The **factor theorem** can be used to help factorize *cubics* and higher order polynomials, i.e. equations in integer powers of a variable x. Consider the cubic below in factorized form.

$f(x)=(x-1)(x-2)(x-3)$ Since (x-1) is a factor, *x=1 will make the expression zero*. Similarly, *f(2) = 0* and *f(3) = 0*.

This leads us to a general and somewhat obvious result (!)

$$f(x)=(x-a)g(x)+R$$
$$\therefore f(a)=R$$

i.e. we write *f(x)* as the product of a factor *(x-a)* and another function *g(x)* plus a constant *R*. The first part becomes zero when *x* = *a*, leaving a remainder *R*.

**The Factor Theorem therefore states:**

*(x – a)* is a factor of *f(x)* if *f(a) = 0*

**Example 1:** Confirm *(x+1)* is a factor of $f(x)=x^3-7x-6$ then find all three roots of the cubic $y=x^3-7x-6$

$$f(-1)=(-1)^3-7(-1)-6$$
$$f(-1)=-1+7-6$$
$$f(-1)=0$$

So by the Factor Theorem *(x+1)* must be a factor of *f(x)*

Hence:

$$f(x)=x^3-7x-6$$
$$f(x)=(x+1)(x^2+Ax-6)$$
$$f(x)=x^3+x^2+Ax^2+Ax-6x-6$$
$$f(x)=x^3+(1+A)x^2+(A-6)x-6$$

Comparing coefficients: $x^3-7x-6=x^3+(1+A)x^2+(A-6)x-6$

$x^2$: $1+A=0$

$\therefore A=-1$

$x$: $A-6=-7$

$\therefore A=-1$

check!

$$f(x)=(x+1)(x^2-x-6)$$
$$f(x)=(x+1)(x-3)(x+2)$$

Therefore *y = 0* when *x = -2,-1,3*

**Example 2:** Is *(x-2)* a factor of $f(x)=x^3-8x+11$ ? If not, what is the remainder and quotient?

$$f(x)=x^3-8x+11$$
$$f(x)=(x-2)(x^2+Ax+B)+R$$
$$f(2)=R$$
$$f(2)=(2)^3-8(2)+11$$
$$f(2)=8-16+11$$
$$f(2)=3$$
$$\therefore R=3$$

The *quotient* is

$$g(x)=x^2+Ax+B$$

i.e. $f(x)=(x+2)g(x)+R$

$$f(x)=x^3-8x+11$$
$$f(x)=(x-2)(x^2+Ax+B)+3$$
$$f(x)=x^3-2x^2+Ax^2-2Ax+Bx-2B+3$$
$$f(x)=x^3+(A-2)x^2+x(B-2A)+3-2B$$

$x^2$: $A-2=0 \Rightarrow A=2$

$x^1$: $B-2A=-8 \Rightarrow B=-4$

$x$ : $3-2B=11 \Rightarrow B=-4$

Comparing coefficients of powers of *x* in *f(x)*

$$f(x)=(x-2)(x^2+2x-4)+3$$

$$\therefore g(x)=x^2+2x-4$$