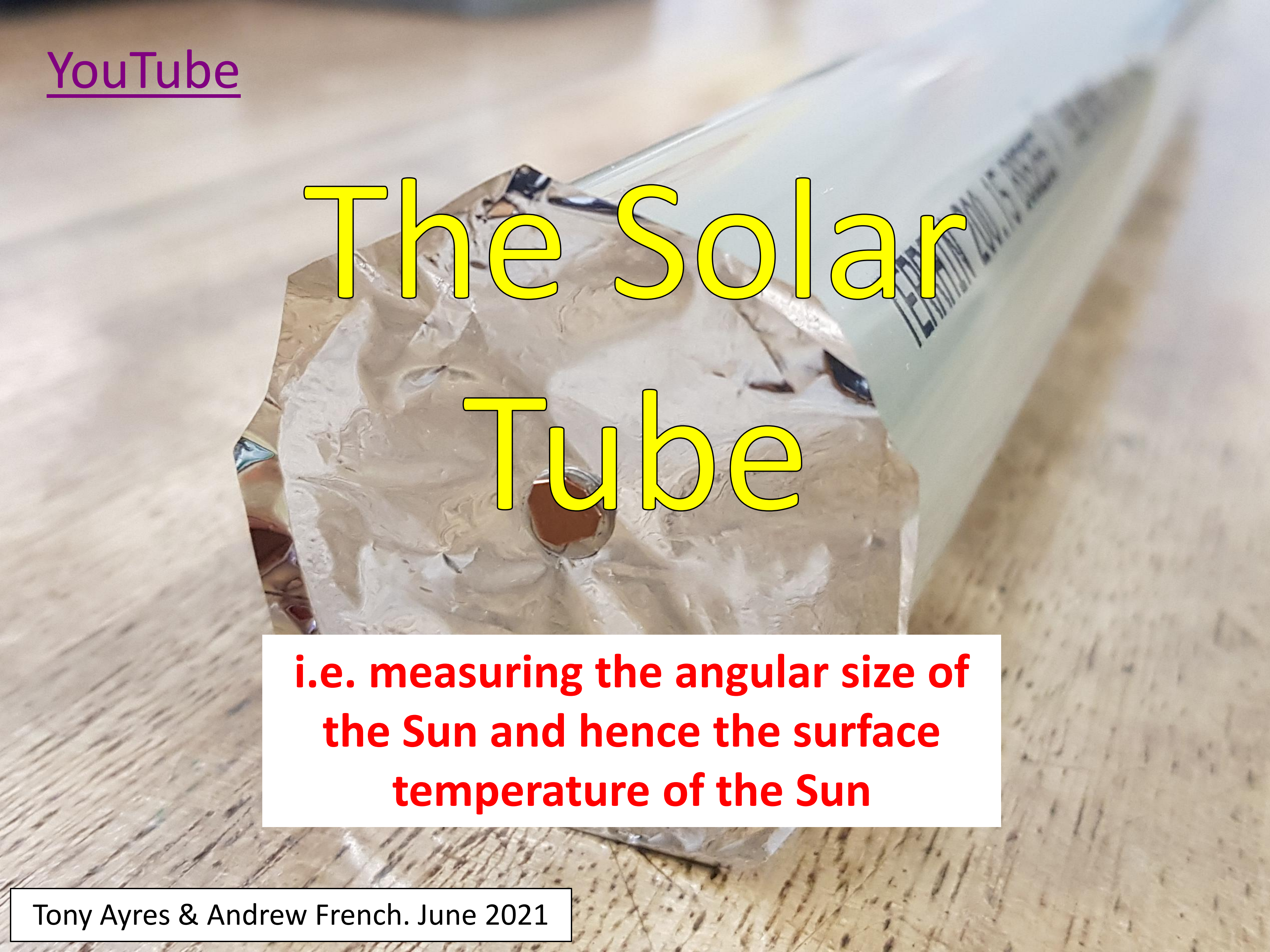
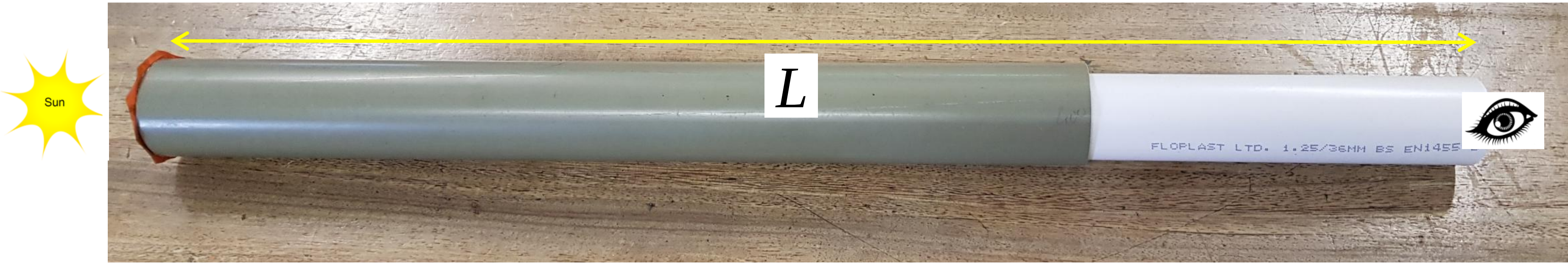
YouTube



