NUCLEAR PHYSICS

1) Definitions * Nucleus consists of Z protons N neutrons. Mass # $A = Z + N$. Denote nuclide by ${}^{A}_{Z}X$, X = chemical symbol.

→ Isotopes – same Z
→ Isobars – same A
→ Isotones – same N

* Spin of nuclei: odd A ⇒ $\frac{1}{2}$ integer spin; Even A ⇒ integer spin; Even Z + Even N J = 0 always (J = total nuclear spin)

* Parity of a system of two nuclei $P = P_1 P_2 (-1)^{L_{12}}$ P_1, P_2 are the respective parities, L_{12} is the total orbital A.M of the system. PARITY CONSERVED IN NUCLEAR PROCESSES

* Binding Energy – Energy B required to split nucleus into constituent nucleons $B = c^2\left(\sum_{\text{nucleons}} m_n - m_{\text{nucleus}}\right)$

* Cross sections – several useful def^n.

① – $\sigma = \dfrac{\#\text{ particles scattered/unit time}}{\text{incident flux}}$

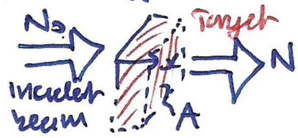
② – σ = Effective area seen by incoming particles i.e totally reflective nuclear area.

③ – (Differential cross section) $\dfrac{d\sigma}{d\Omega} = \dfrac{\#\text{ particles scattered/unit time INTO } d\Omega}{\text{INCIDENT FLUX} \cdot d\Omega}$

④ – $\sigma = \dfrac{\text{Reaction rate } \Gamma_{i\to j}}{\text{incident flux}}$

② useful for calculating attenuation coefficients

" Fraction of particle beam scattered = Fraction of area A blocked "



particles scattered in thickness dx (dN -ve)

$$-\frac{dN}{N} = \frac{n_0 A dx}{A}\sigma$$

density of nuclei in target

$\therefore \int_{N_0}^{N} -\frac{dN}{N} = \int_0^x \sigma n_0 \, dx \Rightarrow N = N_0 e^{-\sigma n_0 x}$

i.e for finite thickness of target x.

Quote σ in units of BARNS $1b = 10^{-28} m^2$

* Electric moments $E_n = \int \psi^* r^n P_n \psi \, d^3\underline{r}$ legendre polynomial.

$E_0 = \int \psi^* \psi \, d^3\underline{r} = Ze$ (charge) $E_1 = \int \psi^* z \psi \, d^3\underline{r}$ Electric dipole moment zero for all nuclei

$E_2 = \int \psi^* (3z^2 - r^2) \psi \, d^3\underline{r} = Q_0 e$ ELECTRIC QUADROPOLE MOMENT If spherically sym.

$z^2 = r^2/3 \Rightarrow Q_0 = 0$. All J = 0 nuclei $Q = 0$

Q +ve PROLATE

Q -ve OBLATE

← More energetically favourable. Spins line up with major axis.

2) Radioactivity * Decay law

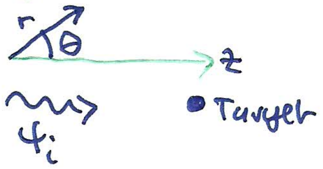
– Probability of decay in time dt = λdt (λ = decay constant). If N(t) nuclei in existence $-\frac{dN}{N} = \lambda dt$. If more than one decay mode $-\frac{dN}{N} = \sum_i \lambda_i dt$

Define MEAN LIFETIME τ = Σ all lifetimes / Total # of atoms. Now $N = N_0 e^{-\lambda t}$

So $\tau = \frac{1}{N_0} \sum_{N=0}^{N_0} \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right) \longrightarrow \frac{1}{N_0 \lambda} \int_0^{N_0} \ln\left(\frac{N_0}{N}\right) dN = 1/\lambda$ So $\tau = 1/\lambda$

Define ACTIVITY $\left|\frac{dN}{dt}\right| = A(t)$

3) Nucleon - Nucleon Scattering



Incoming particle has wavefunction $\psi_i = e^{ikz}$ (ignore time dependence)

$\psi_i = e^{ikr\cos\theta}$ $k = \frac{1}{\hbar}\sqrt{2mE}$

→ expand ψ_i in spherical harmonics and take $l = 0$ term (assume low energies)

$\therefore \psi_i \simeq \dfrac{e^{ikr} - e^{-ikr}}{2ikr}$ (incoming, outgoing spherical waves) Now scattering due to nuclear potential can only change phase since it is deemed to scatter elastically.

