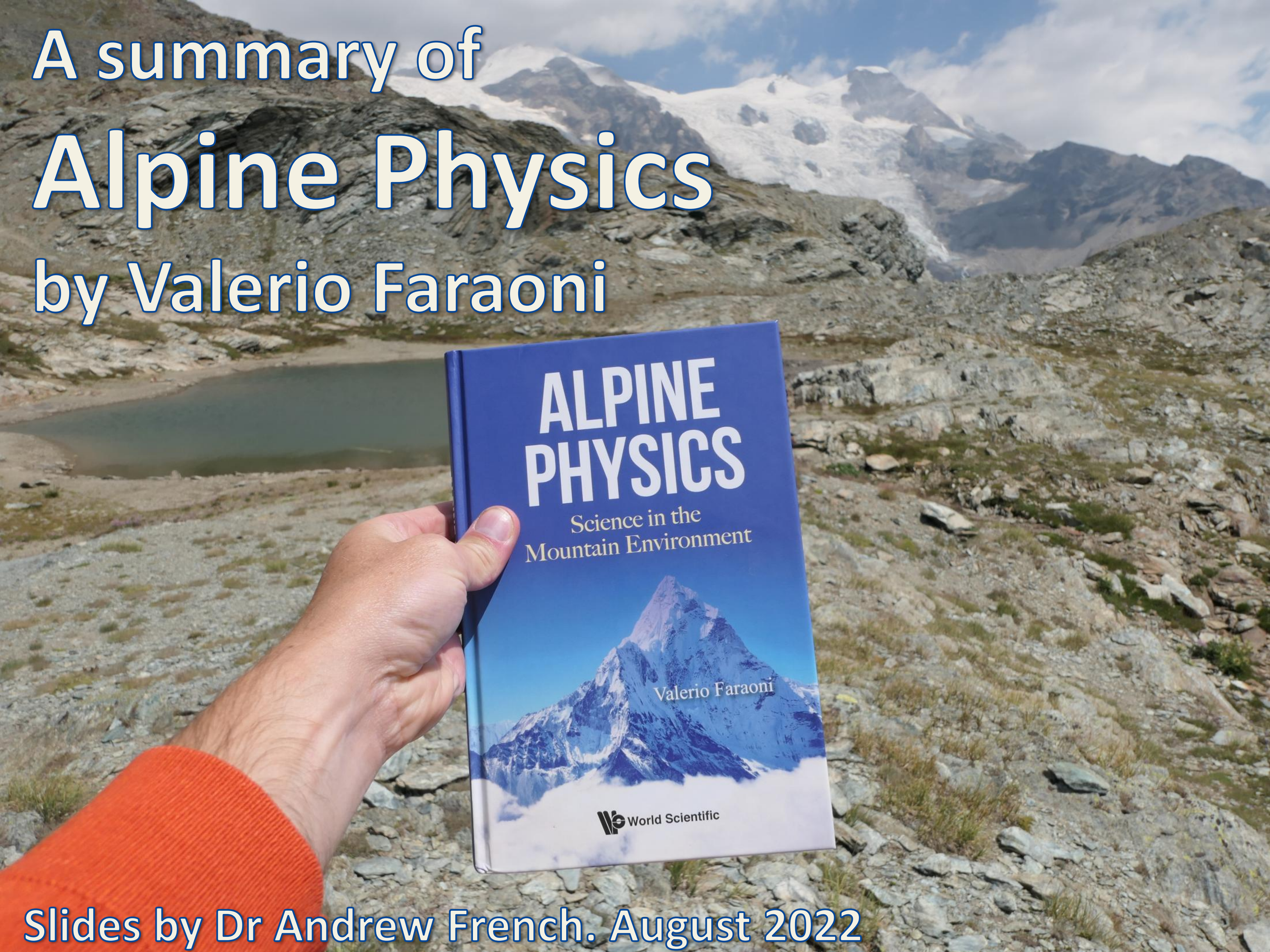
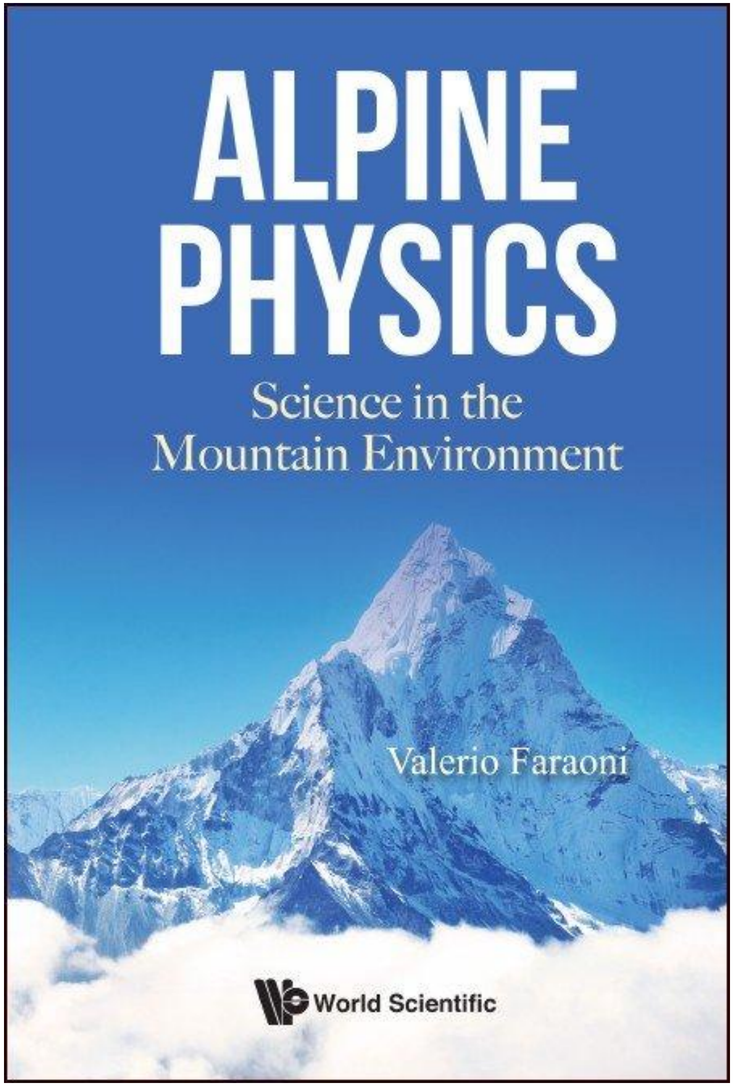
A summary of

Alpine Physics

by Valerio Faraoni



Slides by Dr Andrew French. August 2022

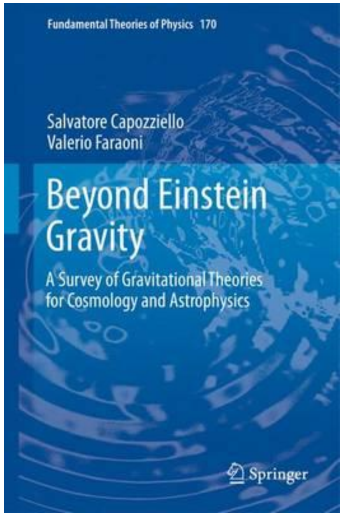


ALPINE PHYSICS

Science in the Mountain Environment

Valerio Faraoni

World Scientific



Fundamental Theories of Physics 170

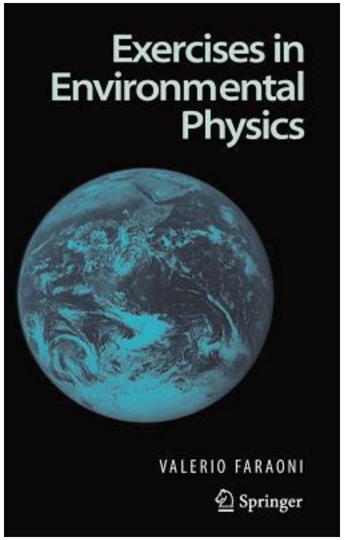
Salvatore Capozziello
Valerio Faraoni

Beyond Einstein Gravity

A Survey of Gravitational Theories for Cosmology and Astrophysics

Springer

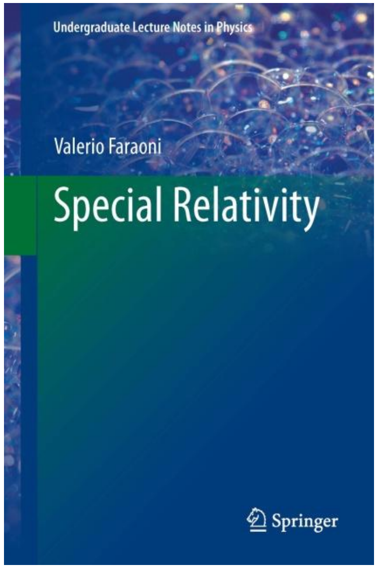
[Dr Valerio Faraoni]



Exercises in Environmental Physics

VALERIO FARAONI

Springer



Undergraduate Lecture Notes in Physics

Valerio Faraoni

Special Relativity

Springer

UNIVERSITÉ
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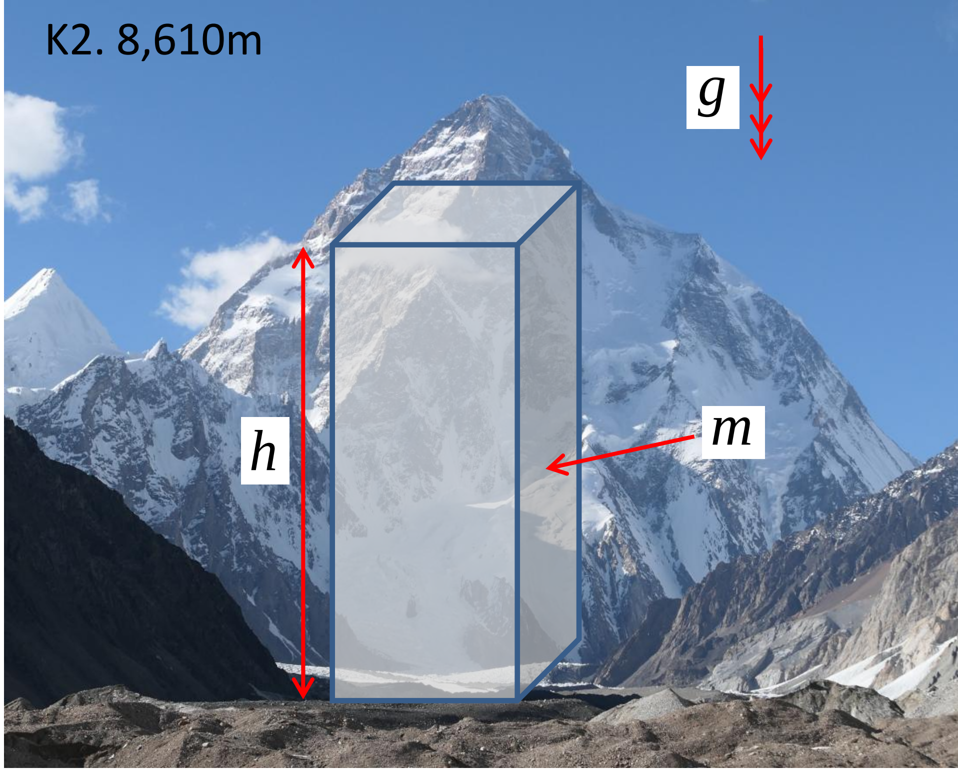
UNIVERSITY

1. Mountains high or low, hard or soft
2. Gravity rules
3. Water, snow and ice
4. Glacier puzzles
5. Heat, cold and air
6. Rock climbing
7. Miscellaneous

1a. How high can a mountain be?

The Gravitational Potential Energy (GPE) of a vertical column of mountain rock of mass m must be less than the latent heat of fusion L_{fus} of the rock.

Otherwise the weight of the mountain would cause the base to melt.



K2. 8,610m

$$mgh < \eta L_{fus} m$$

$$\therefore \boxed{h < \frac{\eta L_{fus}}{g}}$$

Weisskopf guessed empirical factor $\eta \approx \frac{1}{3}$

Earth:

For SiO_2 $\quad L_{fus} \approx 2.0 \times 10^5 \mathrm{Jkg}^{-1} \Rightarrow \frac{\eta L_{fus}}{g} = 6{,}800\mathrm{m}$

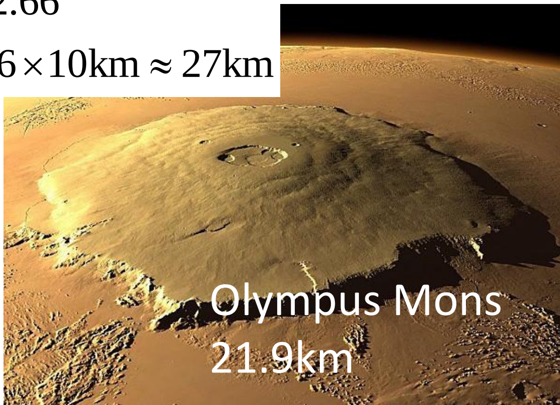
For granite $\quad L_{fus} \approx 4.2 \times 10^5 \mathrm{Jkg}^{-1} \Rightarrow \frac{\eta L_{fus}}{g} = 14{,}300\mathrm{m}$

So about 10km is about the maximum height of mountains on Earth. Mount Everest is 8,849m.

Mars:

$g_{\oplus} / g_{mars} \approx 2.66$

$\therefore h_{\max} \approx 2.66 \times 10\mathrm{km} \approx 27\mathrm{km}$



Olympus Mons
21.9km

Weisskopf, V.F., "Physics of Mountains." *American Journal of Physics* **54**, 871 (1986).

1b. What is the smallest radius of a spherical rocky planet?

Using Weisskopf's argument, $h < \eta L_{fus} / g$ prevents the base of a mountain of height *h* from melting at its base. So if most rocky planets are made from the same material (and therefore have the *same latent heat of fusion*), a condition for them to be *spherical* implies **large imperfections of order the radius of the planet are melted away due to the strength of gravity at the surface.**

$$\boxed{\therefore Rg > h_{\oplus} g_{\oplus}}$$

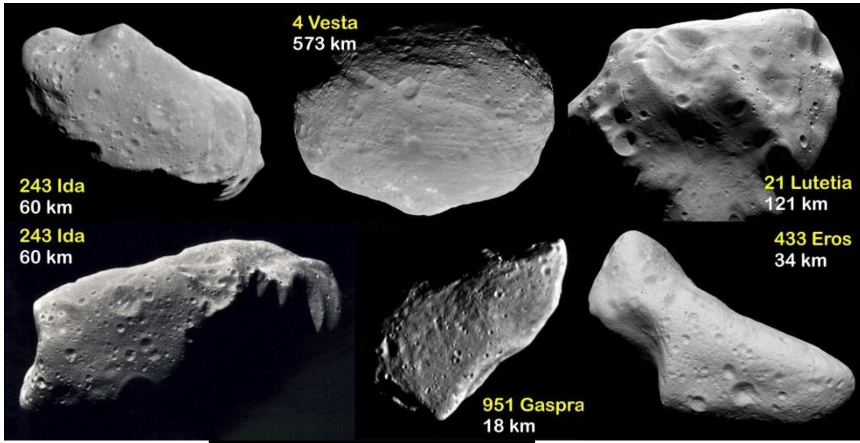
From Newton's law of Universal Gravitation: $g = \dfrac{GM}{R^2} = \dfrac{G\frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4}{3}\pi G \rho R$

$$\therefore R \times \tfrac{4}{3}\pi G \rho R > h_{\oplus} g_{\oplus}$$

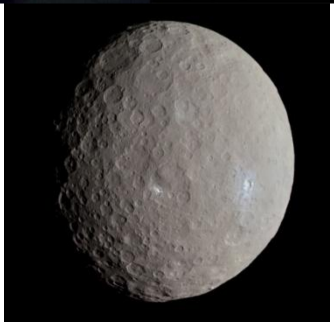
$$\therefore R > \sqrt{\frac{h_{\oplus} g_{\oplus}}{\frac{4}{3}\pi G \rho}}$$

$$\therefore R > \sqrt{\frac{10{,}000 \times 9.81}{\frac{4}{3}\pi \times 6.67 \times 10^{-11} \times 5{,}500}} \quad \text{(m)}$$

$$\therefore \boxed{R > 250\text{km}}$$



250km is the 'potato radius', i.e. the maximum radius of an irregular asteroid. Beyond this limit, gravity and melting will tend to form spherical bodies.

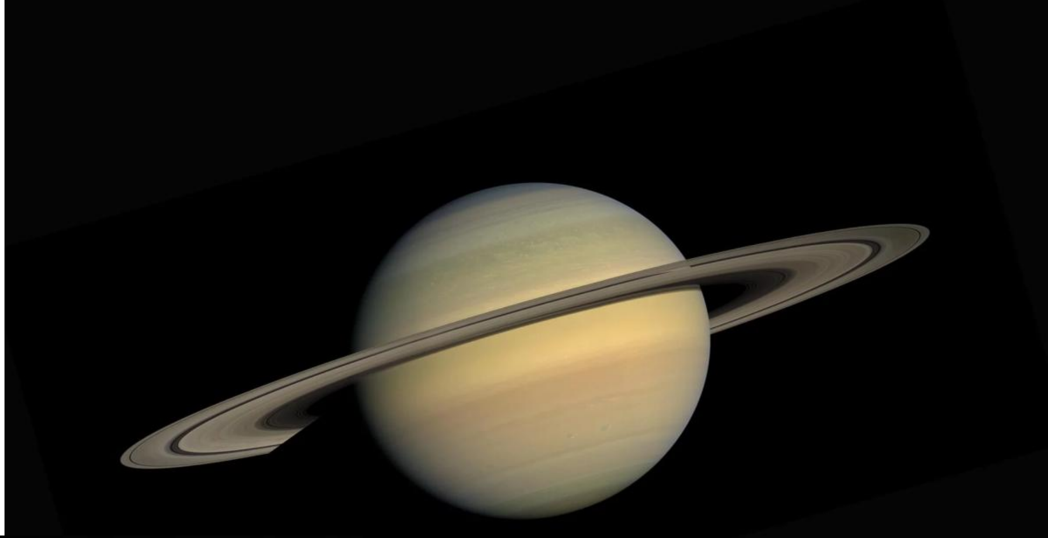


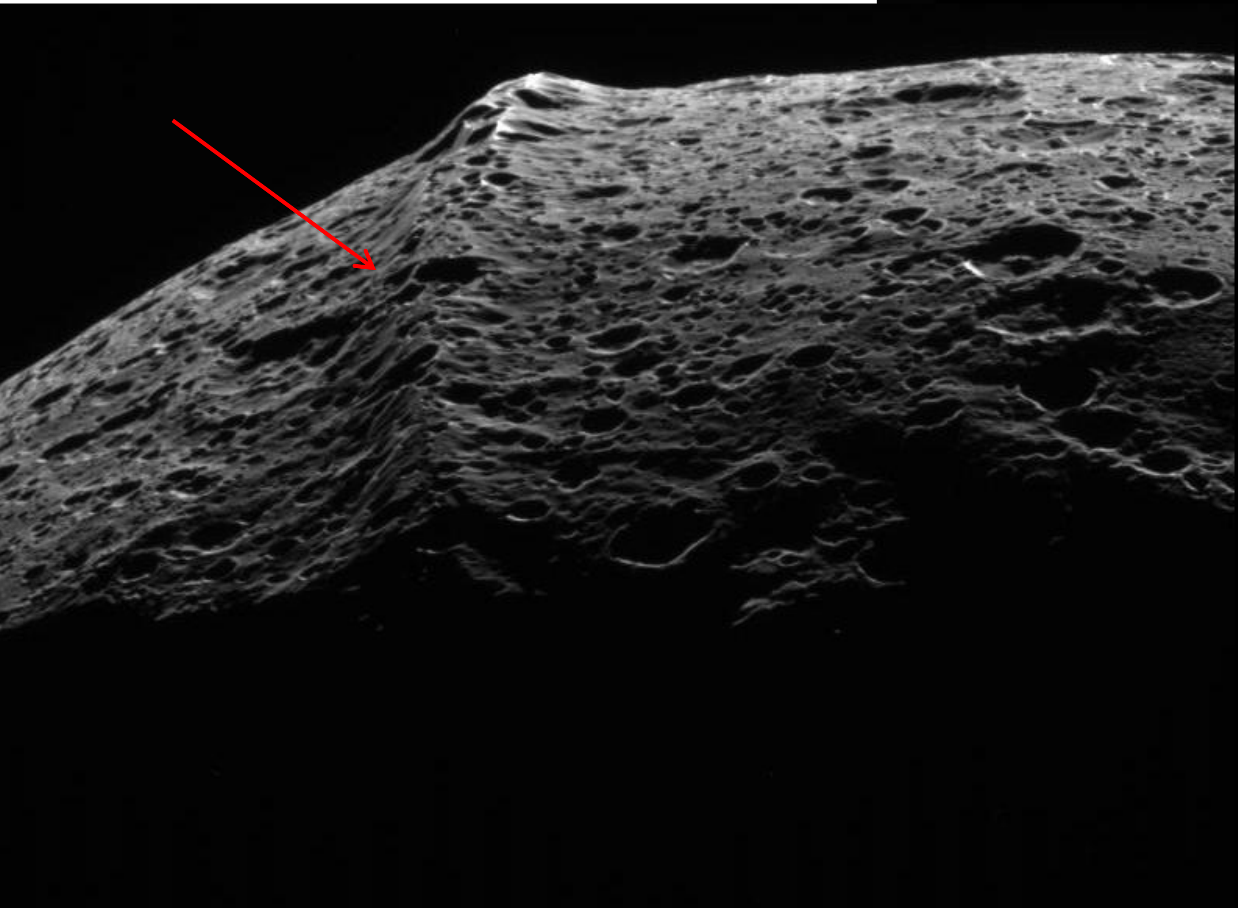
Ceres

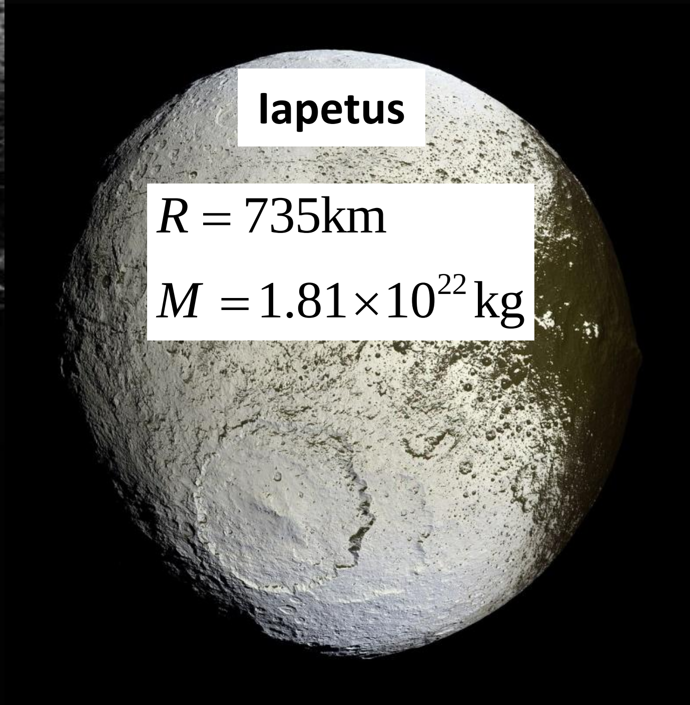
$R \approx 470\text{km}$

1c. Mountains falling from the sky

20km high equatorial ridges on **Iapetus** (and possibly **Rhea**), both moons of Saturn, are thought to be the result of a **collapsed orbiting ring**, perhaps disturbed by **tidal forces** resulting from the competing gravity from Saturn and other nearby moons.







Iapetus

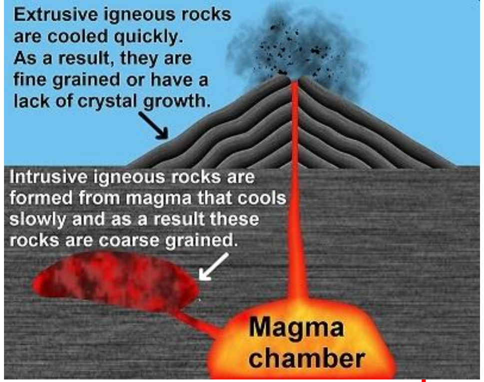
$R = 735\text{km}$

$M = 1.81\times10^{22}\,\text{kg}$

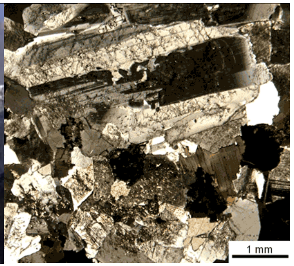
1d. Igneous rocks

Plutonic

Magma cools slowly below surface of Earth then exposed due to erosion and/or tectonic uplift when cooled.



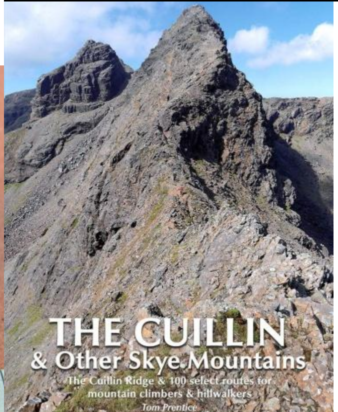
Large crystals, cracks, steps, good for climbing!



Granite

Diorite

Gabbro



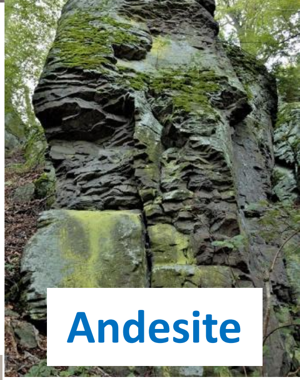
Volcanic

Rapid cooling of magma following a volcanic eruption. Small crystals, often porous, can be low density.

Basalt (forms ocean floors, 2/3 of Earth's crust). Can form hexagonal columns.

