

Linear programming relates to the use of *several linear inequalities* in variables x and y to define a *region* of the x,y plane. In a practical problem x and y might relate to, typically integer, values of quantities that are subject to certain constraints. The goal of linear programming is to work out the valid subset of x,y values. Often a 'cost' function of x and y will also be defined. This can further select 'optimal' x,y coordinates if cost is to be minimized, maximized etc.

Example:

$$x + y \leq 11$$

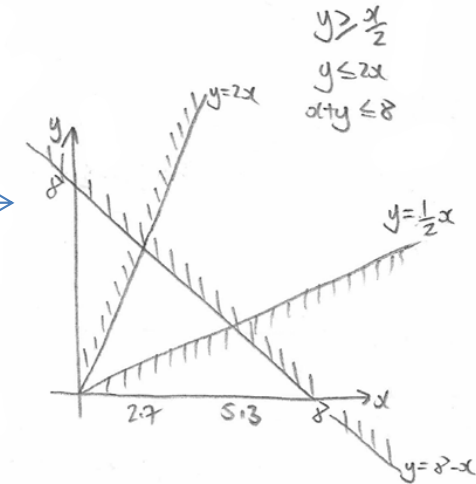
$$y \geq 3$$

$$y \leq x$$

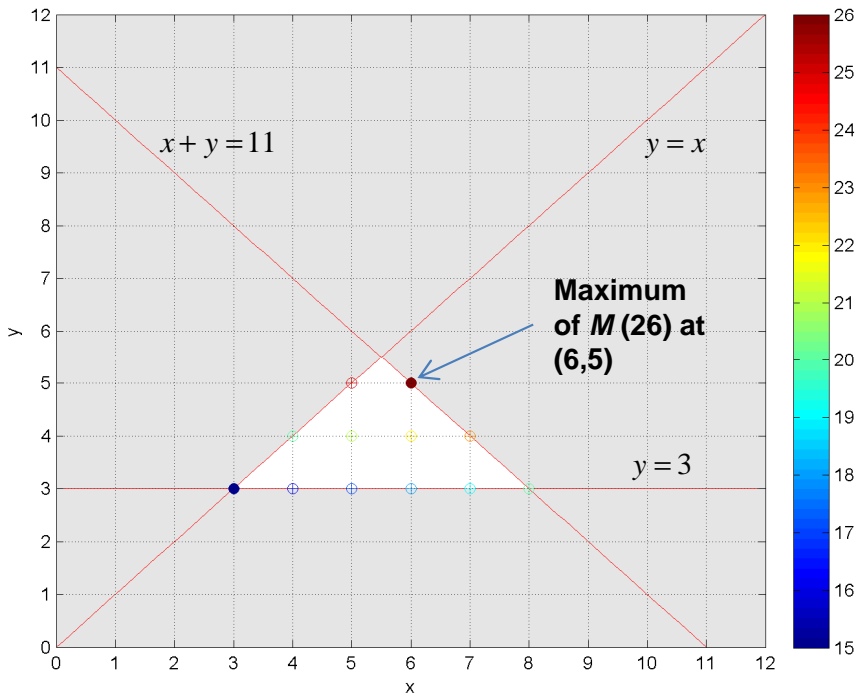
$$M = x + 4y$$

$$m = 3x + y$$

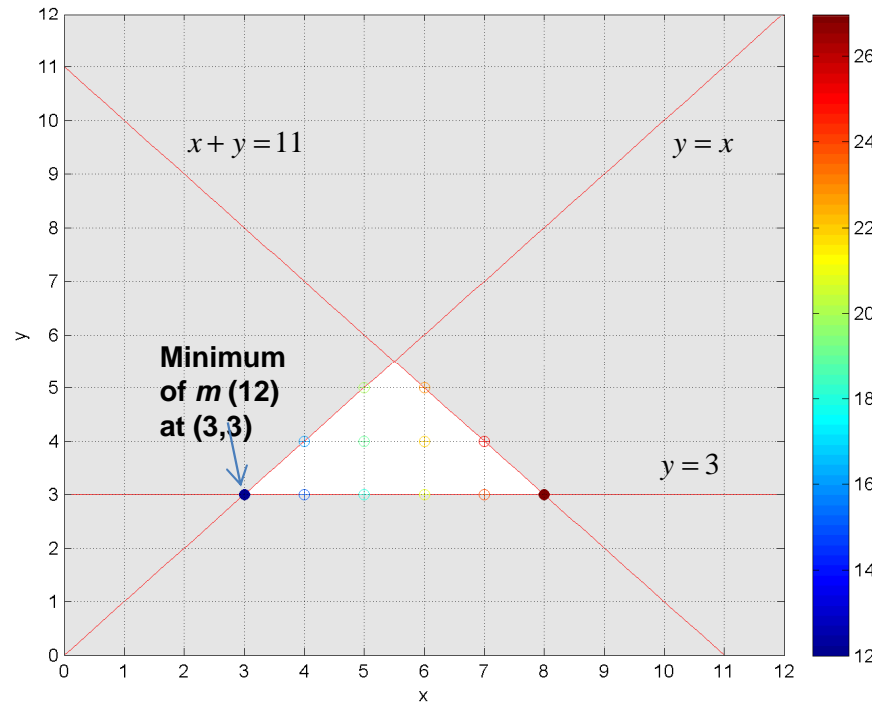
1. To solve a linear programming problem, firstly turn all the inequalities into *equations* and plot them on an (x,y) grid. Take some care to work out an appropriate *scale* to encompass the valid region.
2. Use the *inequalities* to shade above or below the lines to indicate *invalid* regions. This does not have to be the entire space! (See example on the right).
3. If integer values of x,y are required, use the graph paper grid to mark the valid coordinates. Only use the boundary lines if the inequality does indeed permit an $=$ as well as $>$ or $<$.
4. Tabulate the cost function, if required, and hence work out the 'optimum' x,y pair for the problem.



Colour coding of valid x,y coordinates by $M = x + 4y$



Colour coding of valid x,y coordinates by $m = 3x + y$



x	y	$x+4y$	$3x+y$
3	3	15	12
4	3	16	15
4	4	20	16
5	3	17	18
5	4	21	19
5	5	25	20
6	3	18	21
6	4	22	22
6	5	26	23
7	3	19	24
7	4	23	25
8	3	20	27