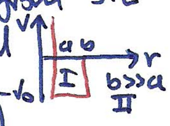
$\therefore$ outgoing wave of particle $\psi_0 = \dfrac{e^{i(kr + 2\delta_0)} - e^{-ikr}}{2ikr}$ phase change (change phase of incoming part – e^{-ikr} can't be affected before it reaches target potential)

$\therefore \psi_{\text{scattered}} = \psi_0 - \psi_i = \frac{1}{kr}\frac{1}{2i} e^{i(kr + \delta_0)}\left(e^{i\delta_0} - e^{-ikr-ikr-2i\delta_0} - e^{ikr-ikr-2i\delta_0} + e^{ikr-ikr-i\delta_0}\right)$

$= \frac{1}{kr} e^{i(kr + \delta_0)} \sin\delta_0$

$\therefore \dfrac{d\sigma}{d\Omega} = \dfrac{|\psi_{\text{scattered}}|^2 r^2 d\Omega}{d\Omega}$ ← # particles scattered / unit time into dΩ / incident flux · dΩ $= \dfrac{\sin^2\delta_0}{k^2}$

Find δ_0 from S.E for potential



I – $\psi = A\sin k'r$ II – $\psi = A\sin[k(r + \delta)]$

$k' = \frac{1}{\hbar}\sqrt{2m(V_0 + E)}$
$k = \frac{1}{\hbar}\sqrt{2mE}$

$k'\cot k'b = k\cot(kb + \delta)$

* Define Scattering length a by $\lim_{k\to 0} \sigma = 4\pi a^2$ or $a = -\lim_{k\to 0} \frac{\sin\delta_0}{k}$

Note $\lim_{k\to 0,\ \delta_0\to 0} \psi_{scattered} = a\frac{e^{ikr}}{r}$

Note for COHERANT scattering $\frac{2\pi}{k} \sim$ atomic nuclear dimension find total cross section by adding scattering lengths and then squaring.

i.e, $\sigma \propto \left(\sum_i a_i\right)^2$ For incoherent scattering can ignore cross terms (no interference). QED QCD

→ To do scattering properly need to use Born Approximation (QM ⑤) and Matrix elements and Fermi's Golden rule in general.

4) Liquid Drop Model for nuclear masses and SEMI EMPIRICAL MASS FORMULA
- nuclear force short range (cf to intermolecular forces)
- Density independent of nuclear size
- Binding energy/A largely constant (cf to heat required to evaporate fixed mass independent of "drop" size).

SEMF $M_X = \sum m_p + m_n - B = Zm_p + Nm_n - B$

$M_X = Zm_p + Nm_n - a_V A + a_S A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_A\frac{(A-2Z)^2}{A} + \delta(A,Z)$ $= -B$

Easier to think of $B = \ldots\ldots$ and what terms do to B.

Volume term $B/A \approx$ constant same sign as B

Surface term Near surface nucleons not bound so much ∴ depends on S.A $\sim R^2 \sim A^{2/3}$ - reduces B so opposite sign

Proton coulomb repulsion (opposite sign to B) $V(R) = \frac{Z^2e^2}{R}$ $\sim \frac{Z^2}{A^{1/3}}$

Stable nuclei when N=Z less stable when N>Z (so reduce B) Effect reduces as A increases hence 1/A term.

Pairing term reflects stability of paired nucleons.
$+\delta$ Z even, N even
0 even, odd
$-\delta$ odd odd

5) Shell Model - obtain energy levels by solving S.E for a spherically symmetric potential which is short range i.e of nuclear scale
↳ Then add spin orbit interaction as a perturbation i.e extra term in energy $\propto \langle\psi|\underline{L}\cdot\underline{S}|\psi\rangle \propto \frac{1}{2}(j(j+1) - l(l+1) - s(s+1))$ for ONE nucleon of l, s.

→ Predicts stability associated with "Magic #'s" $Z = 2, 8, 20, 28, 50, 82$
$N = 2, 8, 20, 28, 50, 82, 126$

↳ use w.f to predict Q_0 (Quadrupole moments) though obviously no good since assumed spherical nuclei!