Porphyry

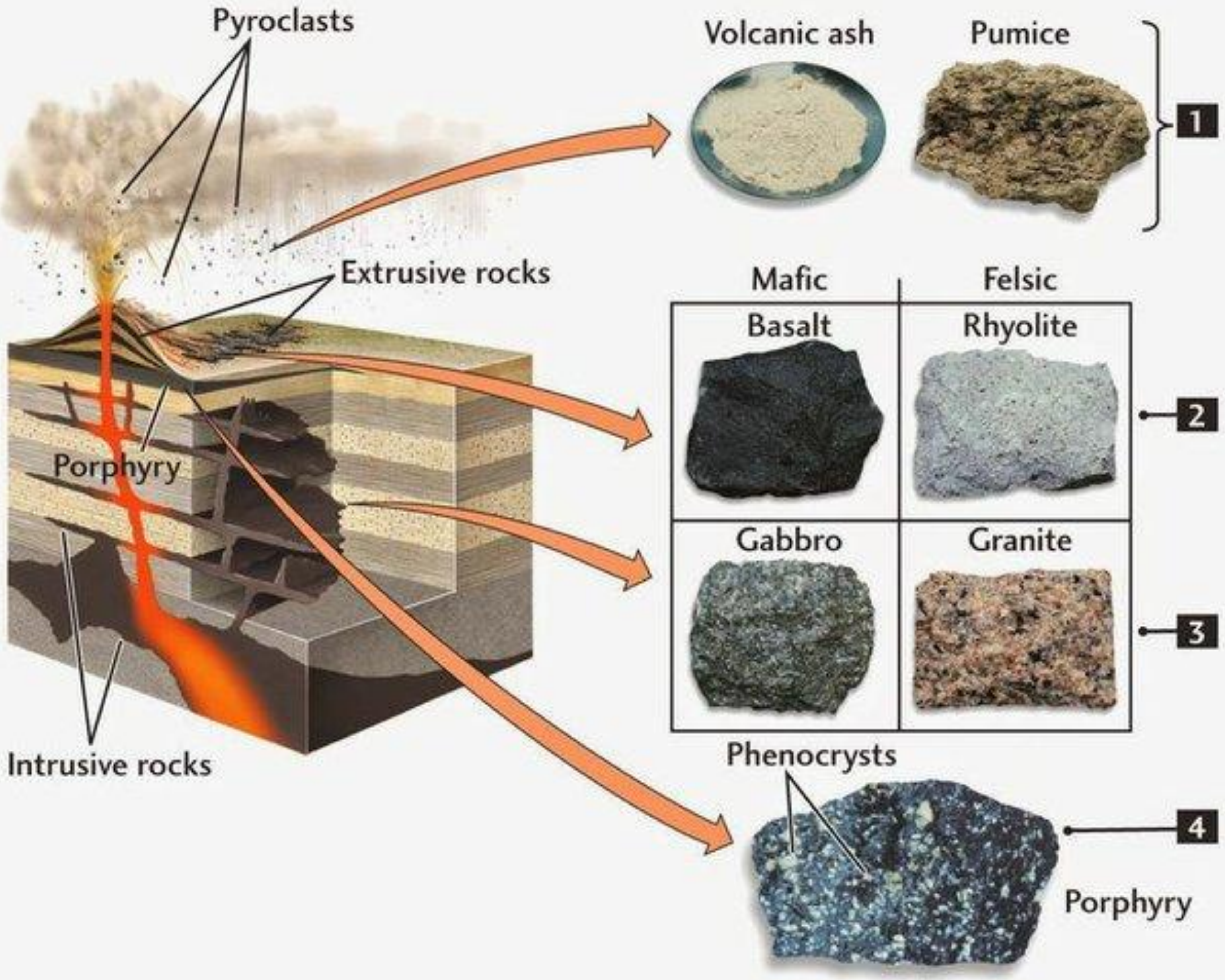
Rhyolite



Andesite

Obsidian

Pumice


Pyroclasts
Volcanic ash
Pumice
1
Extrusive rocks
Mafic
Felsic
Basalt
Rhyolite
2
Porphyry
Gabbro
Granite
3
Intrusive rocks
Phenocrysts
4
Porphyry

Igneous Rock

Examples


Basalt


Pumice


Obsidian


Rhyolite


Scoria


Dacite


Granite


Gabbro


Diabase

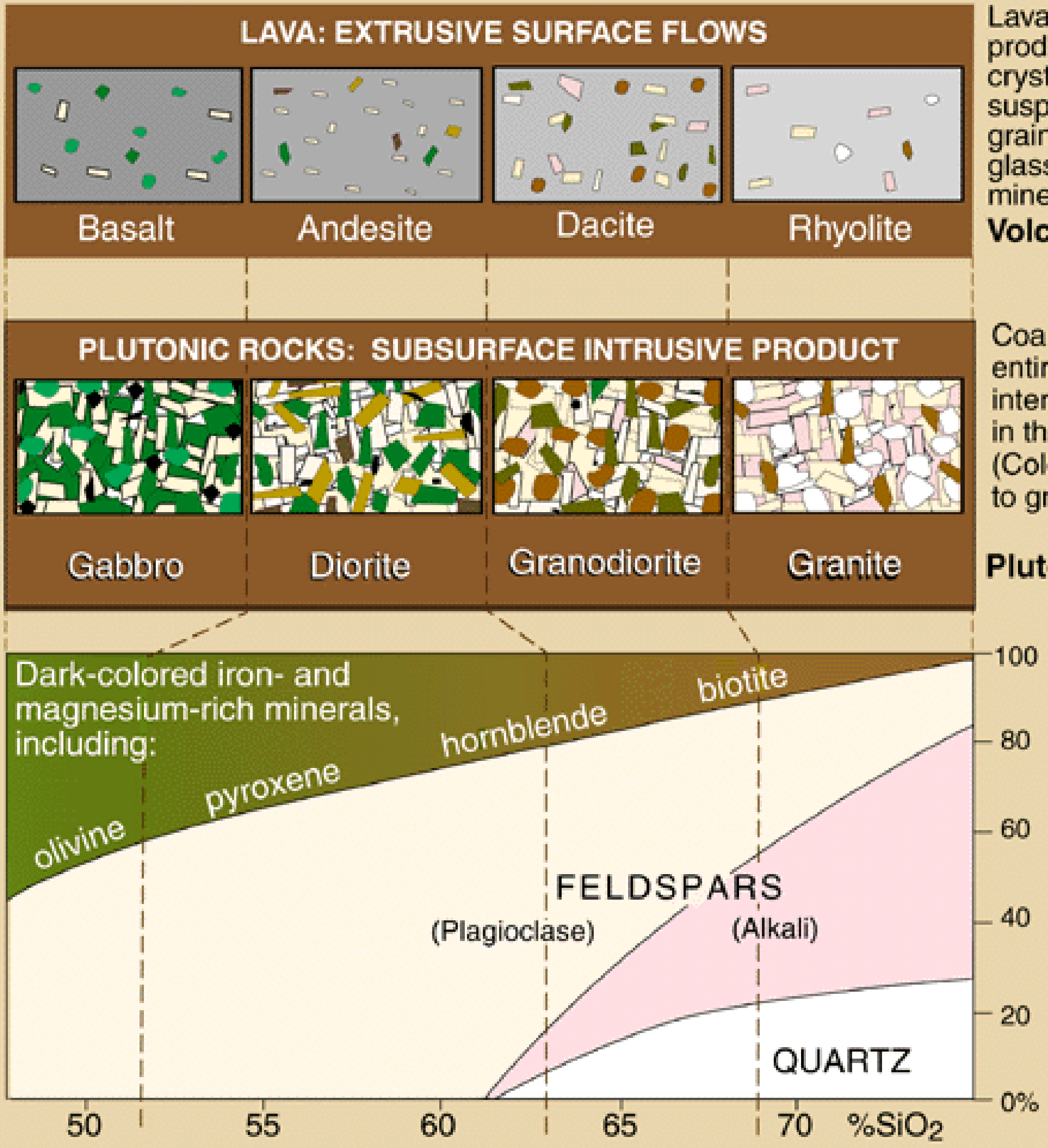

Diorite


Pegmatite


Peridotite

ScienceFacts.net

COMPONENTS OF IGNEOUS ROCKS



Lava flows typically produce rocks with 0-50% crystals (minerals) suspended in a fine-grained groundmass of glass and/or microscopic minerals.

Volcanic rock name

Coarse-grained rock; entirely crystalline with interlocking minerals in the proportions below. (Colors roughly correlate to graph below.)

Plutonic rock name

Volume percent of minerals present in igneous rocks.

This generalized guide shows proportions of common minerals likely to be present in an igneous rock. (See text below on how to calculate.)

Plutonic rocks are entirely crystalline and have a larger variety of unusual minerals than volcanic rocks.

1e. Sedimentary rocks

Formed by erosion of pre-existing rocks, combined with sand, plants, shells, pebbles.

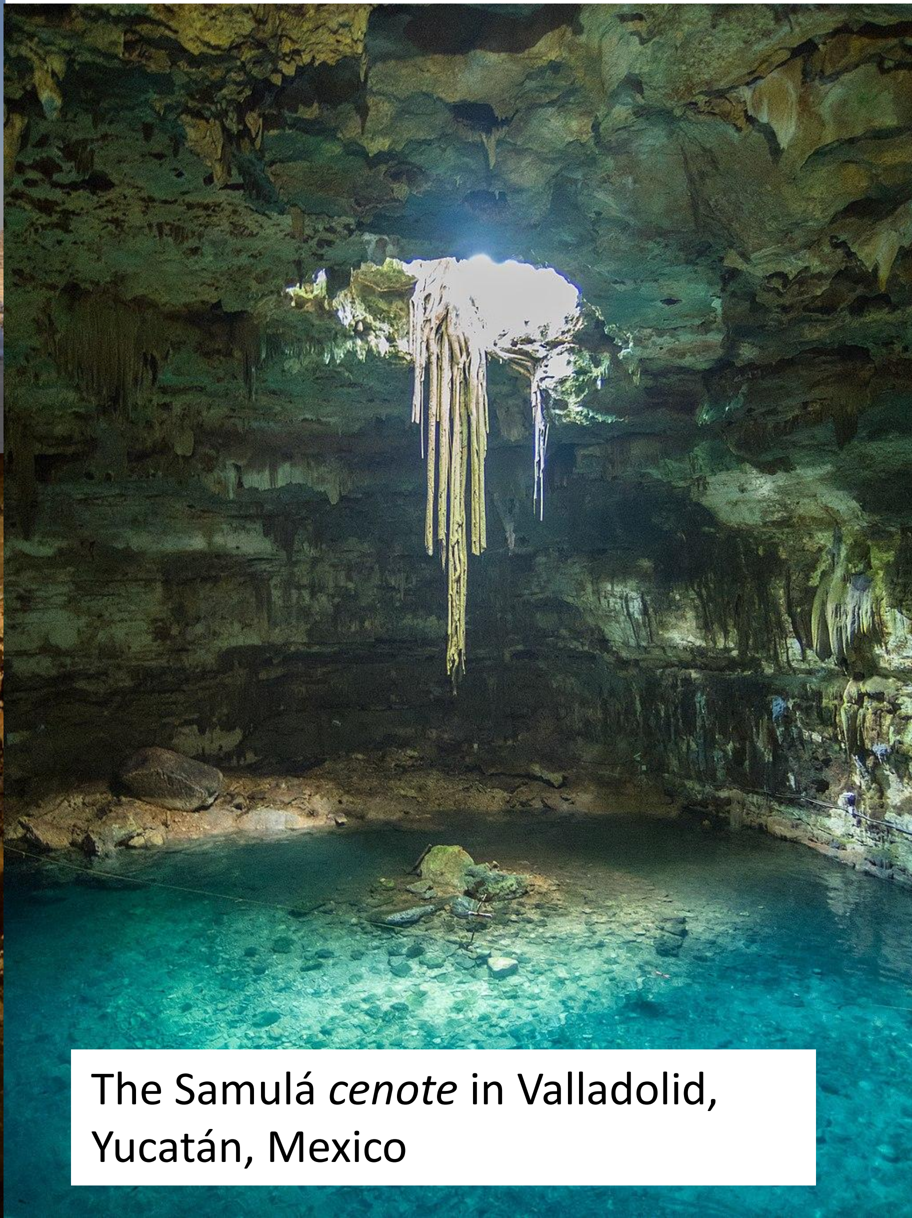
Mount Everest, Nepal/China

The Wave.
Sandstone, Utah.

Limestone $CaCO_3$

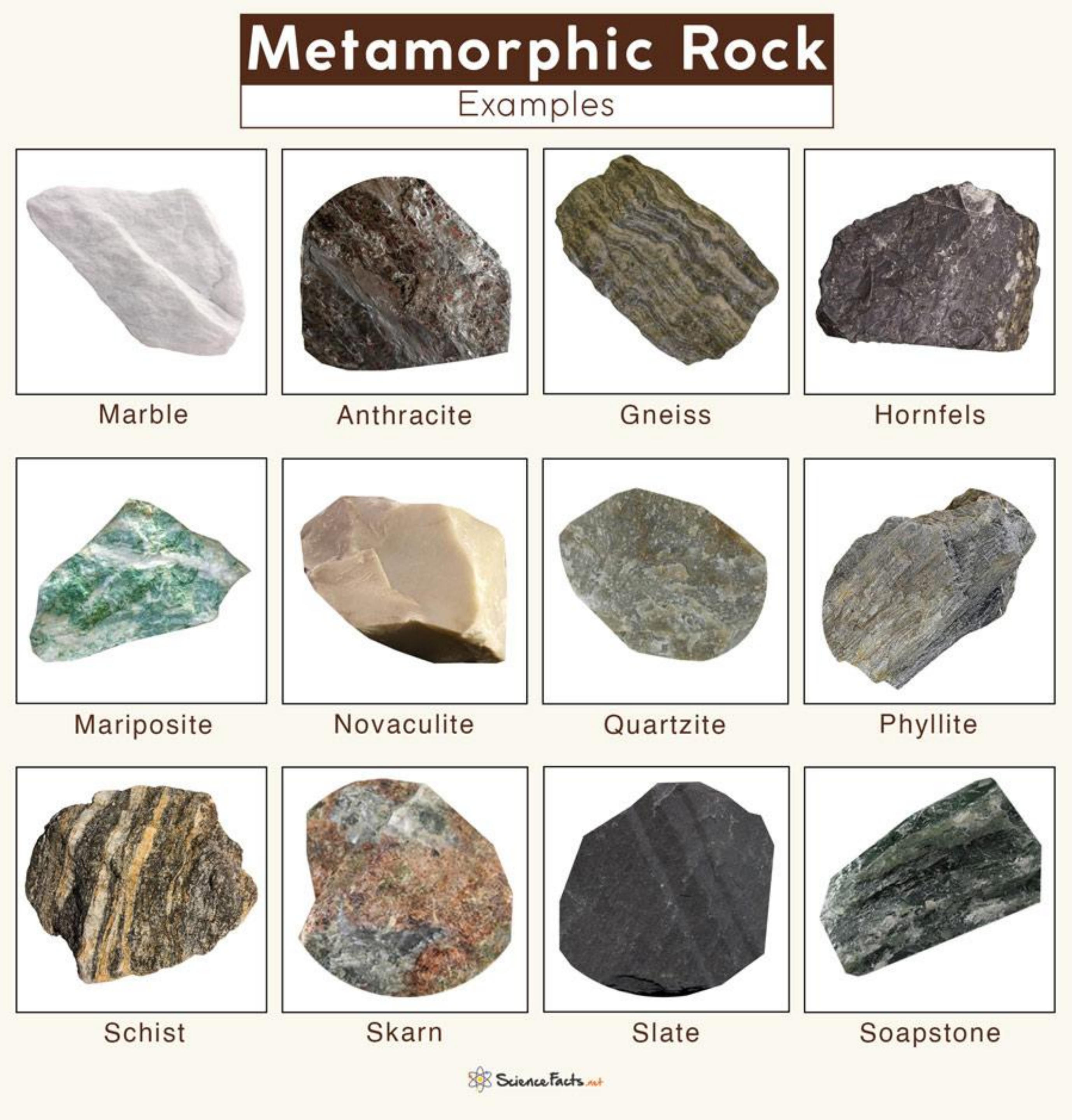

Great Pyramid of Giza

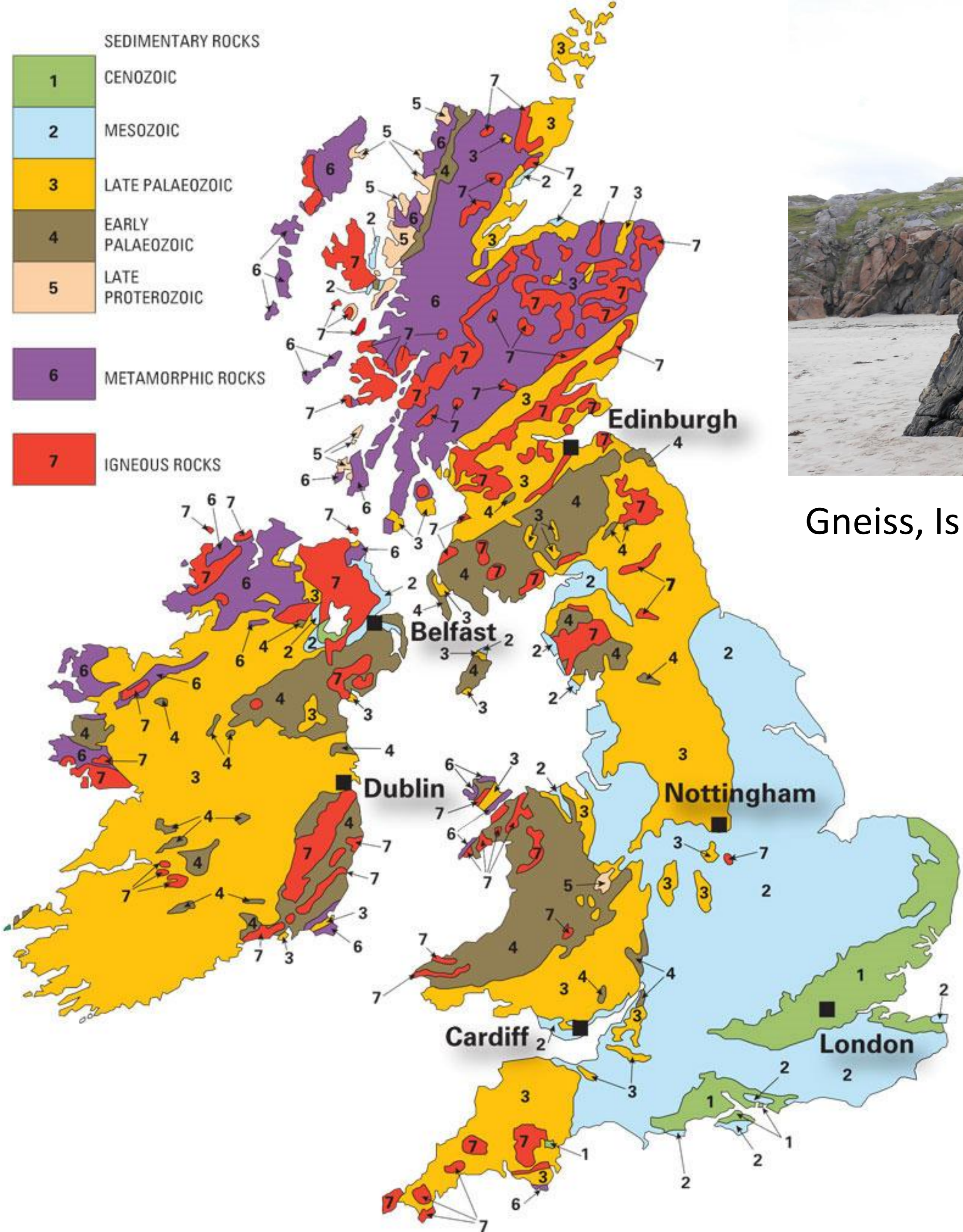

Cave limestone formations in the Luray Caverns of the northern Shenandoah Valley


The Samulá *cenote* in Valladolid, Yucatán, Mexico

1f. Metamorphic rocks

Reprocessing and transformation of igneous or sedimentary rocks under high pressure and temperature conditions deep in the Earth.



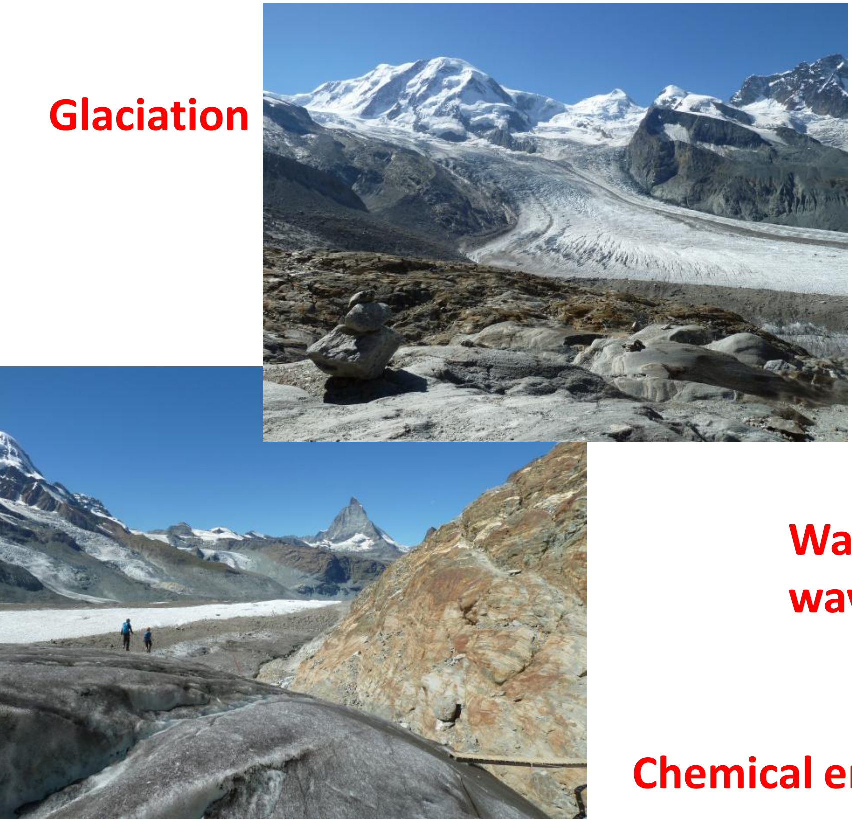


Gneiss, Isle of Lewis

Slate mine, Wales

1g. Destroying mountains - erosion

Glaciation



Wind

Water/rainfall
wave action

Chemical erosion e.g. Karstic rock

$$H_2O + CO_2 \rightarrow H_2CO_3$$ carbonic acid

$$CaCO_3 + H_2CO_3 \rightarrow Ca\left(HCO_3\right)_2$$

limestone

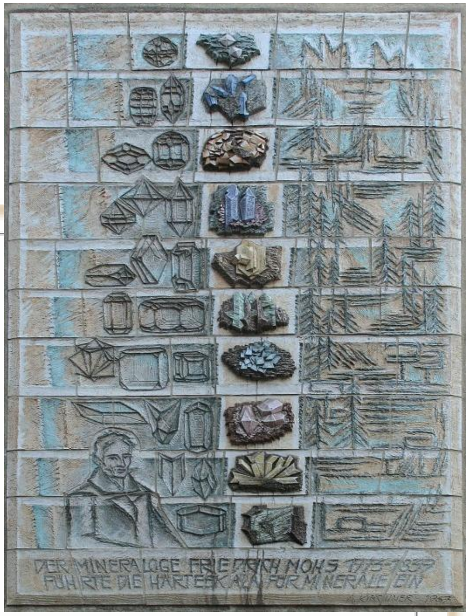
Frost shatter

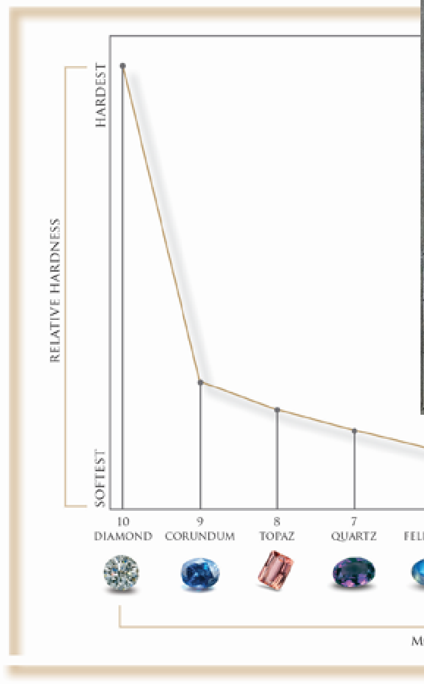
Glyder Fawr, Snowdonia

1h. Mohs hardness scale

GeologyIn.com

Friedrich Mohs
1773-1832

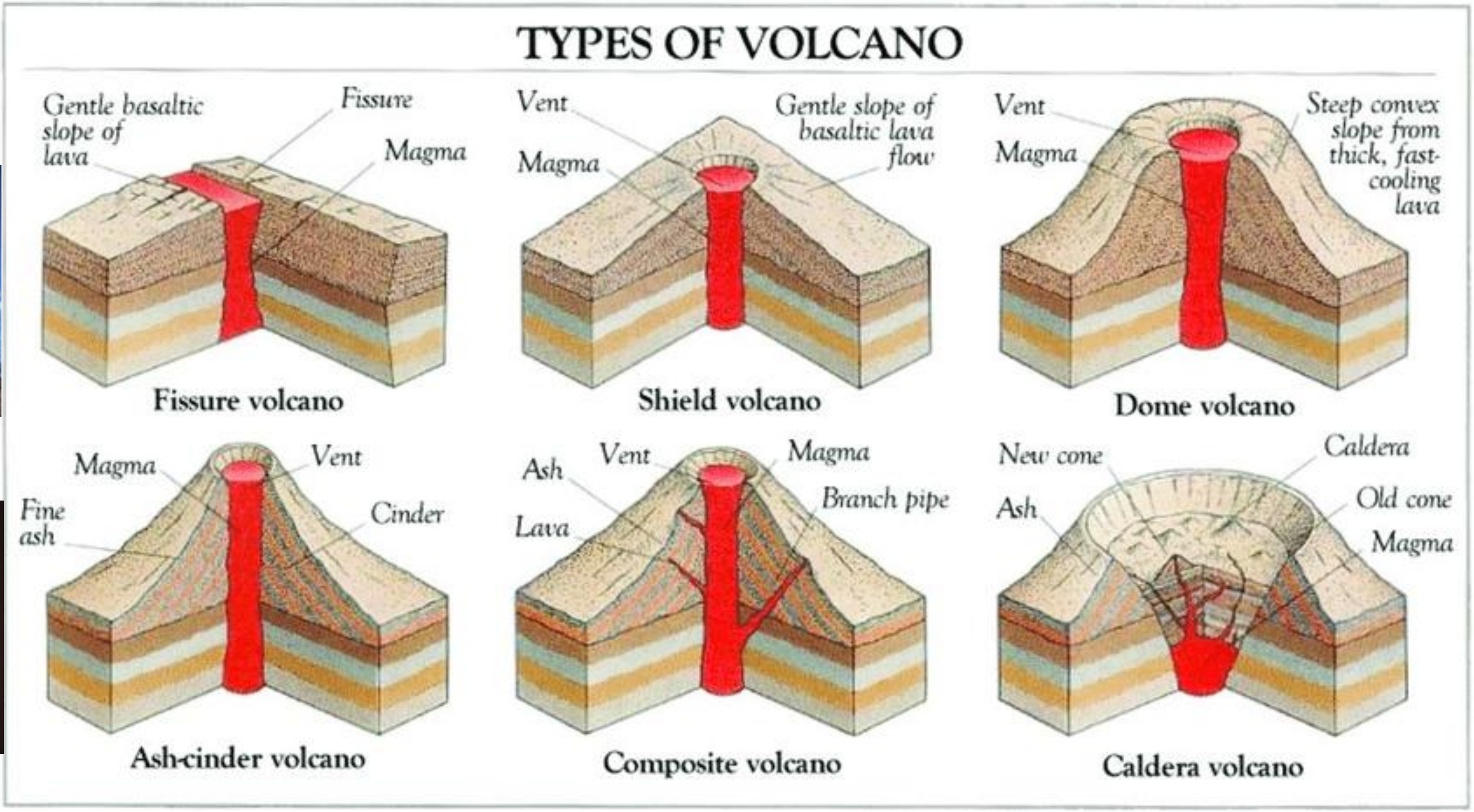




1i. Volcanism

Pinatubo, Philippines

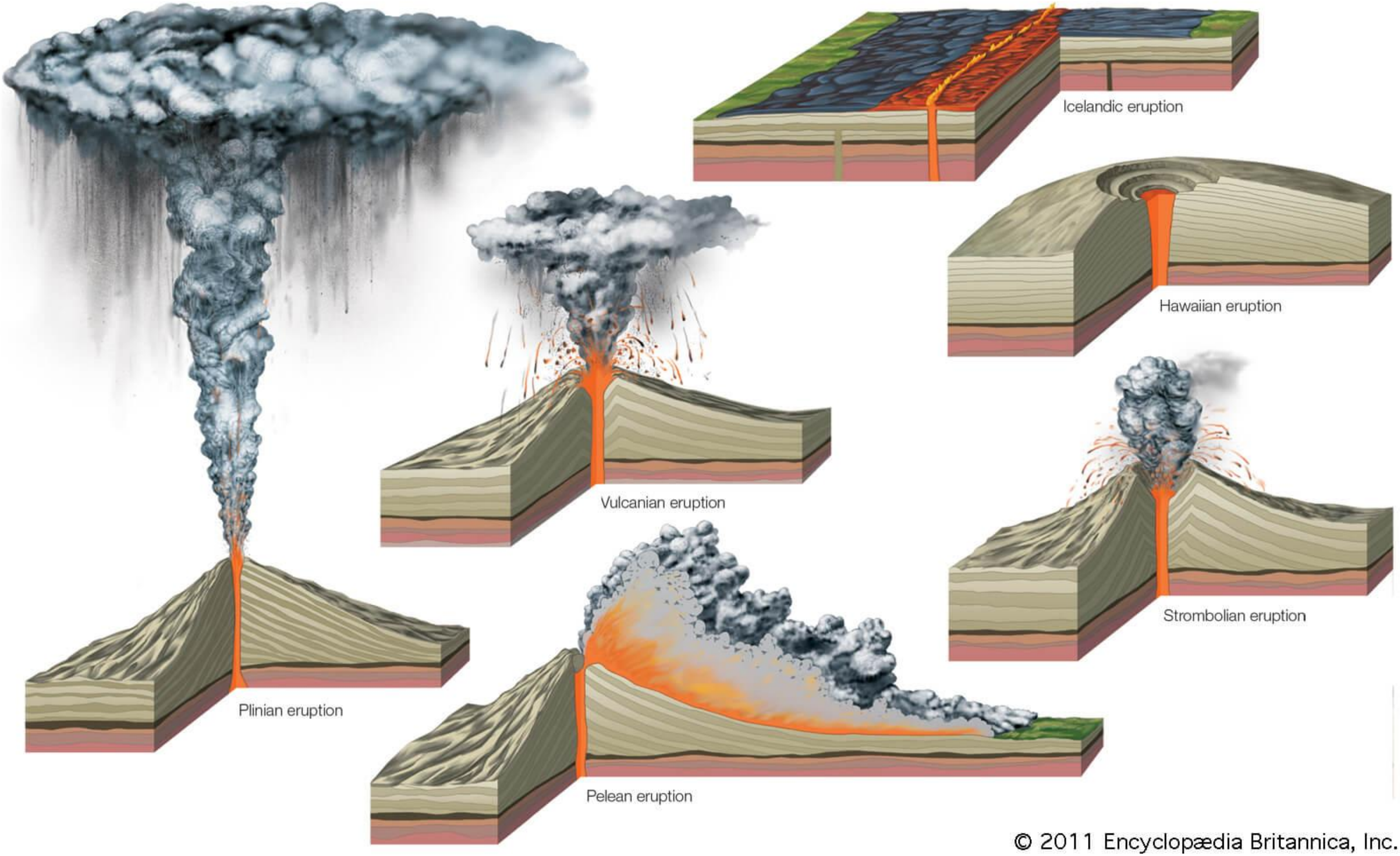
Etna, Sicily



Chimborazo, Ecuador

Kilimanjaro, Tanzania

Fuego, Guatemala


Icelandic eruption
Hawaiian eruption
Vulcanian eruption
Strombolian eruption
Plinian eruption
Pelean eruption

