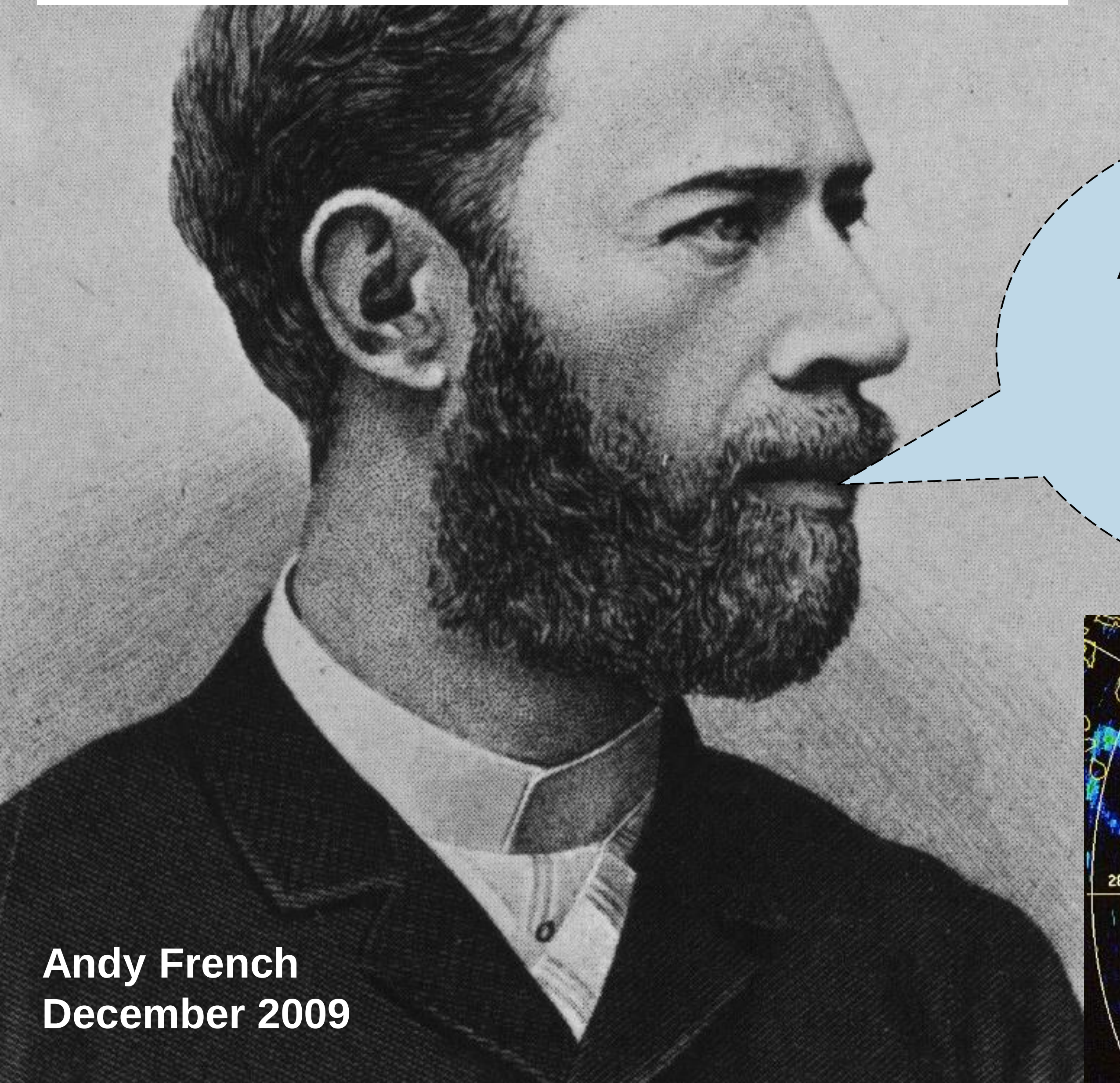
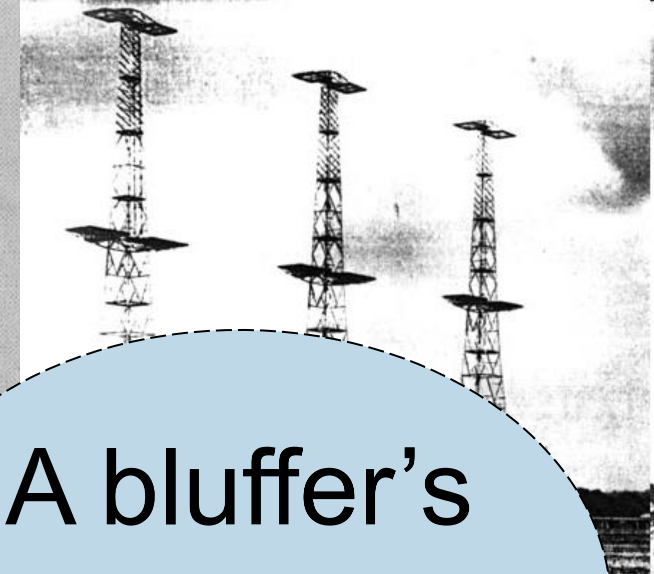
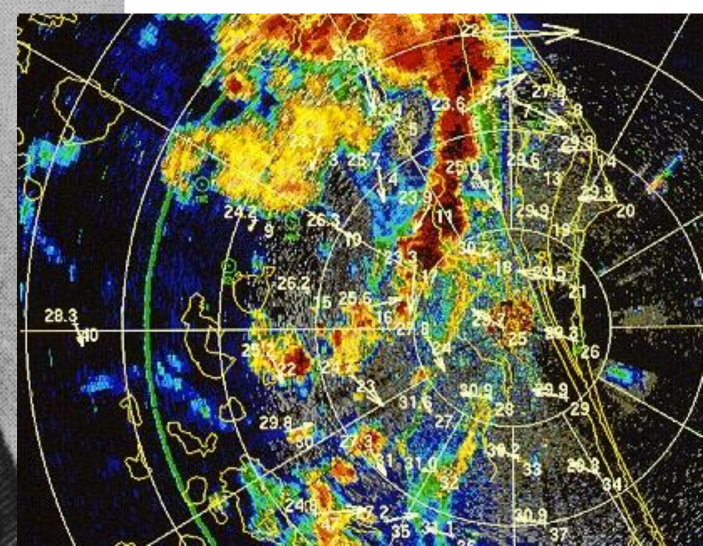
$$S/N = \frac{P_r}{P_{noise}} \frac{nE_i(n)B\tau}{L_{sp}} = \frac{A^2\eta^2 nE_i(n)\tau P_t \sigma f_{Tx}^2 \left|F^2\right|^2}{4\pi c^2 L k_B T_o N_f R^4}$$

