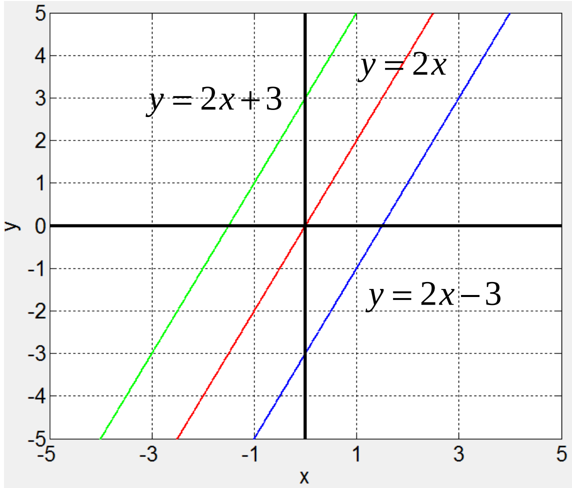
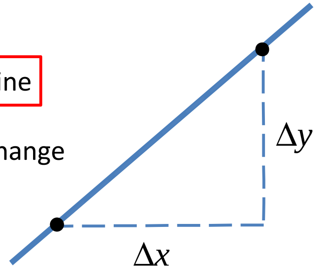
y = mx + c is the equation of a straight line, when drawn on a *Cartesian x,y* grid.



m is the *gradient* of the line

gradient = y change / x change

$$m=\frac{\Delta y}{\Delta x}$$

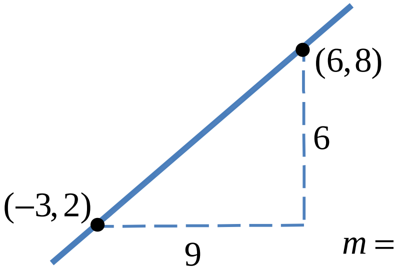


c is the *y-intercept*. i.e. the coordinate (0,*c*) where the line cuts the *y* axis.

Lines with the same gradient are **parallel**. i.e. *y = 2x, y = 2x -3* and *y = 2x + 3* are all parallel.

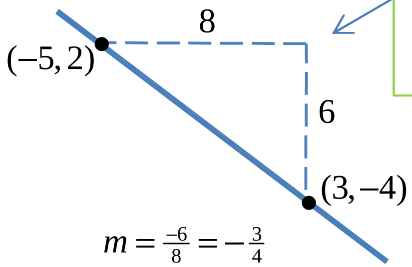
A unique straight line can be drawn between any two *different* coordinates. To work out the equation, firstly arrange the coordinates such that a right angled triangle can be drawn between them.

You can now determine the gradient of the line and hence *m*.



$m=\frac{6}{9}=\frac{2}{3}$

$y=\frac{2}{3}x+c$



$m=\frac{-6}{8}=-\frac{3}{4}$

$y=-\frac{3}{4}x+c$

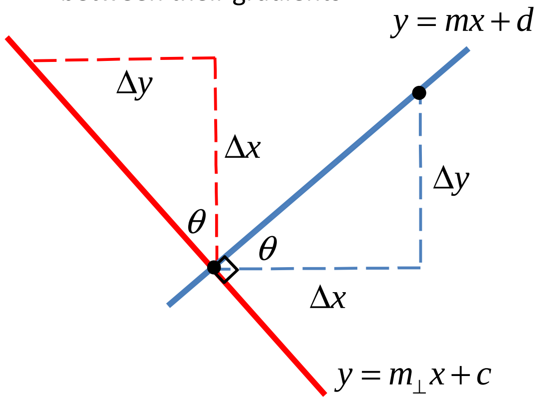
Always arrange the coordinates like this as the gradient could be negative!

To find *c*, use one of the coordinates. It doesn't matter which, since both are on the line.

$$y=\frac{2}{3}x+c$$
$$8=\frac{2}{3}(6)+c$$
$$8-4=c$$
$$\boxed{y=\frac{2}{3}x+4}$$

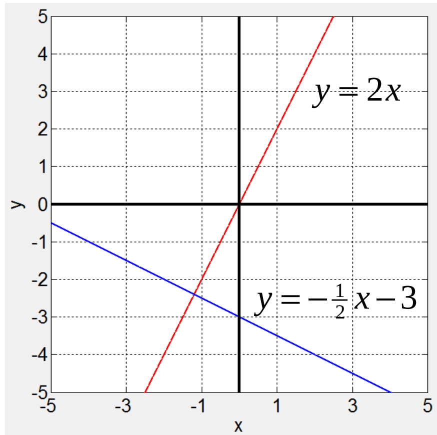
$$y=-\frac{3}{4}x+c$$
$$2=-\frac{3}{4}(-5)+c$$
$$2-\frac{15}{4}=c$$
$$\frac{8}{4}-\frac{15}{4}=c$$
$$\boxed{y=-\frac{3}{4}x-1\frac{3}{4}}$$