1. Mountains high or low, hard or soft
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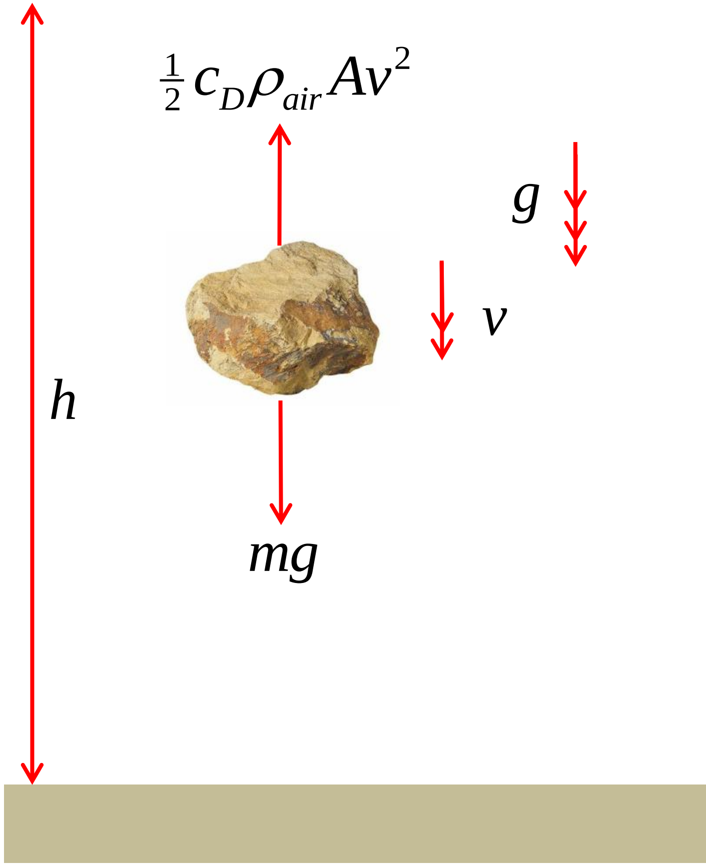
2a. Falling rocks (speed)

For short falls, where rock speed is insufficient to incur much air resistance, balance KE with GPE loss to calculate rock impact speed.

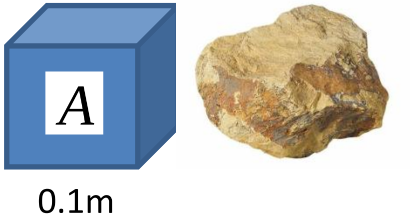
$$mgh = \tfrac{1}{2}mv^2 \quad \therefore v = \sqrt{2gh} = \sqrt{2\times 9.81}\sqrt{h}$$

$$\therefore \left(\frac{v}{\mathrm{ms^{-1}}}\right) \approx 4.4\sqrt{\left(\frac{h}{\mathrm{m}}\right)}$$



For longer falls, the rock may attain terminal speed where drag balances weight.

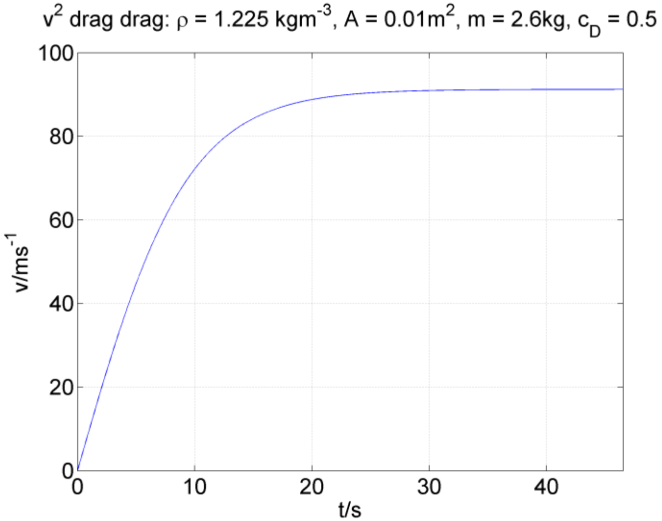
$$\tfrac{1}{2}c_D\rho_{air}Av^2 = mg = \rho A\sqrt{A}g$$



$$\therefore v = \sqrt{\frac{\rho g\sqrt{A}}{\tfrac{1}{2}c_D\rho_{air}}}$$

$$\therefore v = \sqrt{\frac{2,600\times 9.81\times 0.1}{\tfrac{1}{2}0.5\times 1.225}}$$

$$\therefore v \approx 91\mathrm{ms^{-1}}$$



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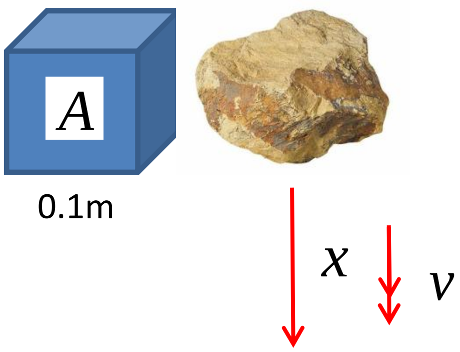
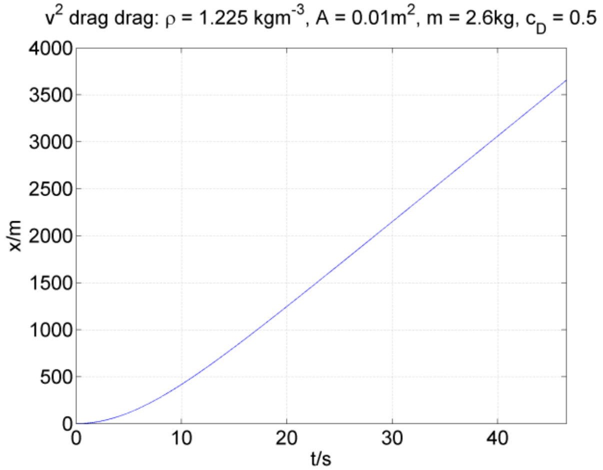
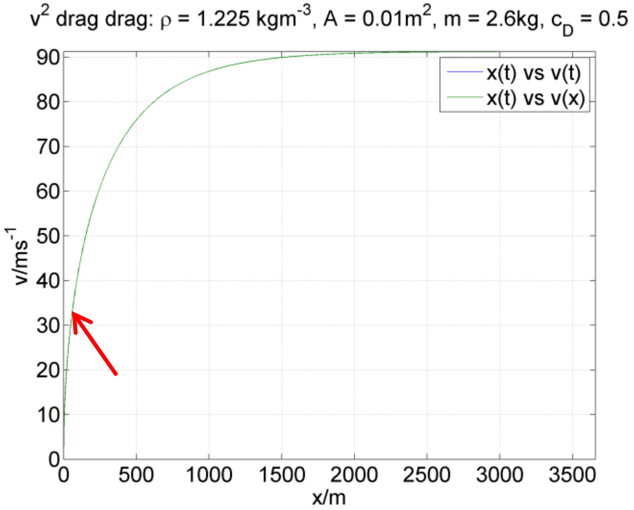
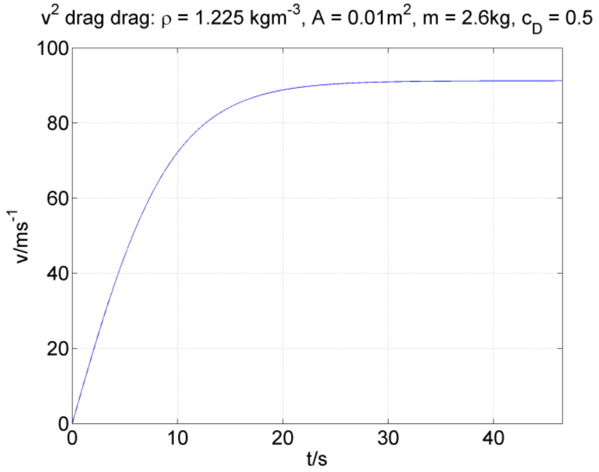




Note for these parameters the rock won't reach terminal speed from a drop from a typical cliff. For 100m it might be more like 30 to 40m/s.





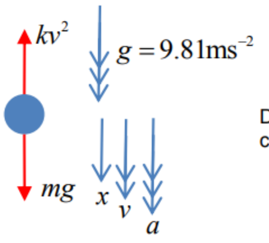
$$\boxed{x = \tfrac{m}{k}\ln\left(\frac{\cosh\left(\sqrt{\tfrac{k}{mg}}\,gt + \tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)\right)}{\cosh\left(\tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)\right)}\right)}$$

$$\boxed{v = \sqrt{\frac{mg}{k}}\left(1-\left(1-\frac{ku^2}{mg}\right)e^{-\frac{2kx}{m}}\right)^{\frac{1}{2}}}$$

$$\boxed{k = \tfrac{1}{2}c_D\rho A}$$

u is initial downward velocity

Subsonic drag $\boxed{F = kv^2}$



$$k = \tfrac{1}{2} c_D \rho A$$

cross sectional area of object perpendicular to velocity

Drag coefficient

fluid density (air is about 1kgm-3)

Newton II

$$m\frac{dv}{dt} = mg - kv^2$$

$$\int_u^v \frac{dv}{g - \frac{k}{m}v^2} = \int_0^t dt$$

$$\int_u^v \frac{dv}{1 - \frac{k}{mg}v^2} = gt$$

$$z^2 = \frac{k}{mg}v^2 \Rightarrow z = v\sqrt{\tfrac{k}{mg}}$$

$$\int \frac{a}{1-a^2x^2} = \tanh^{-1} ax + c$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$dv = \sqrt{\tfrac{mg}{k}}dz$$

$$\int_{u\sqrt{\frac{k}{mg}}}^{v\sqrt{\frac{k}{mg}}} \sqrt{\tfrac{mg}{k}} \frac{dz}{1-z^2} = gt$$

$$\left[\sqrt{\tfrac{mg}{k}} \tanh^{-1} z\right]_{u\sqrt{\frac{k}{mg}}}^{v\sqrt{\frac{k}{mg}}} = gt$$

$$\tanh^{-1} v\sqrt{\tfrac{k}{mg}} = \sqrt{\tfrac{k}{mg}}gt + \tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)$$

$$\boxed{v = \sqrt{\tfrac{mg}{k}} \tanh\left(\sqrt{\tfrac{k}{mg}}gt + \tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)\right)}$$

$$x = \int_0^t v dt = \sqrt{\tfrac{mg}{k}} \int_0^t \tanh\left(\sqrt{\tfrac{k}{mg}}gt + \tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)\right)$$

$$x = \tfrac{1}{g}\sqrt{\tfrac{mg}{k}}\sqrt{\tfrac{mg}{k}}\left[\ln\left(\cosh\left(\sqrt{\tfrac{k}{mg}}gt + \tanh^{-1}\left(u\sqrt{\tfrac{k}{mg}}\right)\right)\right)\right]_0^t$$

$$\boxed{x = \tfrac{m}{k}\ln\left(\frac{\cosh\left(\sqrt{\frac{k}{mg}}gt + \tanh^{-1}\left(u\sqrt{\frac{k}{mg}}\right)\right)}{\cosh\left(\tanh^{-1}\left(u\sqrt{\frac{k}{mg}}\right)\right)}\right)}$$

Note asymptotic behaviour is for velocity to tend to towards 'terminal velocity'

$$\boxed{v_\infty = \sqrt{\tfrac{mg}{k}}}$$

Alternative derivation using $\boxed{a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt} = \dfrac{vdv}{dx}}$

$$m\frac{vdv}{dx} = mg - kv^2$$

Newton II

$$\int_u^v \frac{vdv}{g - \frac{k}{m}v^2} = \int_0^x dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$-\tfrac{m}{2k}\int_u^v \frac{\left(-\frac{2k}{m}\right)vdv}{g - \frac{k}{m}v^2} = \int_0^x dx$$

$$\left[-\tfrac{m}{2k}\ln\left|g - \tfrac{k}{m}v^2\right|\right]_u^v = x$$

$$\ln\left|\frac{g - \frac{k}{m}v^2}{g - \frac{k}{m}u^2}\right| = -\tfrac{2kx}{m}$$

$$\ln\left|\frac{1 - \frac{k}{mg}v^2}{1 - \frac{k}{mg}u^2}\right| = -\tfrac{2kx}{m}$$

$$\left|\frac{1 - \frac{k}{mg}v^2}{1 - \frac{k}{mg}u^2}\right| = e^{-\frac{2kx}{m}}$$

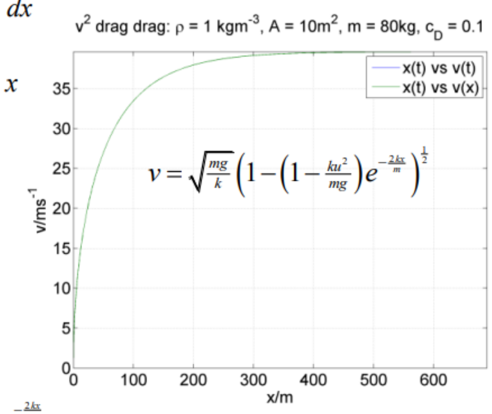
$$1 - \tfrac{k}{mg}v^2 = \left(1 - \tfrac{k}{mg}u^2\right)e^{-\frac{2kx}{m}}$$

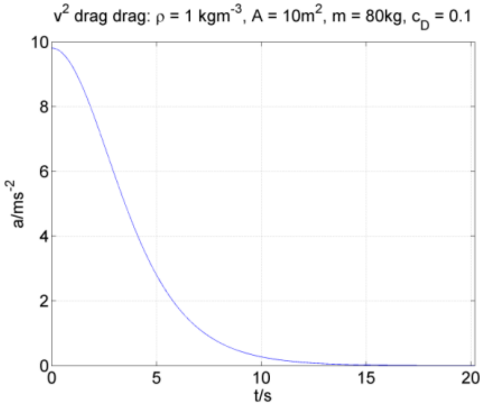
$$\boxed{v = \sqrt{\tfrac{mg}{k}}\left(1 - \left(1 - \tfrac{ku^2}{mg}\right)e^{-\frac{2kx}{m}}\right)^{\frac{1}{2}}}$$

Assume drag always less than weight i.e.

$$kv^2 \le mg$$

$$ku^2 \le mg$$


v^2 drag drag: ρ = 1 kgm^{-3}, A = 10m^2, m = 80kg, c_D = 0.1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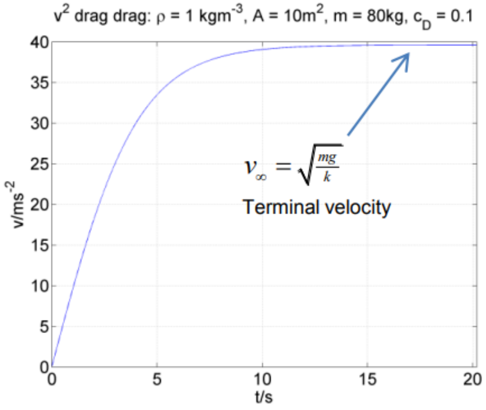
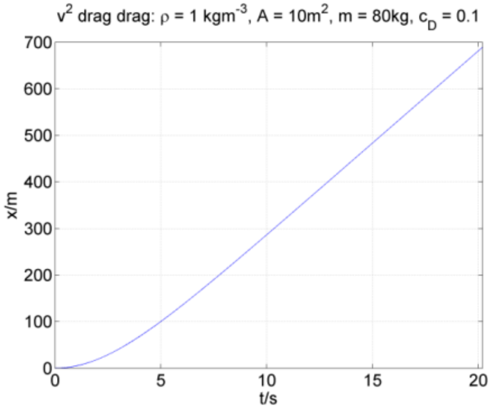

v^2 drag drag: ρ = 1 kgm^{-3}, A = 10m^2, m = 80kg, c_D = 0.1


v^2 drag drag: ρ = 1 kgm^{-3}, A = 10m^2, m = 80kg, c_D = 0.1

$$\int \tanh(ax+b)dx = \tfrac{1}{a}\ln(\cosh(ax+b)) + c$$

2b. Falling rocks (force)

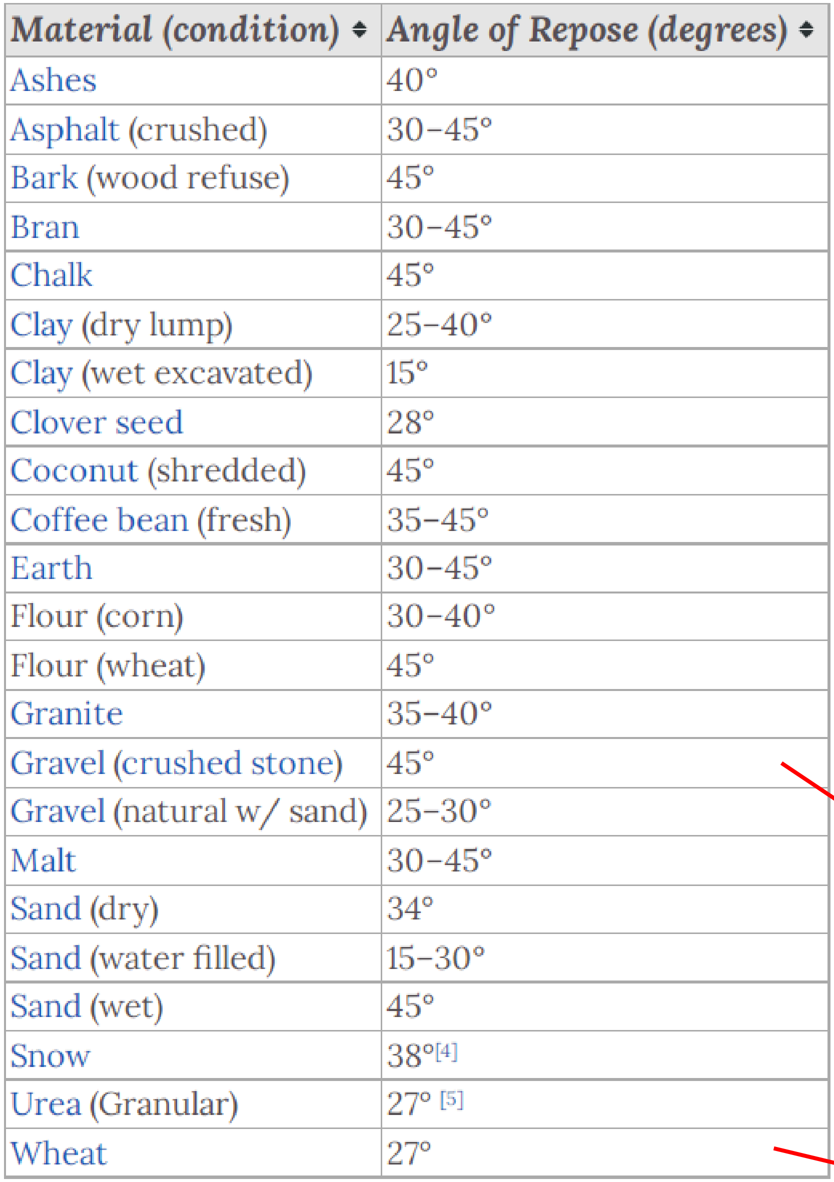
The force of a falling rock is the **rate of change of momentum when it collides with something.**

$(0.1\text{m})^3$ rock of 2,600kgm-3 density travelling at 30m/s, being stopped in 0.005s will result in force:

$$F = \frac{mv}{\Delta t} = \frac{0.1^3 \times 2,600 \times 30}{0.005} \ (\text{N}) = 15,600\text{N}$$

Note EN 12492 requires climbing helmets to be able to resist an impact of 10kN.
(5kg rock dropped from 2m).

So alas *this* rock will probably break most climbing helmets.



Material (condition) ◆	Angle of Repose (degrees) ◆
Ashes	40°
Asphalt (crushed)	30–45°
Bark (wood refuse)	45°
Bran	30–45°
Chalk	45°
Clay (dry lump)	25–40°
Clay (wet excavated)	15°
Clover seed	28°
Coconut (shredded)	45°
Coffee bean (fresh)	35–45°
Earth	30–45°
Flour (corn)	30–40°
Flour (wheat)	45°
Granite	35–40°
Gravel (crushed stone)	45°
Gravel (natural w/ sand)	25–30°
Malt	30–45°
Sand (dry)	34°
Sand (water filled)	15–30°
Sand (wet)	45°
Snow	38°[4]
Urea (Granular)	27° [5]
Wheat	27°

2c. Talus cones (a 'scree slope') on north shore of Isfjord, Svalbard, Norway, showing angle of repose for coarse sediment

https://www.wikiwand.com/en/Angle_of_repose

2d. Capillary forces – rising sap

$$\pi r^2 h\rho g = 2\pi r\sigma\cos\theta$$

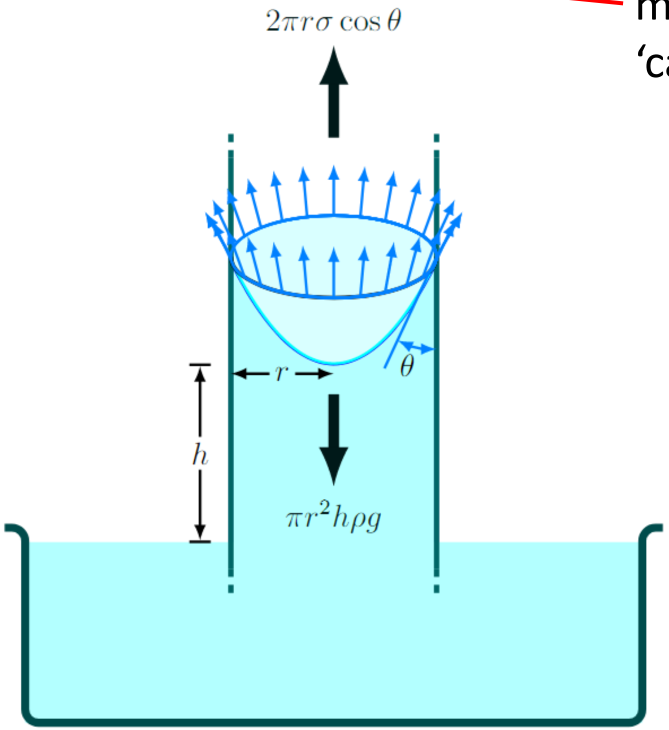
surface tension

$$\therefore h = \frac{2\sigma\cos\theta}{r\rho g}$$

$$\therefore h = \frac{2\times 7.28\times 10^{-2}\times\cos 0^{\circ}}{0.02\times 10^{-3}\times 1.05\times 10^{3}\times 9.81}\ \text{(m)}$$

$$\therefore h \approx 0.71\text{m}$$

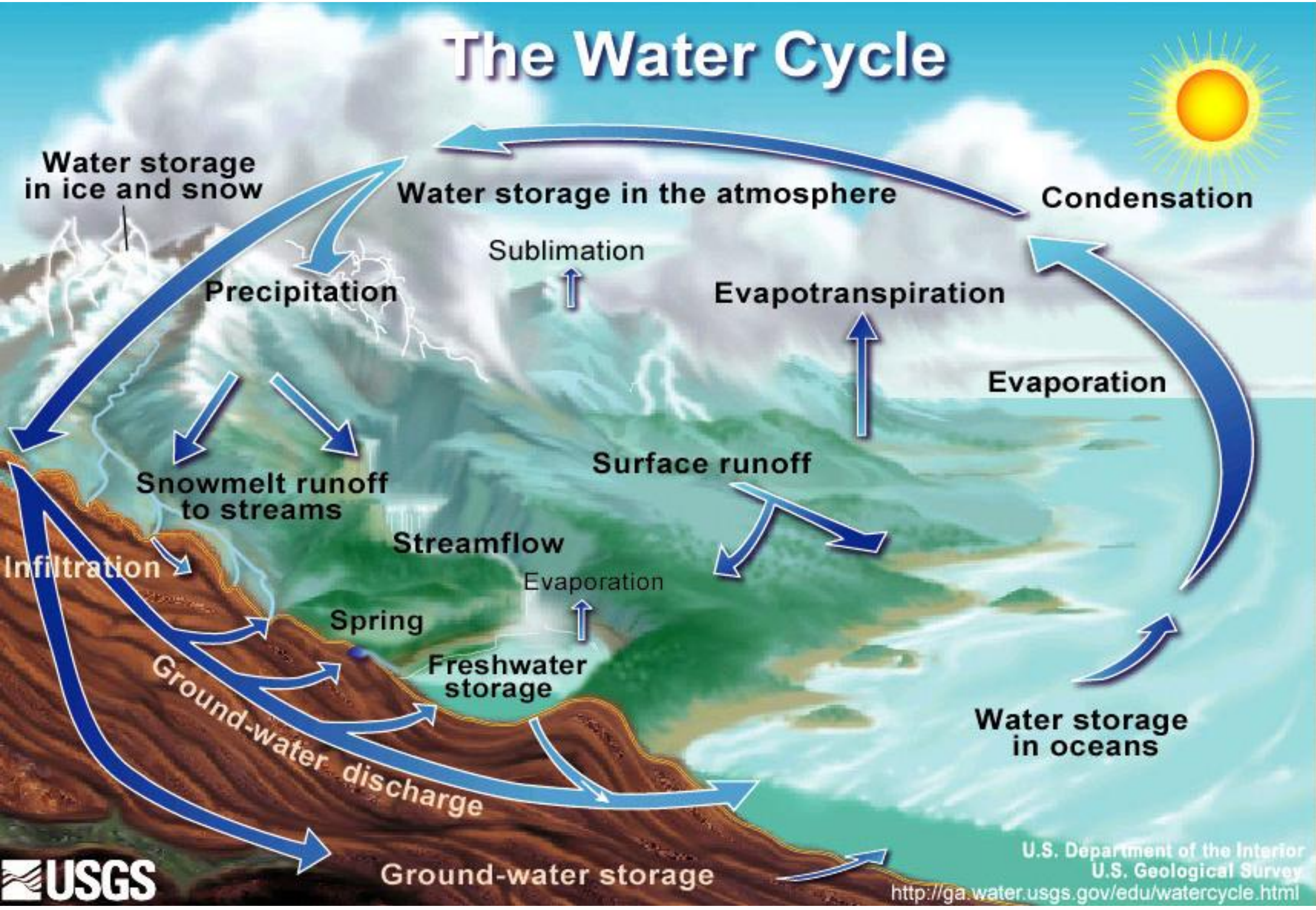
max height of
‘capilliary tube’



So capillary forces are terefore *not* the mechanism for transport of sap to the 80m tall crown of a Giant Sequoia tree! The energy to overcome the GPE is provided by the **latent heat** released by evaporation from leaves.