6) Improvements on the shell model: Rotations and vibrations

* Rotations Energy of rigid rotor $E = \frac{1}{2}I\omega^2$ A.M $L = I\omega$ so $E = \frac{L^2}{2I} = \frac{J(J+1)\hbar^2}{2I}$ using J^2 for QM.

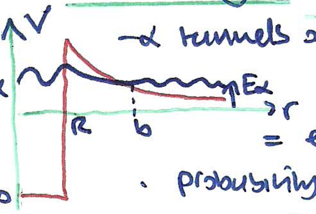
* Vibrations 3 types radial oscillation COM vibrates

Get "Giant dipole resonance" when n, p vibrate against each other.

Quadrupole vibration - Treat as phonon modes. Two units of A.M.

7) α-Decay $^A_Z X \to {}^{A-4}_{Z-2}Y + \alpha$ $+ Q$

Energy released, $Q = (B_Y + B_\alpha) - B_X$ (To show draw mass cycle)
$(Q = m_Xc^2 - m_Yc^2 - m_\alpha c^2$
$= -B_X + B_Y + B_\alpha$
using $M_i = \sum n,p$ masses $- B_i)$



- α tunnels out of V(r)

Decay probability $= \lambda$
= escape trial frequency (f)
· probability of tunnelling through barrier (P).

Now transmission prob for V_0 barrier of width Δ: $P_\Delta \sim e^{-2k\Delta}$ where $k = [2m(V_0 - E)]^{1/2}/\hbar$

m = reduced mass $\frac{M_\alpha M_Y}{M_\alpha + M_Y} = m$

∴ approx as $\prod$ ∴ $P = \prod e^{-2k_i\Delta}$

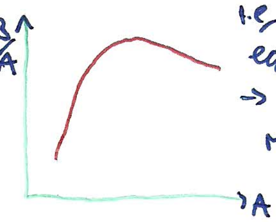
$= \exp\left[-2\int_R^b \left[\frac{2m(V(r) - E_\alpha)}{\hbar^2}\right]^{\frac{1}{2}} dr\right] = e^{-2G}$ Gamow factor

NP②

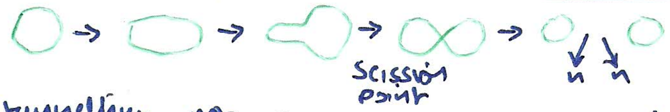
Now $f \sim \frac{v}{R}$ where v = speed in nucleus ↳ classical model $v = \sqrt{2E_\alpha/m_\alpha}$

So $\lambda = \frac{1}{R}\sqrt{\frac{2E_\alpha}{m_\alpha}}\, e^{-2G}$ if $R/b \ll 1 \Rightarrow G \sim \frac{1}{\sqrt{E_\alpha}}$ $\Rightarrow \ln\left(\frac{1}{\lambda}\right) \approx E_\alpha^{-\frac{1}{2}} + \text{const.}$ GEIGER NUTTAL LAW

8) Nuclear Fission $X \to Y + Z + \nu n$ ← prompt neutrons. Will occur if energy released > 0 i.e. $B_Y + B_Z + \nu B'_n - B_X > 0$ B'_n is K.E of n s.t. $m_n + \frac{B'_n}{c^2} = m'_n$ ← effective mass of prompt neutron.
i.e. want to minimise B_X for fission to occur



i.e. use largest abundant nuclei. Now SEMF ⇒ fission into equal fragments energetically possible for $A \gtrsim 90$ but this is not observed.
→ FISSION BARRIER. No spontaneous fission for $A \lesssim 240$.
Mechanism:



No fission if nucleus single or small deformations

Now QM tunnelling not a good mechanism for fission since $\lambda \sim e^{-2G}$ and $G \sim m^{\frac{1}{2}}$. Since m large for fission ⇒ prob v. low. Use SEMF instead. Consider deformation

if $a = R(1+\varepsilon)$ $b = \frac{R}{(1+\varepsilon)^{\frac{1}{2}}}$ (using volume of an ellipsoid $= \frac{4}{3}\pi abc$ where $c = b$ in this case) i.e. volume constant.

