Introduction to quadratic equations

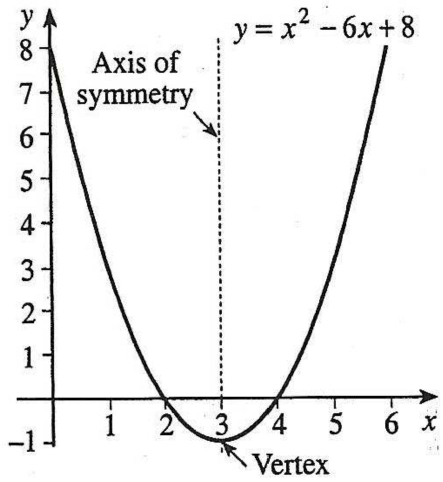
A *quadratic equation* is one which contains, in general, a term in x^2, a linear term in x plus a constant

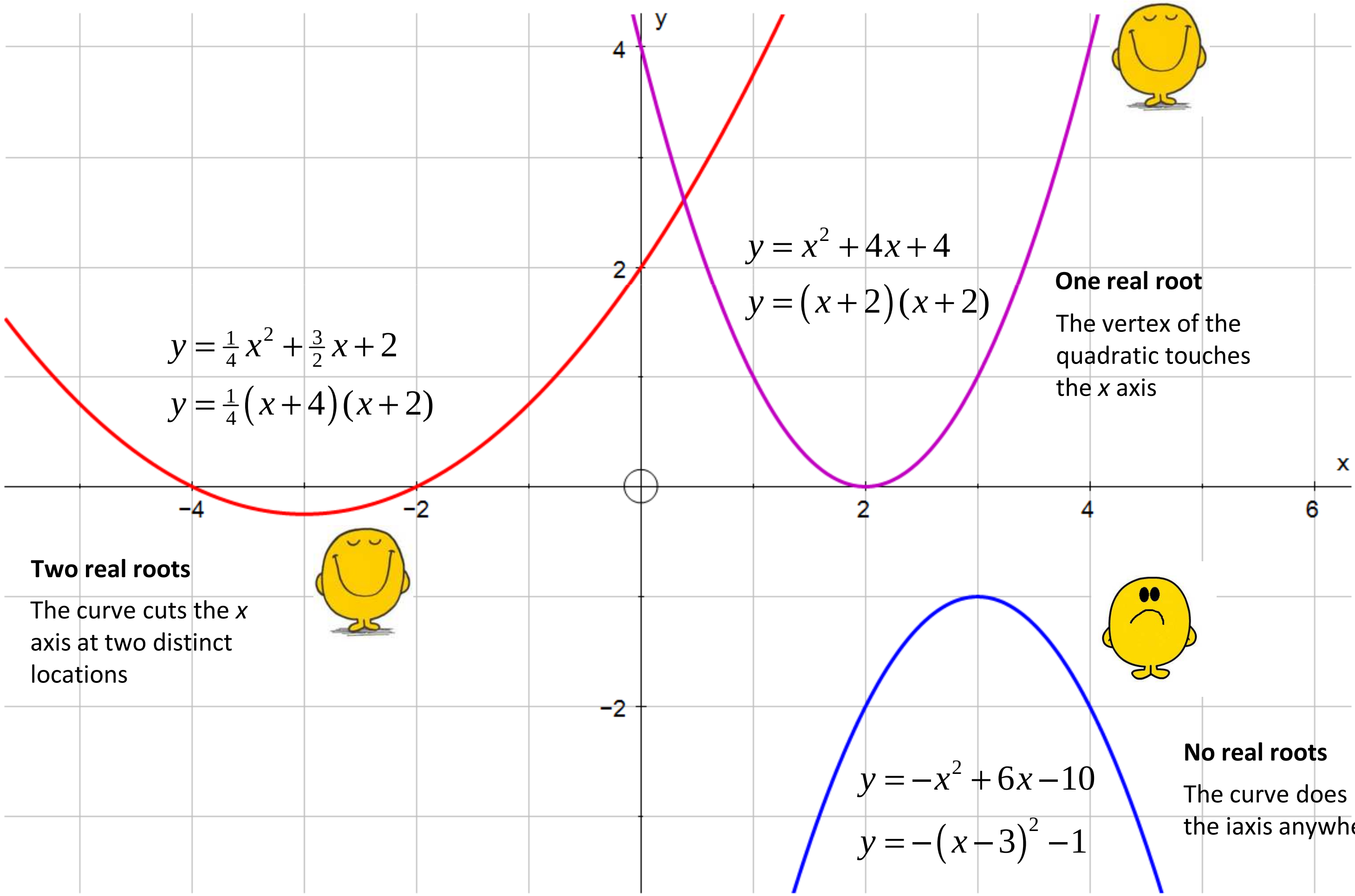
$$y = ax^2 + bx + c$$

is the **expanded form** of the quadratic. The sign of the x^2 *coefficient, a* tells us whether the curve is a ‘smile’ ($a > 0$) or a ‘frown’ ($a < 0$). The constant *c* tells us where the curve crosses the *y* axis i.e. when $x = 0$.