1. Mountains high or low, hard or soft
2. Gravity rules
3. Water, snow and ice
4. Glacier puzzles
5. Heat, cold and air
6. Rock climbing
7. Miscellaneous

The Water Cycle

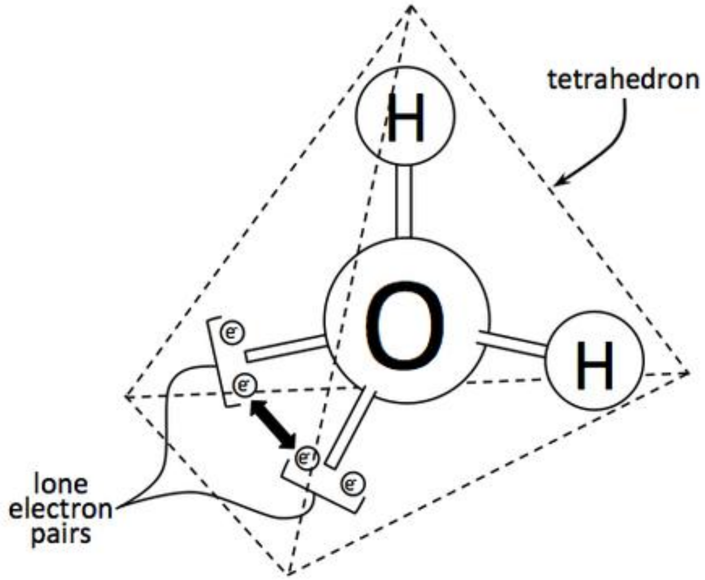


U.S. Department of the Interior
U.S. Geological Survey
http://ga.water.usgs.gov/edu/watercycle.html

Water is an amazing molecule!

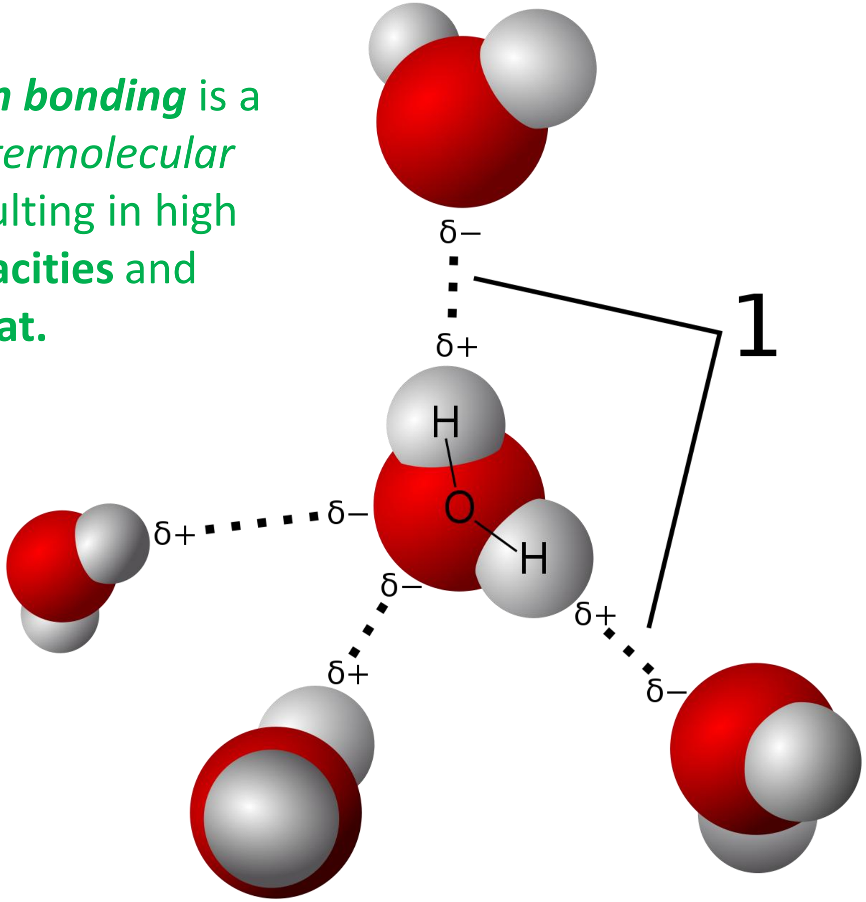
Liquid water EXPANDS as it freezes

Water vapour is about 1,600 times larger than liquid water

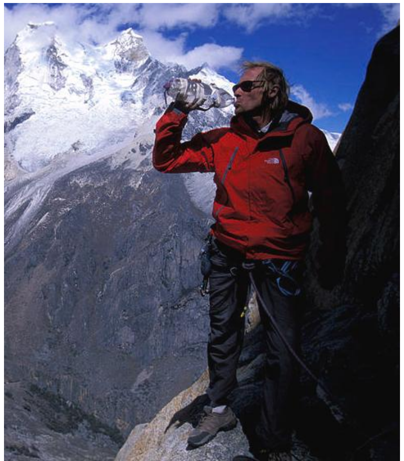


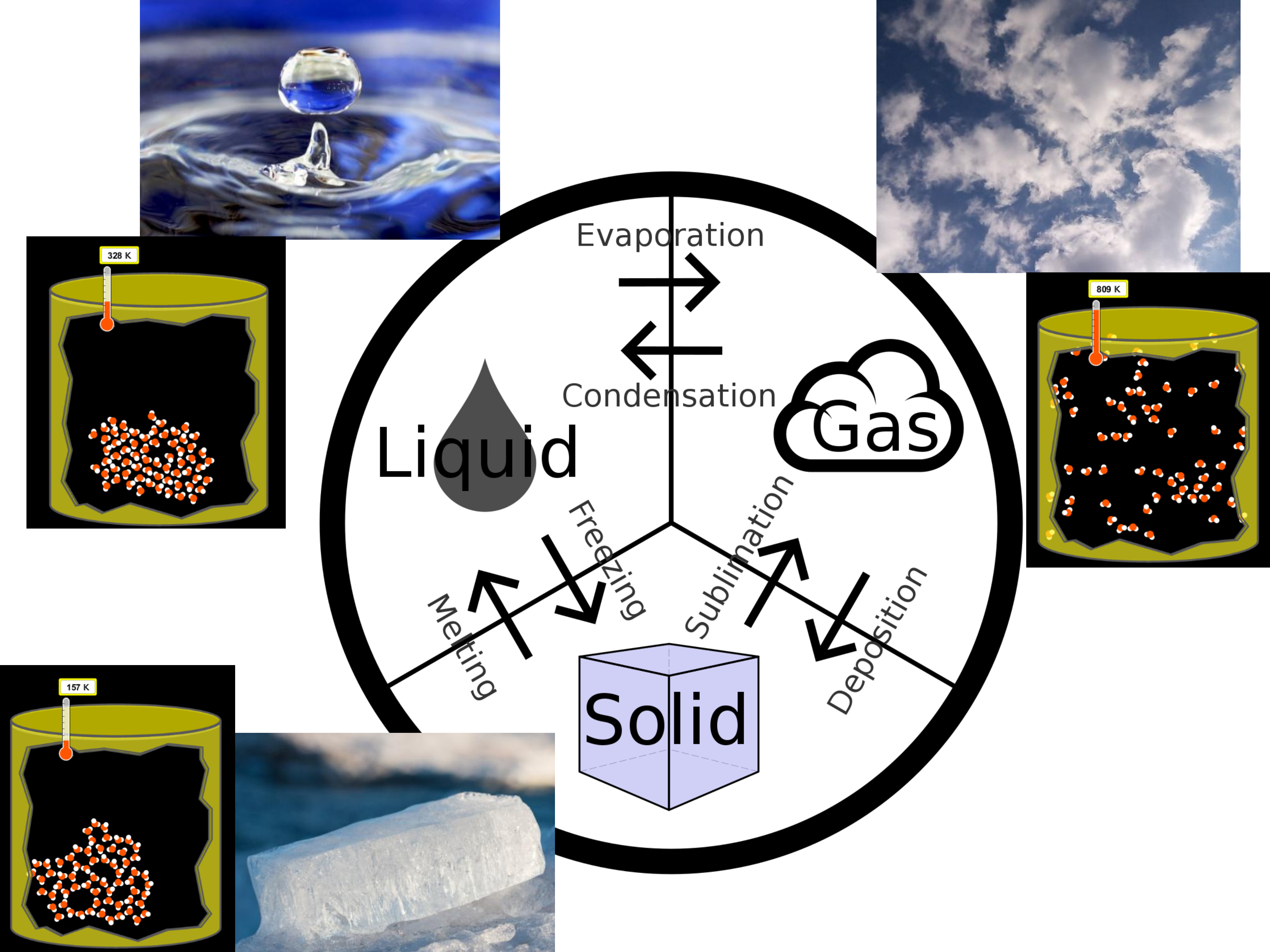
The bent structure of a water molecule.

Hydrogen bonding is a strong *intermolecular force* resulting in high **heat capacities** and **latent heat.**



The polar nature of water molecules makes water a very effective solvent for substances involving biochemical reactions ... such as nutrients and waste products in plants and animals! **This is why you must drink to stay hydrated in the mountains!**





Evaporation

Condensation

Liquid

Gas

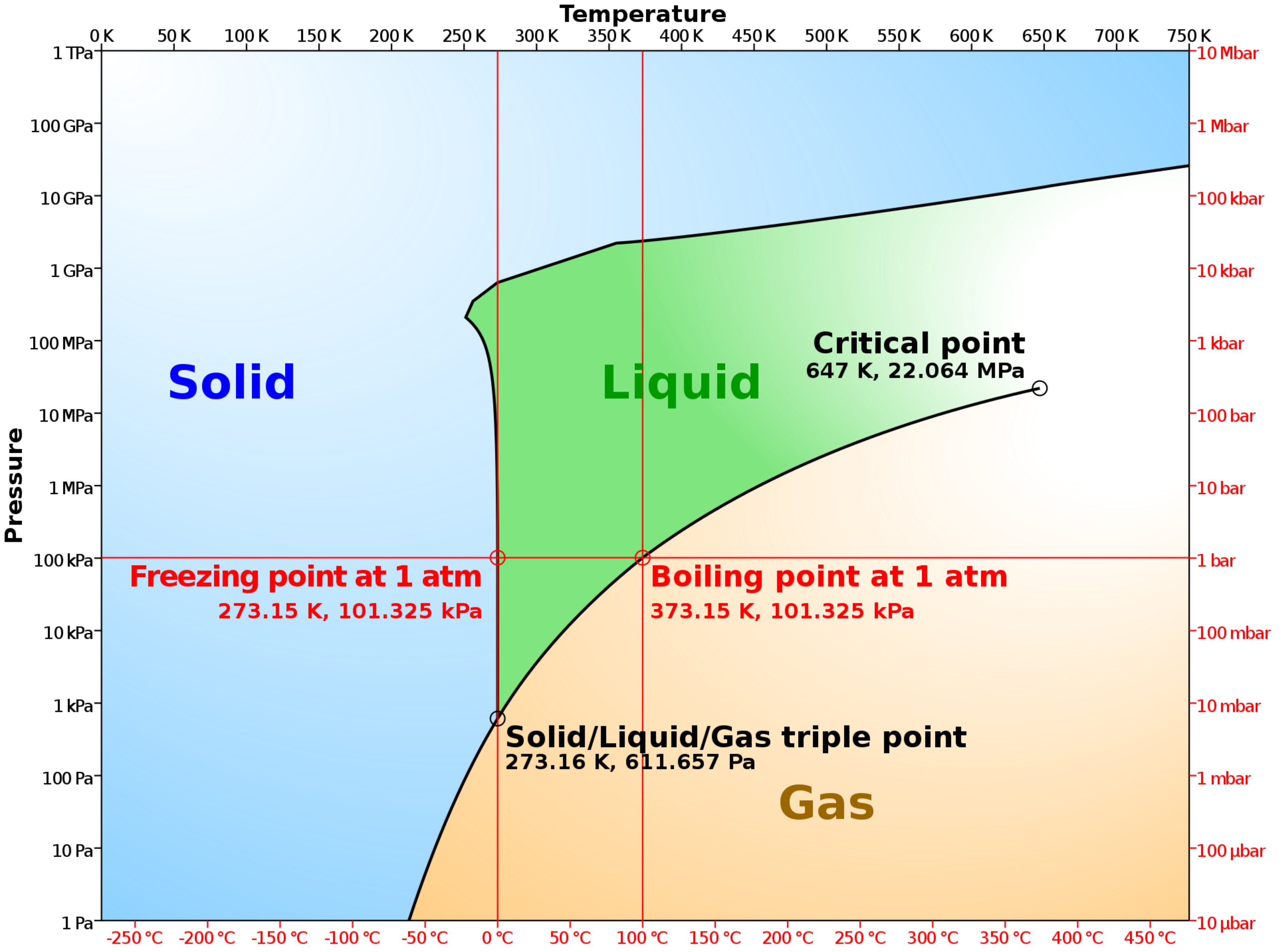
Freezing

Melting

Sublimation

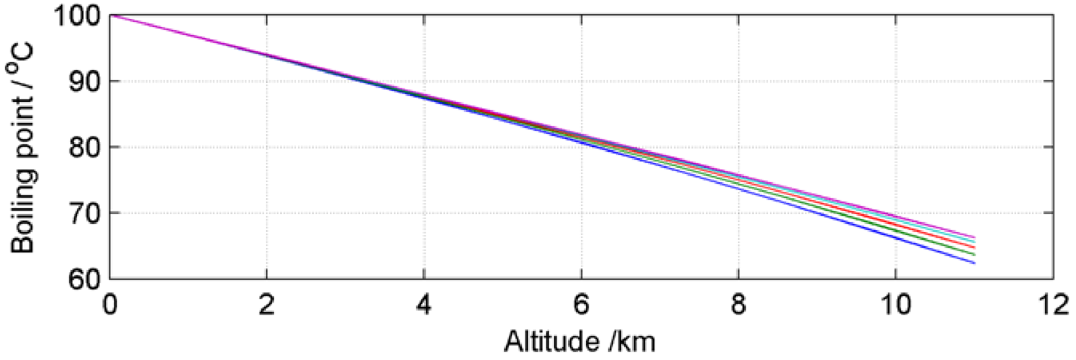
Deposition

Solid


Temperature
0 K
50 K
100 K
150 K
200 K
250 K
300 K
350 K
400 K
450 K
500 K
550 K
600 K
650 K
700 K
750 K
Pressure
1 TPa
100 GPa
10 GPa
1 GPa
100 MPa
10 MPa
1 MPa
100 kPa
10 kPa
1 kPa
100 Pa
10 Pa
1 Pa
10 Mbar
1 Mbar
100 kbar
10 kbar
1 kbar
100 bar
10 bar
1 bar
100 mbar
10 mbar
1 mbar
100 µbar
10 µbar
Solid
Liquid
Gas
Critical point
647 K, 22.064 MPa
Freezing point at 1 atm
273.15 K, 101.325 kPa
Boiling point at 1 atm
373.15 K, 101.325 kPa
Solid/Liquid/Gas triple point
273.16 K, 611.657 Pa
-250 °C
-200 °C
-150 °C
-100 °C
-50 °C
0 °C
50 °C
100 °C
150 °C
200 °C
250 °C
300 °C
350 °C
400 °C
450 °C

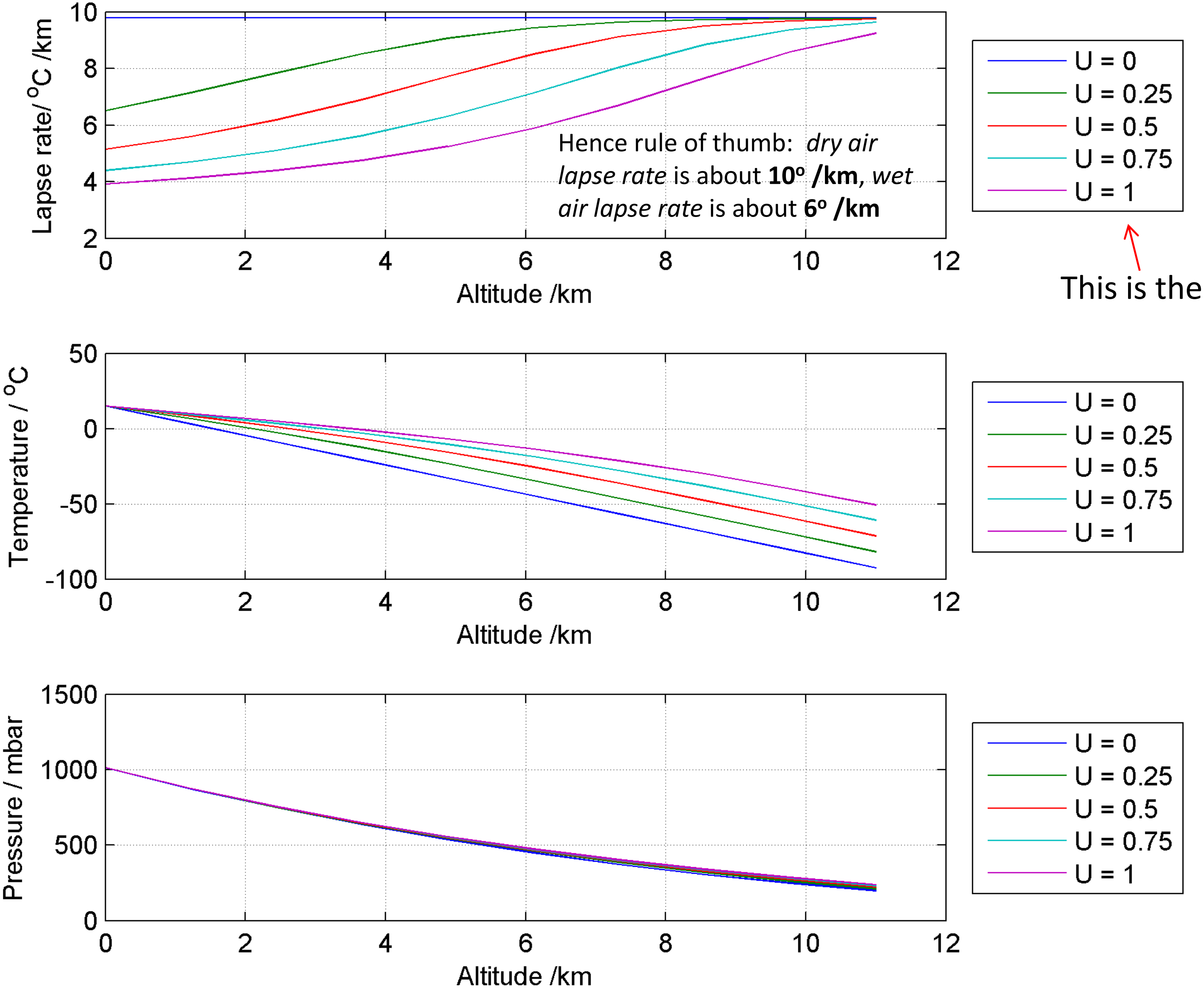
The boiling point of water *decreases* as atmospheric pressure *decreases*.
You can observe this on a mountaineering expedition as you ascend in altitude.

Note one litre of water becomes 1,600 litres of steam!



K2. August 2018

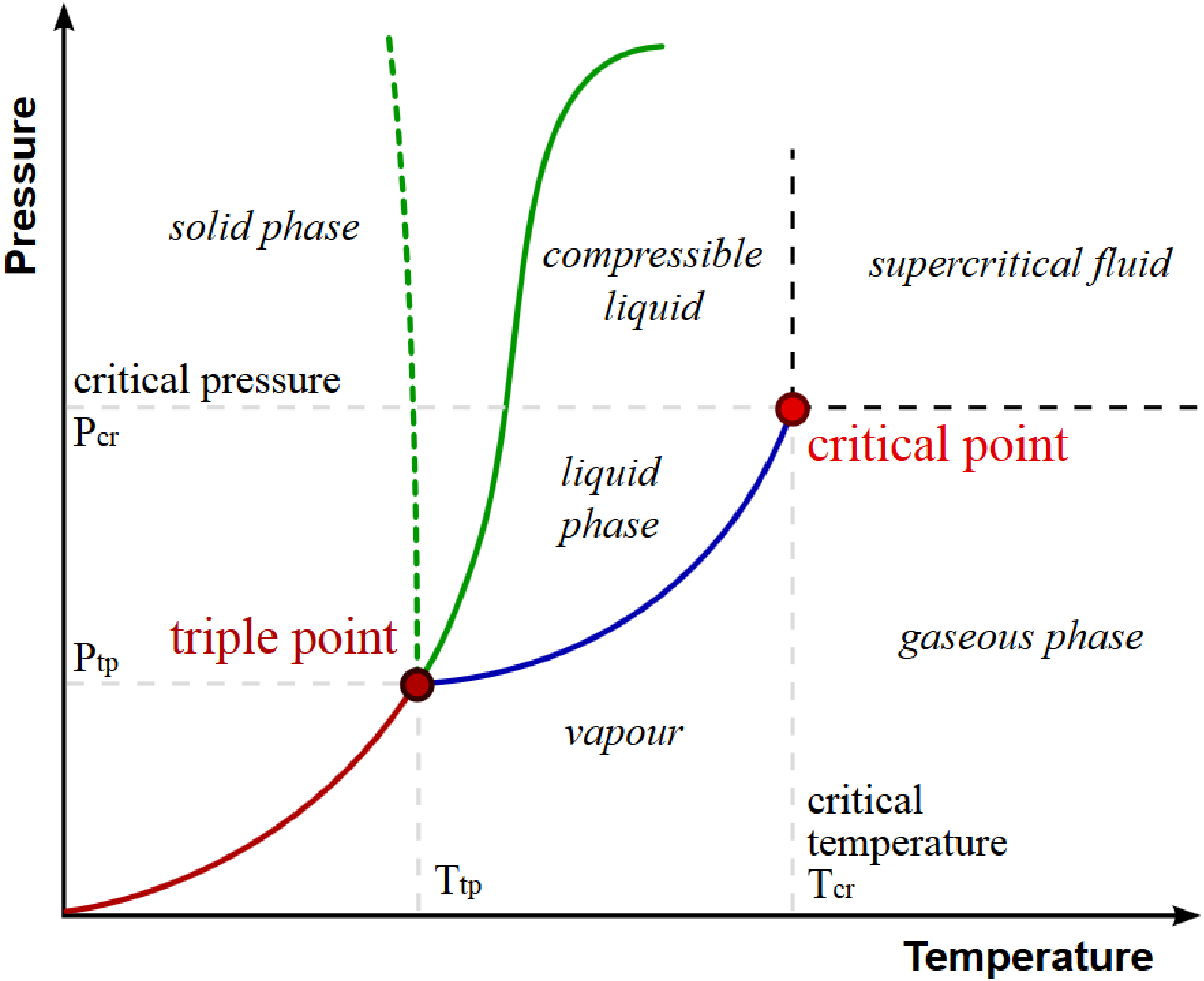
Lapse rates for different relative humidities



Hence rule of thumb: *dry air lapse rate* is about **10° /km**, *wet air lapse rate* is about **6° /km**

This is the humidity

To **boil** water, it must undergo a *phase transition* from liquid to gas. This requires a certain amount of heat, *the latent heat of vaporization*, to break the inter-molecular bonds inherent in the water.



The gradient of a phase transition line in the p,T diagram is given by the *Clausius-Clapeyron equation:*

Pressure in Pascals (Pa)

Latent heat /Jmol^{-1}

$$\boxed{\frac{dp}{dT} = \frac{L_{vap}}{T\Delta V}}$$

Temperature in Kelvin

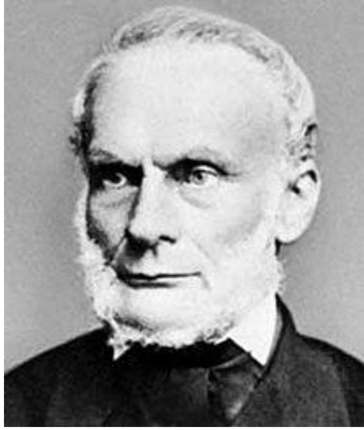
Volume change of 1 mole of substance during the phase transition

In a **liquid to gas** transition we can assume the volume change is sufficiently large as to ignore the original fluid volume. If one assumes the resulting gas is ***ideal***:

$$\Delta V = \frac{RT}{p}$$

Hence:

$$\boxed{\frac{dp}{dT} = \frac{L_{vap}}{RT^2}p}$$



Rudolph Clausius
1822-1888

Benoît
Clapeyron
1799-1864

We can use this relationship to determine the liquid-to-gas line in the p,T diagram if the latent heat of vaporization is assumed to be temperature independent. (In reality this is not the case, but is less of an issue at low temperatures – see next slide).

$$\frac{dp}{dT} = \frac{L_{vap}}{RT^2}\, p$$

$$\int_{P_*}^{P} \frac{1}{p}\, dp = \frac{L_{vap}}{R} \int_{T_*}^{T} \frac{1}{T^2} dT$$

$$\ln\left(\frac{p}{p_*}\right) = -\frac{L_{vap}}{R}\left(\frac{1}{T} - \frac{1}{T_*}\right)$$

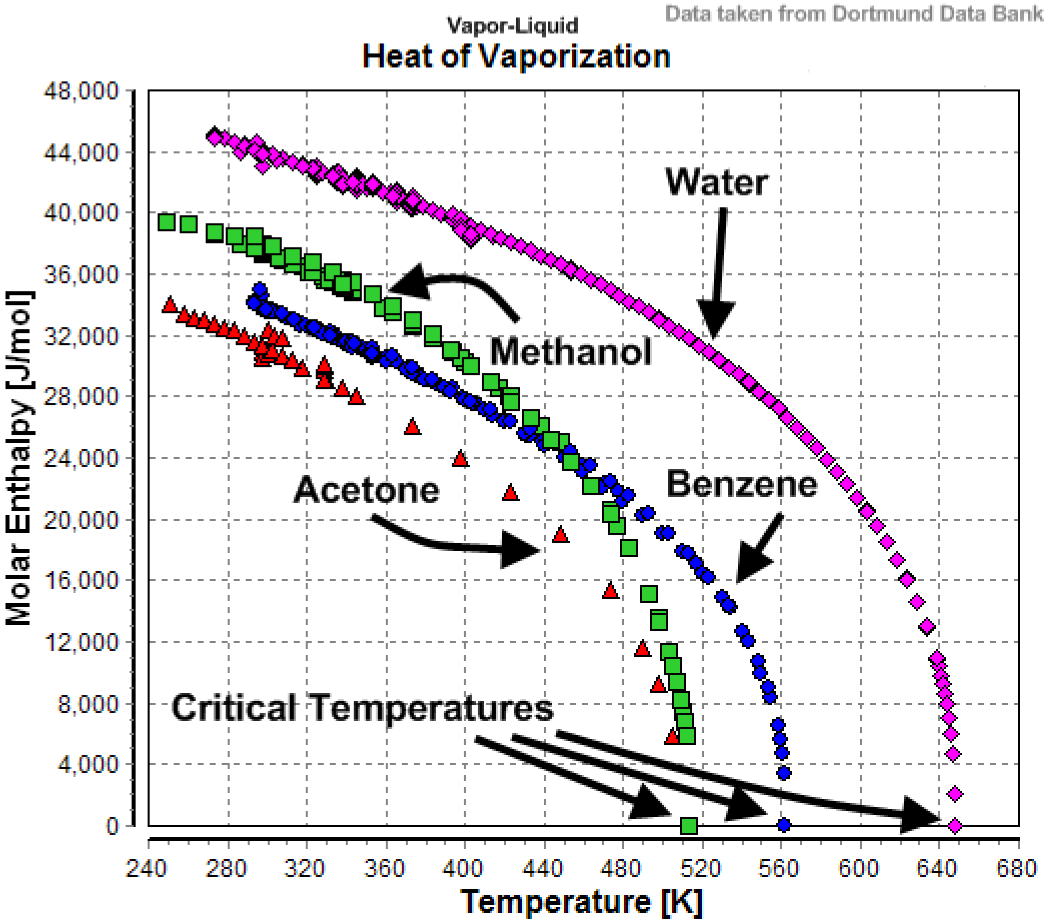
$$p = p_* e^{-\frac{L_{vap}}{R}\left(\frac{1}{T} - \frac{1}{T_*}\right)}$$

$$\boxed{T = \left(\frac{1}{T_*} - \frac{R}{L_{vap}} \ln\left(\frac{p}{p_*}\right)\right)^{-1}}$$

Since the temperature corresponds to the liquid-to-gas transition line, we can therefore derive an expression for the **boiling point** of a liquid relative to ambient temperature T and pressure p, as long as one fixed boiling point T_* and corresponding pressure p_* is known.

$$\boxed{T_{boil} = \left(\frac{1}{T_*} - \frac{R}{L_{vap}} \ln\left(\frac{p}{p_*} \right) \right)^{-1}}$$

Boiling point of water at different atmospheric pressures /mbar.