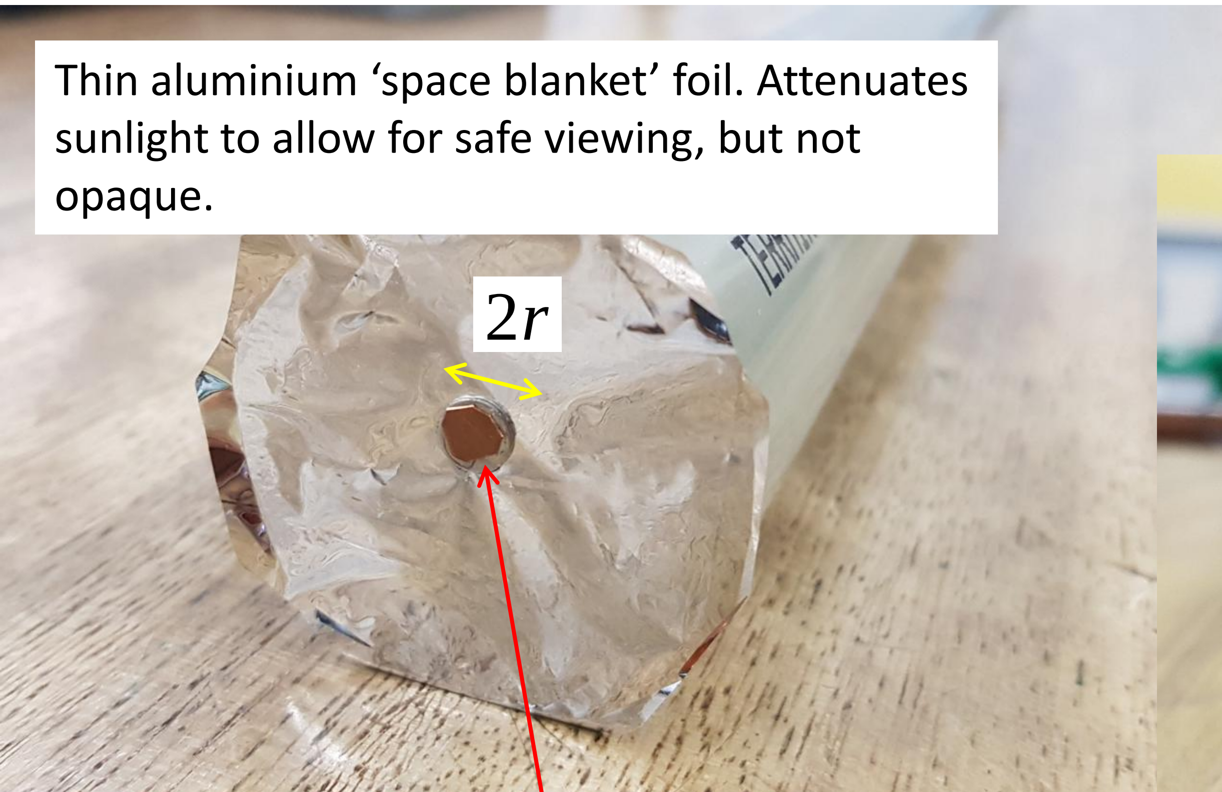
The Solar Tube

i.e. measuring the angular size of the Sun and hence the surface temperature of the Sun

