Factorizing quadratics

A quadratic equation in factorized form is given by a 'double-bracket' product of linear functions $y = a(Ax+B)(Cx+D)$ where a, A, B, C, D are constants. Although *all* quadratics can be written in this way, to say a quadratic is 'factorizable' typically means these values are *integers*.

Factorized form is useful because the *roots* of the quadratic (i.e. the x solutions of y = 0) can be found simply by *looking* at the expression.

$$y = -3(2x+1)(7x-5)$$
$$y = 0$$
$$\therefore \;\; 2x+1=0 \;\; \Rightarrow \;\; x = -\tfrac{1}{2}$$
$$\therefore \;\; 7x-5=0 \;\; \Rightarrow \;\; x = \tfrac{5}{7}$$

$$\boxed{y = a(Ax+B)(Cx+D)}$$

Factorized form of a quadratic equation

Where x = 0 is a root. i.e x is a factor

$$y = -2x^2 + 8x$$
$$y = -2x(x-4)$$
$$y = 0$$
$$\therefore \;\; x = 0$$
$$\therefore \;\; x-4=0 \;\; \Rightarrow \;\; x = 4$$

A perfect square
In this case both brackets are the same and therefore give the same solution

$$y = 4x^2 - 12x + 9$$
$$y = (2x-3)^2$$
$$y = 0$$
$$\therefore \;\; 2x-3=0 \;\; \Rightarrow \;\; x = \tfrac{3}{2}$$

Difference of two squares

$$y = 144x^2 - 81$$
$$y = (12x-9)(12x+9)$$
$$y = 0$$
$$\therefore \;\; 12x-9=0 \;\; \Rightarrow \;\; x = \tfrac{3}{4}$$
$$\therefore \;\; 12x+9=0 \;\; \Rightarrow \;\; x = -\tfrac{3}{4}$$

$$\boxed{\begin{aligned} y &= a^2x^2 - b^2 \\ y &= (ax-b)(ax+b) \end{aligned}}$$

Coefficient of x² is one

$$\boxed{\begin{aligned} y &= (x+a)(x+b) \\ y &= x^2 + (a+b)x + ab \end{aligned}}$$

$$y = x^2 - 10x + 21$$
$$y = (x-3)(x-7)$$
$$y = 0$$
$$\therefore \;\; x = 3, 7$$

i.e. we need two numbers that multiply to give 21 and add to give -10

Coefficient of x² is *not* one
To factorize we can either (i) guess, or (ii) convert into a quadratic in 'ax' which has a (ax)² coefficient of one.*

$$y = 6x^2 - x - 35$$
$$6y = (6x)^2 - (6x) - 210$$

i.e. this is a 'disguised' quadratic in 6x

$$6y = (6x-15)(6x+14)$$
$$6y = (3)(2x-5)(2)(3x+7)$$
$$y = (2x-5)(3x+7)$$
$$\therefore \;\; x = -\tfrac{7}{3}, \tfrac{5}{2}$$

*Note there are other methods given in textbooks which are just as effective. I like this one!

Various *disguised* quadratics (that factorize)

Rearranged basic quadratics

$$\frac{15}{x}+\frac{56}{x^2}=-1$$
$$15x+56=-x^2$$
$$x^2+15x+56=0$$
$$(x+7)(x+8)=0$$
$$\therefore\ x=-8,-7$$

Powers of x

$$x^4-14x^2+45=0$$
$$(x^2-9)(x^2-5)=0$$
$$x^2=9\quad\therefore\ x=\pm 3$$
$$x^2=5\quad\therefore\ x=\pm\sqrt{5}$$

Surds

$$x-20\sqrt{x}+91=0$$
$$(\sqrt{x}-7)(\sqrt{x}-13)=0$$
$$\sqrt{x}=7\quad\therefore\ x=49$$
$$\sqrt{x}=13\quad\therefore\ x=169$$

Reciprocals

$$\frac{1}{x^2}+\frac{4}{x}-96=0$$
$$(x^{-1}+12)(x^{-1}-8)=0$$
$$x^{-1}=-12\quad\therefore\ x=-\tfrac{1}{12}$$
$$x^{-1}=8\quad\therefore\ x=\tfrac{1}{8}$$

Basic trigonometry (with no identities)

$$0^\circ\le\theta<360^\circ$$

$$4\sin^2\theta+4\sin\theta+1=0$$
$$(2\sin\theta+1)(2\sin\theta+1)=0$$
$$\sin\theta=-\tfrac{1}{2}\quad\therefore\ x=210^\circ,330^\circ$$

Basic trigonometry in terms of $n\theta$

$$0^\circ\le\theta<360^\circ$$

$$2\cos^2 2\theta+9\cos 2\theta-5=0$$
$$(2\cos 2\theta-1)(\cos 2\theta+5)=0$$
$$\cos 2\theta=\tfrac{1}{2}\quad\therefore\ \theta=30^\circ,150^\circ,210^\circ,330^\circ$$

Power equations

$$\left(2^x\right)^x\times\sqrt[10]{1024^x}=16^5$$
$$2^{x^2}\times\left(2^{10x}\right)^{\frac{1}{10}}=\left(2^4\right)^5=2^{20}$$
$$2^{x^2+x-20}=2^0$$
$$x^2+x-20=0$$
$$(x-4)(x+5)=0$$
$$\therefore\quad x=-5,4$$

A double disguise!

$$\frac{7^x}{343^{\sqrt{x}}}=\frac{1}{49}$$
$$7^{x-3\sqrt{x}+2}=7^0$$
$$x-3\sqrt{x}+2=0$$
$$(\sqrt{x}-1)(\sqrt{x}-2)=0$$
$$\sqrt{x}=1\quad\therefore\ x=1$$
$$\sqrt{x}=2\quad\therefore\ x=4$$