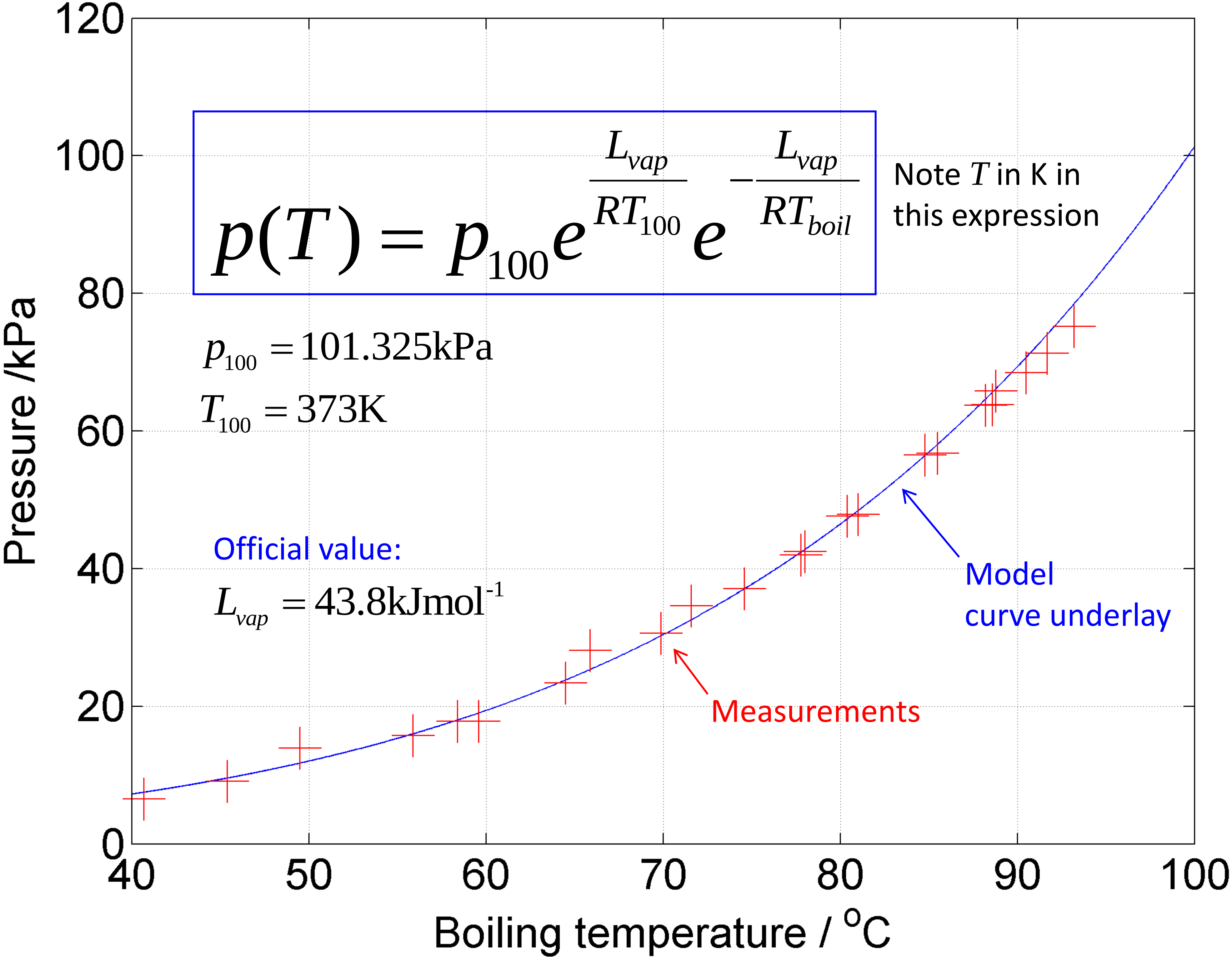


Latent heat of vaporization of water is
L_{vap} = 43.8 kJ mol^{-1} at 100°C and 1013.25 mbar ambient air pressure.

At ‘typical’ troposphere temperatures, a higher value of 45.07 kJ mol^{-1} is used in some calculations, such as the lapse rate model.

http://en.wikipedia.org/wiki/Lapse_rate
http://en.wikipedia.org/wiki/Enthalpy_of_vaporization
http://en.citizendium.org/wiki/Heat_of_vaporization

Pressure vs boiling temperature. L_{vap} = (42.7 +/-0.8) kJ/mol



$$p(T) = p_{100} e^{\frac{L_{vap}}{RT_{100}}} e^{-\frac{L_{vap}}{RT_{boil}}}$$

Note *T* in K in this expression

$$p_{100} = 101.325\text{kPa}$$

$$T_{100} = 373\text{K}$$

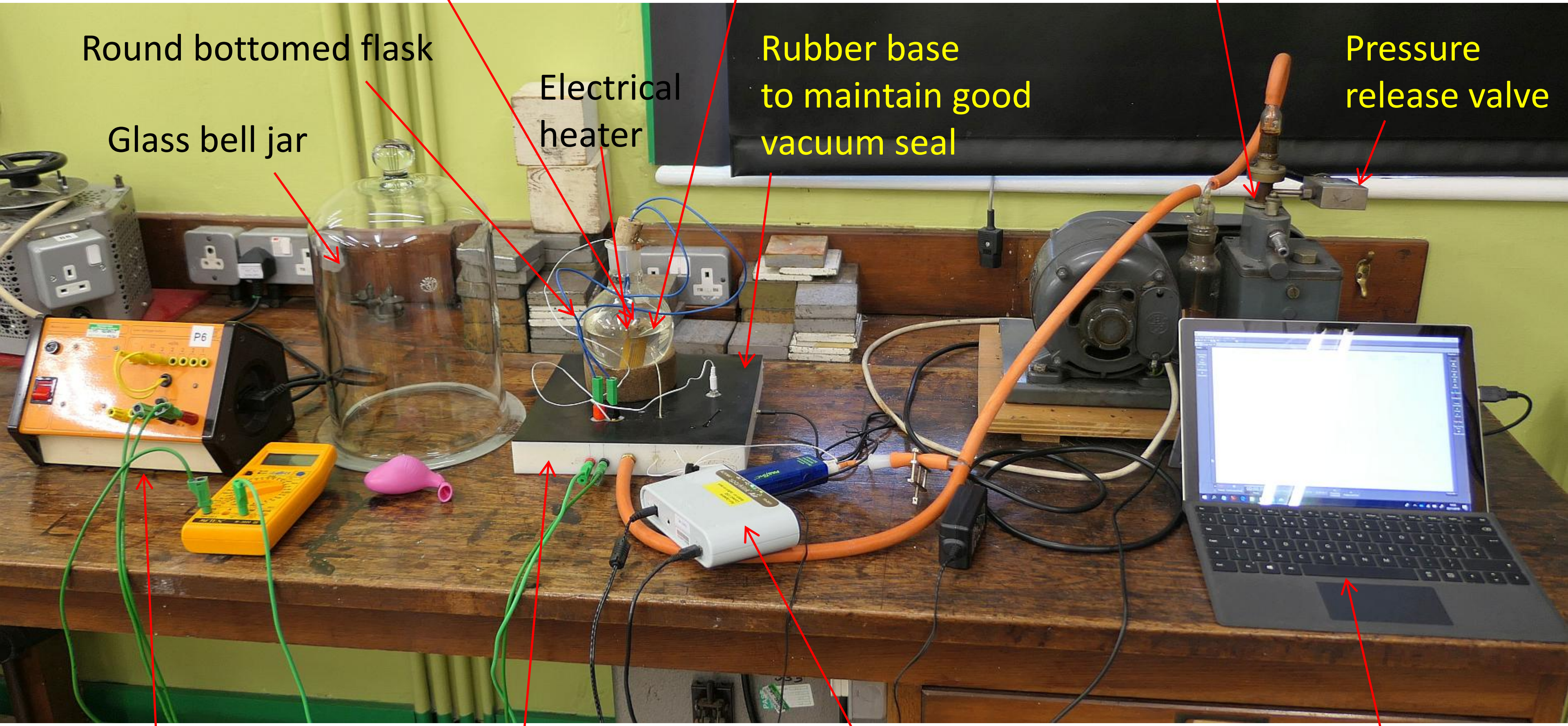
Official value:

$$L_{vap} = 43.8\text{kJmol}^{-1}$$

Model curve underlay

Measurements

Equipment for water boiling temperature vs pressure experiment



About 400ml of water

Thermocouple in water

Vacuum pump

Round bottomed flask

Electrical heater

Glass bell jar

Rubber base to maintain good vacuum seal

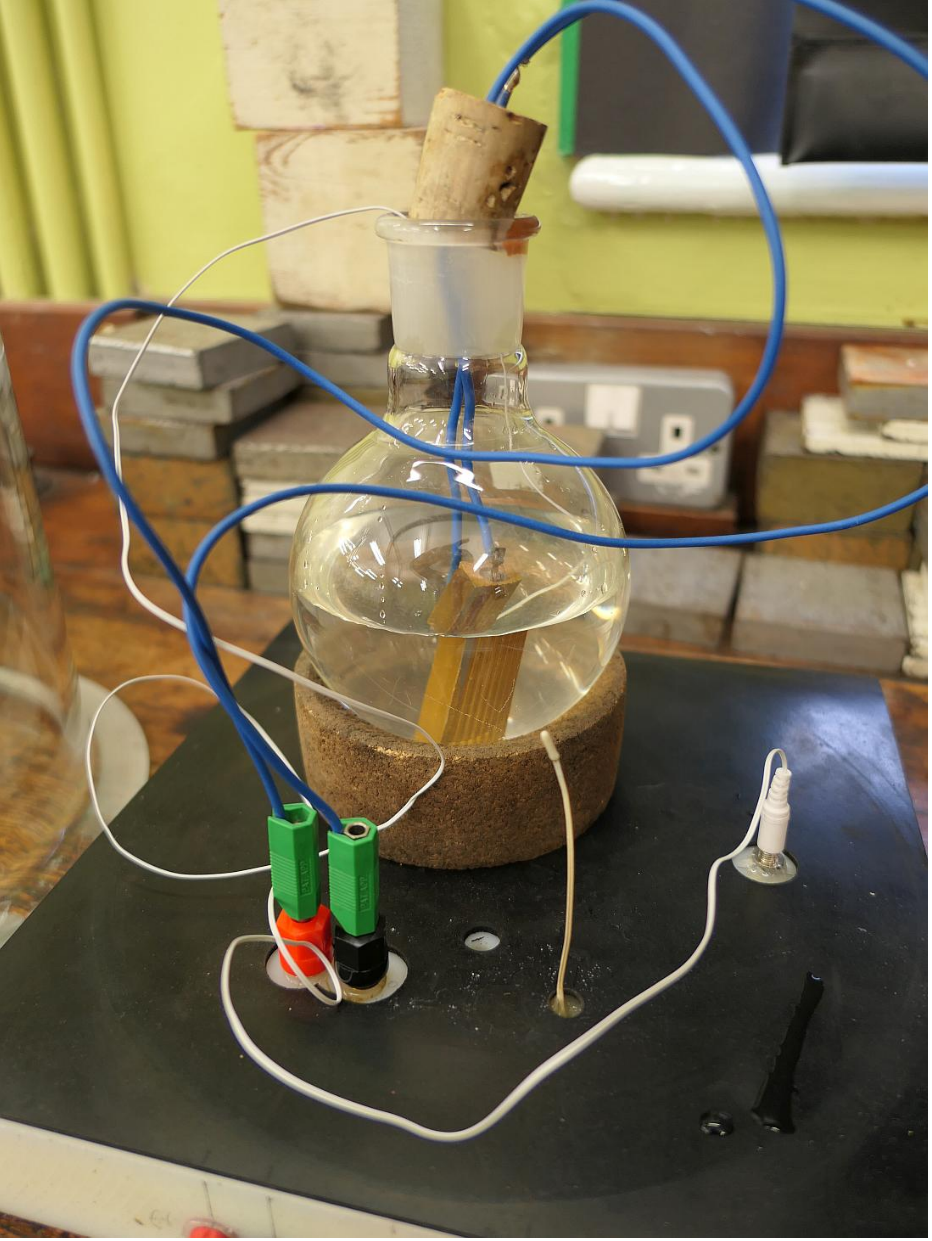
Pressure release valve

12V DC power supply to electrical heater (about 4.8A)

Pump valve, +/- power socket + 2x thermocouple connections drilled into plastic base

PASCO datalogger USB hub (pressure unit + 2x thermocouple connections)

Laptop running CAPSTONE connected to USB hub



Heat water up to a desired temperature using electrical heater. (It is pretty slow, i.e about 10s per deg C, so you can leave it on)

Then pump out air till water spontaneously boils. At this point turn off the pump and record pressure and temperature using the datalogger display on the PC

Determining water thermal properties in a laboratory

Heat a known mass of liquid water and record temperature vs time using a *thermocouple*. If kettle electrical power input is known, you can calculate the **specific heat capacity of liquid water: 4,180J/kg/K**

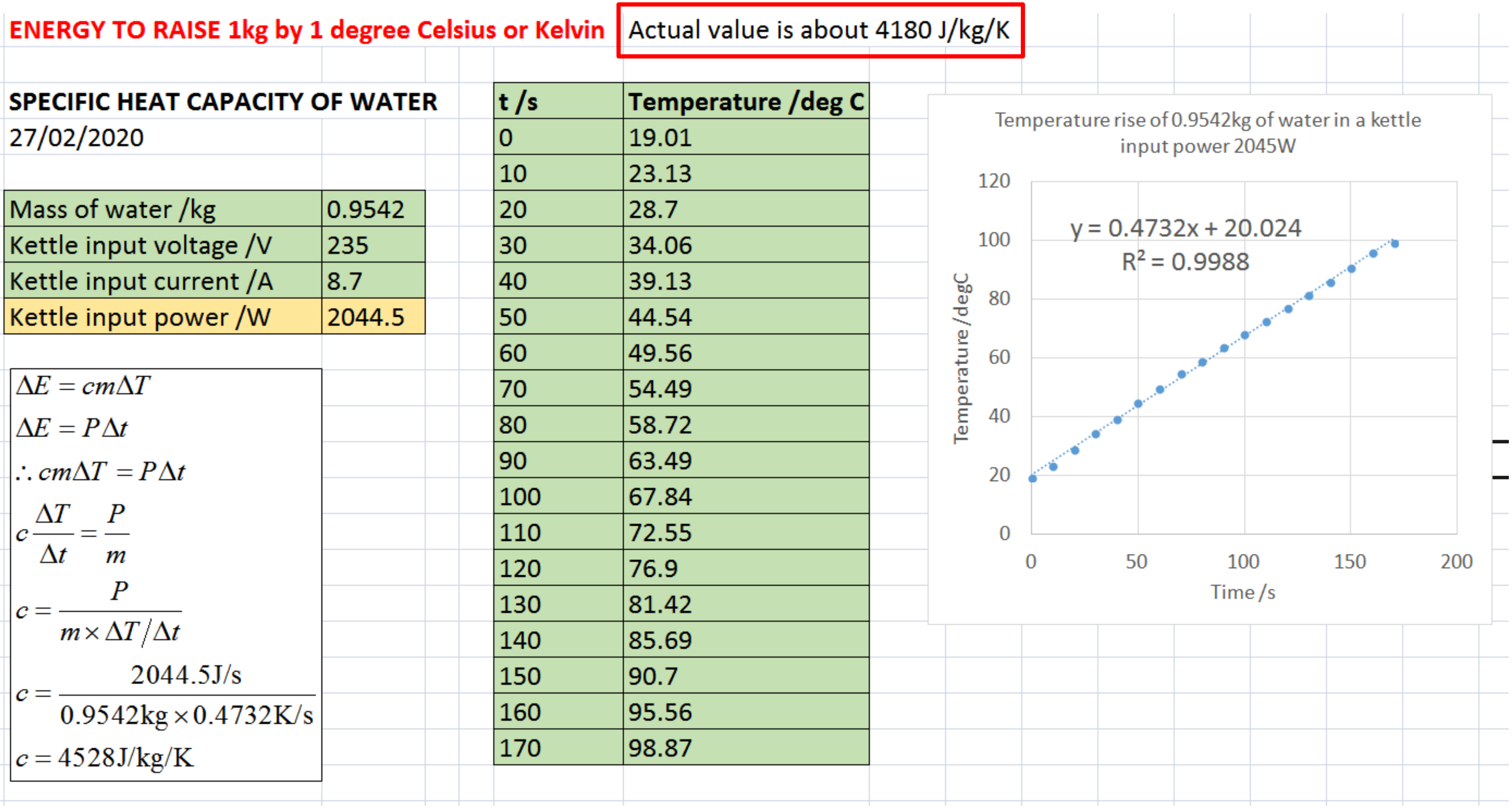
Place an ice cube into hot water in an insulated cup.
Use a thermocouple to record the temperature drop.
Use this to find the **specific latent heat of fusion:**

334 kJ/kg

Record mass lost vs time (open the lid of the kettle) once water is boiling.
The electrical power input can then
be used to determine the **specific latent heat of vaporization:**

2,265kJ/kg

SPECIFIC HEAT CAPACITY OF WATER: 4,180 J/kg/K



ENERGY TO RAISE 1kg by 1 degree Celsius or Kelvin

Actual value is about 4180 J/kg/K

SPECIFIC HEAT CAPACITY OF WATER
27/02/2020

Mass of water /kg	0.9542
Kettle input voltage /V	235
Kettle input current /A	8.7
Kettle input power /W	2044.5

$$\Delta E = cm\Delta T$$

$$\Delta E = P\Delta t$$

$$\therefore cm\Delta T = P\Delta t$$

$$c\frac{\Delta T}{\Delta t} = \frac{P}{m}$$

$$c = \frac{P}{m \times \Delta T/\Delta t}$$

$$c = \frac{2044.5\text{J/s}}{0.9542\text{kg} \times 0.4732\text{K/s}}$$

$$c = 4528\text{J/kg/K}$$

t /s	Temperature /deg C
0	19.01
10	23.13
20	28.7
30	34.06
40	39.13
50	44.54
60	49.56
70	54.49
80	58.72
90	63.49
100	67.84
110	72.55
120	76.9
130	81.42
140	85.69
150	90.7
160	95.56
170	98.87

The kettle will absorb some of the heat from the heating element, and there will also be resistive losses. So input power P is likely to be less than 2,044W, which is consistent with our *overestimate* of **specific heat capacity**.

Energy to convert 1kg of liquid water to vapour, at 100 degC.

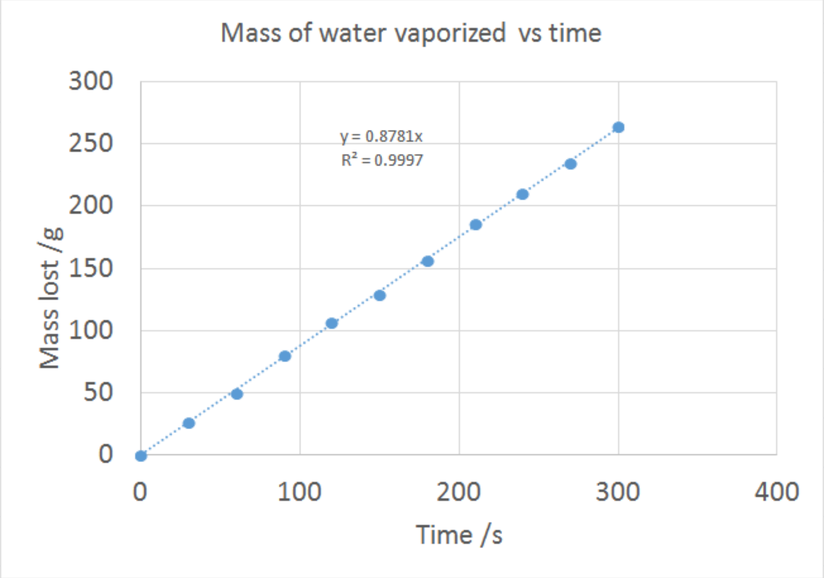
Actual value is about 2265 kJ/kg/K

SPECIFIC LATENT HEAT OF VAPORIZATION

27/02/2020

Kettle input voltage /V	235
Kettle input current /A	8.7
Kettle input power /W	2044.5

Mass lost /g	time /s
0	0
26.2	30
49.5	60
80.2	90
106.7	120
129.8	150
157.1	180
186.1	210
210.9	240
235.5	270
264.8	300



$$\Delta E = L\Delta m$$

$$\Delta E = P\Delta t$$

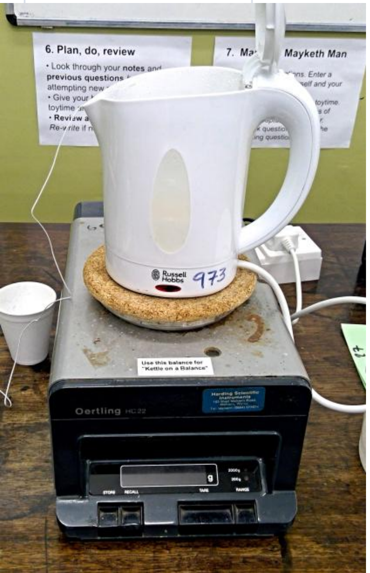
$$\therefore L\Delta m = P\Delta t$$

$$L\frac{\Delta m}{\Delta t} = P$$

$$L = \frac{P}{\frac{\Delta m}{\Delta t}}$$

$$L = \frac{2044.5\text{J/s}}{0.8781\times10^{-3}\text{kg/s}}$$

$$L = 2,328\text{kJ/kg}$$



Record mass lost vs time (open the lid of the kettle) once water is boiling. The electrical power input can then be used to determine the **specific latent heat of vaporization.**

SPECIFIC LATENT HEAT OF VAPORIZATION OF WATER: 2,265 kJ/kg

Energy to convert 1kg of ice to liquid, at 0 degC.

