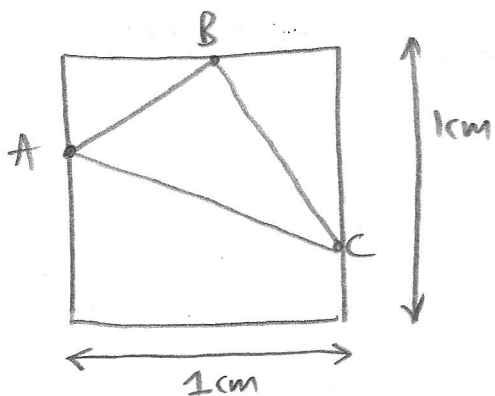


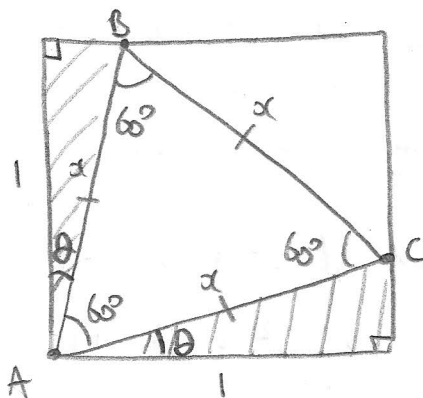
STEP I 2001

1/



Points A, B, C lie on different sides of a 1 cm x 1 cm square and form a triangle ABC.

The largest possible value of the smallest side is when all three sides of ABC are equal i.e. ABC is an equilateral triangle. Let the 'largest, smallest side' = α

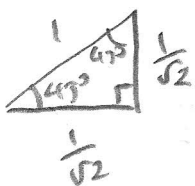


Since $AB = AC$ then shaded triangles must be congruent.

$$\begin{aligned} \therefore 2\theta + 60^\circ &= 90^\circ \\ \Rightarrow \theta &= 15^\circ \end{aligned}$$

Hence $\alpha \cos 15^\circ = 1 \quad \therefore \alpha = \frac{1}{\cos 15^\circ}$

$$\begin{aligned} \text{Now } \cos 15^\circ &= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \end{aligned}$$



$$\therefore \alpha = \frac{2\sqrt{2}}{1 + \sqrt{3}} = \frac{2\sqrt{2}(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} = \frac{2\sqrt{2} - 2\sqrt{6}}{1 - 3} = \frac{2\sqrt{6} - 2\sqrt{2}}{2}$$

$$\therefore \alpha = \sqrt{6} - \sqrt{2} \text{ as required}$$

2/ (i) $1+2x-x^2 > \frac{2}{x}$ ($x \neq 0$)

Since $x^2 > 0$, multiplying both sides by x^2 doesn't change the sign of the inequality

$$x^2(1+2x-x^2) > 2x$$

$$x^2 + 2x^3 - x^4 > 2x$$

$$0 > x^4 - 2x^3 - x^2 + 2x$$

consider $f(x) = x^4 - 2x^3 - x^2 + 2x$ [\therefore inequality above $\Rightarrow f(x) < 0$]

(*) $f(x) = x(x^3 - 2x^2 - x + 2)$ so $x=0$ is a factor of $f(x)$

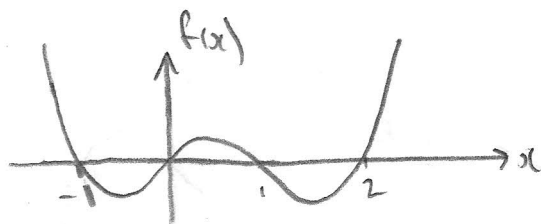
$f(1) = 1(1-2-1+2) = 0 \therefore x-1$ is a factor of $f(x)$

$$\begin{aligned} \therefore f(x) &= x(x-1)(x^2 + ax - 2) \\ &= x(x^3 - x^2 + ax^2 - ax - 2x + 2) \\ &= x(x^3 + x^2(a-1) - x(a+2) + 2) \end{aligned}$$

comparing coefficients with (*) $a-1 = -2$ [x^2] $\Rightarrow a = -1$
 $a+2 = 1$ [x] $\Rightarrow a = -1$

$$f(x) = x(x-1)(x^2 - x - 2)$$

$$f(x) = x(x-1)(x+1)(x-2)$$



$f(x) < 0$ means

$$\begin{cases} -1 < x < 1 \\ 1 < x < 2 \end{cases}$$

which shows the inequality

[Note $f(x) > 0$ as $|x| \gg 1$ since $f(x) \propto x^4$ as $|x|$ large]

(iii) $\sqrt{3x+6} > 2 + \sqrt{x+4}$ $x \geq -\frac{10}{3}$

$$\therefore 3x+6 > (2 + \sqrt{x+4})^2$$

$$3x+6 > 4 + 4\sqrt{x+4} + x+4$$

$$2x+2 > 4\sqrt{x+4}$$

$$x+1 > 2\sqrt{x+4} \leftarrow \text{if } x > -1$$

$$x^2 + 2x + 1 > 4(x+4)$$

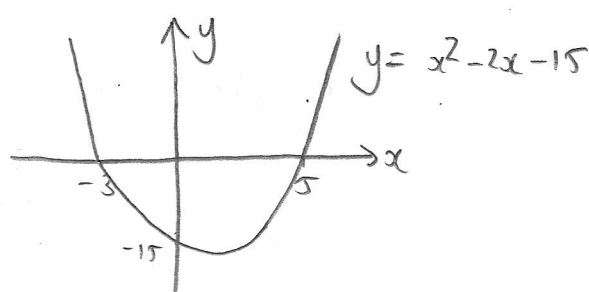
$$x^2 - 2x - 15 > 0$$

Note both sides +ve if $x > -1$

$$(x+3)(x-5) > 0$$

So from graph:

$$\boxed{x < -3}$$
$$\boxed{x > 5}$$



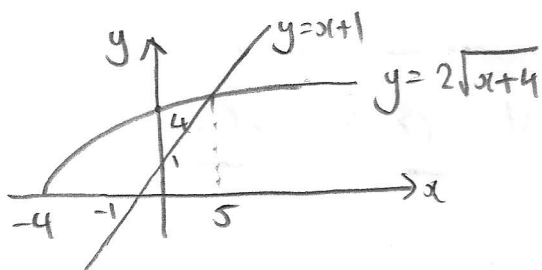
check lower limit of $x = -3$

$$\sqrt{3x+10} = 1$$

$$2 + \sqrt{x+4} = 3$$

So clearly $\sqrt{3x+10} > 2 + \sqrt{x+4}$
is not true here....

↳ go back to $x+1 > 2\sqrt{x+4}$



clearly $x > 0$ and only one
intersection is $x=5$, since $2\sqrt{x+4} \geq 0$

So overall solution to $\sqrt{3x+10} > 2 + \sqrt{x+4}$ is $\boxed{x > 5}$