New B $(= B')$ $= -Zm_p - Nm_n + a_V A - a_S A^{2/3}\left(1 + \frac{2}{5}\varepsilon^2\right)$ ↓ change in S.Area $- a_C \frac{Z^2}{A^{1/3}}\left(1 - \frac{\varepsilon^2}{5}\right)$ ↓ change in average radius $- a_A \frac{(A-2Z)^2}{A} \pm \delta(A,B)$

→ Fissile nucleus occurs when $B' - B < 0$

i.e. $\frac{\varepsilon^2}{5}\frac{a_C Z^2}{A^{1/3}} > a_S A^{2/3}\frac{2}{5}\varepsilon^2$ ⇒ $\frac{Z^2}{A} > 2a_S/a_C$ ⇒ $\frac{Z^2}{A} > 50$ To get nucleus e.g. U^{235}_{92} to fission bombard with thermal neutrons. However, U^{235}_{92} rare compared to U^{238}_{92}.
However U^{238} can be used with 'FAST BREEDER' reactions. $^{238}U + n \to {}^{239}U$ γ to create $\xrightarrow[\beta]{} {}^{239}Np \xrightarrow[\beta]{} {}^{239}Pu$ ^{239}Pu is fissile. Use fuel 20% ^{239}Pu and 80% ^{238}U
In each reaction release $\geq$ n produced in each reaction relative to n

chain reaction. (i.e. at least one n put in). classify $\frac{\#n_{out}}{\#n_{in}} \equiv N$

$N < 0$ subcritical
$N = 0$ critical
$N > 0$ supercritical

– control speed of n using MODERATOR (light nucleus ^{12}C, D_2O) – control # n using high σ rods of Cd.

- Design reactor to be subcritical to prompt n so can operate control rods mechanically in good time to avoid $N > 0$. ⇒ explosion!

9) β decay (β⁻) $^A_Z X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$ (β⁺) $^A_Z X \to {}^{A}_{Z-1}Y + e^+ + \nu_e$

and outside nucleus.... $n \to p + e^- + \bar{\nu}$ Also inside: $p + e^- \to n + \nu_e$
Fermi theory. Start from F.G.R $\lambda = \frac{2\pi}{\hbar}|M_{if}|^2 g(E)$ ⇒ $^A_Z X \to {}^{A}_{Z-1}Y + \nu_e$ Electron capture E.C
Assume matrix element $M_{if} = \langle \psi_i | H' | \psi_f \rangle = G\langle \psi_i | \psi_f \rangle$ (G constant)
→ get SARGENT RULE for leptonic decay $\lambda_{e^\pm} = \frac{G_F^2 m_{e^\pm}^5}{192\pi^3}$

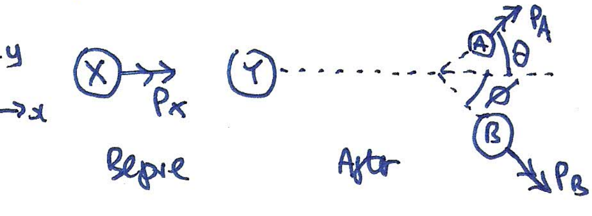
Gamow Teller transitions occur when total spin of $e^\pm$ $\nu_e = 1$. Fermi transitions occur when total spin $= 0$.

so for "Allowed": Fermi $\Delta L = 0$ $S = 0$ $\Delta J = 0$
GT $\Delta L = 0$ $S = 1$ $\Delta J = \pm 1$, $0 \not\to 0$

If $\Delta L \neq 0$ get 1st forbidden, 2nd forbidden transitions – i.e. increasingly unlikely. ($\Delta L = 1$ ⇒ parity change ......)

10) γ Decay → see electric dipole transitions in QM.

11) Nuclear reactions – treat classically, conserve * energy * linear momentum * A.M * charge * # nucleons use Q = energy released $= (M_{reactants} - M_{products})c^2$ = change in K.E i.e. $T_{products} - T_{reactants}$. consider reaction $X + Y \to A + B$ in frame where Y is initially stationary.