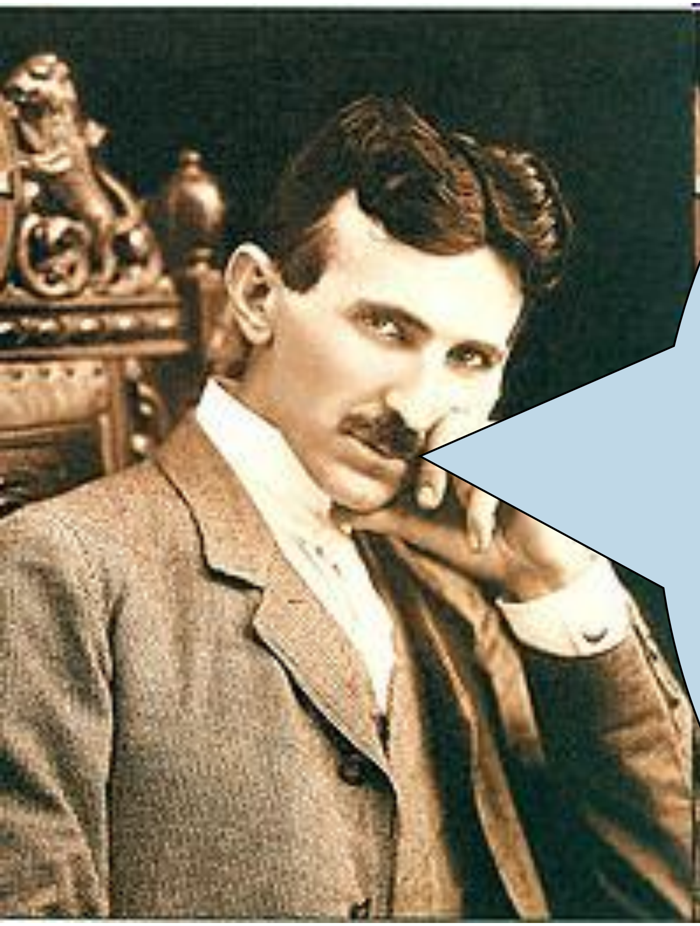




A bluffer's guide to Radar

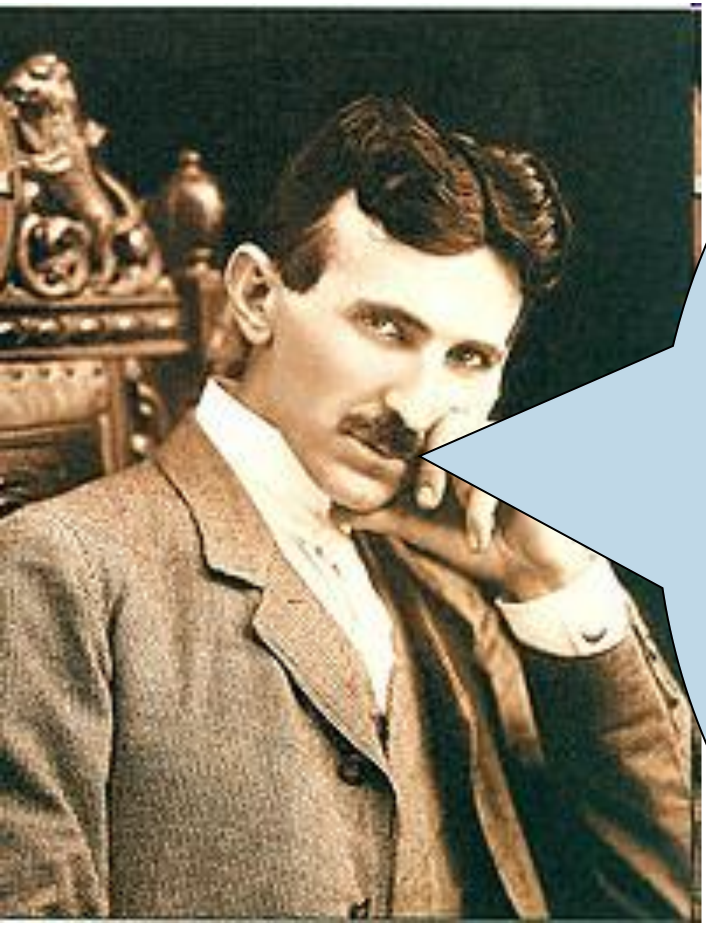


Andy French
December 2009



“We may produce at will, from a ***sending station****, an* ***electrical effect*** *in any particular region of the globe; (with which) we may determine the relative* ***position*** *or* ***course*** *of a moving object, such as a vessel at sea, the* ***distance*** *traversed by the same, or its* ***speed****.”*

Nikola Tesla (1856-1943)



"And yes, my wig is very nice"

Nikola Tesla (1856-1943)

RAdio Detection And Ranging

Radars detect the presence of a physically remote object via the reception and processing of **backscattered electromagnetic waves**.

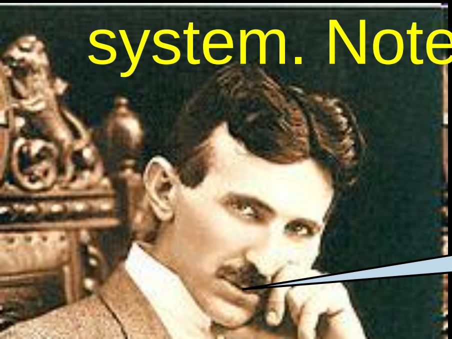
Unlike optical systems, (which are responsive to frequencies ≈10^{15}Hz), Radar is typically associated with frequency bands ranging from a **few MHz** (High Frequency or HF band) up to **hundreds of GHz** (mm wave).

- Most targets of interest (especially those constructed from **metal**) are **highly reflective** at Radar frequencies.

