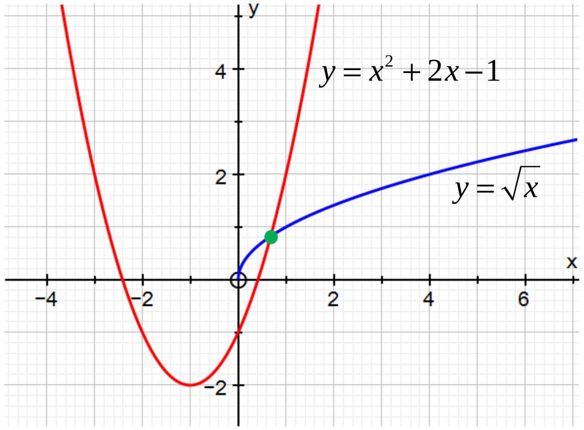
Solving equations using graphs

The solution of an equation such as $x^2 + 2x - 1 = \sqrt{x}$ can be re-interpreted *geometrically*. It is the *intersection* of the lines in a Cartesian coordinate *(x,y)* coordinate system described by the equations:

$$y = x^2 + 2x - 1$$

$$y = \sqrt{x}$$



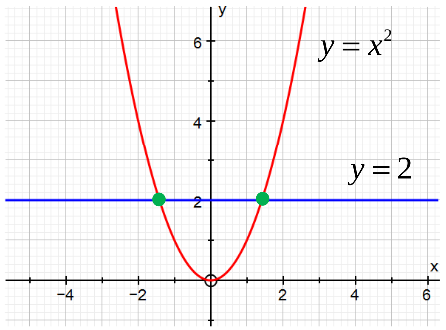
The intersection of the two curves above occurs, approximately, at $x = 0.6808$

All linear, quadratic, cubic and quartic *polynomial* equations can be solved *exactly*. Many variants of exponential functions can be solved in terms of logarithms, and many trigonometric and hyperbolic equations can be 'solved' in terms of their respective inverse functions. However, the toolbox of exact equation solving methods (i.e. 'analytical methods') is *paltry* compared to the *infinite variety* of possible equations one could devise. If applied maths, the numerical solution of an equation is often of real importance. 'Exactness' is not typically a requirement as long as the solution *can* be found to the required *accuracy*. Numerical methods such as *iteration* can provide an answer scheme. **These essentially exploit the geometric idea of a intersection of lines being equivalent to a particular equation.**

The geometric interpretation is also useful to explain certain features of equations that can be solved exactly

$x^2 = 2$

$\therefore \boxed{x = \pm\sqrt{2}}$

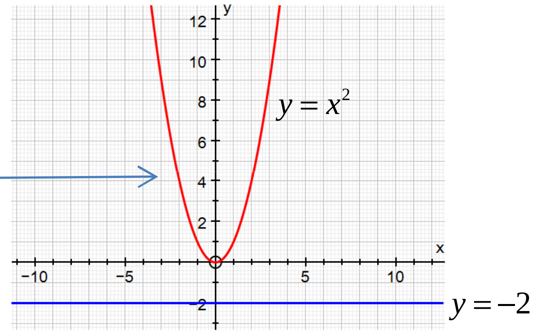


The two +/- roots can easily be explained by considering the equation to be the intersection of the lines

$y = 2$

$y = x^2$

This also explains why $x^2 = -2$ has no real solutions, i.e. there is no intersection of the quadratic curve and the horizontal line



Graphs are also an essential feature for solving quadratic inequalities. For example: $x^2 - x - 6 \geq 0$

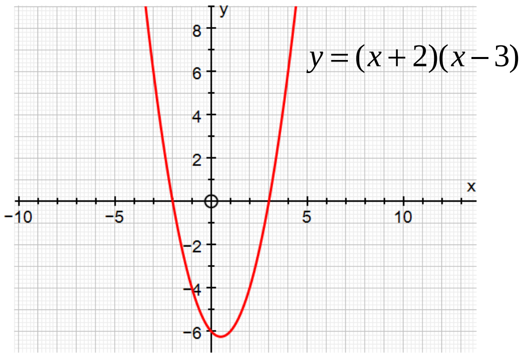
To solve, consider:

$y = x^2 - x - 6$

$y = (x+2)(x-3)$

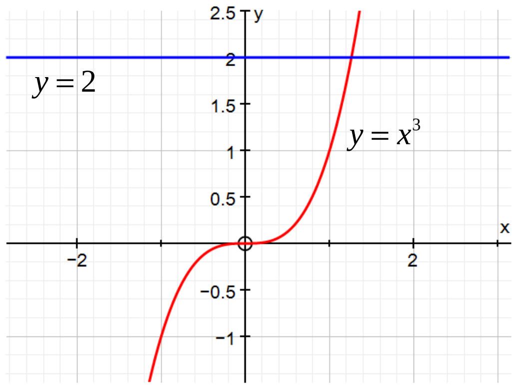
The original inequality can now be interpreted *geometrically*:

"The x range such that the graph is above the axis axis. i.e. y is greater or equal to zero".



