

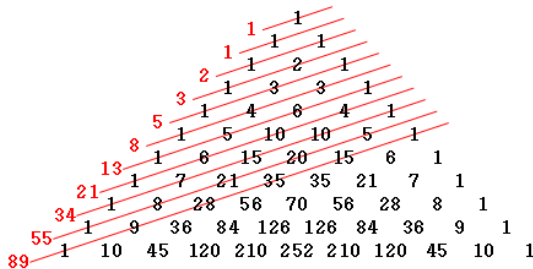
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Row n
Column k

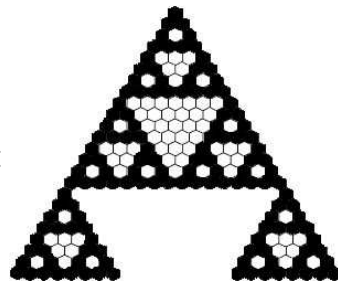


[Blaise Pascal](#) 1623-1662

- Concatenate row digits. Answer is 11^n . e.g. $1331 = 11^3$ $14641 = 11^4$
- If first (not 1) number in row n is prime, all other numbers are divisible by it
- Sum of numbers in rows = 2^n
- Generate the square numbers: $1+3 = 2^2$, $3+6 = 3^2$, $6+10 = 4^2$
- Sum of 'shallow diagonals' gives the Fibonacci numbers $F_n = F_{n-1} + F_{n-2}$



- Odd numbers for the Sierpinski Gasket



Binomial expansion

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

$$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$$

Pascal's Triangle yields **Binomial Coefficients**

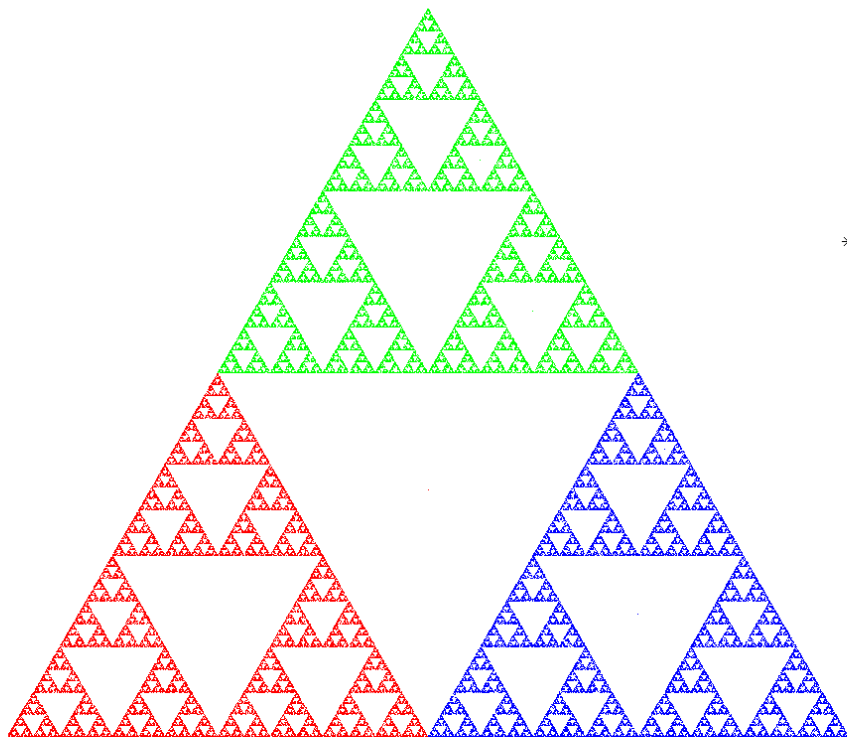
$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

$$\begin{aligned} (3x-4)^3 &= (3x)^3 + 3(3x)^2(-4) + 3(3x)(-4)^2 + (-4)^3 \\ &= 27x^3 - 12(9)x^2 + 9(16)x - 64 \\ &= 27x^3 - 108x^2 + 144x - 64 \end{aligned}$$

$$\begin{aligned} (2x+1)^4 &= (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x) + 1 \\ &= 16x^4 + 32x^3 + 24x^2 + 8x + 1 \end{aligned}$$

$$\begin{aligned} \left(2x - \frac{3}{x}\right)^6 &\text{ has constant term} \\ &= 20(2x)^3 \left(-\frac{3}{x}\right)^3 \\ &= -8x^3(27)(20) \frac{1}{x^3} = -4320 \end{aligned}$$

Sierpinski triangle via chaos game.
 N = 100000, p = [0.33333 0.33333 0.33333].