- Radar can be used in **darkness** and can **penetrate haze, fog, snow and rain.**

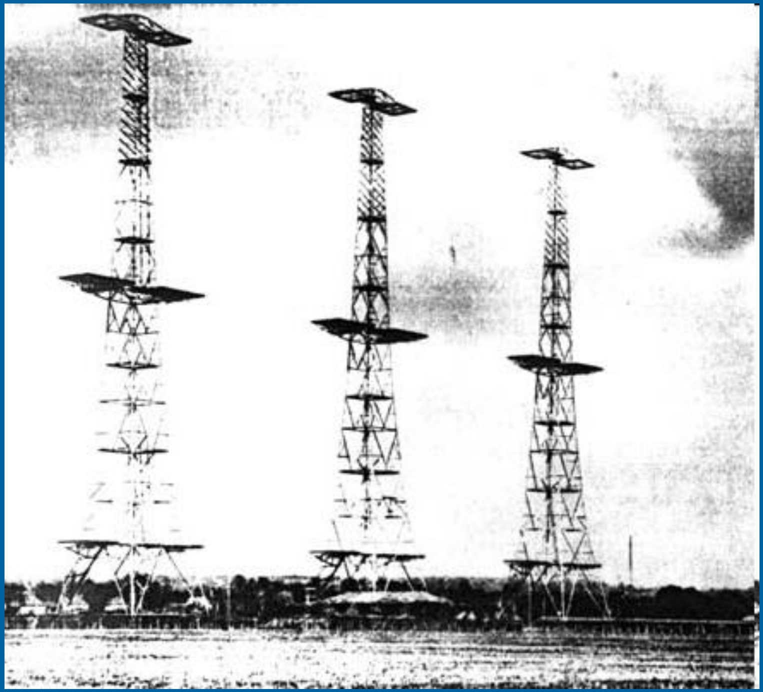
- **Atmospheric propagation attenuation is much less severe** for Radar than higher frequency electromagnetic disturbances. This means Radar can be used for **long range surveillance**. A **military** air defence system may have an operational range of **hundreds of km**.

- Radar has been used to successfully measure the distance between the Earth and other planets in the solar system. Note Mars is 56 million km from Earth!



- The technology to generate, receive and process Radar signals has been continuously refined for nearly **100 years**

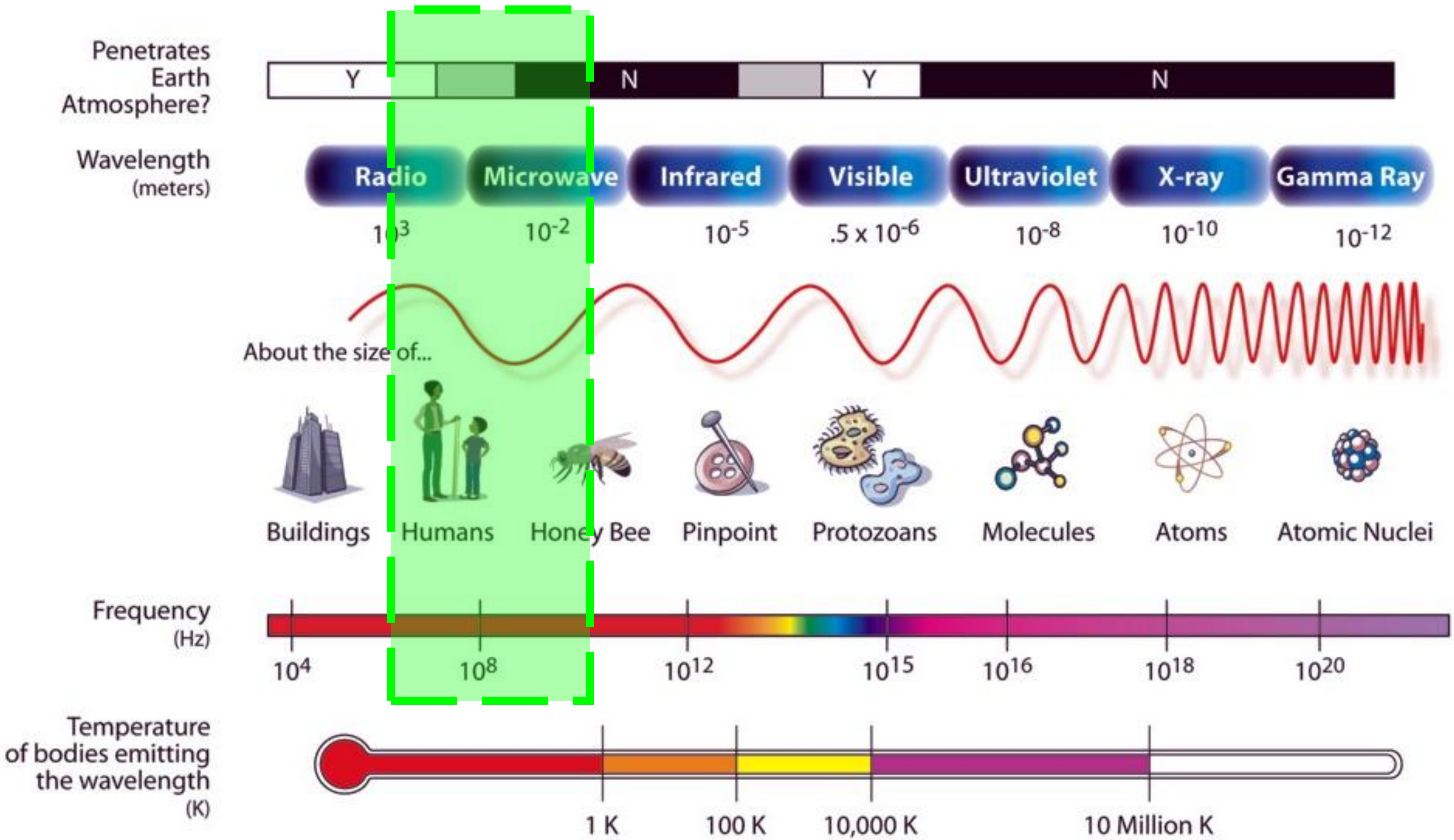
- Military and civilian air traffic control have employed Radar as a key sensor extensively since the Second World War.

