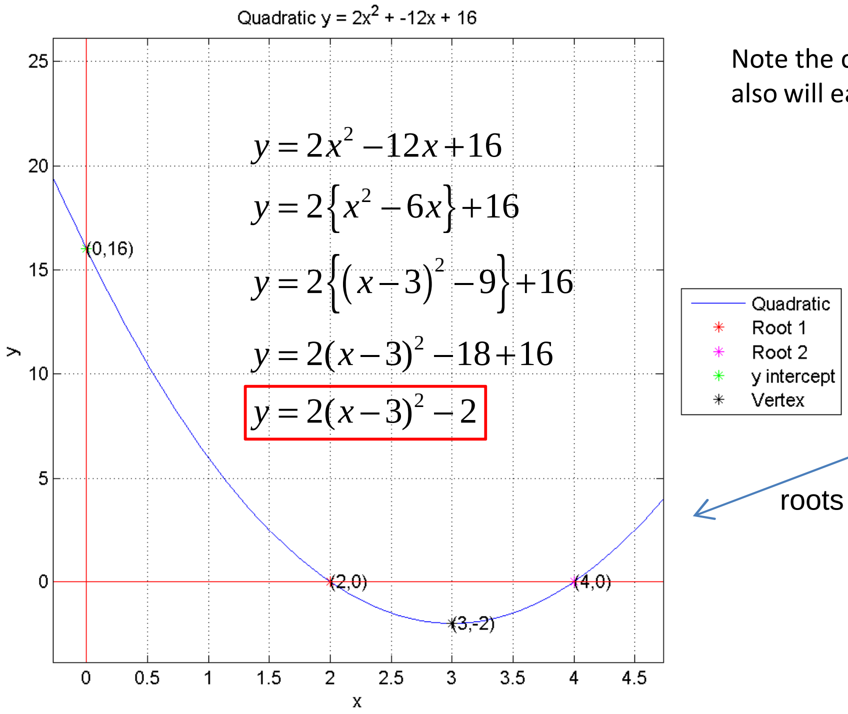
Completing the square

A quadratic equation in *completed square form* looks like $y = a\left(x - A\right)^2 + C$ where *a*, *A* and *C* are constants. The advantage of this way of writing the quadratic is that the coordinates of the *vertex* (i.e. the highest point if $a < 0$ or lowest point if $a > 0$) can be determined without any further calculation. Since the squared quantity $(x - A)^2$ *must be positive*, when *x* is not equal to *A*, it will (depending on the sign of *a*) only add or only subtract from *C*. Hence *(A,C)* is the vertex coordinate.



$$y = 2x^2 - 12x + 16$$
$$y = 2\left\{x^2 - 6x\right\} + 16$$
$$y = 2\left\{\left(x-3\right)^2 - 9\right\} + 16$$
$$y = 2(x-3)^2 - 18 + 16$$
$$\boxed{y = 2(x-3)^2 - 2}$$

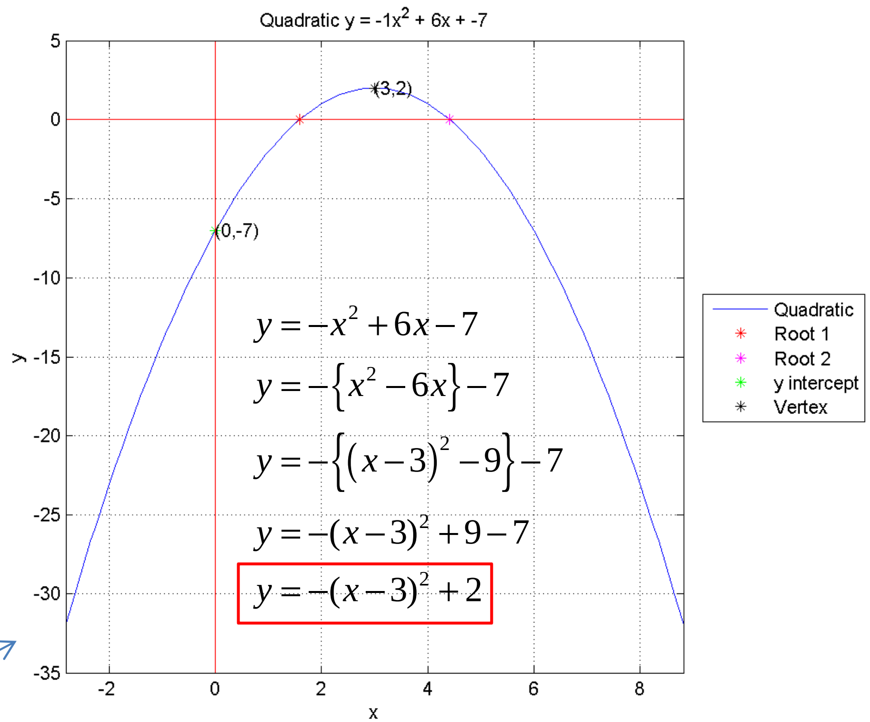
Note the completed square form also will easily yield the *roots* of the quadratic

$$y = 2x^2 - 12x + 16 = 0$$
$$0 = 2(x-3)^2 - 2$$
$$2 = 2(x-3)^2$$
$$\pm 1 = x - 3$$
$$x = 3 \pm 1$$
$$\boxed{x = 2, 4}$$

roots

$$y = -x^2 + 6x - 7$$
$$0 = -(x-3)^2 + 2$$
$$(x-3)^2 = 2$$
$$x - 3 = \pm\sqrt{2}$$
$$\boxed{x = 3 \pm \sqrt{2}}$$

roots



$$y = -x^2 + 6x - 7$$
$$y = -\left\{x^2 - 6x\right\} - 7$$
$$y = -\left\{\left(x-3\right)^2 - 9\right\} - 7$$
$$y = -(x-3)^2 + 9 - 7$$
$$\boxed{y = -(x-3)^2 + 2}$$

To 'complete the square' we consider only the $ax^2 + bx$ part of the quadratic.

$$y = ax^2 + bx + c$$
$$y = a\left\{x^2 + \frac{b}{a}x\right\} + c$$
$$y = a\left\{\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right\} + c$$
$$\boxed{y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c}$$

$$\left(x+b\right)^2 = x^2 + 2bx + b^2$$
$$\therefore \boxed{x^2 + 2bx = \left(x+b\right)^2 - b^2}$$

Subtract b^2 from the 'perfect square' $(x - b)^2$ as we would have an **extra** b^2 term if we multiplied it out.

We **halve** the coefficient of x

When $y = 0$, this gives the *quadratic formula*

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$