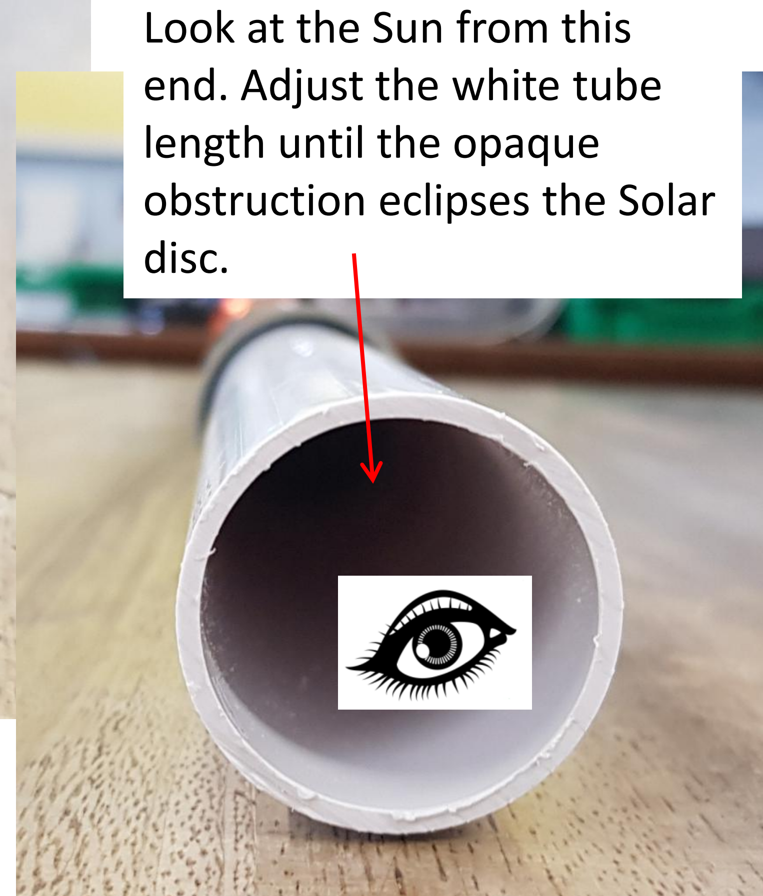
Tony Ayres & Andrew French. June 2021



WARNING: <u>NEVER</u> STARE DIRECTLY AT THE SUN WITH A NAKED EYE



Opaque obstruction of radius r

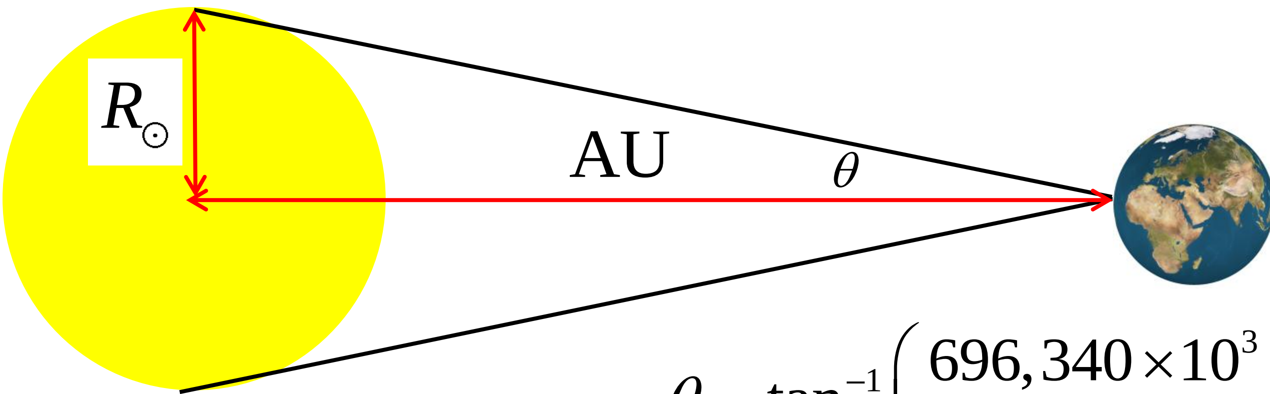


Diagrams NOT to scale!

$$R_\odot = 696,340\text{km}$$

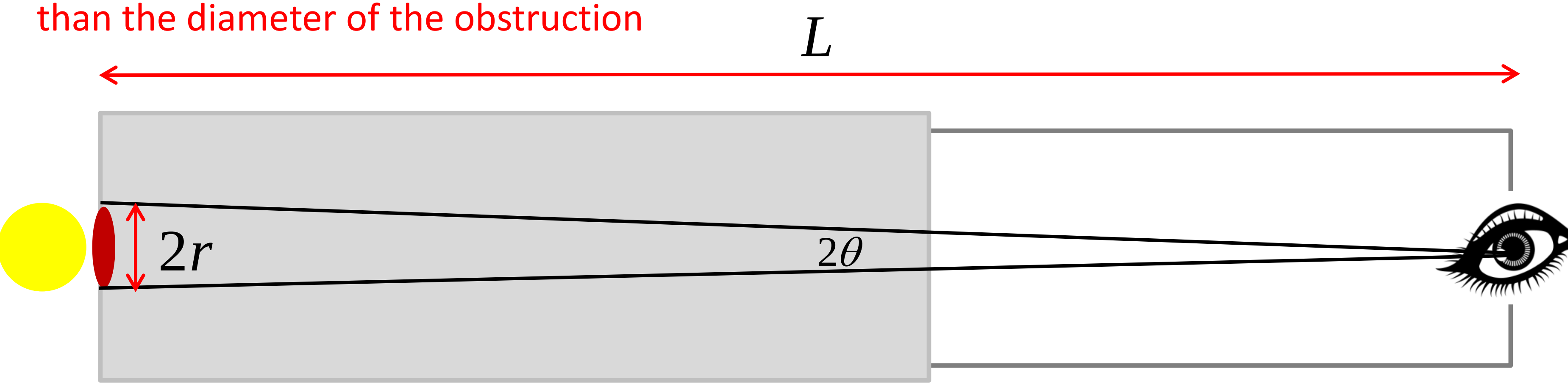
$$\text{AU} = 1.496\times10^{11}\text{m}$$

$$\boxed{\frac{\text{AU}}{R_\odot} \approx 215}$$



$$\theta = \tan^{-1}\left(\frac{696,340\times10^{3}}{1.496\times10^{11}}\right) = 0.267^{\circ}$$

So L should be **108** times larger than the diameter of the obstruction



Without the opaque obstruction, the apparent size of the solar disc would look like:

