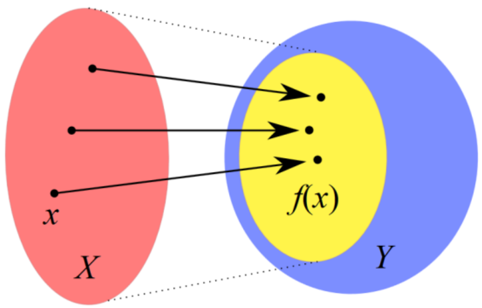
Functions

The set of possible inputs to a function is the **domain**
The set of possible values that the outputs of a function lies is the **codomain**
The actual possible outputs of a function is the **range**.
The range is a *subset* of the co-domain, and depends on the inputs chosen.

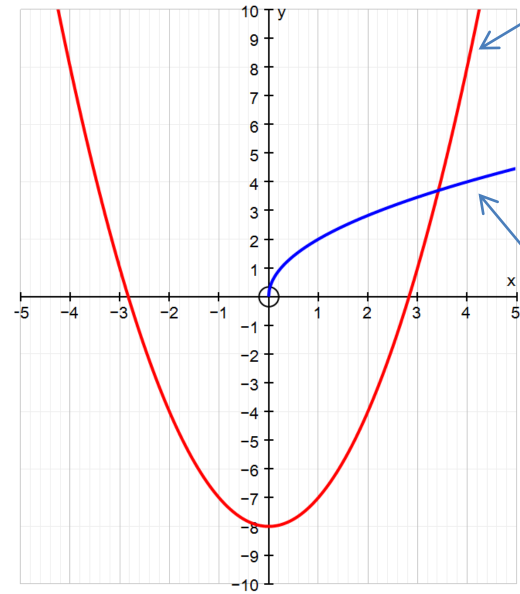


$$f : X \to Y$$

red: domain
blue: codomain
yellow: range

Combining functions

$$f(x) = x^2 + 2x + 1$$
$$g(x) = \sqrt{x}$$
$$fg(x) = f(g(x))$$
$$= \left(\sqrt{x}\right)^2 + 2\sqrt{x} + 1$$
$$= x + 2\sqrt{x} + 1$$
$$gf(x) = g(f(x)) = \sqrt{x^2 + 2x + 1}$$
$$gg(x) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$$



$y = f(x) = x^2 - 8$
domain: $-\infty < x < \infty$
codomain: $-8 \le f(x) < \infty$

if $-3\sqrt{2} \le x \le 3\sqrt{2}$
range: $-8 \le f(x) \le 10$

$y = g(x) = \sqrt{x}$
domain: $0 \le x < \infty$
codomain: $0 \le g(x) < \infty$

if $0 \le x \le 5$
range: $0 \le g(x) \le \sqrt{5}$

The **inverse** of a function is its *reflection* in the line y = x.

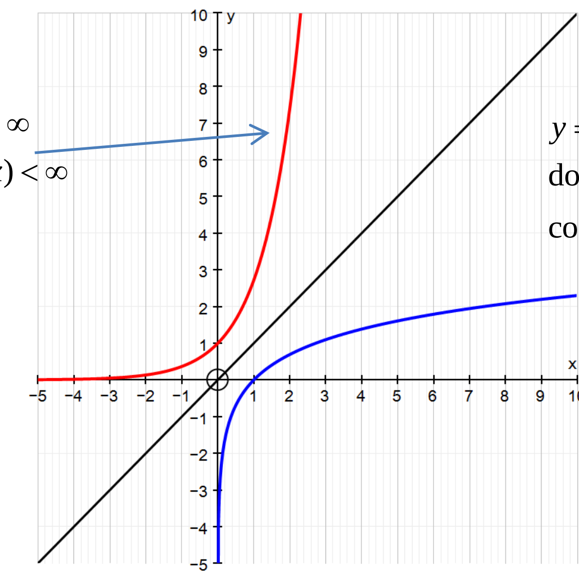
To find the inverse of a function, re-arrange to make *x* the subject, then set the formula in terms of *f* to be the inverse.

$$f(x) = \sqrt{x+3}$$
$$\therefore f^2 - 3 = x$$
$$\therefore f^{-1}(x) = x^2 - 3$$

Note range and codomain often have the same meaning, if one uses the *fullest extent of the domain*. However, we can *reduce the range by reducing the domain* (e.g only use integer values). This will *not* change the codomain, which is the set of possible outputs. So *we* set the domain, but the function determines the codomain!

$y = f(x) = e^x$
domain: $-\infty < x < \infty$
codomain: $0 \le f(x) < \infty$

if $-5 \le x \le \ln 10$
range: $e^{-5} \le f(x) \le 10$



$y = x$

$y = f^{-1}(x) = \ln x$
domain: $0 < x < \infty$
codomain: $-\infty < f^{-1}(x) < \infty$

if $0 < x \le 10$
range: $-\infty < f(x) \le \ln 10$