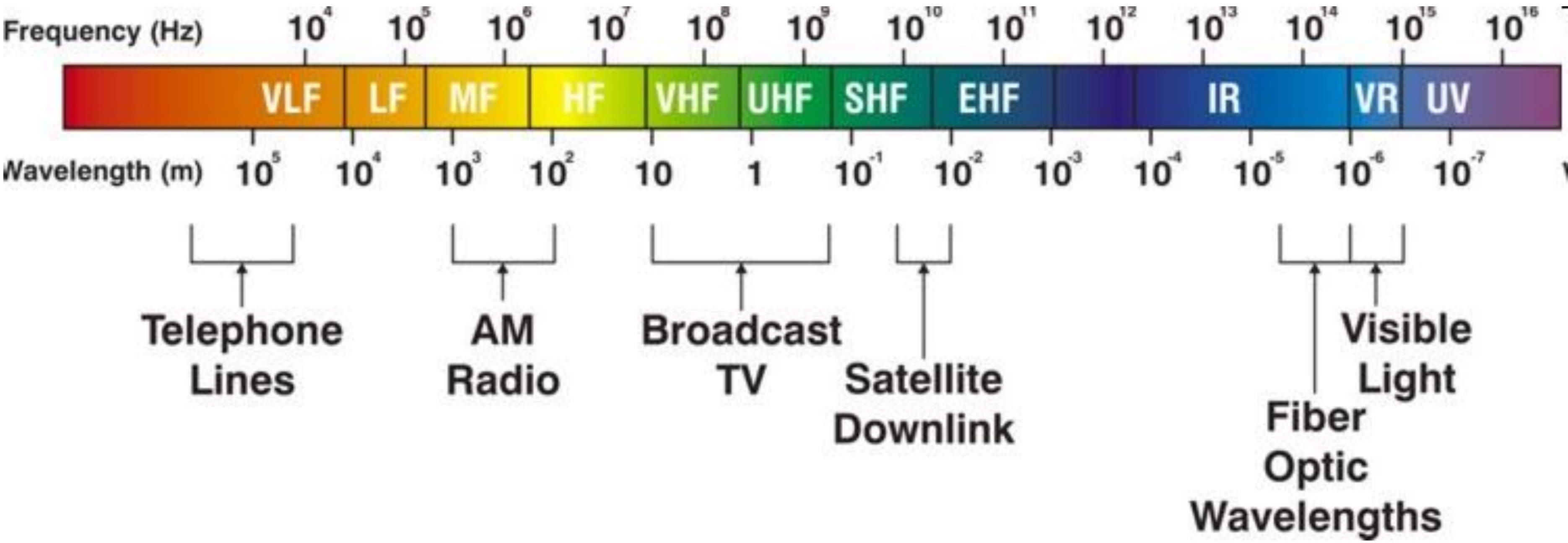


- Magnetron transmitters, which are stable sources of microwaves (0.1 - 100 GHz approximately) are ubiquitous as a fundamental element of modern domestic ovens.

- Given the size of a Radar antenna roughly scales with the wavelength it transmits / receives; Radars (with modest directivity, i.e. a beamwidth of a few degrees) tend to be of dimensions well suited to human use i.e. of the order of a few metres.

THE ELECTROMAGNETIC SPECTRUM



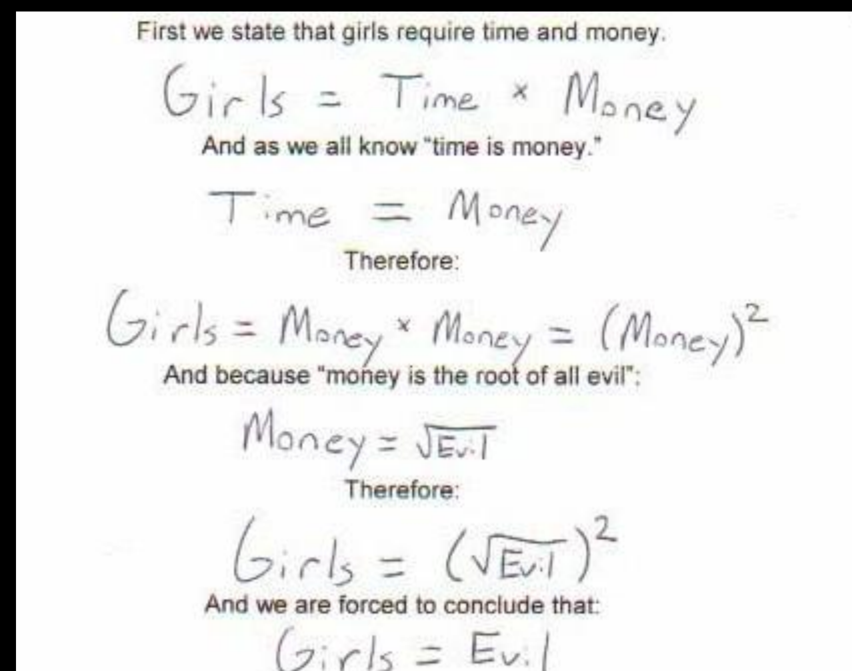

Frequency (Hz)
10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16
VLF LF MF HF VHF UHF SHF EHF IR VR UV
Wavelength (m)
10^5 10^4 10^3 10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7
Telephone Lines
AM Radio
Broadcast TV
Satellite Downlink
Visible Light
Fiber Optic Wavelengths

Radar bands

30-300Hz	Extremely low frequency ELF
3 - 30kHz	Very low frequency VLF
30 - 300kHz	Low frequency LF
300 - 3000kHz	Medium frequency MF
3 - 30MHz	High frequency HF
30 - 300MHz	Very high frequency VHF
0.3 - 3GHz	Ultra high frequency UHF
3 - 30GHz	Super high frequency SHF
30 - 300GHz	Extremely high frequency EHF
300GHz - 429THz	Infrared IR
429 - 750THz	Visible Light
>750THz	Ultraviolet UV