$a > 0$

$a < 0$





Two real roots

The curve cuts the *x* axis at two distinct locations

One real root

The vertex of the quadratic touches the *x* axis

No real roots

The curve does not cut the iaxis anywhere

There are two other ‘forms’ of a quadratic equation

The **factorized form** tells us what the *x* values are when $y = 0$. These are called the *roots* of the quadratic.

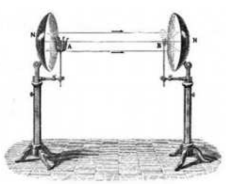
$$y = \tfrac{1}{4}(x+4)(x+2)$$
$$y = 0$$
$$x + 4 = 0 \Rightarrow x = -4$$
$$x + 2 = 0 \Rightarrow x = -2$$

The **completed square form** tells us the location of the *vertex*

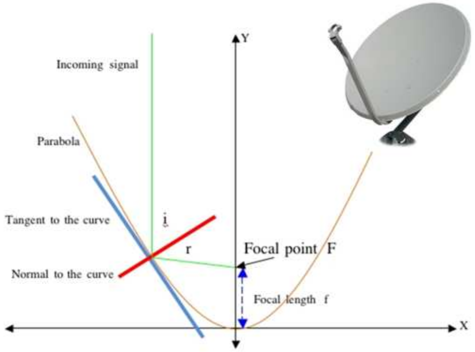
$$y = -(x-3)^2 - 1$$

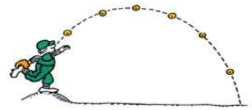
vertex is at coordinate (3,-1)

Some applications of Quadratic Equations



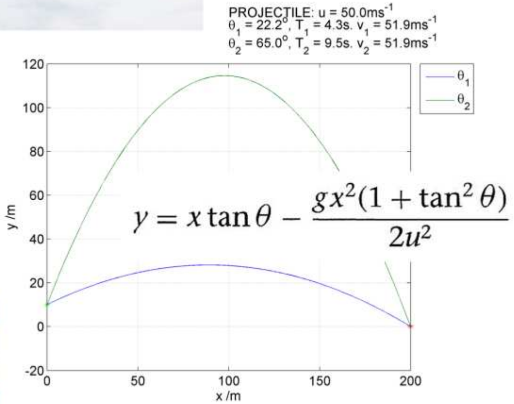
Parabolic reflectors



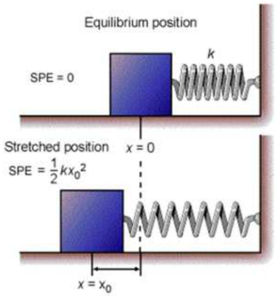


Projectile motion

$g = 9.81\text{ms}^{-2}$

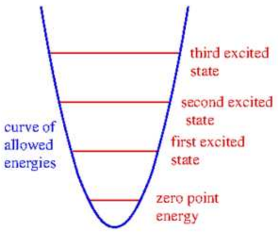


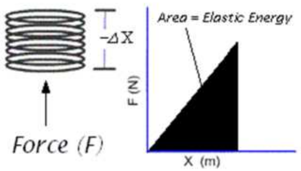
$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$$



Potential energy in springs

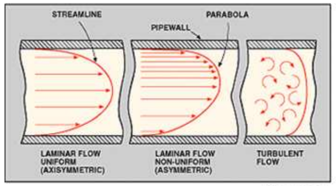
From car suspension to vibration of molecules!





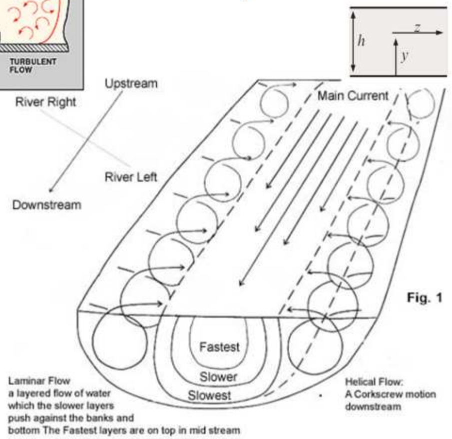
$$F = kx$$

$$E = \int F\,dx = \int kx\,dx = \tfrac{1}{2}kx^2 + c$$



Velocity profile of fluids in a river or pipe

$$v_z(y) = \frac{1}{2\eta}y(h-y)\frac{\partial p}{\partial z}$$



Bernoulli's equation (incompressible flow)

$$\frac{1}{2}\rho v^2 + p + \rho g z = \text{constant}$$