SPECIFIC LATENT HEAT OF FUSION

27/02/2020

Actual value is about 334 kJ/kg

Mass of beaker /g	1.93
Mass of water added /g	85.40
Mass of ice added /g	26.98
Melting temperature of ice /degC	0.00
Temperature of ice /degC	0.00
Initial temperature of water /degC	79.37
Temperature of water + melted ice /degC	39.41

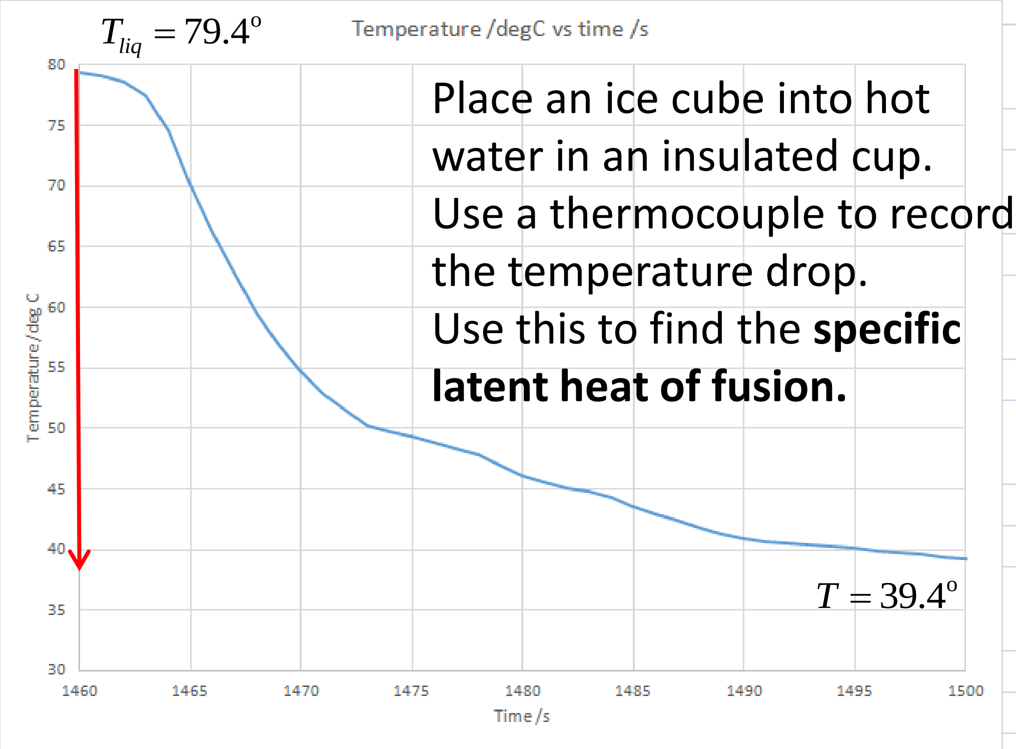
SPECIFIC LATENT HEAT OF VAPORIZATION (L) /kJ/kg	**364**

Energy to raise ice to melting temperature /kJ	0.00
Energy to melt ice /kJ	9.82
Energy to raise ice to final liquid temperature /kJ	4.44
Energy loss from hot water /kJ	14.26

Specific heat capacity of liquid water /kJ/kg/K	4.18
Specific heat capacity of ice /kJ/kg/K	2.11

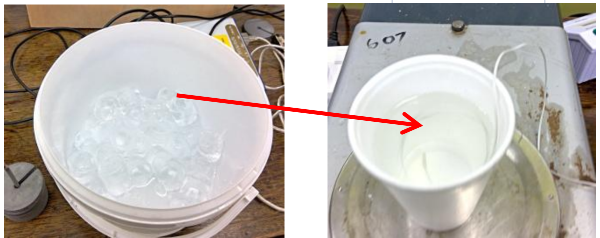
$$c_{liq}m_{liq}\left(T_{liq}-T\right)=c_{ice}m_{ice}\left(T_m-T_{ice}\right)+Lm_{ice}+c_l m_l\left(T-T_m\right)$$

$$L=\frac{c_{liq}m_{liq}\left(T_{liq}-T\right)-c_{ice}m_{ice}\left(T_m-T_{ice}\right)-c_l m_l\left(T-T_m\right)}{m_{ice}}$$



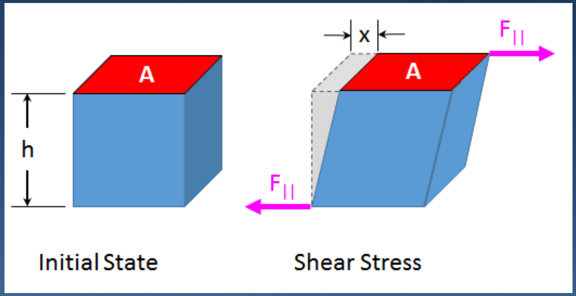
Place an ice cube into hot water in an insulated cup. Use a thermocouple to record the temperature drop. Use this to find the **specific latent heat of fusion.**

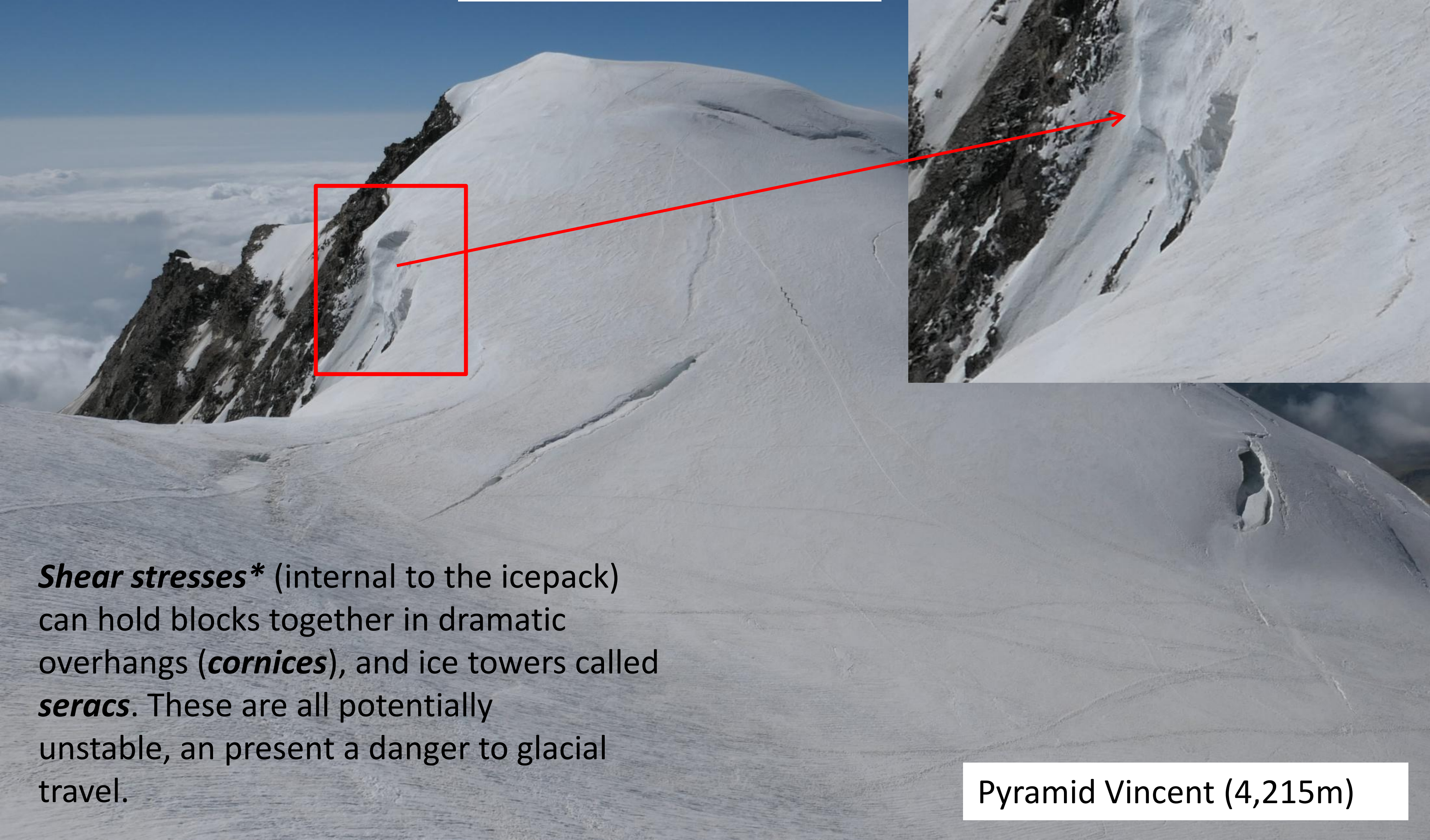
$T_{ice}=0.0^o$



SPECIFIC LATENT HEAT OF FUSION OF WATER: 334 kJ/kg

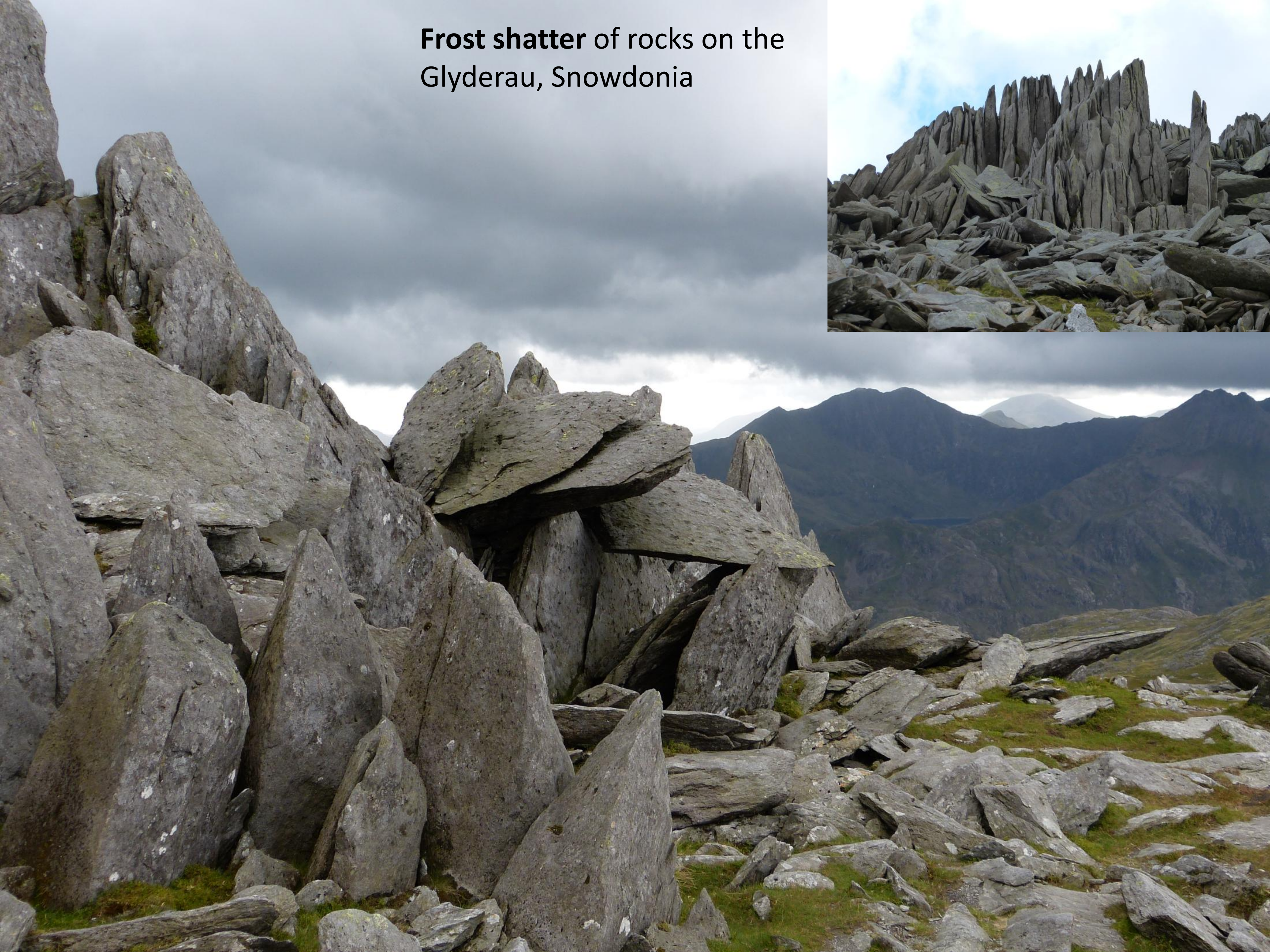
***shear stress** is a surface effect: the force per unit area that is *parallel* to the force.





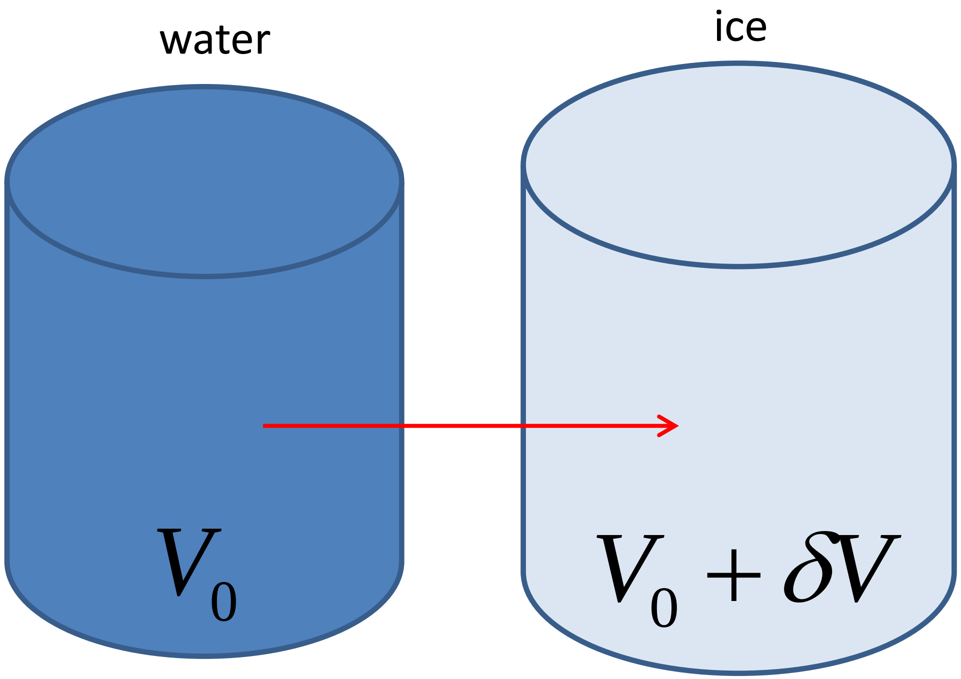
Shear stresses* (internal to the icepack) can hold blocks together in dramatic overhangs (***cornices***), and ice towers called ***seracs***. These are all potentially unstable, an present a danger to glacial travel.

Pyramid Vincent (4,215m)



Frost shatter of rocks on the Glyderau, Snowdonia

Water expands when freezing, which can lead to *frost shatter* of rocks as water expands in cracks



pressure

bulk modulus of ice

$$p = B\frac{\delta V}{V_0}$$

$$m = \rho_w V_0 = \rho_{ice}\left(V_0 + \delta V\right)$$

$$\therefore \frac{\rho_w}{\rho_{ice}} = 1 + \frac{\delta V}{V_0}$$

$$\therefore \frac{\delta V}{V_0} = \frac{\rho_w}{\rho_{ice}} - 1$$

Find the fractional volume change of liquid water as it freezes to ice, assuming no loss of water mass.

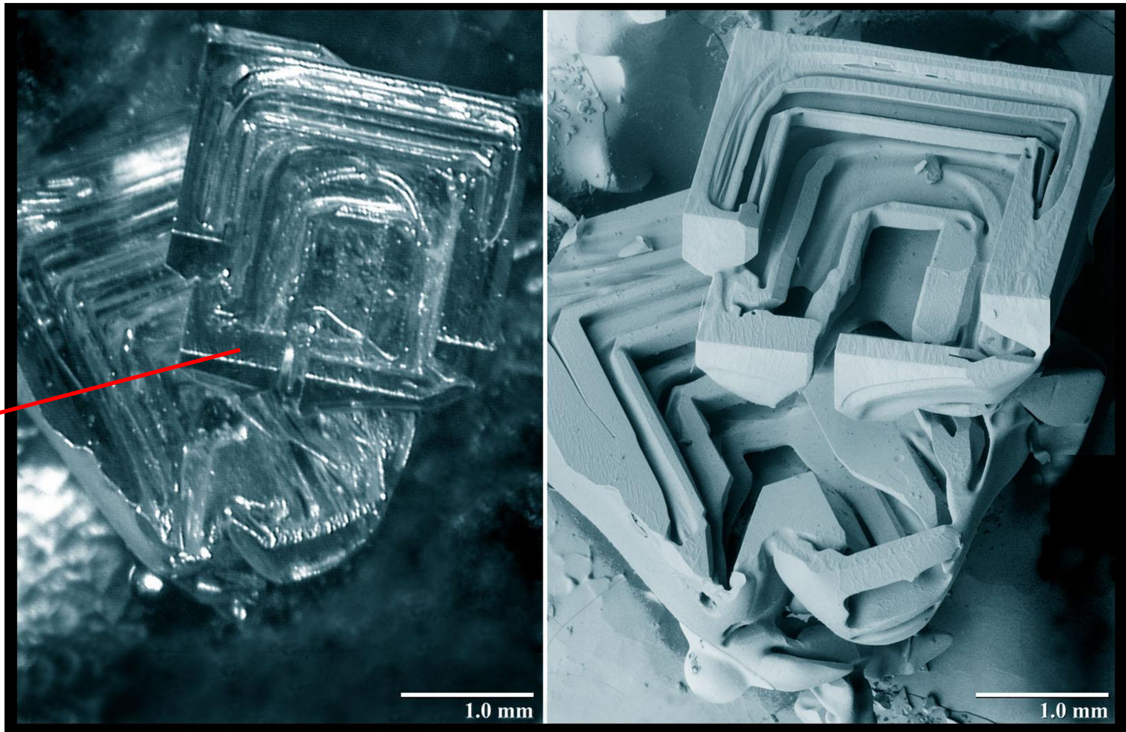
$$p = 1.13 \times 10^6\,\mathrm{Nm^{-2}} \times \frac{1{,}000\,\mathrm{kgm^{-3}}}{917\,\mathrm{kgm^{-3}}}$$

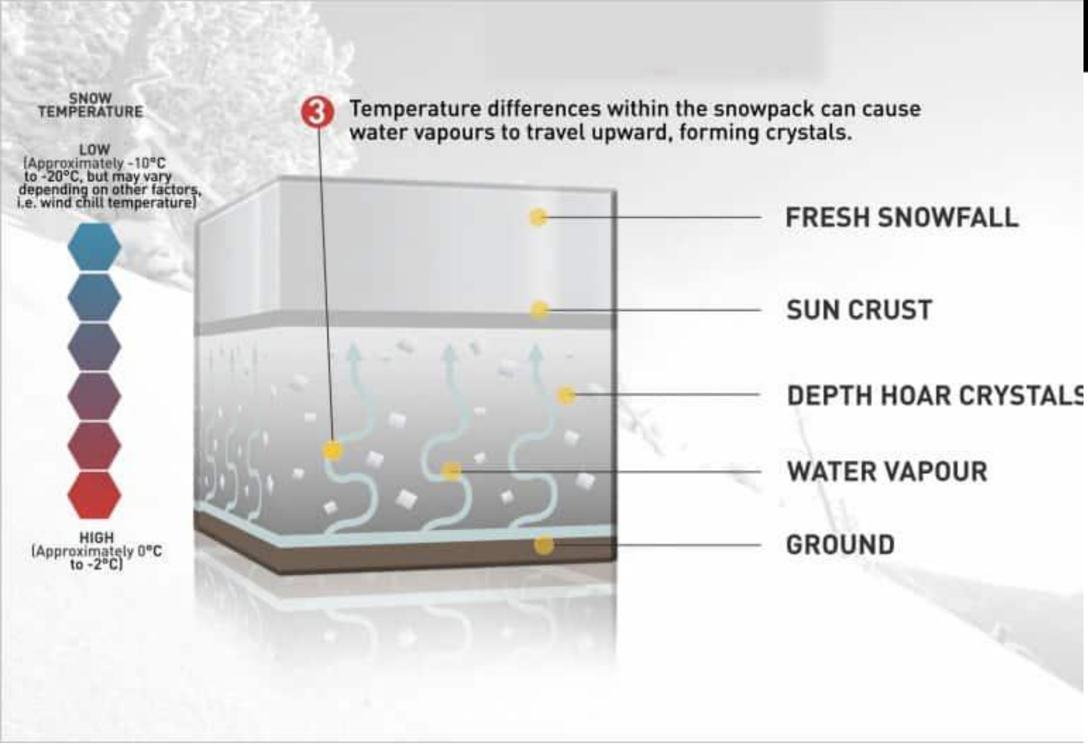
$$p = 1.02 \times 10^5\,\mathrm{Nm^{-2}} \approx 1\,\mathrm{atm}$$

So a *significant* pressure, although not immediately clear if sufficient to fracture rock! i.e. equivalent to 10m of water depth.

Verglas
Solidification of water vapour

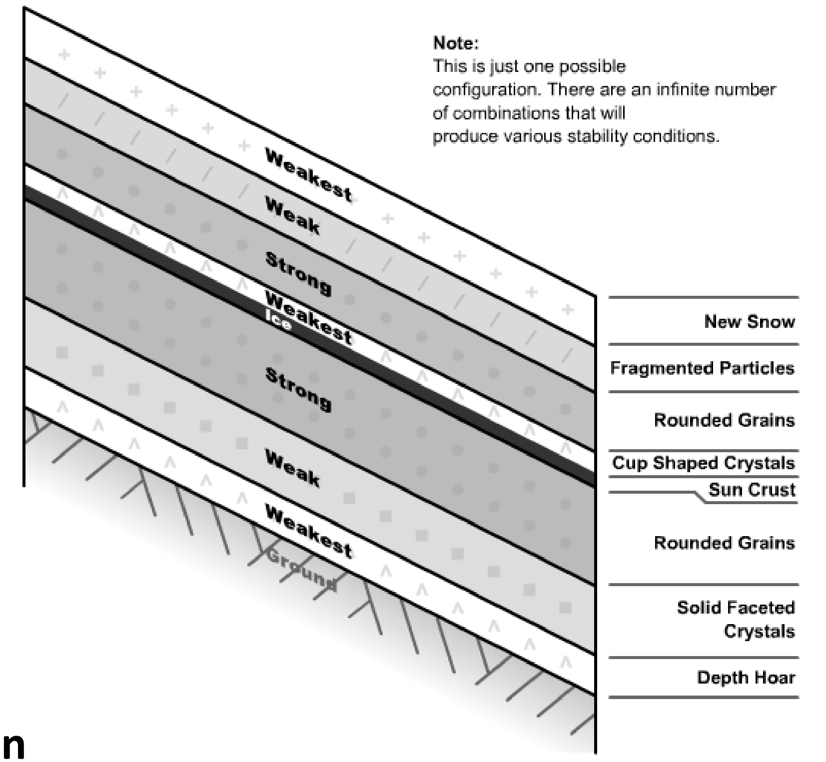
Depth hoar – a layer of powdery, sugary snow that can result in an **avalanche** as the (harder) snow layers slide over each other.





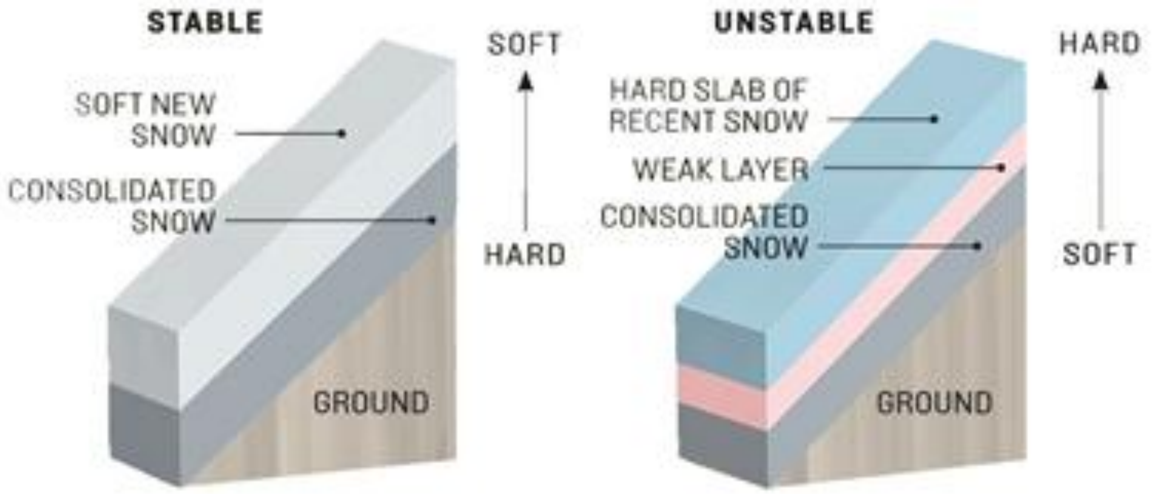
③ Temperature differences within the snowpack can cause water vapours to travel upward, forming crystals.

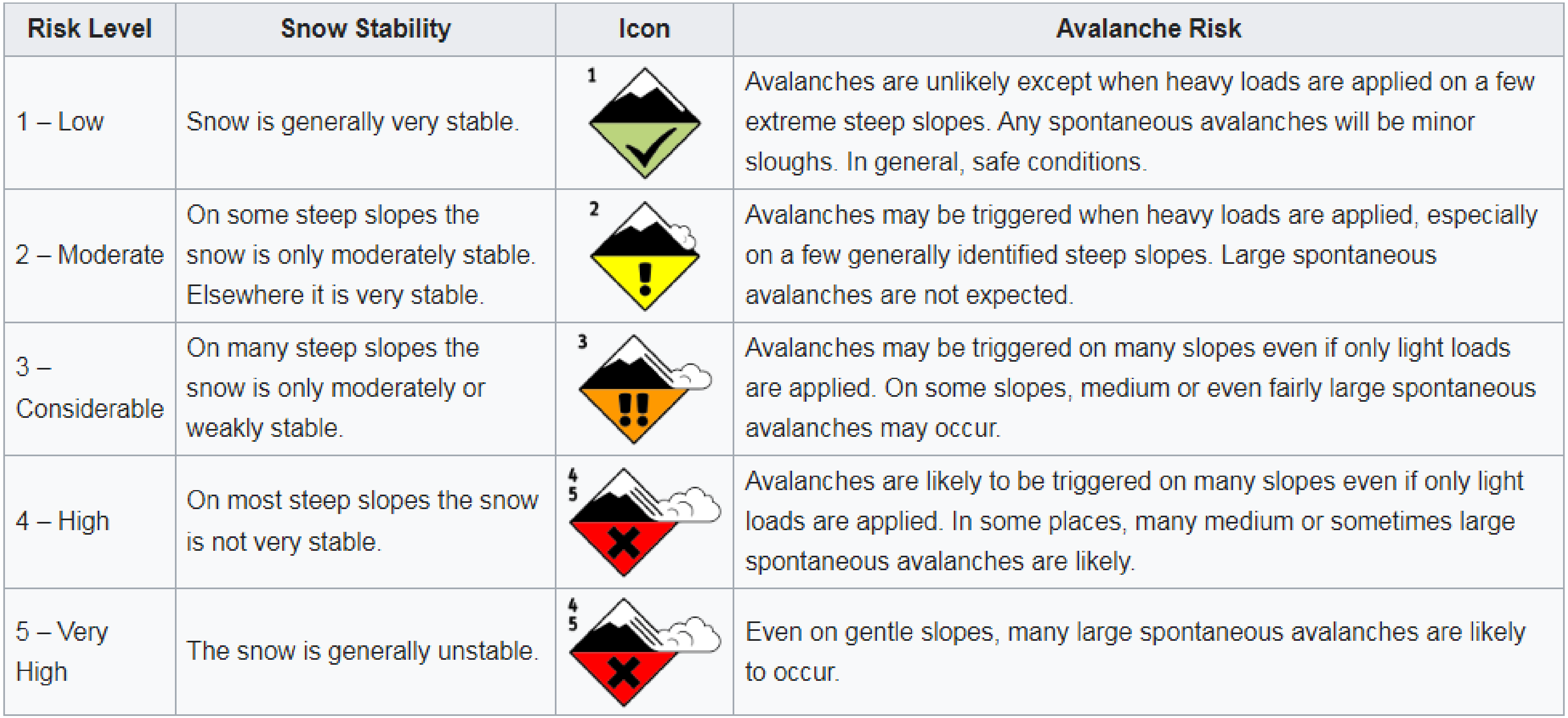
EXAMPLE OF LAYERS IN SNOWPACK:



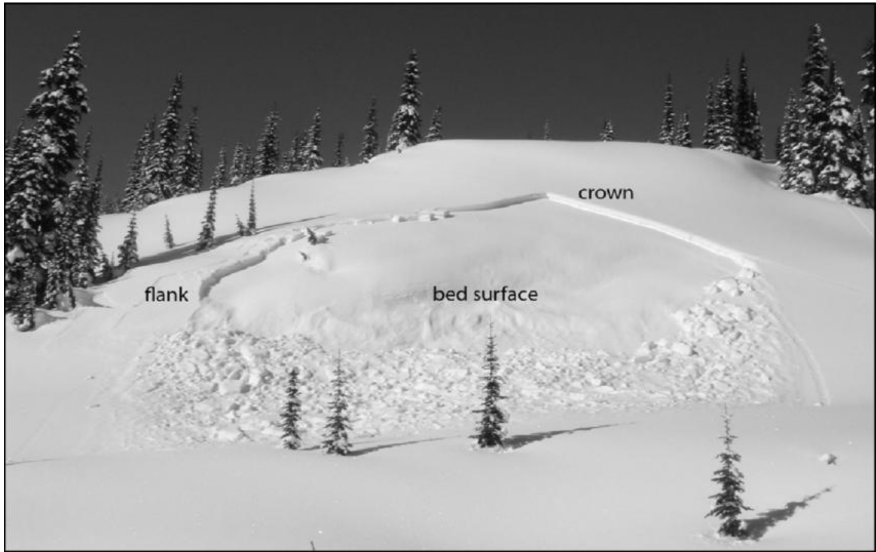
Note:
This is just one possible configuration. There are an infinite number of combinations that will produce various stability conditions.

