Laws of indices

$$x^a x^b = x^{a+b}$$

e.g. $10^3 \times 10^6 = 10^9$

$$(x^a)^b = x^{ab}$$

e.g. $(2^3)^2 = 2^6 = 64$

$$x^{-a} = \frac{1}{x^a}$$

e.g. $10^6 / 10^{-3} = 10^9$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\sqrt{10} \equiv 10^{\frac{1}{2}}$$

$$\sqrt[3]{17} \equiv 17^{\frac{1}{3}}$$

$$\left(\sqrt[4]{2}\right)^{12} = 2^{\frac{12}{4}} = 2^3 = 8$$

Powers of 2	Powers of 3	Powers of 5	Squares	Cubes
$2^0 = 1$	$3^0 = 1$	$5^0 = 1$	$1^2 = 1$	$1^3 = 1$
$2^1 = 2$	$3^1 = 3$	$5^1 = 5$	$2^2 = 4$	$2^3 = 8$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$	$3^2 = 9$	$3^3 = 27$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$	$4^2 = 16$	$4^3 = 64$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$	$5^2 = 25$	$5^3 = 125$
$2^5 = 32$	$3^5 = 243$		$6^2 = 36$	$6^3 = 216$
$2^6 = 64$	$3^6 = 729$		$7^2 = 49$	$7^3 = 343$
$2^7 = 128$			$8^2 = 64$	$8^3 = 512$
$2^8 = 256$			$9^2 = 81$	$9^3 = 729$
$2^9 = 512$			$10^2 = 100$	$10^3 = 1000$
$2^{10} = 1024$			$11^2 = 121$	$11^3 = 1331$
			$12^2 = 144$	$12^3 = 1728$
			$13^2 = 169$	
$2^{11} = 2048$			$14^2 = 196$	
$2^{12} = 4096$			$15^2 = 225$	
$2^{16} =$ 65,536			$16^2 = 256$	
$2^{24} =$ 16,777,216			$17^2 = 289$	
$2^{32} =$ 4,294,967,296			$18^2 = 324$	
			$19^2 = 361$	
			$20^2 = 400$	

Equations involving powers

To solve these *make the base the same* and then *equate the powers*.
If the base cannot be made to be the same using known squares, cubes etc then on has to use *logarithms*.

$$243^{3x-1} = 81^{5-x}$$
$$\left(3^5\right)^{3x-1} = \left(3^4\right)^{5-x}$$
$$3^{5(3x-1)} = 3^{4(5-x)}$$
$$\therefore 15x - 5 = 20 - 4x$$
$$19x = 25$$
$$\boxed{x = \frac{25}{19}}$$

$$2^{3x-1} = 3^{5-x}$$
$$(3x-1)\log 2 = (5-x)\log 3$$
$$x(3\log 2 + \log 3) = 5\log 3 + \log 2$$
$$x = \frac{5\log 3 + \log 2}{3\log 2 + \log 3}$$
$$x = \frac{\log\left(3^5 \times 2\right)}{\log\left(2^3 \times 3\right)}$$
$$\boxed{x = \frac{\log 486}{\log 24}}$$

Simplifying surds

$$\frac{\sqrt{98}}{7} = \frac{\sqrt{70+28}}{7} = \frac{\sqrt{14 \times 7}}{7} = \frac{\sqrt{2 \times 7^2}}{7} = \frac{7\sqrt{2}}{7} = \boxed{\sqrt{2}}$$

$$\left(2\sqrt{5}\right)^6 = 2^6 \times 5^{\frac{6}{2}} = 2^6 \times 5^3$$
$$= \left(2^2\right)^3 \times 5^3 = (4 \times 5)^3$$
$$= 20^3 = \boxed{8{,}000}$$

$$\left(7 + 4\sqrt{2}\right)\left(5 - 3\sqrt{2}\right)$$
$$= 35 + 20\sqrt{2} - 21\sqrt{2} - 12\sqrt{2}\sqrt{2}$$
$$= 35 - \sqrt{2} - 24$$
$$= \boxed{11 - \sqrt{2}}$$

$$\frac{\sqrt{5}+3}{\sqrt{5}-1} = a\sqrt{5} + b$$

Find *a* and *b*

$$\frac{\sqrt{5}+3}{\sqrt{5}-1} = \frac{\left(\sqrt{5}+3\right)\left(-\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(-\sqrt{5}-1\right)} = \frac{-5-3\sqrt{5}-\sqrt{5}-3}{-5+1} = \frac{-8-4\sqrt{5}}{-4} = 2 + \sqrt{5}$$

$\boxed{a = 1 \text{ and } b = 2}$