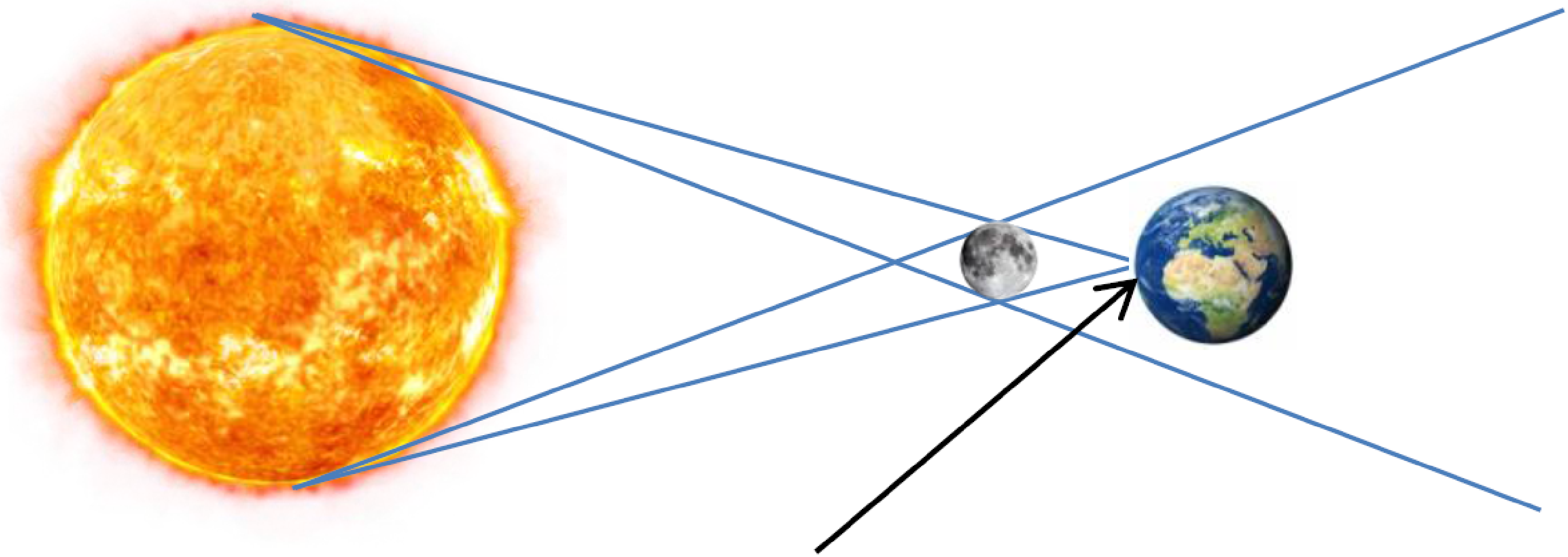
So if we extend L until the disc is *just eclipsed* by the obstruction, we can find angle θ, if we measure r and L.

$$\theta = \tan^{-1}\left(\frac{r}{L}\right)$$

Solar and lunar eclipses

A total lunar eclipse is when the moon is positioned in the umbra of the Sun's light, obscured by the Earth. i.e. Light cannot reach the surface of the moon from the Sun as the Earth is in the way. This means no light is reflected back to Earth from the Moon's surface, which is why it appears black during a lunar eclipse.

Solar Eclipse



'*Totality*' i.e. at this point
on Earth the Moon blocks
the sun. Only the *solar corona* will be seen.

Solar flares and corona observed from Earth during a total solar eclipse

Looking at the Sun through our Solar Tube should reveal something similar.
(Although we won't see a corona!)

Radiation power per square metre

$$I = \varepsilon \sigma T^4$$

Absolute temperature /K

Emmisivity

Stefan-Boltzmann constant

$$\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-1}}$$

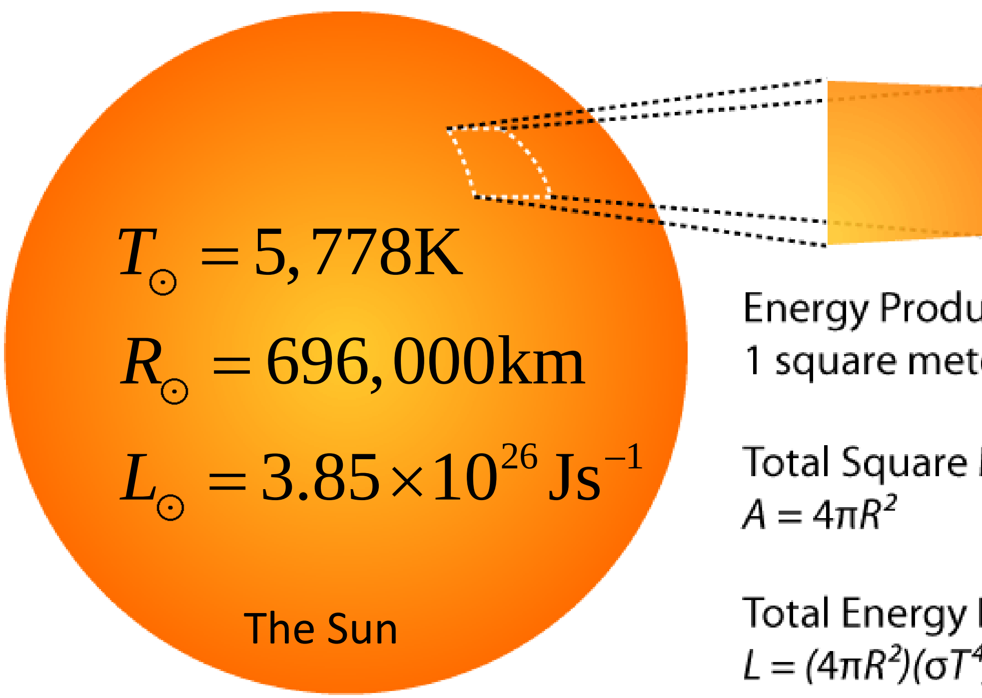
$$\varepsilon = 1$$

A ‘**Black Body**’. i.e. all incident radiation is absorbed and then re-radiated

For a ‘Black Body’ at 20°C = 293K

$$\boxed{I = 418\,\mathrm{Wm^{-2}}}$$

It is interesting to compare this to the maximum solar energy incident upon the Earth, which is on average about 1,361 $\mathrm{Wm^{-2}}$



