Direct & Inverse proportion

If two quantities are ***proportional***, this means that their ***ratio*** is a constant. In most of the examples below, this ***constant of proportionality*** shall be denoted by *k*.

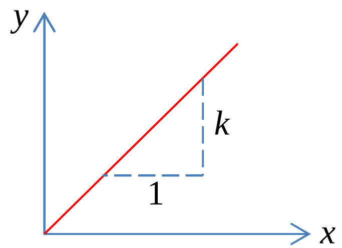
$$y \propto x \quad \Rightarrow \frac{y}{x} = k \quad \boxed{y = kx}$$

$$y \propto x^2 \quad \Rightarrow \frac{y}{x^2} = k \quad \boxed{y = kx^2}$$

$$y \propto 1+x^2 \quad \Rightarrow \frac{y}{1+x^2} = k \quad \boxed{y = k\left(1+x^2\right)}$$

$$y \propto \frac{\sqrt{x}-1}{x} \quad \Rightarrow \boxed{y = k\frac{\sqrt{x}-1}{x}}$$

If *y* is proportional to *x*, this describes a **straight line of gradient *k* which passes through the origin.**



Practical examples from Physics:
Ohm's Law $V = IR$
(V is voltage, I is current and R is current.
In this case R is the constant)

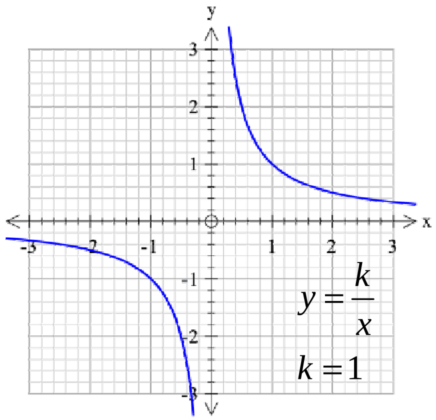
Hooke's Law $F = kx$
(F is force due to a stretched spring, k is the spring constant and x is the displacement from equilibrium).

If two quantities are ***inversely proportional***, this means that their ***product*** is a constant.

$$y \propto \frac{1}{x} \quad \Rightarrow xy = k \quad \boxed{y = \frac{k}{x}}$$

$$y \propto \frac{1}{x^2} \quad \Rightarrow yx^2 = k \quad \boxed{y = \frac{k}{x^2}}$$

$$y \propto \frac{1}{1+x^2} \quad \Rightarrow y\left(1+x^2\right) = k \quad \boxed{y = \frac{k}{1+x^2}}$$



If *y* is inversely proportional to *x*, then the graph of *y* vs *x* is a *hyperbola*. It tends to zero as the magnitude of *x* becomes large, and has an *asymptote* of the y axis as *x* tends to zero. It is undefined at *x* = 0.

Example 1: Force (F) in Newtons is directly proportional to the extension (x) Of a spring. 10N will extend the spring by 2cm. What extension will 15N result?

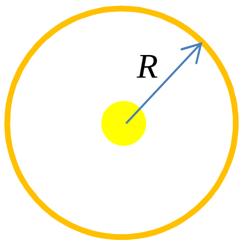
1. Find constant of proportionality *k* first $\quad \boxed{F = kx} \quad k = \frac{10\text{N}}{2\text{cm}} = 5\text{Ncm}^{-1}$
2. Then solve the problem using an *equation* $\quad x = \frac{F}{k} \quad \therefore x = \frac{15\text{N}}{5\text{Ncm}^{-1}} = \boxed{3\text{cm}}$

Example 2: The maximum power that could possibly be extracted From Sunlight via a photovoltaic cell on Earth is about **1kW**. Assuming an *inverse-square law*, what is the maximum power that could be extracted on Mars? Mars is approximately 1.5 times the distance from the Sun as the Earth.

$$P = \frac{k}{R^2} \qquad \text{Earth: } P = 1\text{kW/m}^2,\ R = 1\text{AU} \quad \therefore k = 1\text{AU}^2\text{kW/m}^2$$

$$\therefore P = \frac{1}{R^2} \qquad \text{Mars: } P = \frac{1}{\left(\frac{3}{2}\right)^2} = \frac{4}{9} \approx \boxed{0.44\text{kW/m}^2}$$

Note: If the Sun radiates *isotropically* (i.e. equally in all directions) *E* joules of energy *every second*, the total energy per second passing through a shell of radius *R* from the sun **must also be *E***, since *energy is conserved*.



The **power per square metre** passing through this shell is therefore:

$$P = \frac{E}{4\pi R^2}$$ since the surface area of the shell is $4\pi R^2$

This explains why the solar power per square metre is likely to follow an inverse-square law.