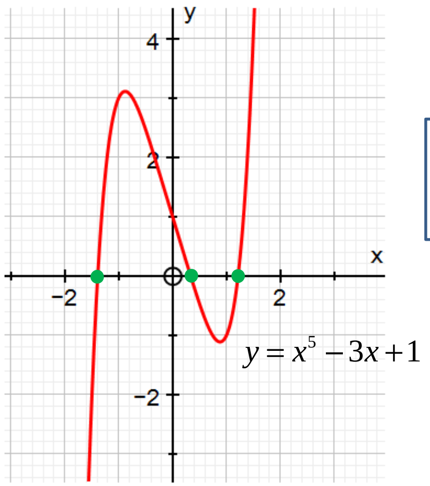
From the graph (and the factorized form) we can clearly see the solution is:

$\boxed{x \leq -2 \quad x \geq 3}$



The geometric approach also explains why $x^3 = 2$ only has *one* solution.

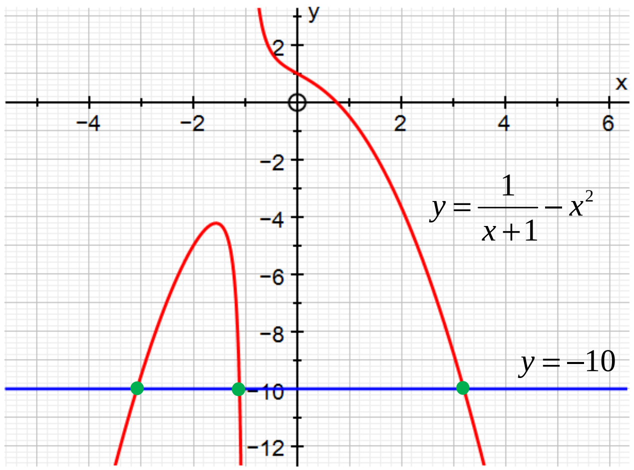
Example 1: Use a graphical method to solve the equation $x^5 - 3x + 1 = 0$



Approximate solutions are:

x = -1.389
x = 0.3347
x = 1.215

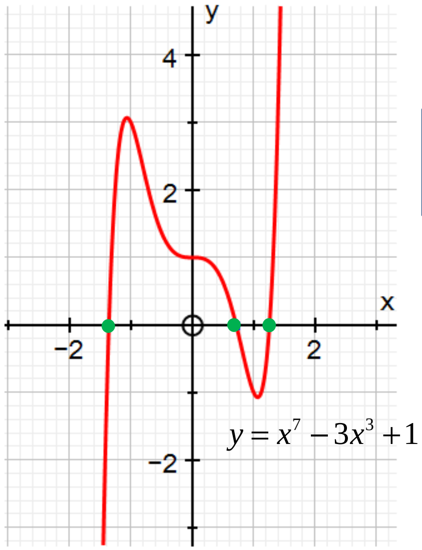
Example 2: Use a graphical method to solve the equation $\dfrac{1}{x+1} - x^2 + 10 = 0$



Approximate solutions are:

x =-3.086
x = -1.114
x = 3.2

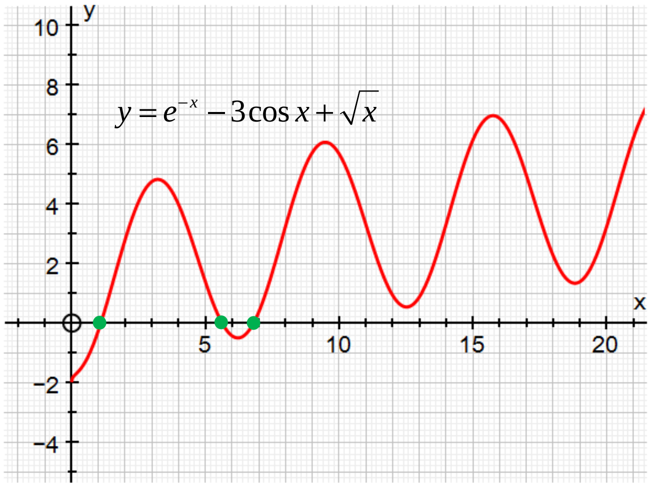
Example 3: Use a graphical method to solve the inequality $x^7 - 3x^3 + 1 < 0$



Approximate solutions are:

$x < -1.358$
$0.7147 < x < 1.257$

Example 4: Use a graphical method to solve the equation $e^{-x} - 3\cos x + \sqrt{x} = 0$



Approximate solutions are:

x =1.093
x = 5.626
x = 6.8