Radar bands

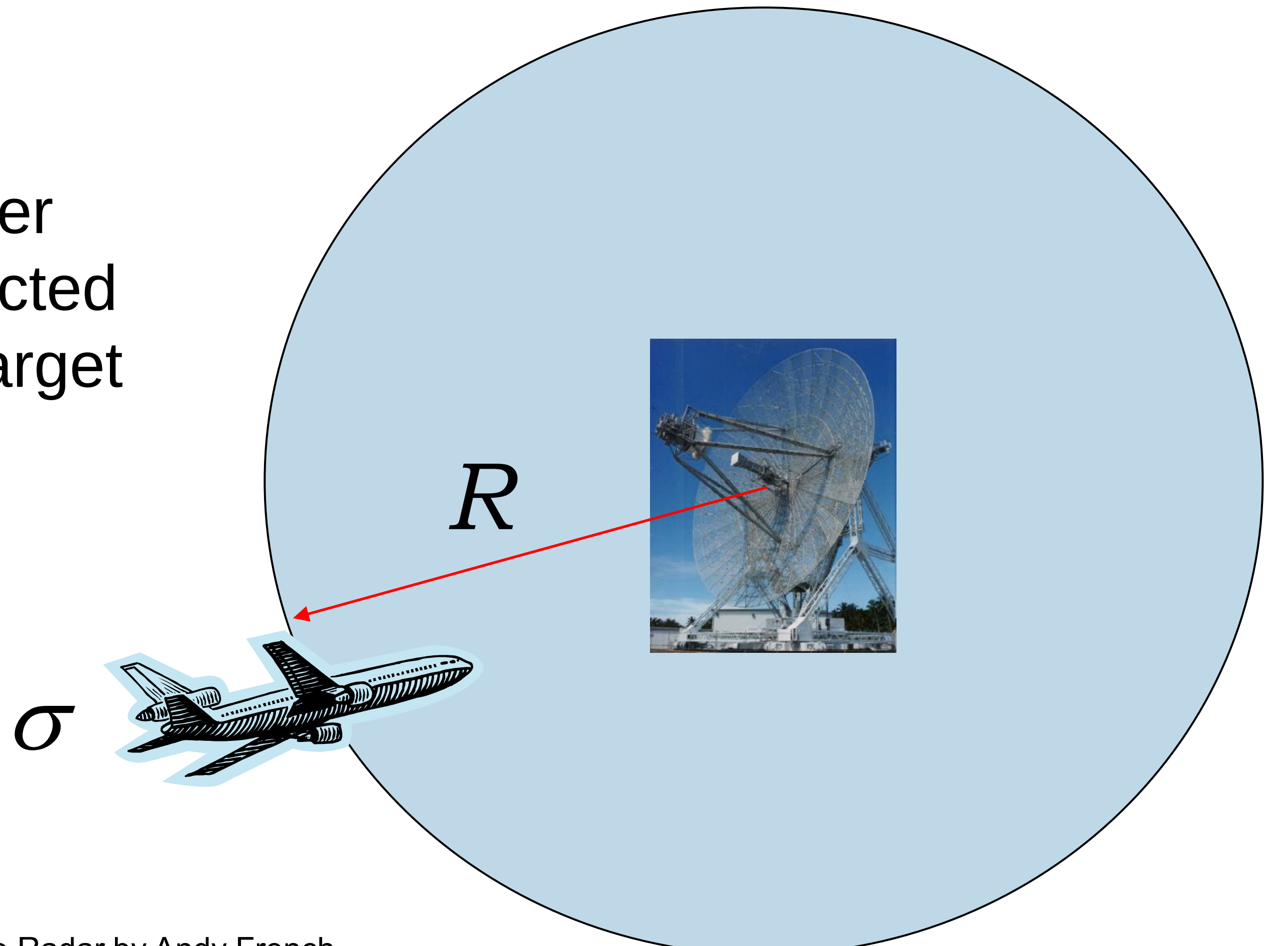
The Radar Equation



Transmitted power

$$P_s = G\frac{\sigma}{4\pi R^2}\frac{P_t}{L_t} = \frac{A\eta}{c^2}f_{Tx}^2\frac{\sigma}{R^2}\frac{P_t}{L_t}$$

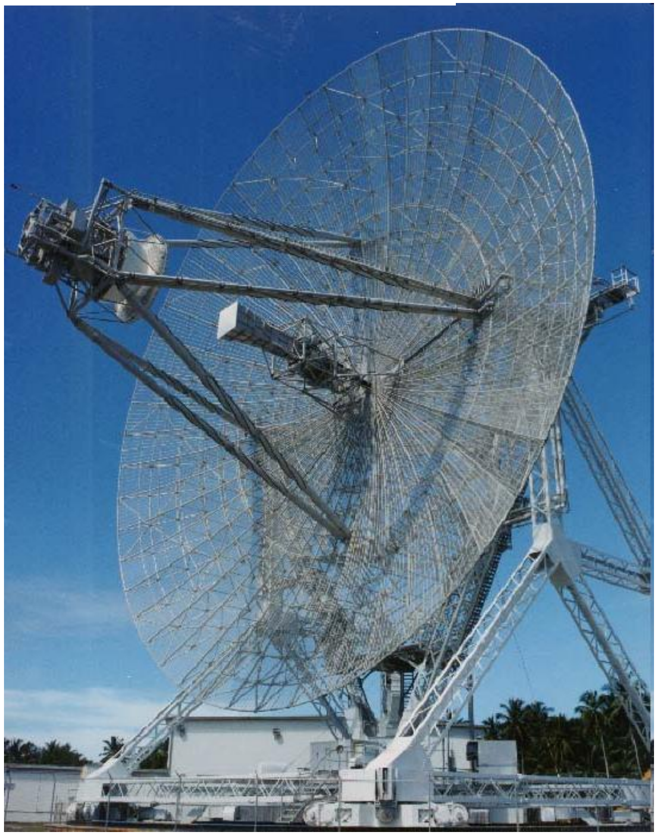
Power reflected off target



Antenna gain

$$G = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi A\eta}{c^2} f_{Tx}^2$$

$$G \approx \frac{26,000}{(\Delta\epsilon/\text{deg})\,(\Delta\phi/\text{deg})}$$



$$G = \frac{4\pi A\eta}{\lambda^2} = \frac{4\pi A\eta}{c^2} f_{Tx}^2$$

$$G \approx \frac{26,000}{(\Delta\epsilon/\text{deg})(\Delta\phi/\text{deg})}$$

$$P_r = \frac{P_s}{4\pi R^2}\frac{A\eta}{L_r}\left|F^2\right|^2 = \frac{A^2\eta^2\sigma f_{Tx}^2}{4\pi R^4 c^2}\frac{P_t}{L_t L_r}\left|F^2\right|^2$$

