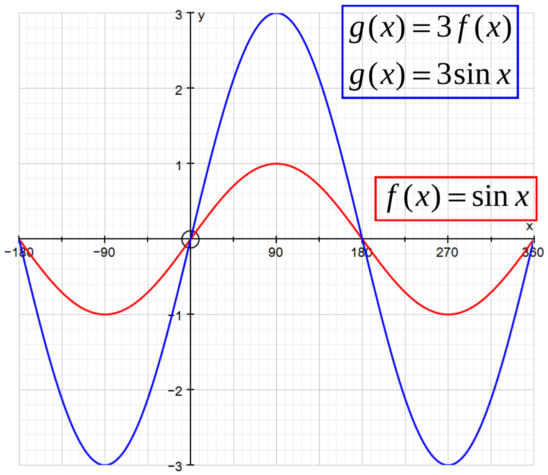
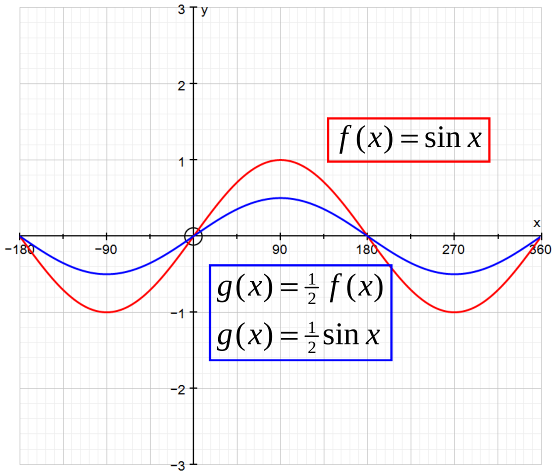
Function transformations

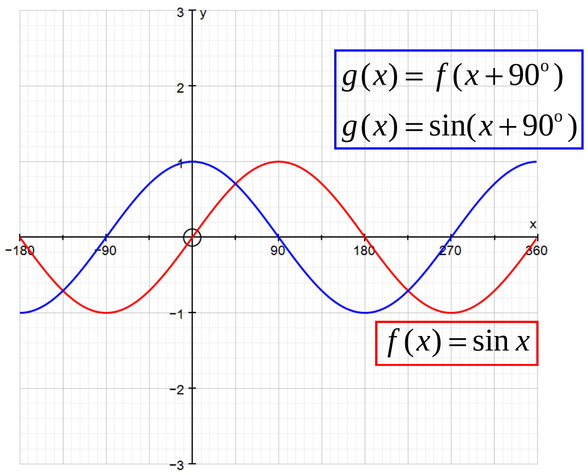
The (*x,y*) curve of a wide variety of functions can be readily sketched by considering successive *transformations* of more basic functions. Transformations are of the *geometric* kind, i.e. a *stretch* in either x or y direction, a *translation* in the x or y direction or a *reflection* in the lines *x = 0*, *y = 0* or *y = x*. (The latter is an *inverse* transformation)