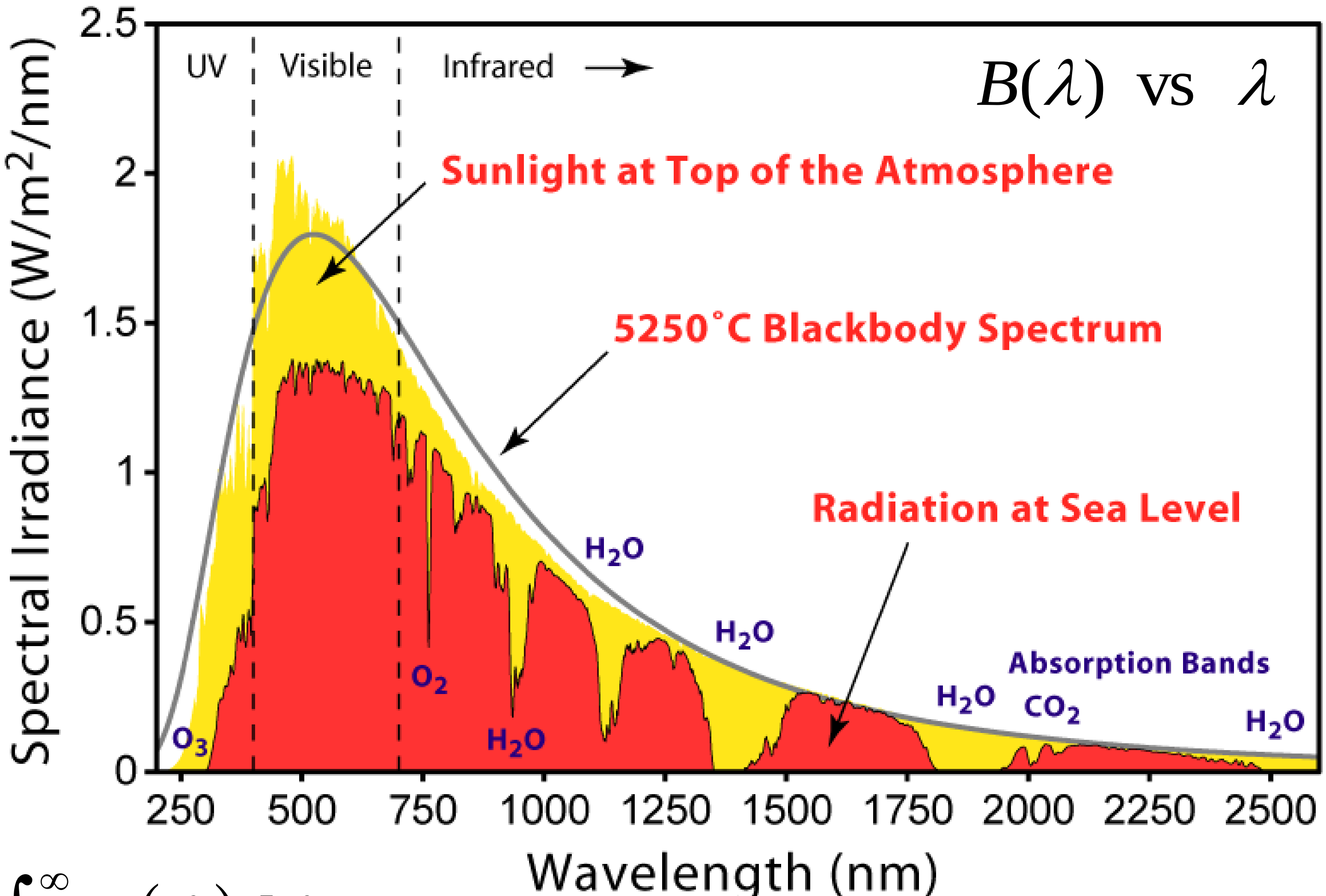
Energy Produced by 1 square meter = σT^4

Total Square Meters
$A = 4\pi R^2$

Total Energy Produced
$L = (4\pi R^2)(\sigma T^4)$

The measured solar irradiance (i.e. power received on Earth per square metre within a wavelength interval

