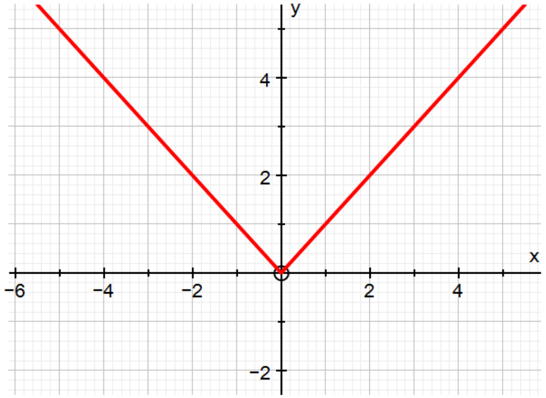
Modulus equations involve the symbol | *f(x)* | This means ignore the sign of *f(x)*.   i.e. |3| = |-3| = 3.



The graph of *y* = |*x*| is *discontinuous* at x = 0. This means the gradient of the graph is *undefined* at *x* = 0.

It is often a good idea to think of the function *f(x)* = |*x*| as being defined in terms of *line segments* as this is helpful when solving equations involving |*x*|.

$$f(x)=|x|$$

$$f(x)=\begin{cases}-x & x\le 0\\ x & x\ge 0\end{cases}$$

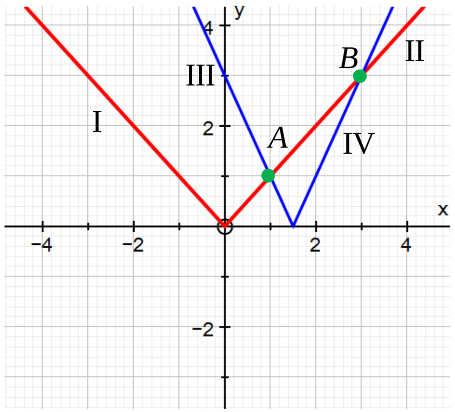
This generalizes to a stretched and translated version:

$$f\left(a(x-b)\right)+c=|a(x-b)|+c$$

$$f\left(a(x-b)\right)+c=\begin{cases}-a(x-b)+c & x\le 0\\ a(x-b)+c & x\ge 0\end{cases}$$

Example 1: Find solutions to $|x|=|2x-3|$

The solution is to break up each modulus term into pairs of line segments I,II, III, IV etc. Following a *graph sketch* we can determine the solution based upon the *intersections* of the appropriate segments.



I: $y=-x$

II: $y=x$

III: $y=-2x+3$

IV: $y=2x-3$

● Intersections are at points *A* and *B*

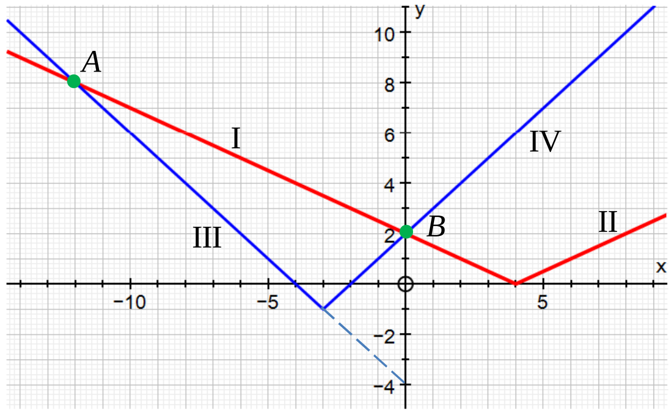
$$A:\ \underbrace{x}_{\text{II}}=\underbrace{-2x+3}_{\text{III}}$$

$$\therefore 3x=3\Rightarrow \boxed{x=1}$$

$$B:\ \underbrace{x}_{\text{II}}=\underbrace{2x-3}_{\text{IV}}$$

$$\therefore \boxed{x=3}$$

Example 2: Find solutions to $\left|\tfrac{1}{2}x-2\right|=|x+3|-1$



I: $y=-\tfrac{1}{2}x+2$

II: $y=\tfrac{1}{2}x-2$

III: $y=-x-4$

IV: $y=x+2$

● Intersections are at points *A* and *B*

$$A:\ \underbrace{-\tfrac{1}{2}x+2}_{\text{I}}=\underbrace{-x-4}_{\text{III}}$$

$$\therefore \tfrac{1}{2}x=-6\Rightarrow \boxed{x=-12}$$

$$B:\ \underbrace{-\tfrac{1}{2}x+2}_{\text{I}}=\underbrace{x+2}_{\text{IV}}$$

$$\therefore \boxed{x=0}$$

A **good sketch** is required if you are to correctly pair up the line segments!

Think about the *gradients* of each segment when deciding which ones will intersect.