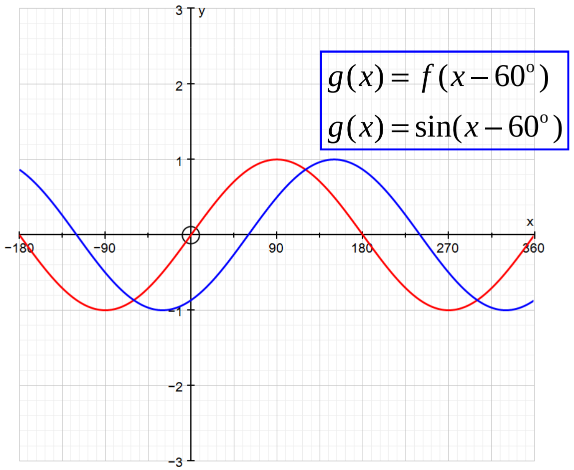




$f(x) \to kf(x)$

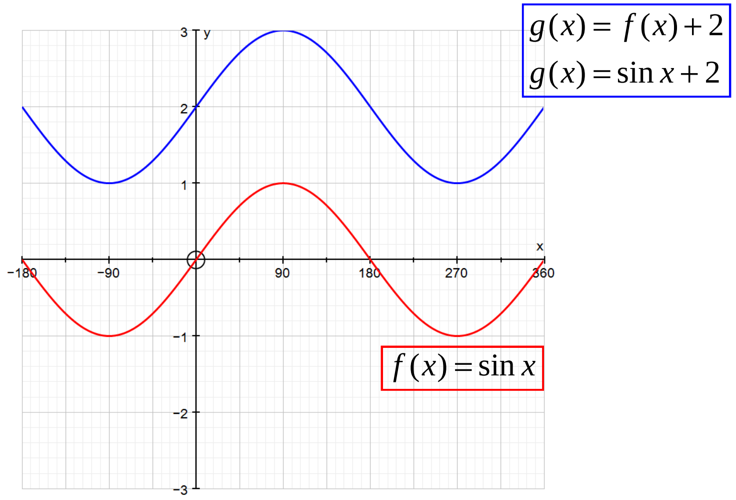
Stretch in *y* direction by *k*





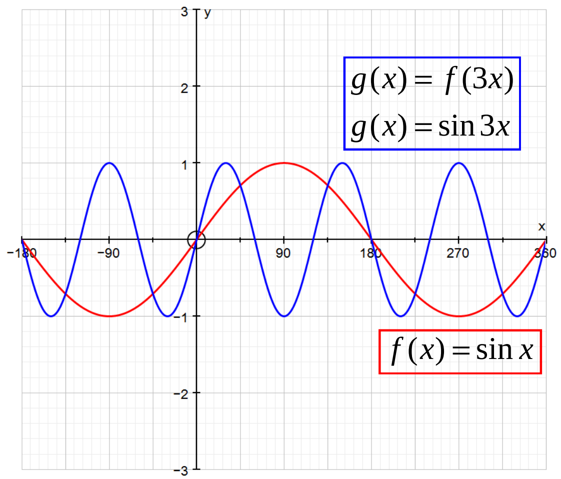
$f(x) \to f(x-a)$

Translate in *x* direction by *a*



$f(x) \to f(x) + b$

Translate in *y* direction by *b*



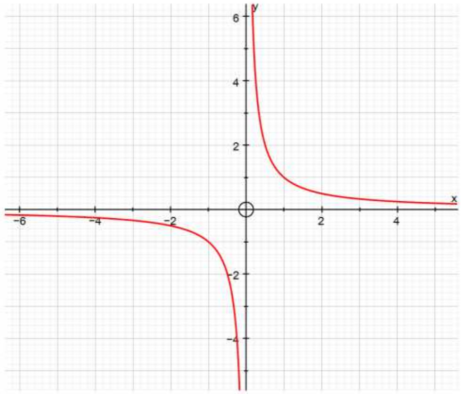
$f(x) \to f(kx)$

Stretch in *x* direction by *1/k*

[Note: For all the examples here we shall evaluate sine and cosine functions of variables measured in *degrees* rather than radians]

Example 1: Write and hence sketch $y=1-\dfrac{1}{x+2}$ as a series of transformations

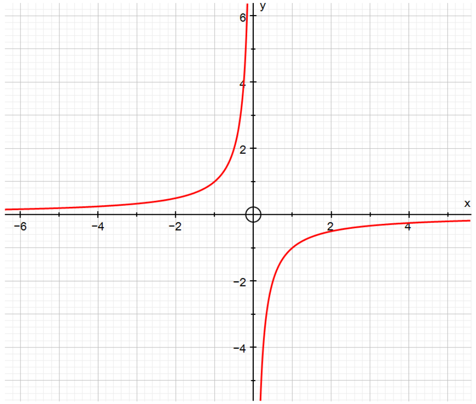
$y=\dfrac{1}{x}$



$f(x)=\dfrac{1}{x}$

Basic function

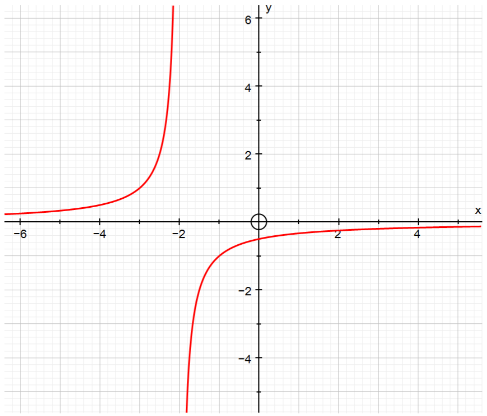
$y=-\dfrac{1}{x}$



$f(x)\to -f(x)$

y stretch by -1
(or reflect in the
x axis)

