D = diameter of objective lens, $\beta = 1.22$ for circular apertures).



Babinet's Principle

In Fraunhofer conditions:



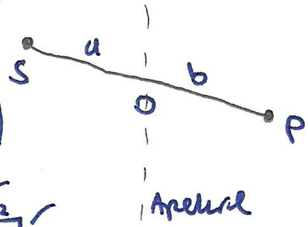
$\psi_a = \psi_0 \int_A e^{-i(px+qy)}\,dx\,dy$

$\psi_b = \psi_0 \left\{ \int_{\text{all space}} e^{-i(px+qy)}\,dx\,dy - \int_A e^{-i(px+qy)}\,dx\,dy \right\} = \delta(p,q) - \psi_a$

So except at the origin ($p=q=0$) $\Rightarrow$ $I_a = I_b$ i.e. same D.P. for complementary apertures.

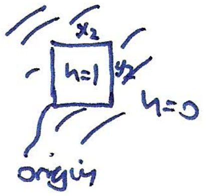
More on <u>Fresnel regime</u> – when SOP is a straight line and $\frac{ky^2}{2}\left(\frac{1}{S}+\frac{1}{R}\right) \geq 2\pi$ s.t. we cannot ignore the quadratic terms in optical path we have Fresnel regime i.e. phase is quadratic in y.

In this case $\psi_p \propto \int_\Sigma h(x,y)\, e^{ik\left(\frac{x^2+y^2}{2R}\right)} dx\,dy \rightarrow \left(R^{-1} = \frac{1}{a}+\frac{1}{b}\right)$



if $K(\theta) \approx 1$ by small angle approx.

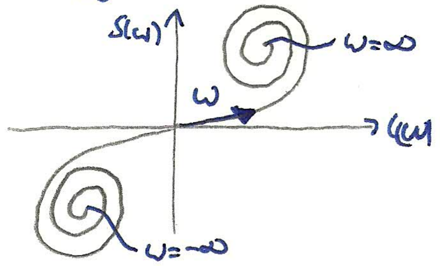
if $h(x) = X(x)\,Y(y)$, letting $u = x\sqrt{\frac{2}{\lambda R}}$ $v = y\sqrt{\frac{2}{\lambda R}}$ for



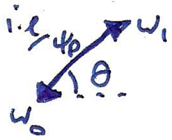
with x_1, y_1 width rectangular aperture:

$\Rightarrow \psi_p \propto \int_0^{x_1} e^{i\pi u^2/2} \int_0^{y_1} e^{i\pi v^2/2}\, du\,dv$ Now $\int_{\omega_0}^{\omega_1} e^{i\pi u^2/2} du = \left[C(\omega) + iS(\omega)\right]_{\omega_0}^{\omega_1}$

= 'Cornu spiral'.



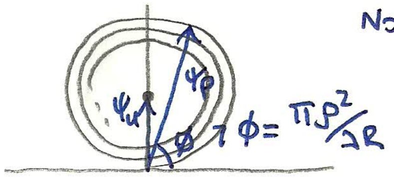
i.e. ψ_p is a spanning vector between ω points on the spiral.* i.e. Cornu spiral is an amplitude / phase diagram. i.e.



gives amplitude of ψ_p phase θ.

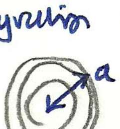
* which correspond to limits of edge of aperture w.r.t origin – note that this may be moveable since SOP must be a straight line for Fresnel condition. Now if aperture is circular:

$x^2+y^2 = \rho^2$, $dx\,dy = 2\pi\rho\, d\rho$ (ρ = aperture radius) $\Rightarrow \psi_p$ is a circularly decaying spanning vector with a circularly spiralling phase variation. ($K(\theta)$ slowly reduces ψ_p).



Note for unobstructed incident waves $\psi_p = \psi_u$ corresponding to $\rho = \infty$. when $\phi = n_{odd}\pi$ $\psi_p = 2\psi_u$. (until $K(\theta)$ becomes significant for high angles) <1.

<u>Poisson's Spot</u> – for a circular obstruction of radius a one still observes a diffracted spot $\simeq \psi_u$ at the 'origin' (SOP // optic axis ⊥ to obstruction) → because integration regime is $\phi = \frac{\pi a^2}{\lambda R}$ to ∞ get a non zero spanning vector



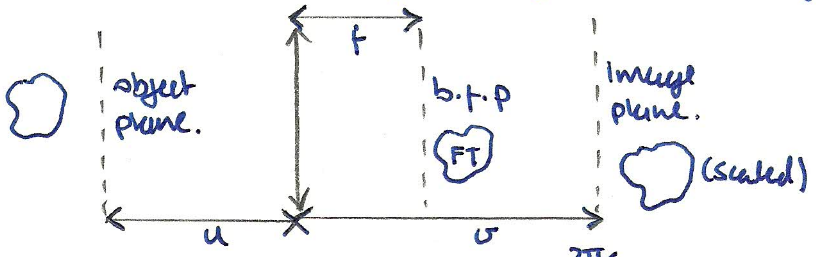
hence intensity. ($\propto |\psi_p|^2$). Fresnel zones occur between integers of phase variation. i.e. nth zone is $(n-1)\pi \leq \phi \leq n\pi$.