Check the snowpack to help with **avalanche prediction**





Risk Level	Snow Stability	Icon	Avalanche Risk
1 – Low	Snow is generally very stable.	1	Avalanches are unlikely except when heavy loads are applied on a few extreme steep slopes. Any spontaneous avalanches will be minor sloughs. In general, safe conditions.
2 – Moderate	On some steep slopes the snow is only moderately stable. Elsewhere it is very stable.	2	Avalanches may be triggered when heavy loads are applied, especially on a few generally identified steep slopes. Large spontaneous avalanches are not expected.
3 – Considerable	On many steep slopes the snow is only moderately or weakly stable.	3	Avalanches may be triggered on many slopes even if only light loads are applied. On some slopes, medium or even fairly large spontaneous avalanches may occur.
4 – High	On most steep slopes the snow is not very stable.	4 5	Avalanches are likely to be triggered on many slopes even if only light loads are applied. In some places, many medium or sometimes large spontaneous avalanches are likely.
5 – Very High	The snow is generally unstable.	4 5	Even on gentle slopes, many large spontaneous avalanches are likely to occur.



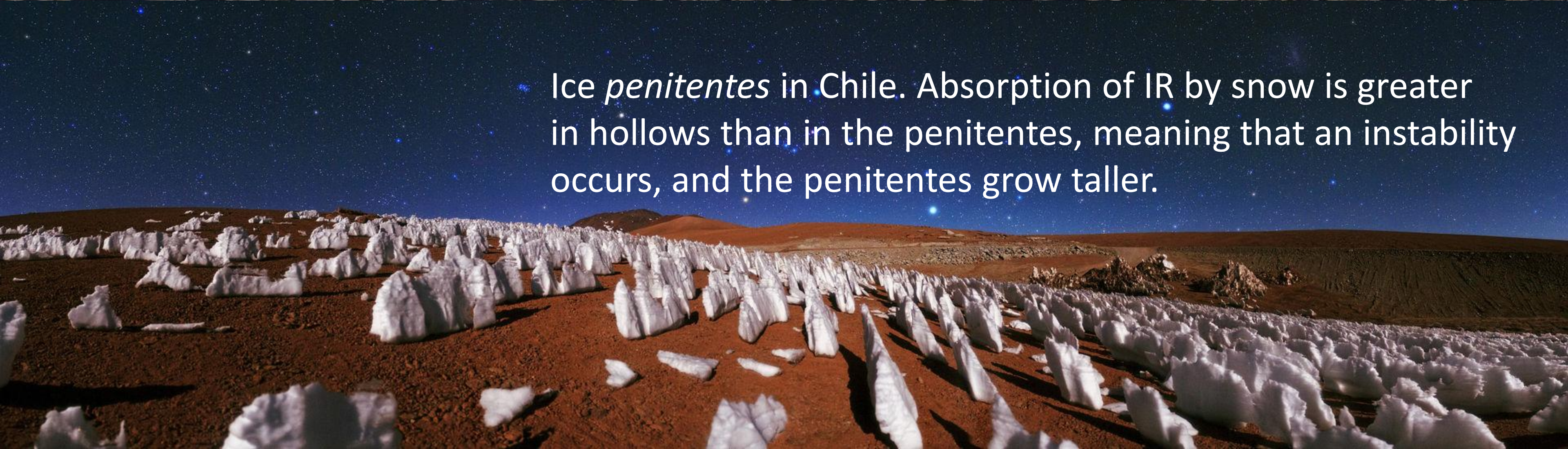
Sastrugi

Snow erosion by wind.

Suncups formed by *ablation.*

i.e. the removal of snow by *sublimation* (solid to vapour) due to solar radiation. Similar process to the formation of *penitentes* (perhaps in drier conditions?)

Penitentes in Cordoba
during Santa Semana
(Easter Holy Week)



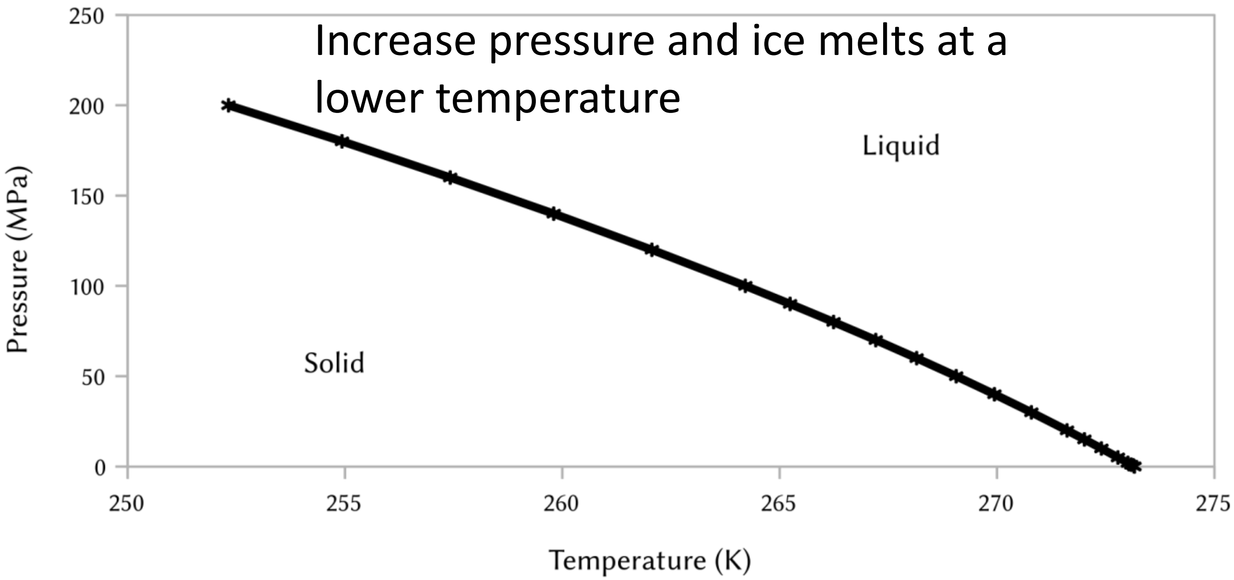
Ice *penitentes* in Chile. Absorption of IR by snow is greater in hollows than in the penitentes, meaning that an instability occurs, and the penitentes grow taller.

Regelation is the phenomenon of ice melting under pressure and refreezing when the pressure is reduced.

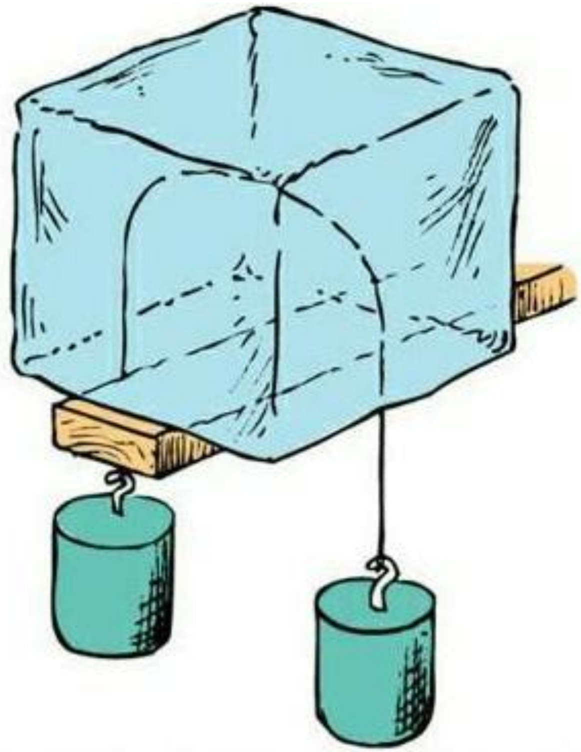
This is *part* (!) of the explanation of how ice skaters can travel on ice with very little friction.

[Video from Veritasium]

Melting curve



Increase pressure and ice melts at a lower temperature



1. Mountains high or low, hard or soft
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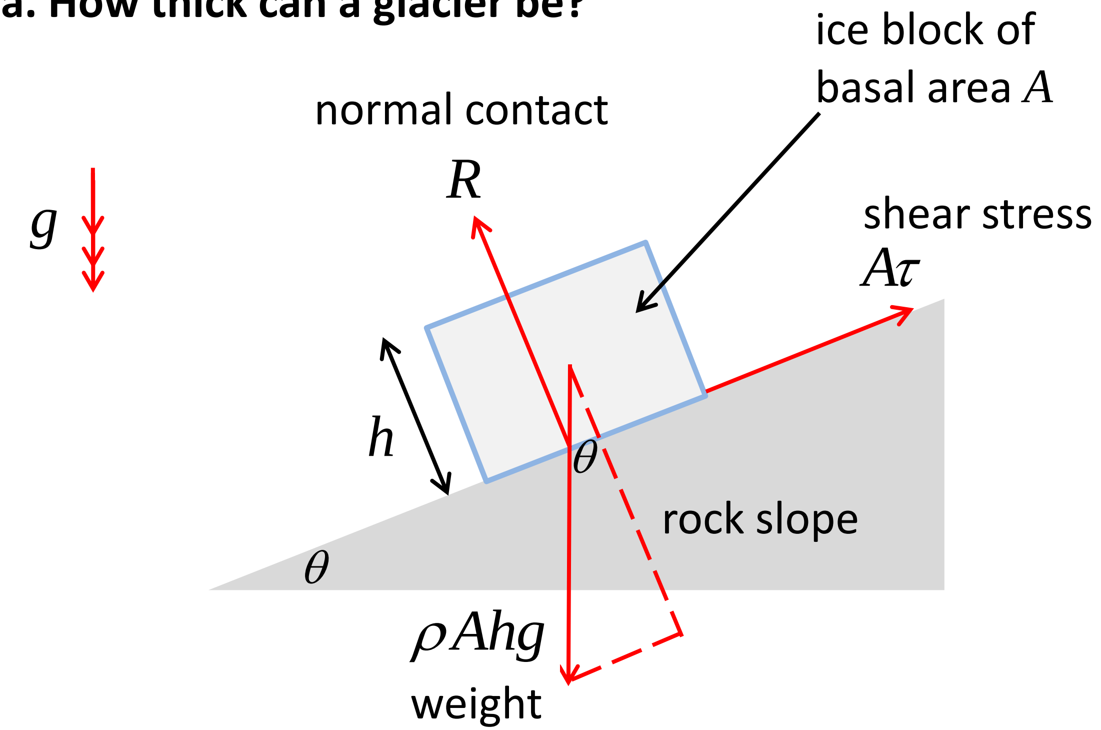
4. Glacier puzzles
5. Heat, cold and air
6. Rock climbing
7. Miscellaneous

4a. How thick can a glacier be?

ice density is

$\rho \approx 917\,\mathrm{kgm}^{-3}$



In equilibrium, balance shear stress with weight acting down the slope:

$$A\tau = \rho Ahg\sin\theta$$

$$\therefore h = \frac{\tau}{\rho g\sin\theta}$$

Incorporating an empirical 'shape factor' *f*, maximum glacier thickness is about:

$$h = \frac{\tau}{f\rho g\sin\theta} \qquad 0.5 < f < 0.9 \qquad \tau \approx 10^5\,\mathrm{Pa},\ \therefore \frac{\tau}{\rho g} \approx 11.1\mathrm{m}$$

$$\boxed{h \approx \frac{11\mathrm{m}}{f\sin\theta}}$$

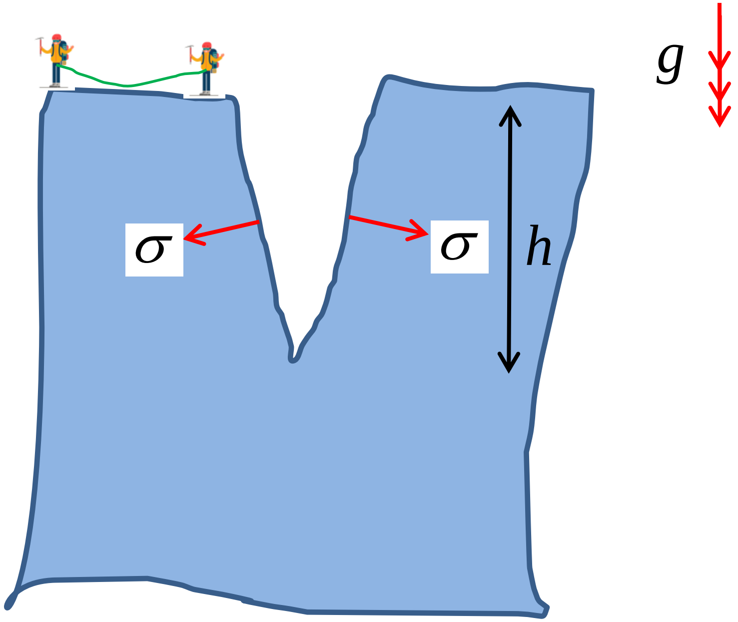
Upper limit for small slope angles is given by Weisskopf's argument:

$$mgh < \tfrac{1}{3}L_{fus}m \quad \therefore\ h < \frac{\frac{1}{3}L_{fus}}{g} \Rightarrow h < \frac{\frac{1}{3}334{,}000}{9.81} \Rightarrow \boxed{h < 11\mathrm{km}}$$

About the right order of magnitude for polar ice sheets (Antarctica: mean 2,160m, max 4,776m)

70% of World's fresh water!

4b. How deep can a crevasse be?




Crevasses on the Lys glacier on Monte Rosa

Maximum crevasse depth when the weight per unit area of the ice column (the ‘cryostatic pressure’) equals the total sideways stress (force per unit area)

$$\rho g h \approx 2\sigma$$

$$\therefore \boxed{h \approx \frac{2\sigma}{\rho g}}$$

$$\therefore h \approx \frac{2\times 10^{5}}{917\times 9.81}\ (\mathrm{m}) \approx \boxed{22\mathrm{m}}$$

ice density is

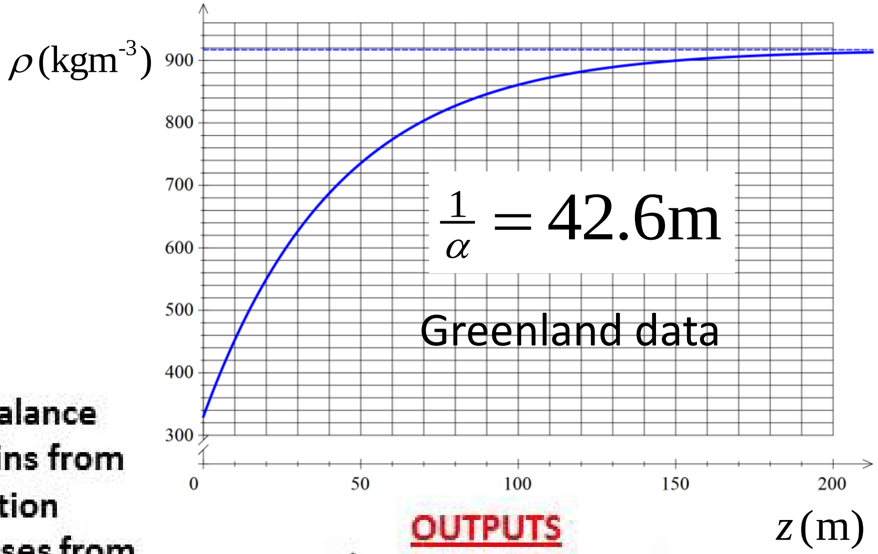
$$\rho \approx 917\mathrm{kgm}^{-3}$$

4c. How long does it take snow to become ice?

Dry snow has density of about ρ_s = 330kgm^{-3}, whereas ice has density of about ρ_i = 917kgm^{-3}
For a glacier that is added to by falling snow, what is the density vs depth profile?

$$\frac{d\rho}{dz} = \alpha\left(\rho_i - \rho\right); \quad \rho(0) = \rho_s,\ \rho(\infty) = \rho_i \quad \Rightarrow \boxed{\rho = \rho_i - \left(\rho_i - \rho_s\right)e^{-\alpha z}}$$

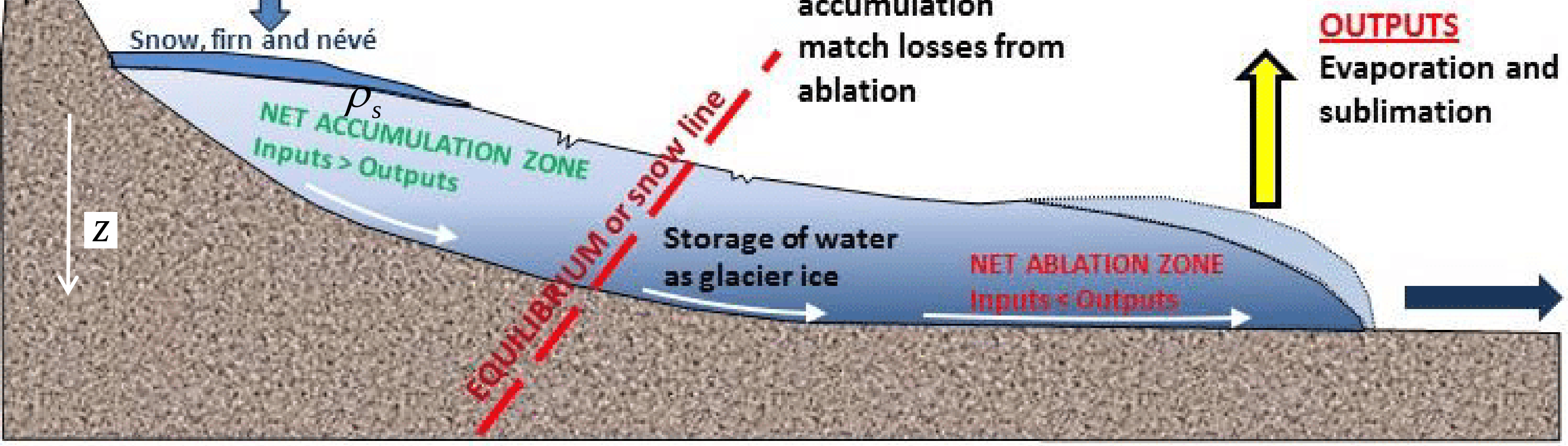
Simple linear model for density change with snow depth z



$$\frac{1}{\alpha} = 42.6\mathrm{m}$$

Greenland data

Inputs –
Snow fall
Avalanches from valley side



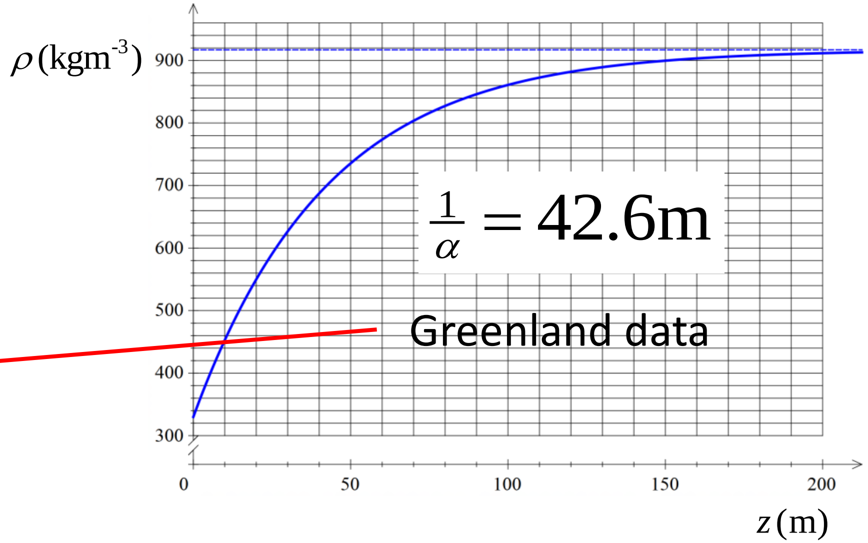
Zone of balance where gains from accumulation match losses from ablation

OUTPUTS
Evaporation and sublimation

OUTPUTS
melt water higher in summer, minimal in winter.

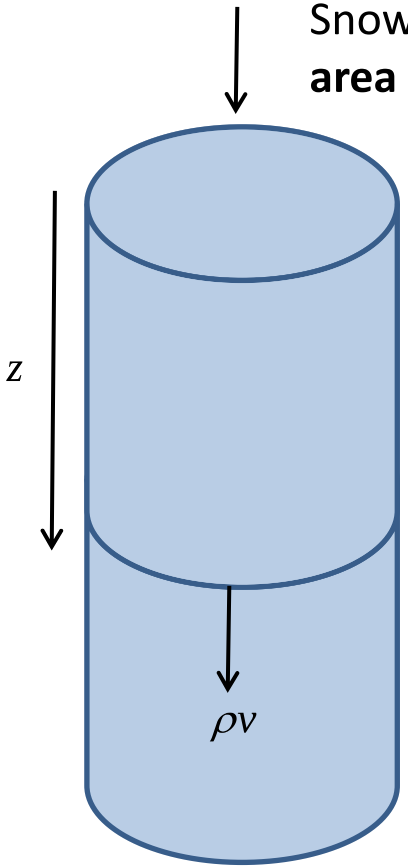
$$\frac{d\rho}{dz} = \alpha(\rho_i - \rho); \quad \rho(0) = \rho_s,\ \rho(\infty) = \rho_i \quad \Rightarrow \boxed{\rho = \rho_i - (\rho_i - \rho_s)e^{-\alpha z}}$$

How *long* does it take snow to form ice?



$$\frac{1}{\alpha} = 42.6\mathrm{m}$$

Greenland data



Snowfall of **mass per unit area per second:**

$$\rho_s v_s \approx 265\,\mathrm{kgm^{-2}yr^{-1}}$$

By continuity of snow:

$$\rho v = \rho_s v_s, \quad v = \frac{dz}{dt} \quad \therefore \rho\frac{dz}{dt} = \rho_s v_s$$

$$\therefore t(z) = \int_0^z \frac{\rho(z')dz'}{\rho_s v_s}$$

$$\therefore t(z) = \tfrac{1}{\rho_s v_s}\int_0^z \left(\rho_i - (\rho_i - \rho_s)e^{-\alpha z'}\right)dz'$$

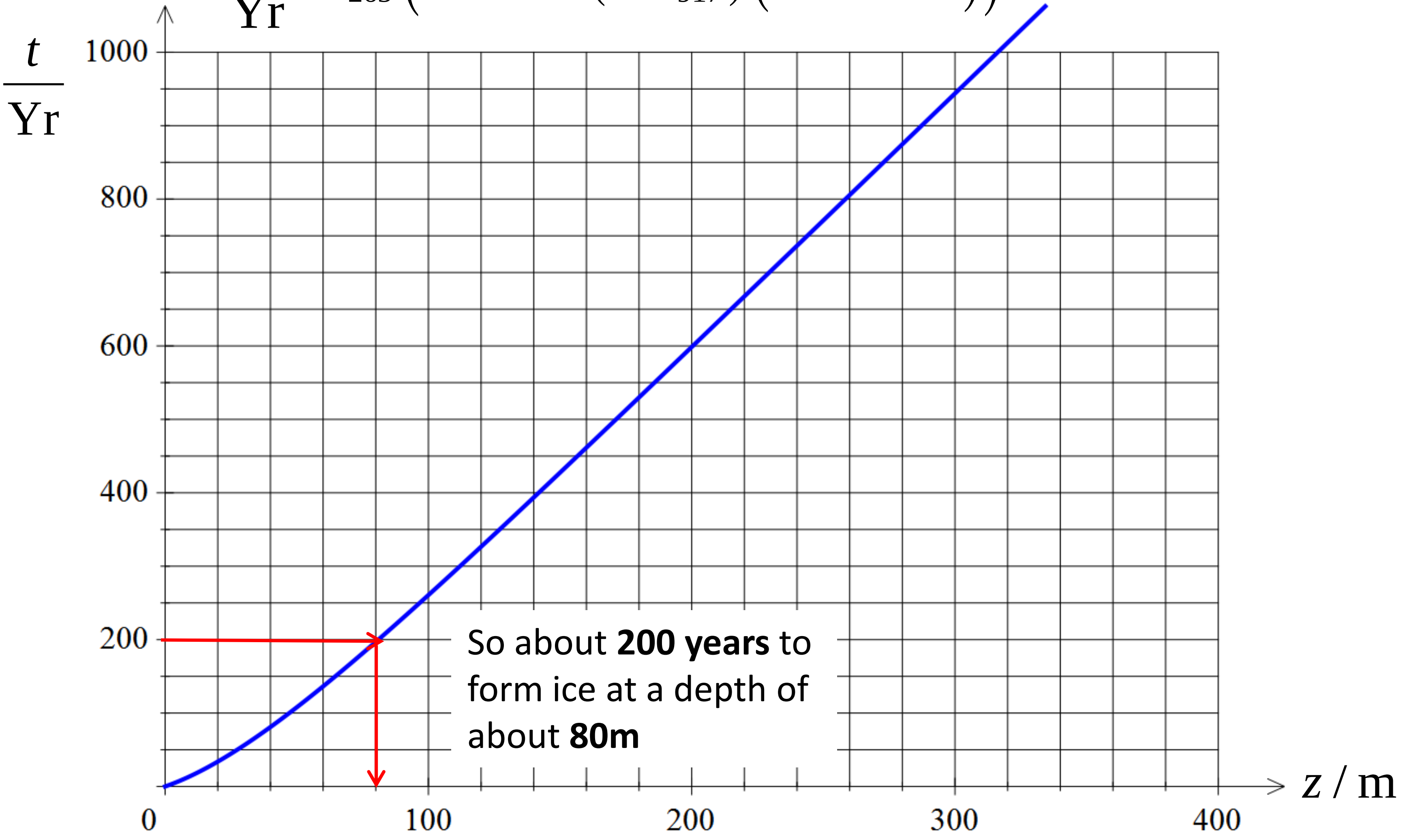
$$\therefore \boxed{t(z) = \tfrac{\rho_i}{\rho_s v_s}\left(z + \tfrac{1}{\alpha}\left(1 - \tfrac{\rho_s}{\rho_i}\right)\left(e^{-\alpha z} - 1\right)\right)}$$

How *long* does it take falling snow to form ice?

$$\boxed{t(z)=\frac{\rho_i}{\rho_s v_s}\left(z+\frac{1}{\alpha}\left(1-\frac{\rho_s}{\rho_i}\right)\left(e^{-\alpha z}-1\right)\right)}$$

$$\frac{t}{\mathrm{Yr}}=\frac{917}{265}\left(z+42.6\left(1-\frac{330}{917}\right)\left(e^{-z/42.6}-1\right)\right)$$

Greenland data



So about **200 years** to form ice at a depth of about **80m**

4d. U shaped glacial valleys

Morgan (2005)

$$x = \pm \frac{C\lambda\sqrt{1-w^2} + C\cos^{-1} w}{\lambda^2 - 1}; \qquad w = \frac{\lambda^2 - 1}{C} y - \lambda$$

$$-\tfrac{1}{\lambda} \le w \le 1; \quad -\tfrac{C}{\lambda} \le y \le \tfrac{C}{\lambda - 1}$$

$$\left(\frac{y'}{y}\right)^2 = \frac{1}{(\lambda y - C)^2} - \frac{1}{y^2}$$

Idea is to determine the curve which **extremizes** the amount of surface friction of ice of depth $y(x)$ subject to the *constraint* that the cross-sectional area A of the glacial valley is a constant (i.e. proportional to a fixed quantity of ice).

$$J = \int_{-a}^{a} \rho g y \sqrt{dx^2 + dy^2}$$

Sum of cryostatic pressure times arc length (?)

$$A = \int_{-a}^{a} y dx$$

Glacier cross section

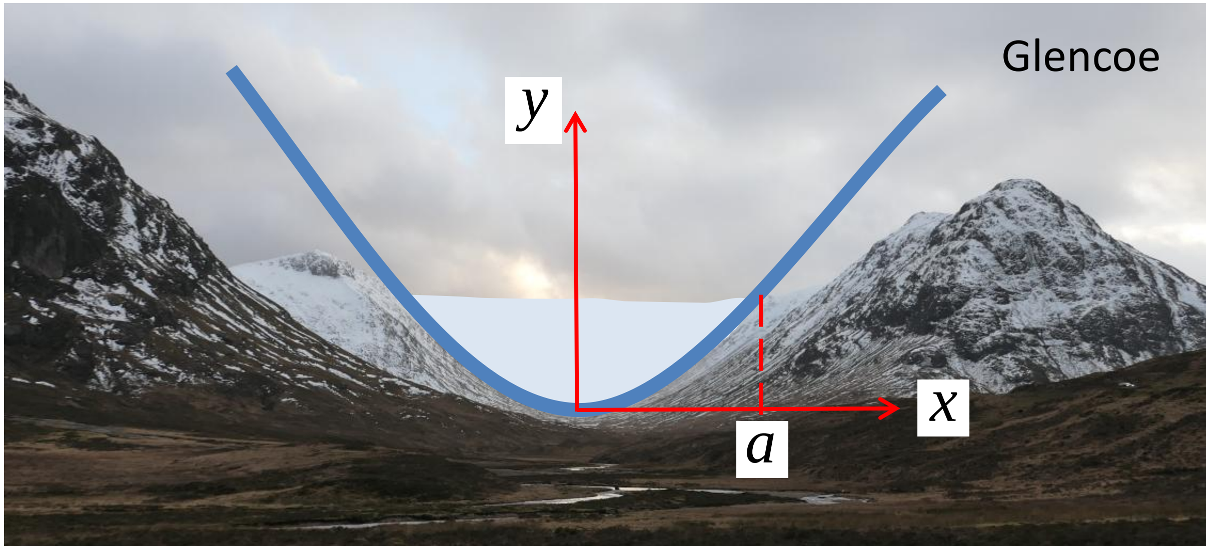
To extremize:

$$L = \int_{-a}^{a} \left\{ y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - \lambda y \right\} dx$$

Lagrange multiplier

Solve **Euler-Lagrange equation**

$$\frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) = \frac{\partial L}{\partial y} \quad ; \quad y' = \frac{dy}{dx}$$



1. Mountains high or low, hard or soft
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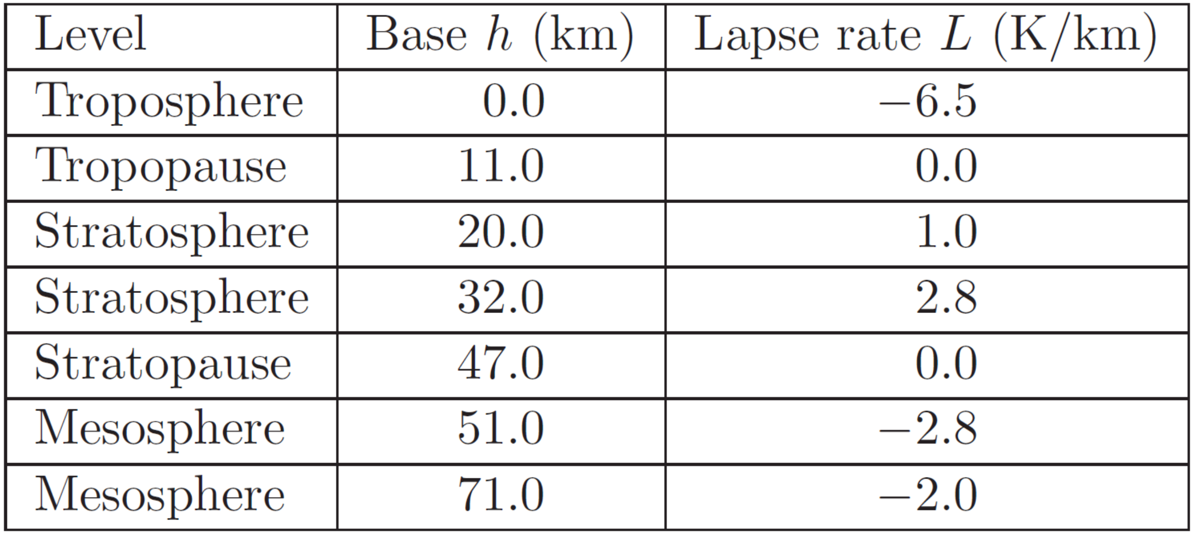
5a. An ideal gas atmosphere

At the base of the *Troposphere*, altitude $h = 0.0$ km, temperature $T_0 = (273 + 15)$ K, pressure $P_0 = 101, 325$ Pa.