i.e. $\lambda \rightarrow \lambda + d\lambda$



$$I = \int_0^\infty B(\lambda) d\lambda$$

Red	620-750nm
Yellow	570-590nm
Green	495-570nm
Blue	450-495nm

Boltzmann's constant $k_B = 1.381\times10^{-23}\,\mathrm{m^2kgs^{-2}K^{-1}}$

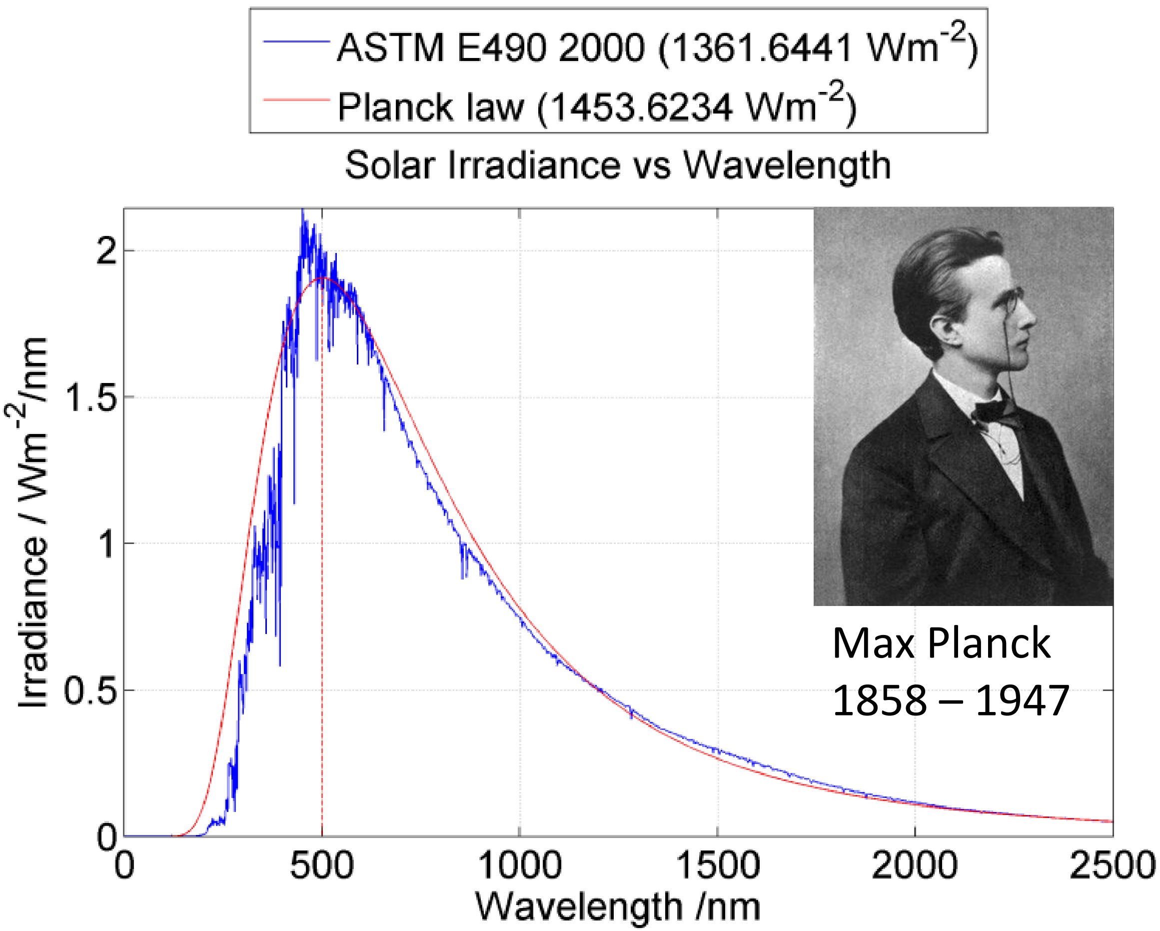
Planck's constant $h = 6.626\times10^{-34}\,\mathrm{m^2kgs^{-1}}$

Speed of light $c = 2.998\times10^{8}\,\mathrm{ms^{-1}}$

$$I = \int_0^\infty B(\lambda,T)\,d\lambda = \sigma T^4$$

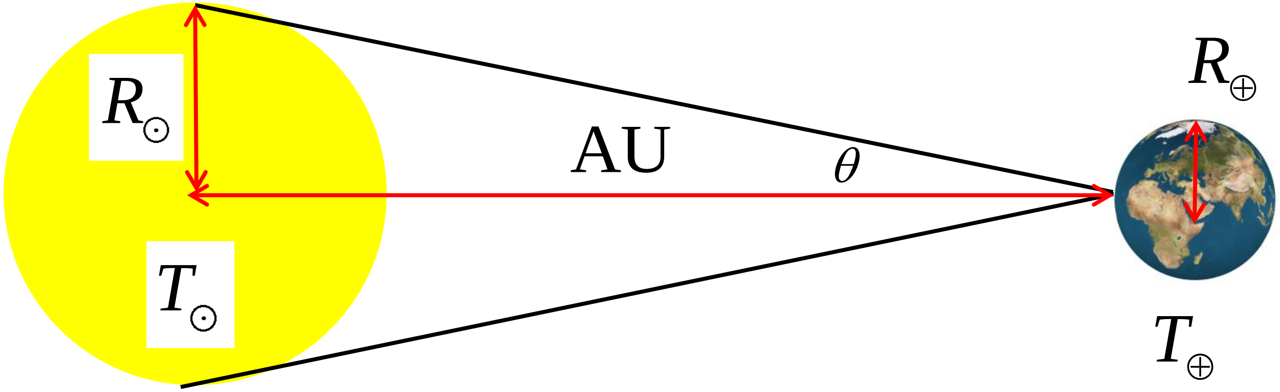
$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}$$

$$B(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}$$



Max Planck
1858 – 1947

Planck Radiation spectrum



Diagrams NOT to scale!