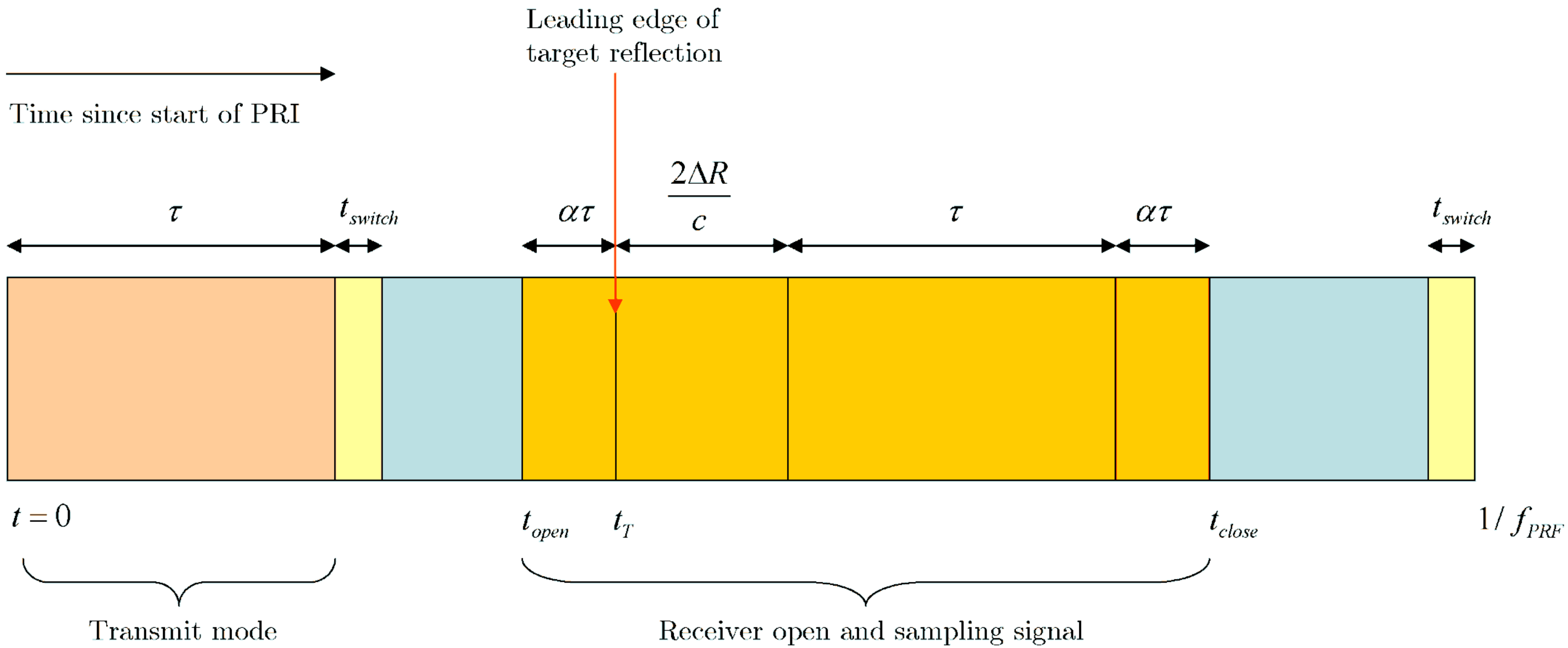
$$S/N = \frac{P_r}{P_{noise}}\frac{nE_i(n)B\tau}{L_{sp}} = \frac{A^2\eta^2 nE_i(n)\tau P_t\sigma f_{Tx}^2\left|F^2\right|^2}{4\pi c^2 L k_B T_o N_f R^4}$$

THE RADAR EQUATION

$$P_{noise} = k_B T_o B N_f$$

$A = 4\text{m}^2$, $\eta = 0.5$, $n = 8$, $E_i(8) = 1$, $\tau = 25.6\mu\text{s}$, $P_t = 12\text{kW}$, $\sigma = 20\text{m}^2$, $f_{Tx} = 3\text{GHz}$, $\left|F^2\right|^2 = 0\text{dB}$, $k_B = 1.38\times10^{-23}\text{JK}^{-1}$, $T_o N_f = 500\text{K}$, $R = 50\text{km}$, $L = 10\text{dB}$

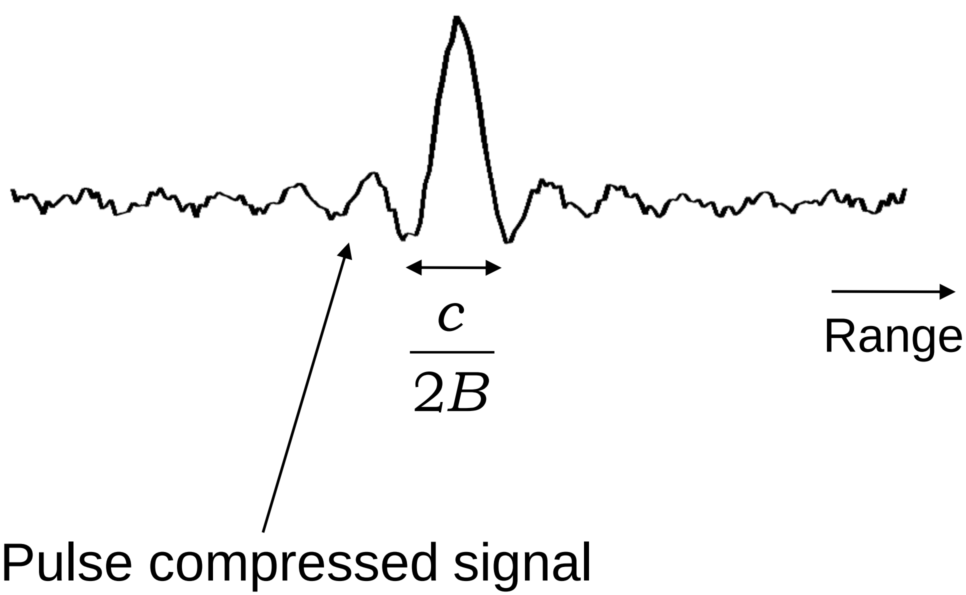
The Radar Equation above yields $S/N = 35.6\text{dB}$

