

## Algebraic fractions

Use of fractions in algebra follows exactly the same rules as numeric fractions, i.e. multiplying by a suitable variant of 1 ( i.e.  $n/n$  where  $n$  is a number not equal to zero) to make the denominator of the fractions the same, enabling fractions to be added or subtracted.

### Example 1

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} \\ = \frac{1}{x} \times \frac{y}{y} + \frac{1}{y} \times \frac{x}{x} \\ = \frac{y+x}{xy} \end{aligned}$$

### Example 2

$$\begin{aligned} \frac{1}{2x} - \frac{3}{y} \\ = \frac{1}{2x} \times \frac{y}{y} - \frac{3}{y} \times \frac{2x}{2x} \\ = \frac{y-6x}{2xy} \end{aligned}$$

### Example 3

$$\begin{aligned} \frac{2}{x+1} - \frac{3}{y-2} \\ = \frac{1}{x+1} \times \frac{y-2}{y-2} - \frac{3}{y-2} \times \frac{x+1}{x+1} \\ = \frac{y-2-3(x+1)}{(x+1)(y-2)} \\ = \frac{y-5-3x}{(x+1)(y-2)} \end{aligned}$$

### Example 4

$$\begin{aligned} x + \frac{3}{y-2} - \frac{1}{x+1} \\ = x + \frac{3}{y-2} \times \frac{x+1}{x+1} - \frac{1}{x+1} \times \frac{y-2}{y-2} \\ = x + \frac{3(x+1)-(y-2)}{(y-2)(x+1)} \\ = x \times \frac{(y-2)(x+1)}{(y-2)(x+1)} + \frac{3(x+1)-(y-2)}{(y-2)(x+1)} \\ = \frac{x(xy-2x+y-2)+3x+3-y+2}{(y-2)(x+1)} \\ = \frac{x^2y-2x^2+yx+x+5-y}{(y-2)(x+1)} \end{aligned}$$

## Rearrangements involving algebraic fractions

For the following questions  $x = f(y)$  is desired:

### Example 1

$$\begin{aligned} y &= \frac{1}{x+1} \\ (x+1)y &= 1 \\ x+1 &= \frac{1}{y} \\ x &= \frac{1}{y} - 1 \end{aligned}$$

### Example 2

$$\begin{aligned} y &= \frac{x}{x-1} \\ (x-1)y &= x \\ xy - y - x &= 0 \\ x(y-1) - y &= 0 \end{aligned}$$

Collecting terms multiplied by  $x$  into a single bracket multiplied by  $x$

$$x = \frac{y}{y-1}$$

## Solving equations involving algebraic fractions

### Example 1

$$\begin{aligned} \frac{x+1}{x-2} &= \frac{x+6}{x} \\ x(x+1) &= (x-2)(x+6) \\ x^2 + x &= x^2 + 4x - 12 \\ 12 &= 3x \\ 4 &= x \end{aligned}$$

### Example 2

$$\begin{aligned} \frac{2}{x^2} - \frac{3}{x} - 2 &= 0 \\ \frac{2}{x^2} - \frac{3}{x} \times \frac{x}{x} - 2 \times \frac{x^2}{x^2} &= 0 \\ \frac{2-3x-2x^2}{x^2} &= 0 \\ 2-3x-2x^2 &= 0 \\ 2x^2+3x-2 &= 0 \\ (2x)^2+3(2x)-4 &= 0 \\ (2x+4)(2x-1) &= 0 \\ (x+2)(2x-1) &= 0 \\ x &= -2, \quad x = \frac{1}{2} \end{aligned}$$

### Example 3

$$\begin{aligned} \frac{3x}{x-1} + 1 &= \frac{x+6}{x-2} \\ \frac{3x}{x-1} + \frac{x-1}{x-1} &= \frac{x+6}{x-2} \\ \frac{4x-1}{x-1} &= \frac{x+6}{x-2} \\ (4x-1)(x-2) &= (x+6)(x-1) \\ 4x^2-9x+2 &= x^2+5x-6 \\ 3x^2-14x+8 &= 0 \\ (3x)^2-14(3x)+24 &= 0 \\ (3x-12)(3x-2) &= 0 \\ (x-4)(3x-2) &= 0 \\ x &= \frac{2}{3}, 4 \end{aligned}$$