- momentum conservation: $P_x = P_A\cos\theta + P_B\cos\phi$
$0 = P_A\sin\theta - P_B\sin\phi$
⇒ $\sin\phi = \frac{P_A\sin\theta}{P_B}$ ⇒ $\cos\phi = \left[1 - \frac{P_A}{P_B}\sin\theta\right]^{\frac{1}{2}}$ Want to remove P_B, T_B φ.....
⇒ $P_B^2 = P_x^2 + P_A^2 - 2P_xP_A\cos\theta$. $Q = T_A + T_B - T_x$

$$\Rightarrow Q = T_A - T_x + \frac{p_B^2}{2m_B} = T_A\left(1+\frac{m_A}{m_B}\right) + T_x\left(\frac{m_x}{m_B}-1\right) - \frac{2\cos\theta}{m_B}\sqrt{m_A m_x T_A T_x}$$

use F.G.R and $\sigma_{i-f} \propto \lambda_{i\to f}$ for the initial / final state of a particle

Now hermiticity of F.G.R matrix elements $\Rightarrow \dfrac{\sigma_{i-f}}{\sigma_{f-i}} = \dfrac{g_f(E)}{g_i(E)}$

Now in k space $g(E_k) = \dfrac{\sigma V}{8\pi^3} 4\pi k^2$

where σ = spin degeneracy factor
V = volume. - note momentum $= \hbar k$

so $\dfrac{\sigma_{i\to f}}{\sigma_{f\to i}} = \dfrac{\sigma_f p_f^2}{\sigma_i p_i^2}$ if $V_i = V_f$.

12) Breit - Wigner consider reaction via an intermediate compound nucleus.

$X + Y \xrightarrow{(1)} Z \xrightarrow{(2)} A + B$ stage (1) independent of stage (2)

$$\therefore \sigma(X+Y\to A+B) = \sigma(X+Y\to Z)\frac{\Gamma(Z\to A+B)}{\Gamma(Z\to \text{Anything else})}$$

Wavefunction for state Z is $\psi(t) = \psi(0)e^{-iE_0t/\hbar}e^{-\Gamma t/2\hbar}$
i.e. Z has energy E_0 and decays at rate $\Gamma/\hbar$

So write $\psi(t)$ as a superposition of stationary states of energy E $\Rightarrow \psi(t) = \int A(E)e^{-iEt/\hbar}\,dE$ - Fourier transform of $A(E)$

$$\Rightarrow A(E) = \int \psi(t)e^{iEt/\hbar}\frac{dt}{\hbar} = \frac{-\psi(0)}{i\left[\frac{(E-E_0)}{\hbar} + \frac{i\Gamma}{2\hbar}\right]\hbar} \Rightarrow |A(E)|^2 = \frac{|\psi|^2}{(E_0-E)^2 + \frac{\Gamma^2}{4}}$$

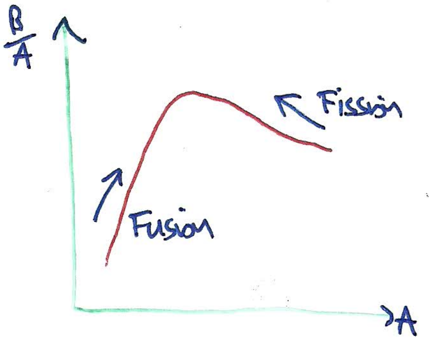
(using t variable $\frac{t}{\hbar}$ $\Rightarrow \frac{1}{\hbar}$ in dt
Note $\psi(\frac{t}{\hbar}) = \psi(t)$)

$$\Rightarrow \sigma(X+Y\to Z\to A+B) = \frac{\pi ƛ^2 g\,\Gamma_{XY}\Gamma_{AB}}{(E_0-E)^2 + \Gamma^2/4} \qquad g = \frac{2J_Z+1}{(2S_X+1)(2S_Y+1)} \qquad ƛ = \frac{\hbar}{p_a^*}$$

in c.o.m system.
density of states F.G.R.

13) Nuclear Fusion

whereas fission aims to increase binding energy of system and thus release energy by breaking up large nuclei - fusion joins small nuclei to achieve the same aim.



$Q = B_{products} - B_{reactants}$
$\therefore$ increase Q by increasing $B_{products}$.

For fusion need to overcome Coulomb barrier
This is ~ 1 MeV
Proof: $E_{coulomb} \sim \dfrac{e^2}{4\pi\varepsilon_0 R}$ let R be $\sim 10^{-15}$ m (nuclear radius)

$\Rightarrow E_{coulomb} \sim \dfrac{(1.6\times10^{-19})^2}{4\pi\times8.85\times10^{-12}.10^{-15}} \sim 1.4$ MeV

In Sun: $T \sim 10^7$ K $\Rightarrow k_BT \sim 1$ keV. So require Q.M tunnelling to fuse nuclei

common fusion reaction $p + p \to {}^2H + e^+ + \nu_e$ $\quad Q = 0.42$ MeV