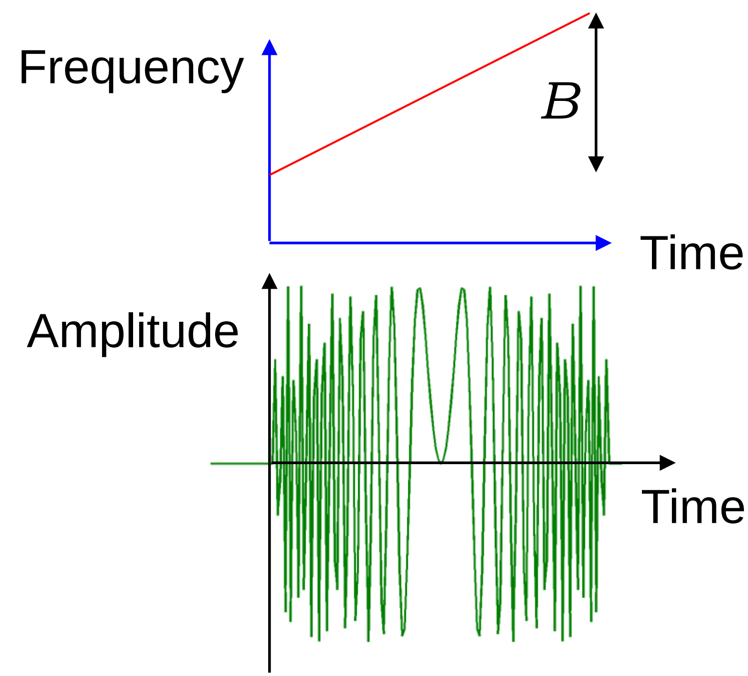


Pulse Repetition Interval

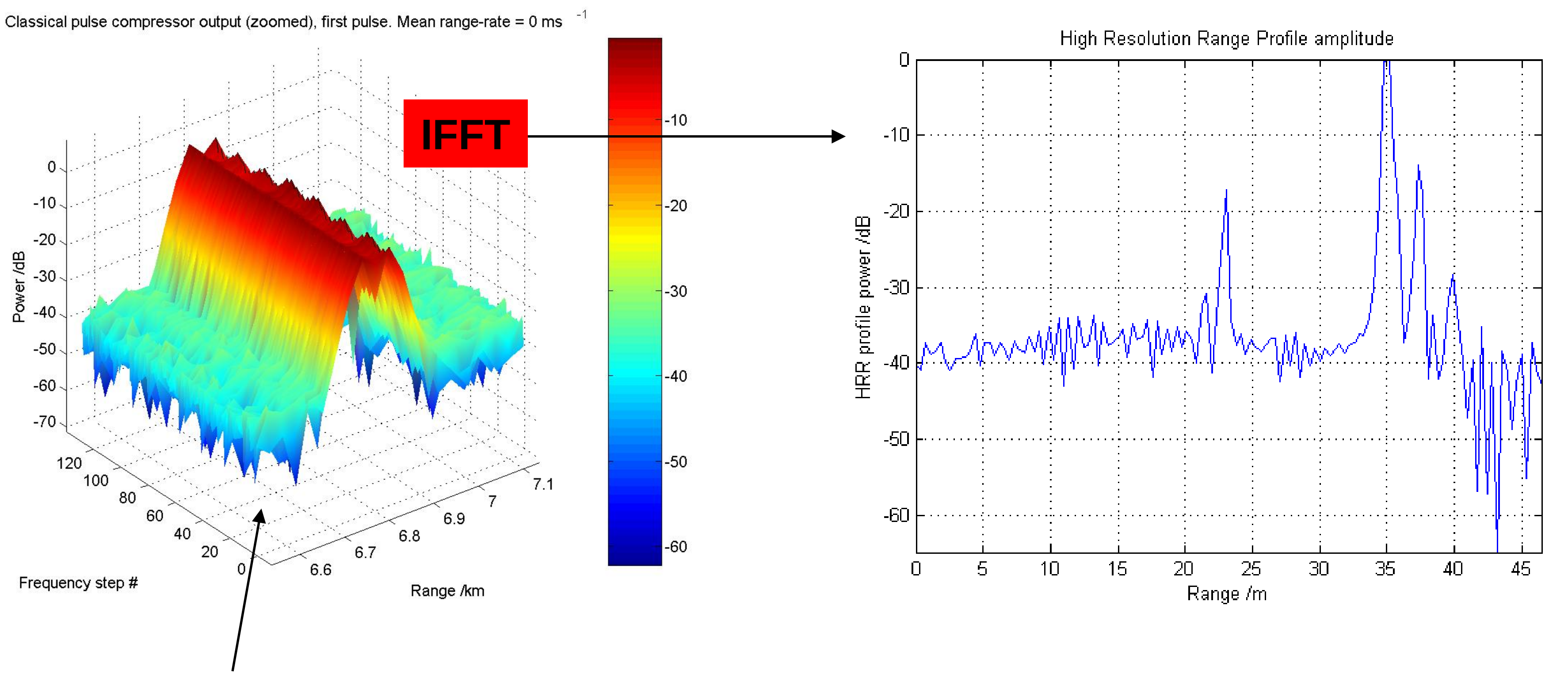
Range processing & pulse compression

- Range profiles obtained by transmitting a frequency coded pulse and correlating received and transmitted signals
- Range resolution inversely proportional to pulse bandwidth B