* Areas are same for each zone since $\pi(\rho_n^2 - \rho_{n-1}^2) = \lambda R$. Now form zone plate by blocking all alternate zones. This increases amplitude as 'amplitude reducing' phase contributions are ommitted in Fresnel integral. Net result is 'zone plate' acting as a LENS. (Highly chromatic as $f \propto \frac{1}{\lambda}$).

i.e. for N zones, amplitude is $2N\psi_u$ ($\Rightarrow I_p \approx 4N^2 I_u$) and if incident rays are ∥, $R = f = \frac{\rho_n^2}{n\lambda}$. Now $N = \frac{\pi a^2}{\lambda R}$ (a = radius of zone plate). So at different R, 'focus point' see's a different zone number. Hence when $R = n_{odd}^{-1}\frac{\pi a^2}{\lambda}$ we have a maximal. i.e. principle maximum at $n_{odd} = 1$, nth at $\frac{\pi a^2}{n}$. Note $K(\theta)$ becomes significant for high N so expect intensity to drop as N becomes n_{odd} high.

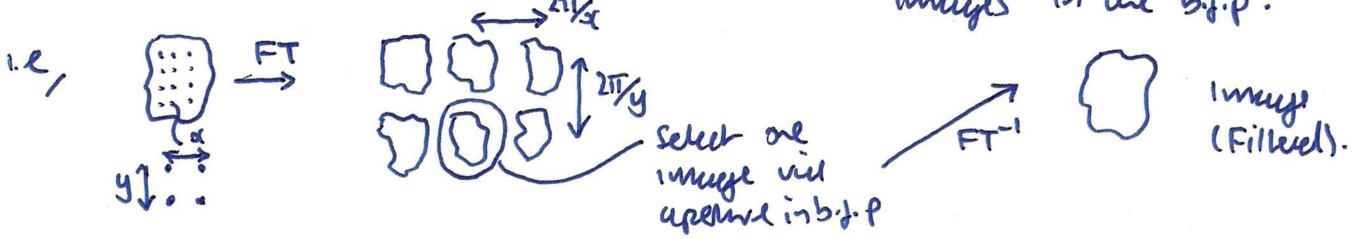
Image Formation – Abbé Theory

– Image formed in back focal plane of lens is Fourier transform of original image.



Note we can exploit this fact by filtering periodic variations in the object i.e, line lines in a TV picture via selecting proportions of constituent images in the b.f.p.

i.e,



Thermodynamics Formulae to remember.

1st law: $dU = dq + dw$

internal energy change of system (dU); heat input to system (dq); work done on system (dw)

U is a state function; q, w are path functions

$dU = C_V dT$
$dw = -pdV$

$\left[C_V = \frac{\partial q}{\partial T}\Big|_V \right.$

Entropy $dS = dq_{rev}/T$

S is a state function

S = k ln g
$U_f = F = U - TS$
↑ free energy. useful in stat mech.

⇒ rewrite 1st law: $C_V dT = TdS - pdV$

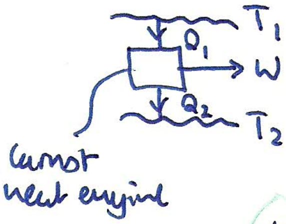
pick reversible paths

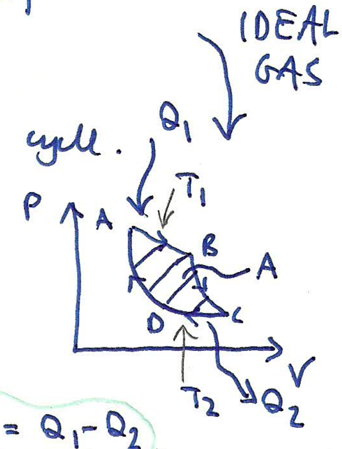
Also: $C_p = \frac{\partial q}{\partial T}\Big|_p$ ⇒ for ideal gas ($PV = nRT$) $C_p = C_V + Rn$.

Carnot cycle is reversible and most efficient heat cycle.

Efficiency (general) $= \eta = W/Q_1$

$= 1 - T_2/T_1$ for Carnot cycle.





A–B : Isothermal expansion @ T_1
B–C : Adiabatic expansion $\pi\left\{ \begin{array}{l} PV^{\gamma} = \text{constant} \\ p^{1-\gamma}T^{\gamma} = \text{constant} \\ TV^{\gamma-1} = \text{constant} \end{array} \right.$
C–D : Isothermal compression
D–A : Adiabatic compression

Find Q_1, Q_2 in terms of T_1, T_2 only. Then find $\eta = \frac{Q_1 - Q_2}{Q_1}$

$\text{Work} = Q_1 - Q_2 = \int_A^A pdV$

No dU overall in process A→A.

Kelvin's statement : "No process whose sole purpose is heat → work" ↕
Clausius " : "No process can transfer heat from hot to cold body"

2nd law: $dS_{universe} \geq 0$ for any spontaneous process.