Assume the Sun radiates isotropically. The power per unit area at distance AU illuminating the Earth is:

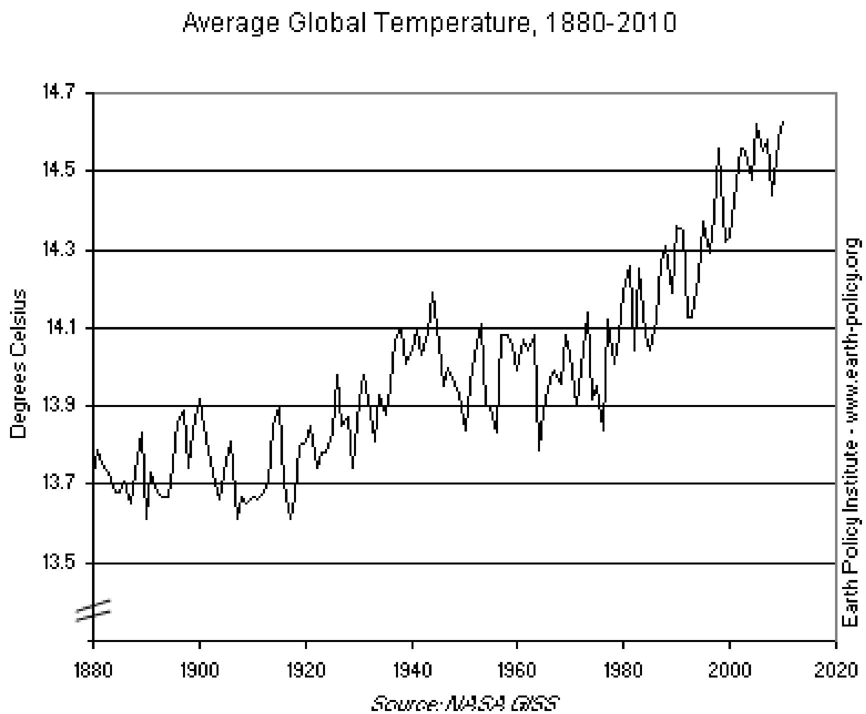
$$\Phi = \frac{4\pi R_\oplus^2 \sigma T_\odot^4}{4\pi \mathrm{AU}^2}$$

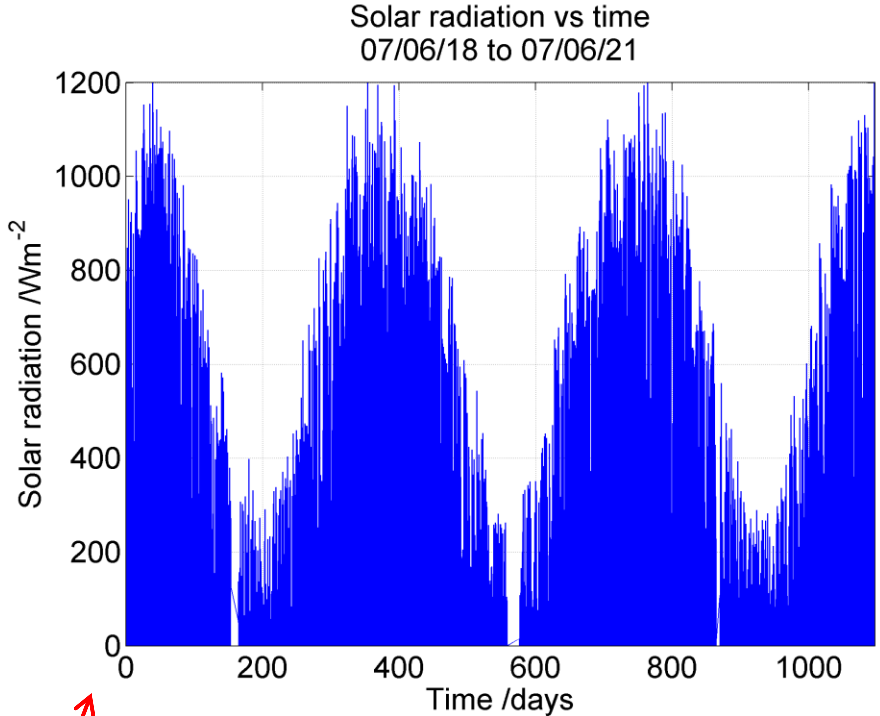
Assume the Earth receives this power over area $\pi R_\oplus^2$ and then, on average, *re-radiates this energy as infra-red radiation*, isotropically, based upon a mean average Earth temperature. By equating power received = power re-radiated:

$$\frac{4\pi R_\odot^2 \sigma T_\odot^4}{4\pi \mathrm{AU}^2} \times \pi R_\oplus^2 = 4\pi R_\oplus^2 \sigma T_\oplus^4$$

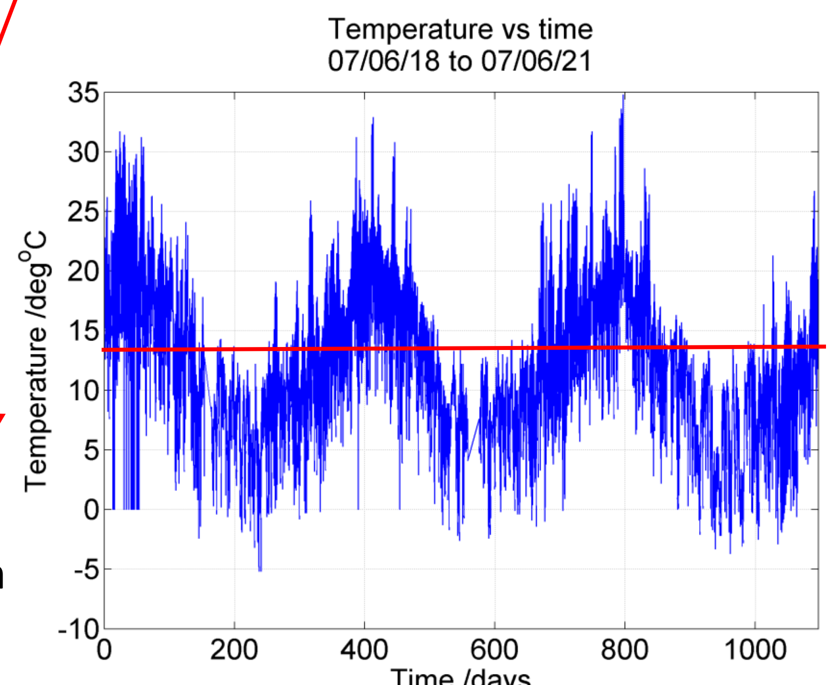
$$\therefore T_\odot = \sqrt{\frac{2\mathrm{AU}}{R_\odot}} T_\oplus = \sqrt{\frac{2}{\tan\theta}} T_\oplus$$