*

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(1+x)^{20} = \binom{20}{0}(1)^{20}x^0 + \binom{20}{1}(1)^{19}x + \binom{20}{2}(1)^{18}x^2 + \binom{20}{3}(1)^{17}x^3 + \dots$$

$$\binom{20}{0} = 1$$

$$\binom{20}{1} = \frac{20!}{1!19!} = 20$$

$$\binom{20}{2} = \frac{20!}{2!18!} = \frac{20 \times 19}{2} = 190$$

$$\binom{20}{3} = \frac{20!}{3!17!} = \frac{20 \times 19 \times 18}{3 \times 2} = 10 \times 19 \times 6 = 60 \times 19 = 1140$$

$$\therefore (1+x)^{20} = 1 + 20x + 190x^2 + 1140x^3 + \dots$$

		1																											
		1	1																										
		1	2	1																									
		1	3	3	1																								
		1	4	6	4	1																							
		1	5	10	10	5	1																						
		1	6	15	20	15	6	1																					
		1	7	21	35	35	21	7	1																				
		1	8	28	56	70	56	28	8	1																			
		1	9	36	84	126	126	84	36	9	1																		
		1	10	45	120	210	252	210	120	45	10	1																	
		1	11	55	165	330	462	462	330	165	55	11	1																
		1	12	66	220	495	792	924	792	495	220	66	12	1															
		1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1														
		1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1													
		1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1												
		1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1											
		1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	2380	680	136	17	1										
		1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	8568	3060	816	153	18	1									
		1	19	171	969	3876	11628	27132	50388	75582	92378	92378	75582	50388	27132	11628	3876	969	171	19	1								
		1	20	190	1140	4845	15504	38760	77520	125970	167960	184756	167960	125970	77520	38760	15504	4845	1140	190	20	1							
		1	21	210	1330	5985	20349	54264	116280	203490	293930	352716	352716	293930	203490	116280	54264	20349	5985	1330	210	21	1						
		1	22	231	1540	7315	26334	74613	170544	319770	497420	646646	705432	646646	497420	319770	170544	74613	26334	7315	1540	231	22	1					
		1	23	253	1771	8855	33649	100947	245157	490314	817190	1144066	1352078	1352078	1144066	817190	490314	245157	100947	33649	8855	1771	253	23	1				
		1	24	276	2024	10626	42504	134596	346104	735471	1307504	1961256	2496144	2704156	2496144	1961256	1307504	735471	346104	134596	42504	10626	2024	276	24	1			
		1	25	300	2300	12650	53130	177100	480700	1081575	2042975	3268760	4457400	5200300	5200300	4457400	3268760	2042975	1081575	480700	177100	53130	12650	2300	300	25	1		
		1	26	325	2600	14950	65780	230230	657800	1562275	3124550	5311735	7726160	9657700	10400600	9657700	7726160	5311735	3124550	1562275	657800	230230	65780	14950	2600	325	26	1	
		1	27	351	2925	17550	80730	296010	888030	2220075	4686825	8436285	13037895	17383860	20058300	20058300	17383860	13037895	8436285	4686825	2220075	888030	296010	80730	17550	2925	351	27	1
1	28	378	3276	20475	98280	376740	1184040	3108105	6906900	13123110	21474180	30421755	37442160	40116600	37442160	30421755	21474180	13123110	6906900	3108105	1184040	376740	98280	20475	3276	378	28	1	
1	29	406	3654	23751	118755	475020	1560780	4292145	10015005	20030010	34597290	51895935	67863915	77558760	77558760	67863915	51895935	34597290	20030010	10015005	4292145	1560780	475020	118755	23751	3654	406	29	1