Collecting terms

In order to rearrange or simplify an algebraic equation one often needs to 'tidy up' an expression involving various terms. A term means a distinct algebraic quantity, i.e. an unknown number which is represented in terms of a power of a particular variable such as x or y, or indeed a more complex expression!

This can be done by writing all terms multiplied by a particular term as that term multiplied by a bracket. In essence, 'collecting terms' is a simple type of *factorization*.

$$y = x^2 + x - 5 - 4x + 2x^2 + 3 - 10x + 5x^2$$

$$y = x^2(1+2+5) + x(1-4-10) + (-5+3)$$

It is good practice to write down all the terms followed by a bracket. Then fill in the terms in the bracket by checking through the expanded expression.

$$\boxed{y = 8x^2 - 13x - 2}$$

When terms are *mixed*, one needs to make a choice which term is 'best' to factor out. 'Best' usually means 'what leads to the simplest expression.'
In the example below, terms have been collected in two ways. In the first example, powers of x. In the second example, powers of y.

$$z = x^2y + y^2x + 3x^2 - 2y + 5x + 1$$

$$\boxed{z = x^2(y+3) + x\left(y^2+5\right) - 2y + 1}$$

$$\boxed{z = y^2x + y\left(x^2-2\right) + 3x^2 + 5x + 1}$$

Example: Find constants a, b and R in the following expression

$$f(x) = x^3 - 2x^2 + 3x - 1$$

$$f(x) = (x-1)\left(x^2 + ax + b\right) + R$$

$$f(x) = x^3 - x^2 + ax^2 - ax + bx - b + R$$

Collect terms in powers of x

$$f(x) = x^3 + x^2(a-1) + x(b-a) - b + R$$

$$\therefore \quad x^2: \quad a - 1 = -2 \Rightarrow \boxed{a = -1}$$

$$\therefore \quad x^1: \quad b - a = 3 \Rightarrow b = 3 + a \Rightarrow \boxed{b = 2}$$

$$\therefore \quad x^0: \quad -b + R = -1 \Rightarrow R = -1 + b \Rightarrow \boxed{R = 1}$$

These steps are called
'comparing the coefficients'
In other words, comparing the *collected terms* of powers of x

$$\therefore \boxed{f(x) = (x-1)\left(x^2 - x + 2\right) + 1}$$

This process enables us to factorize *higher polynomials* such as cubics into the product of simpler functions, e.g. linear and quadratic

When terms are other functions

$$y = x\sin x + x^3\cos x + 3\sin x + e^x \sin x + \frac{\cos x}{x}$$

$$\boxed{y = \sin x\left(x + 3 + e^x\right) + \cos x\left(x^3 + \frac{1}{x}\right)}$$