So if we can calculate angle θ, we can calculate the temperature of the Sun from an average Earth temperature.



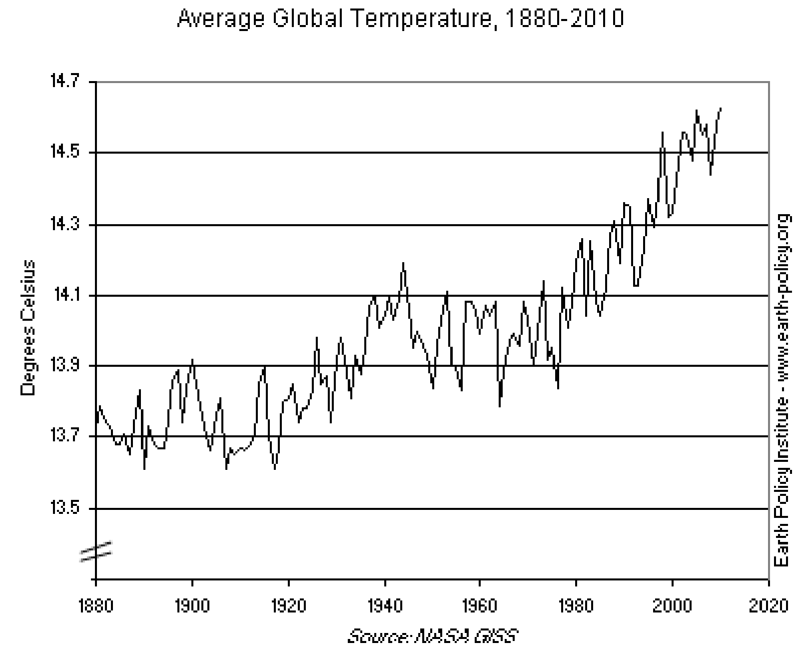


What should we use as $T_\oplus$?



From Wincoll meteorological system
(on the roof of Science School)

To get some idea, let's work backwards from the actual angle θ and the actual temperature of the surface of the Sun.

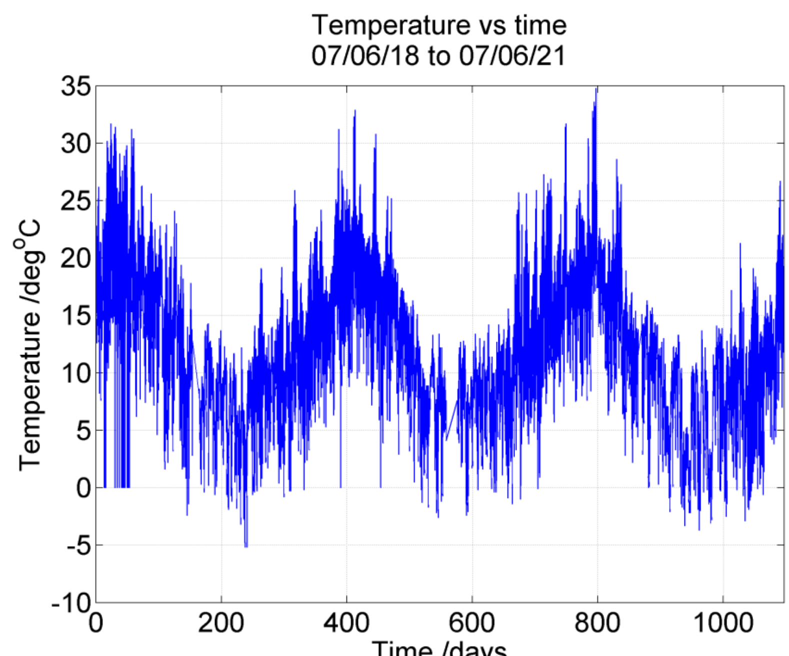


$$T_{\odot} = \sqrt{\frac{2}{\tan\theta}}T_{\oplus}$$

$$T_{\oplus} = \sqrt{\tfrac{1}{2}\tan\theta} \times T_{\odot}$$

$$T_{\oplus} = \sqrt{\tfrac{1}{2}\tan\left(0.267^{\circ}\right)} \times 5,778\text{K}$$

$$T_{\oplus} = 279\text{K} = \boxed{5.9^{\circ}\text{C}}$$



So perhaps a bit too cold to be believable, but remember the Earth will not re-radiate *all* the solar energy. Plenty of energy is locked up in biomass and indeed in the kinetic and potential energies of moving molecules within the troposphere. Including us!