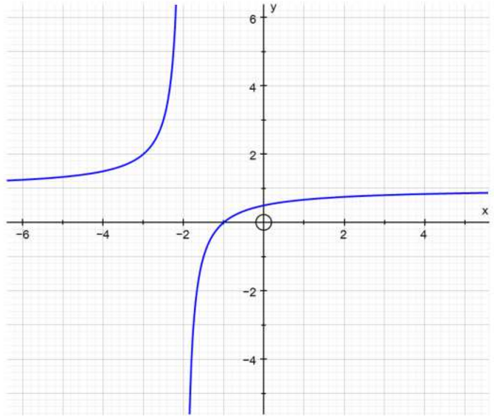
$y=-\dfrac{1}{x+2}$



$-f(x)\to -f(x+2)$

Translate in the
x direction by -2

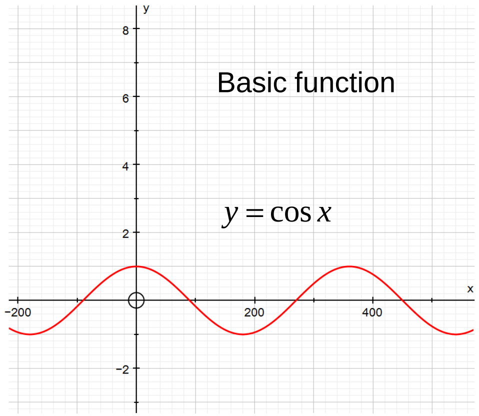
$y=1-\dfrac{1}{x+2}$



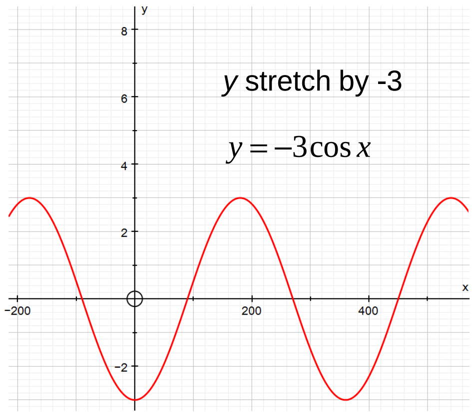
$-f(x+2)\to -f(x+2)+1$

Translate in the
y direction by 1

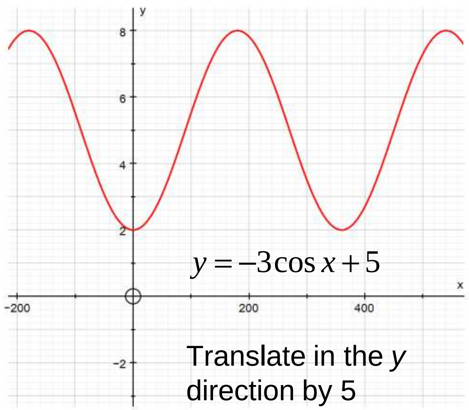
Example 2: Write and hence sketch $y=5-3\cos 2x$ as a series of transformations



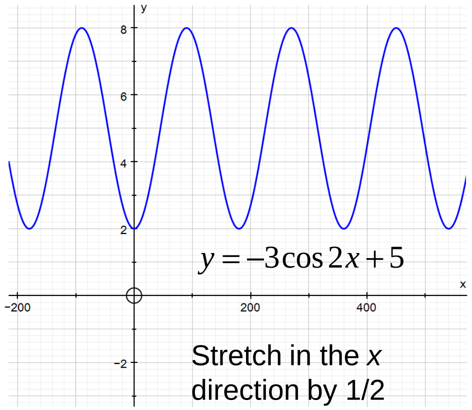
$f(x)=\cos x$



$f(x)\to -3f(x)$



$-3f(x)\to -3f(x)+5$



$-3f(x)+5\to -3f(2x)+5$