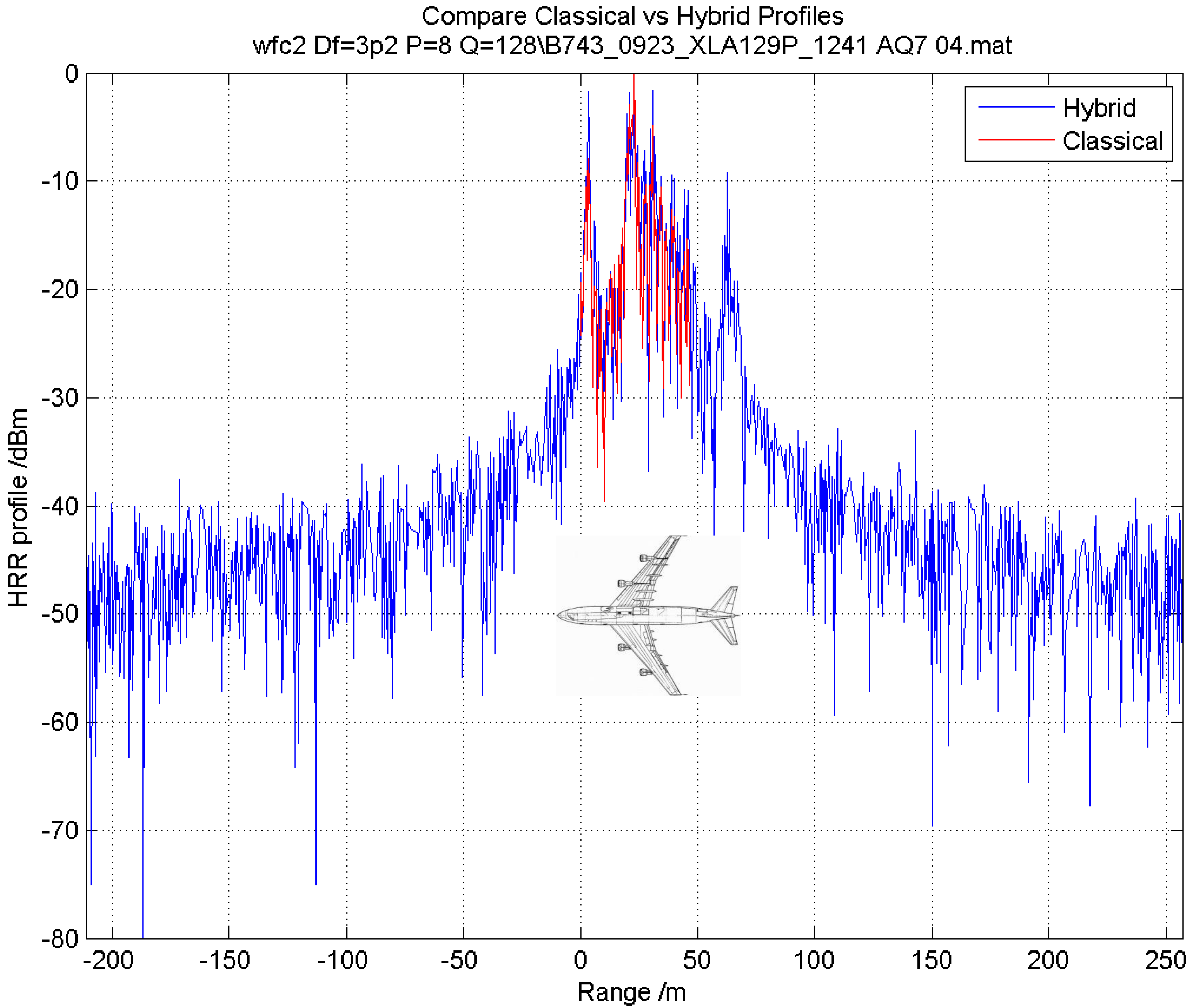




Range samples & high range resolution



Stack of pulse compressor outputs for all frequency steps



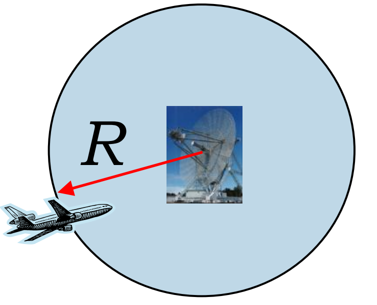
Compare Classical vs Hybrid Profiles
wfc2 Df=3p2 P=8 Q=128\B743_0923_XLA129P_1241 AQ7 04.mat
Hybrid
Classical
HRR profile /dBm
0
-10
-20
-30
-40
-50
-60
-70
-80
-200
-150
-100
-50
0
50
100
150
200
250
Range /m

Doppler shift

$$\phi = 2\pi f_{Tx} t_{delay}$$

$$t_{delay} = 2R/c$$

$$\phi = \frac{4\pi f_{Tx} R}{c}$$



$$f(t) = f_{Tx} - \frac{1}{2\pi}\frac{d\phi}{dt}$$

$$\Delta\phi = \frac{4\pi f_{Tx} \dot{R}}{c f_{PRF}}$$

$$f_D = -\frac{1}{2\pi}\frac{\Delta\phi}{1/f_{PRF}} = -\frac{2\dot{R}}{c} f_{Tx}$$

Doppler filter

$$y = \sum_{p=1}^{P} \psi_p w_p$$

Weights

Samples per pulse

$$\psi_p = e^{2\pi i t_p f}$$

$$|y(f)|^2 = \left| \sum_{p=1}^{P} w_p e^{2\pi i t_p f} \right|^2$$

GROUND CLUTTER FILTER

$$\{w_p\} = \{1, -2, 1\}$$

Doppler filter: DFT

$$w_{p,k} = e^{-\frac{2\pi i(p-1)(k-1)}{P}}$$

DISCRETE FOURIER TRANSFORM

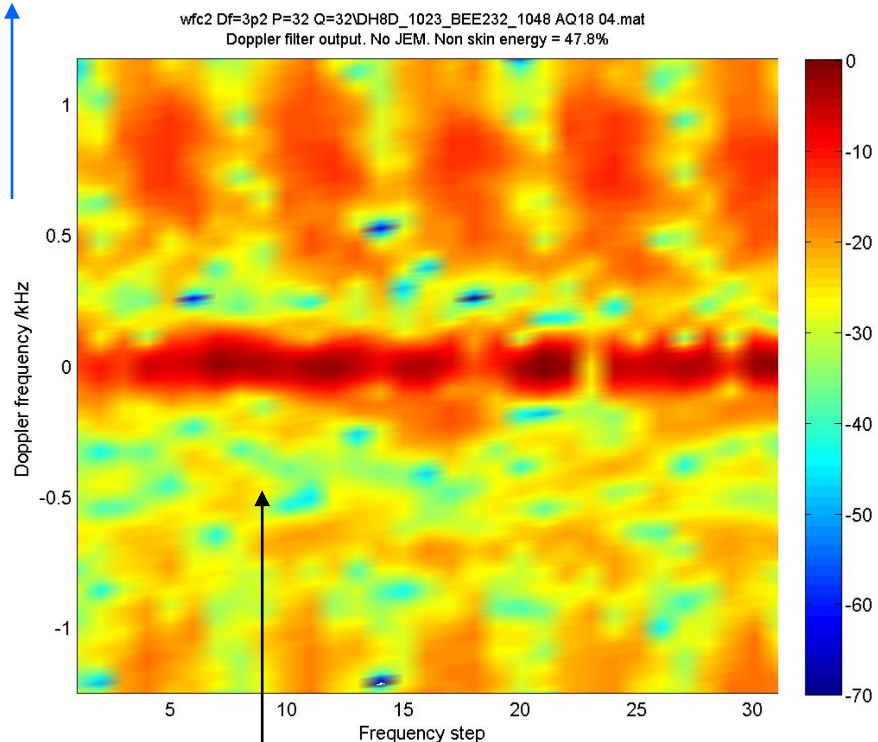
$$y_k(f) = \left| \frac{e^{ix}\sin\left\{\left(1+\frac{1}{P}\right)x\right\}}{\sin\left(\frac{x}{P}\right)} - 1 \right|^2$$

$$\frac{x}{\pi} = 1 - k + \frac{fP}{f_{PRF}}$$

$$f_{\max} = \frac{k-1}{P} f_{PRF}$$

Doppler spectra

DOPPLER FREQUENCY



TIME

Doppler spectrum for 32 pulse, 32 frequency step 2.5kHz PRF

Dash8 six blade propeller aircraft