Note two perpendicular lines (i.e. ones which cross at right angles) have a relationship between their gradients



René Descartes
1596-1650

$$m_{\perp}=-\frac{\Delta x}{\Delta y}$$
$$m=\frac{\Delta y}{\Delta x}$$
$$\therefore\ \boxed{m_{\perp}=-\frac{1}{m}}$$



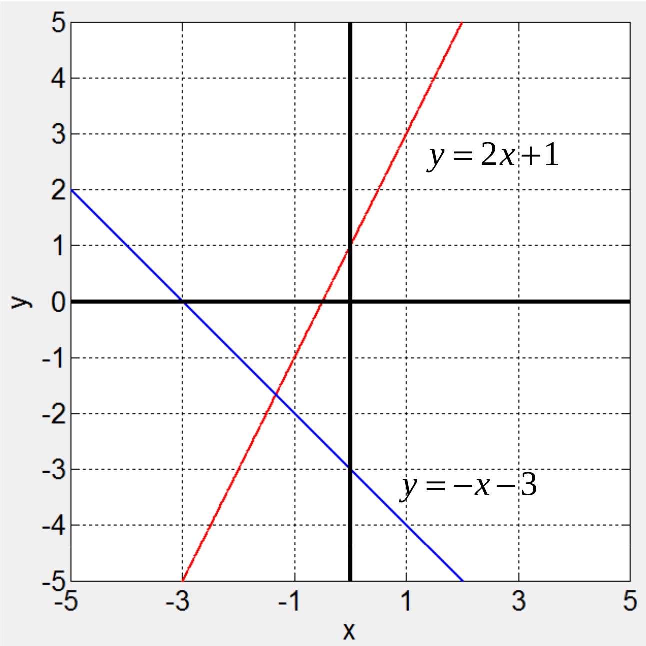
Also, the gradient of the line is $\boxed{m=\tan\theta}$

$\theta=30^{\circ}$

$m=\tan\theta=\frac{1}{\sqrt{3}}$

Simultaneous linear equations

The solution to a system of simultaneous linear equations is equivalent to finding the point where they intersect. Unless a pair of lines are parallel (i.e. have the same gradient) there will always be a coordinate which is the intersection of those lines.



To solve a system of equations we firstly **label them**. Then we add or subtract equations to try an get rid of one of the variables. Once a single equation in one variable is found, this can (always for linear equations) be solved by rearrangement.

$$y = 2x + 1 \qquad (1)$$
$$y = -x - 3 \qquad (2)$$

$$0 = 3x + 4 \qquad (1) - (2)$$
$$-1\tfrac{1}{3} = x$$

Substituting back into the other equations yields the other variable

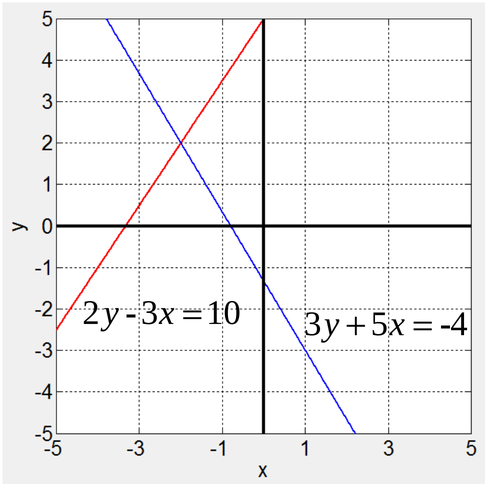
In (1): $y = 2\left(-\tfrac{4}{3}\right) + 1$

$$y = -1\tfrac{2}{3}$$
$$\therefore \boxed{(x, y) = \left(-1\tfrac{1}{3}, -1\tfrac{2}{3}\right)}$$

Another example of a pair of linear equations

$$2y - 3x = 10 \qquad (1)$$
$$3y + 5x = -4 \qquad (2)$$
$$6y + 10x - (6y - 9x) = -8 - 30 \qquad 2(2) - 3(1)$$
$$19x = -38$$
$$\boxed{x = -2}$$

In (1): $y = \dfrac{10 + 3x}{2} = \dfrac{10 + 3(-2)}{2} = \boxed{2}$



We can extend the idea to more than two variables. For *x,y,z* this would be a intersection of 3D lines.

A system of *N* linear equations in *N* unknowns is readily solved via a *matrix method*, since the system of equations can be written as a matrix equation:

$$x + y + z = 1$$
$$2x - 3y + 4z = 2$$
$$-x + y - z = 3$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & -3 & 4 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -3 & 4 \\ -1 & 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}}$$

The tricky bit is finding the inverse of the *N x N* matrix. There are computational recipes for doing this in programs such as MATLAB.