Level	Base h (km)	Lapse rate L (K/km)
Troposphere	0.0	−6.5
Tropopause	11.0	0.0
Stratosphere	20.0	1.0
Stratosphere	32.0	2.8
Stratopause	47.0	0.0
Mesosphere	51.0	−2.8
Mesosphere	71.0	−2.0

Assumptions:

- The atmosphere comprises a number of fixed layers, each with a constant temperature gradient (‘lapse rate’) with altitude.

- The atmosphere consists of a single ideal ‘air’ gas, whose molecular mass takes into account the average compositions of different gases, e.g. nitrogen (78%), oxygen (21%), 0.9% argon, 0.04% carbon dioxide, etc.

Example spreadsheet for the first three ISA layers

Molar gas constant $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$

Molar mass of air $M = 0.02896\,\mathrm{kg\,mol^{-1}}$

Strength of gravity $g = 9.81\,\mathrm{N\,kg^{-1}}$

Standard atmosphere model

	zstart /km	zfinish /km	Tstart /K	Tfinish /K	Lapse rate /K per km	pstart /Pa	pfinish /Pa
Troposphere	0	11	288	216.5	6.5	101,325	22,604
Tropopause	11	20	216.5	216.5	0.0	22,604	5,461
Stratosphere	20	32	216.5	228.5	-1.0	5,461	864

Mg/R
0.034171

Troposphere

	z /km	T /K	p /Pa
0	0	288	101,325
0.05	0.55	284.425	94,885
0.1	1.1	280.85	88,781
0.15	1.65	277.275	82,999
0.2	2.2	273.7	77,525
0.25	2.75	270.125	72,348
0.3	3.3	266.55	67,454
0.35	3.85	262.975	62,832
0.4	4.4	259.4	58,469
0.45	4.95	255.825	54,355
0.5	5.5	252.25	50,479
0.55	6.05	248.675	46,830
0.6	6.6	245.1	43,397
0.65	7.15	241.525	40,171
0.7	7.7	237.95	37,142
0.75	8.25	234.375	34,301
0.8	8.8	230.8	31,638
0.85	9.35	227.225	29,146
0.9	9.9	223.65	26,814
0.95	10.45	220.075	24,636
1	11	216.5	22,604

Power law

Positive lapse rate

$$p(z) = p_0\left(1 - \frac{L(z-z_0)}{T_0}\right)^{\frac{Mg}{LR}}$$

Tropopause

z /km	T /K	p /Pa
11	216.5	22,604
11.45	216.5	21,054
11.9	216.5	19,611
12.35	216.5	18,266
12.8	216.5	17,014
13.25	216.5	15,847
13.7	216.5	14,761
14.15	216.5	13,749
14.6	216.5	12,806
15.05	216.5	11,928
15.5	216.5	11,110
15.95	216.5	10,349
16.4	216.5	9,639
16.85	216.5	8,978
17.3	216.5	8,363
17.75	216.5	7,789
18.2	216.5	7,255
18.65	216.5	6,758
19.1	216.5	6,294
19.55	216.5	5,863
20	216.5	5,461

Exponential

Isothermal

$$p(z) = p_0 e^{-\frac{Mg}{RT_0}(z-z_0)}$$

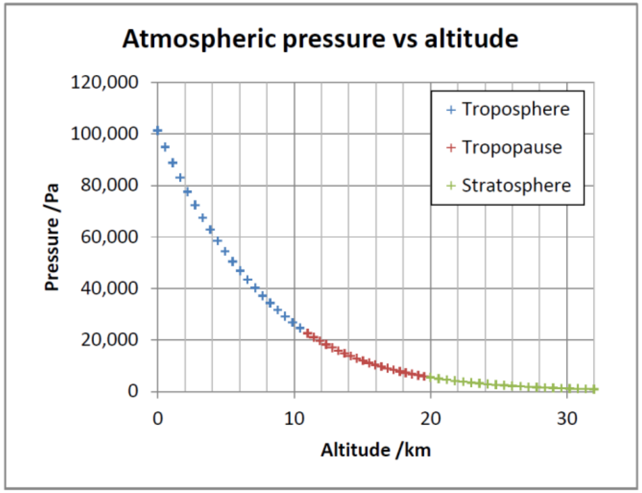
Stratosphere

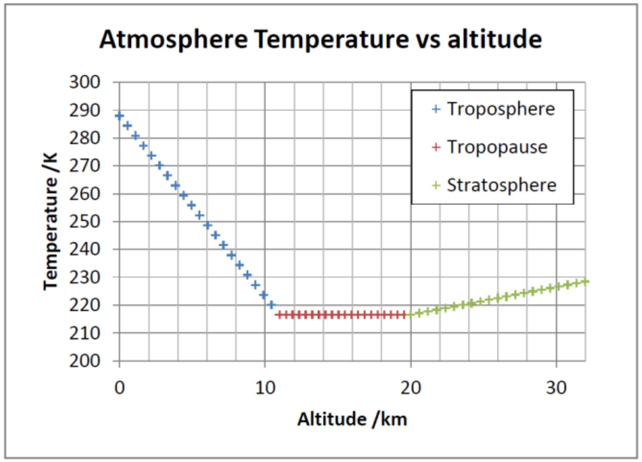
z /km	T /K	p /Pa
20	216.5	5,461
20.6	217.1	4,968
21.2	217.7	4,521
21.8	218.3	4,115
22.4	218.9	3,747
23	219.5	3,412
23.6	220.1	3,108
24.2	220.7	2,832
24.8	221.3	2,581
25.4	221.9	2,353
26	222.5	2,146
26.6	223.1	1,957
27.2	223.7	1,786
27.8	224.3	1,629
28.4	224.9	1,487
29	225.5	1,358
29.6	226.1	1,240
30.2	226.7	1,133
30.8	227.3	1,035
31.4	227.9	946
32	228.5	864

Power law

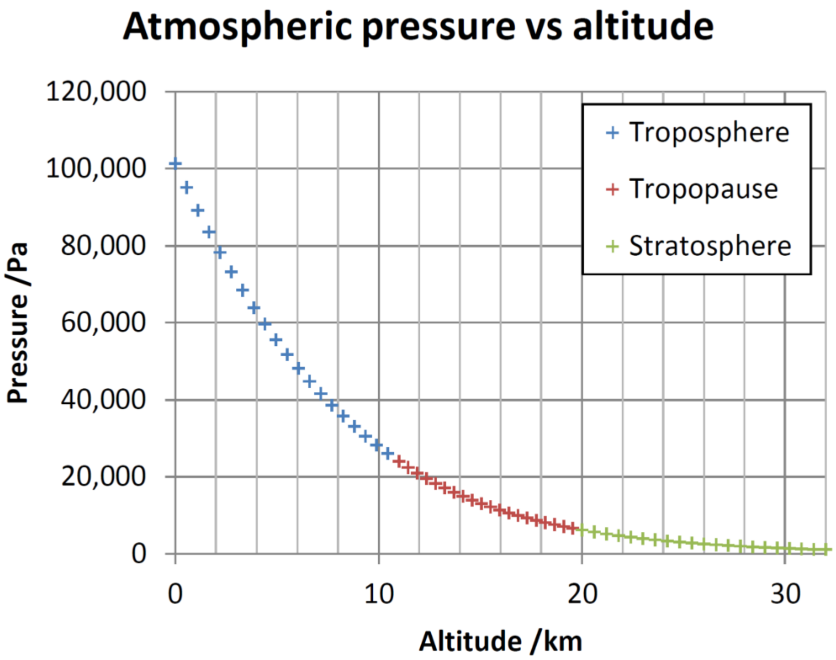
Negative lapse rate

$$p(z) = p_0\left(1 - \frac{L(z-z_0)}{T_0}\right)^{\frac{Mg}{LR}}$$





If we can ignore humidity (i.e. the contribution to air pressure from water vapour), air pressure is simply the weight per unit area of a column of atmosphere.



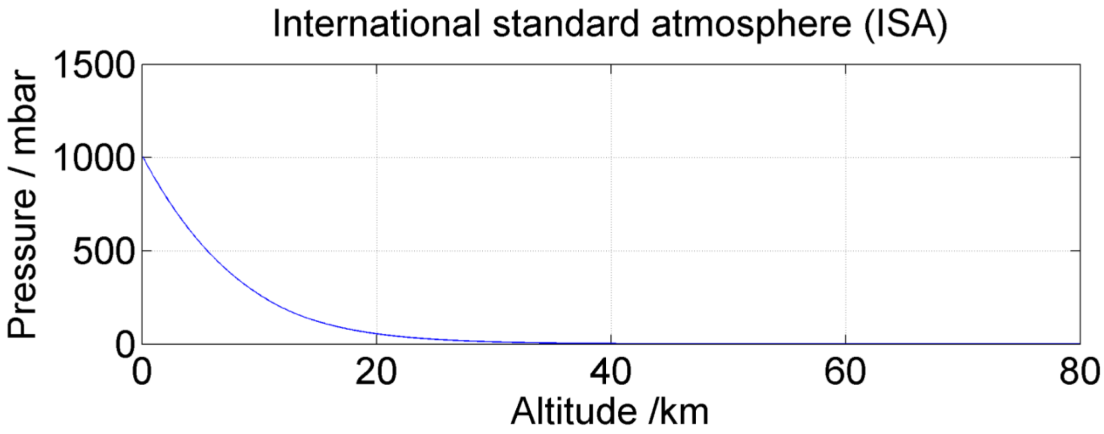
$$P = P_0\left(1 - \frac{L\left(h - h_0\right)}{T_0}\right)^{\frac{Mg}{LR}}$$

$$P = P_0 e^{-\frac{Mg}{RT_0}\left(h - h_0\right)}$$

If L = 0

Note for these models you'll need altitude in **metres** if you use the other constants in standard units. This means you'll need to define the **Lapse rate in K per m, which means dividing the K/km values by 1,000.**

Note P_0 and T_0 correspond to the (Kelvin) temperatures *at the base of the layer*. So work upwards from the base of the Troposphere.



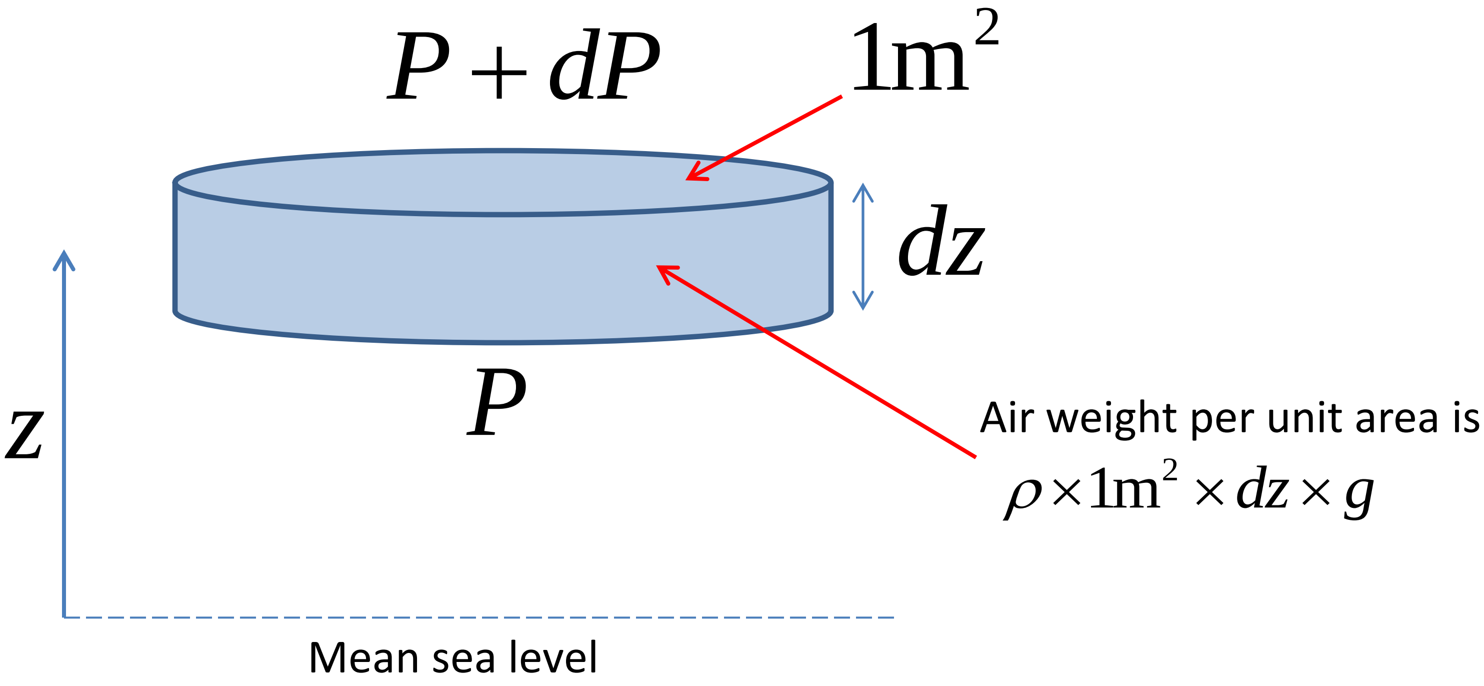
Calculating air pressure

Consider a 1m^2 horizontal cross section parcel of air of density ρ at an altitude z, with vertical width dz. The atmospheric pressure change dP between altitudes z and $z+dz$ resulting from the removal of the air parcel from the total weight of air above is:

$$\boxed{dP = -\rho g dz}$$

Strength of gravity

$$g = 9.81\,\mathrm{Nkg^{-1}}$$



$P + dP$

$1\mathrm{m}^2$

dz

P

z

Air weight per unit area is

$\rho \times 1\mathrm{m}^2 \times dz \times g$

Mean sea level

Determining air pressure in the ISA

Let us assume that the air column is comprised of dry air with molar mass:

$$M = 0.02896\,\mathrm{kgmol}^{-1}$$

Let us assume that the air column is an **ideal gas**. If n moles of gas occupies volume V at pressure P (Pascals) and (absolute) temperature T (Kelvin):

$$PV = nRT$$

Ideal gas equation

$$\therefore \frac{P}{RT} = \frac{n}{V}$$

Molar gas constant

$$R = 8.314\mathrm{Jmol}^{-1}\mathrm{K}^{-1}$$

The density of the air is the mass of n moles divided by the volume V

$$\rho = \frac{nM}{V}$$

Hence:

$$\boxed{\rho = \frac{MP}{RT}}$$

Change in weight of air column

$$\boxed{dP = -\rho g dz}$$

Ideal gas

$$\boxed{\rho = \frac{MP}{RT}}$$

$$\frac{dP}{P} = -\frac{M}{R}\frac{gdz}{T}$$

$$\int_{P_0}^{P} \frac{1}{P} dP = -\frac{M}{R}\int_{z_0}^{z} \frac{g}{T} dz$$

$$\boxed{\therefore \ln\left(\frac{P}{P_0}\right) = -\frac{M}{R}\int_{z_0}^{z} \frac{g}{T} dz}$$

$$\boxed{\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{R}\int_{h_0}^{h}\frac{1}{T}dz}$$

This assumes a **spherical Earth** and therefore *uniform gravitational field strength on the surface of the Earth*. This is quite a good approximation, but in reality the Earth is better modelled as an ***ellipsoid*** + topographic variations such as mountains.

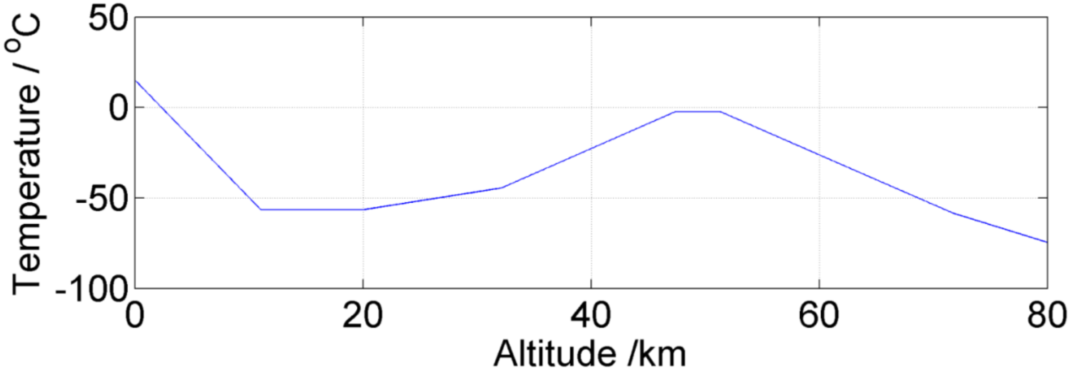
Strength of gravity $g = 9.81\mathrm{Nkg}^{-1}$

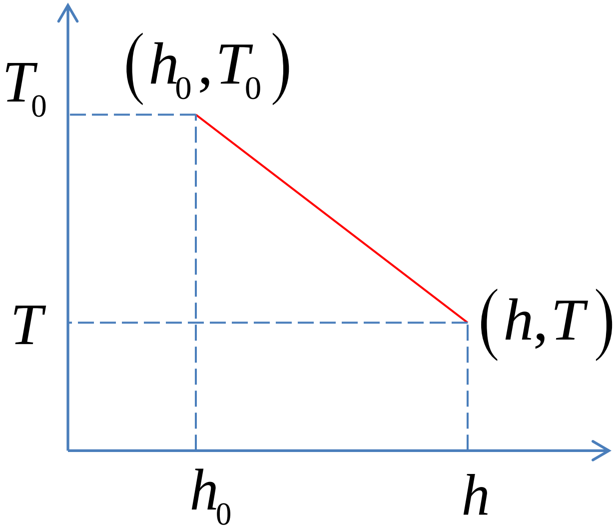
Now let us define the temperature T (in Kelvin) to be a linear function of altitude h. The (negative) gradient is defined to be the **lapse rate** L.

Note this means a *negative* lapse rate implies a *rise* in temperature with height.

$$\boxed{T = T_0 - L(h - h_0)}$$

Effect of lapse rate on ISA temperature



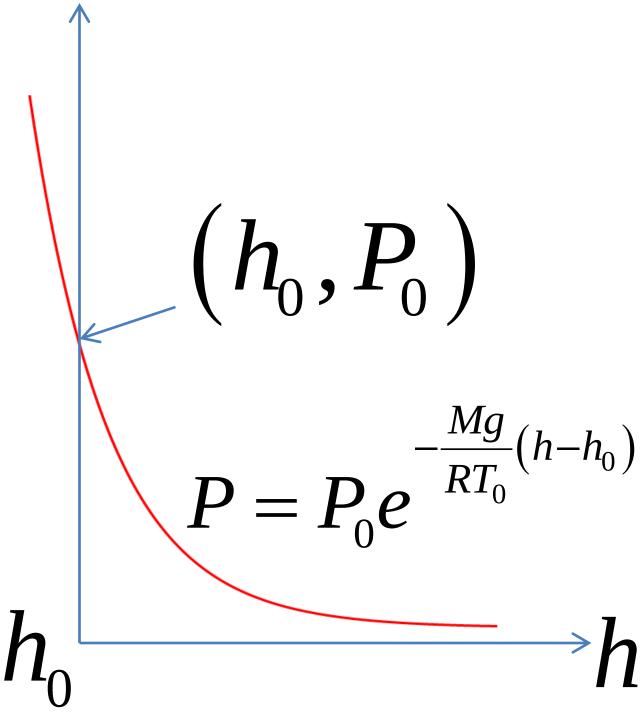


Case 1: Isothermal layer i.e. $L = 0$

Note to compute this in standard units you'll need altitude h in **metres** and temperature in K

$$T = T_0$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{RT_0}\int_{h_0}^{h} dz$$



$$\left(h_0, P_0\right)$$

$$P = P_0 e^{-\frac{Mg}{RT_0}(h-h_0)}$$

$$\boxed{P = P_0 e^{-\frac{Mg}{RT_0}(h-h_0)}}$$

i.e. an *exponential* decay of pressure with altitude.
Notice the **Boltzmann factor** exponent.

Ludwig Boltzmann
1844-1906

Case 2: Constant but non-zero, lapse rate L

Note to compute this in standard units you'll need altitude h in **metres** and lapse rate L in **K per metre** and temperature in K

$$T = T_0 - L(h - h_0)$$

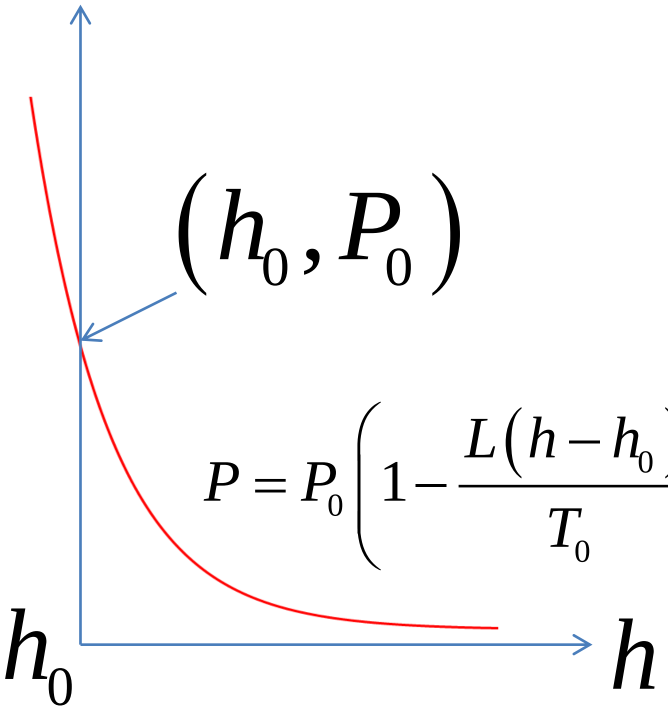
$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{R}\int_{h_0}^{h}\frac{1}{T_0 - L(h-h_0)}\,dh$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{(-L)R}\int_{h_0}^{h}\frac{-L}{T_0 + Lh_0 - Lh}\,dh$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{Mg}{LR}\left[\ln\left(T_0 + Lh_0 - Lh\right)\right]_{h_0}^{h}$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{Mg}{LR}\ln\left(\frac{T_0 + Lh_0 - Lh}{T_0}\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \ln\left(\left(\frac{T_0 + Lh_0 - Lh}{T_0}\right)^{\frac{Mg}{LR}}\right)$$



$$P = P_0\left(1 - \frac{L(h-h_0)}{T_0}\right)^{\frac{Mg}{LR}}$$

$$\boxed{P = P_0\left(1 - \frac{L(h-h_0)}{T_0}\right)^{\frac{Mg}{LR}}}$$

i.e. a **power-law** decay of pressure with altitude.

What about humidity?

i.e. the average pressure, temperature structure of the air column

Any sensible description of *non-arid climatology* will need to take into account the impact of variable amounts of **water vapour** contained within the air column. In the *Troposphere* at least, the presence of water vapour can have a dramatic influence upon thermodynamic variables such as temperature and pressure, and is obviously a fundamental component of weather phenomena such as cloud and fog.

To model the effect of humidity upon temperature and pressure, let us modify our original single ideal gas assumption to consider a *composite* of dry air and water vapour. The molar masses of dry air and water vapour are, respectively:

$$M_d = 0.02896\,\mathrm{kg\,mol}^{-1} \qquad M_v = 0.01802\,\mathrm{kg\,mol}^{-1}$$

The respective ideal gas equations are, for n_d moles of dry air and n_v moles of water vapour:

$$P = P_V + P_d$$

i.e. the overall pressure *P* is the sum of the 'partial pressures' of the component gases

$$P_V = \frac{n_V RT}{V} \qquad P_d = \frac{n_d RT}{V}$$

Molar gas constant

$$R = 8.314\,\mathrm{J\,mol}^{-1}\mathrm{K}^{-1}$$

Let us define ***relative humidity*** U as the ratio of water vapour pressure to that at saturation E_s

$$\boxed{P_V = UE_s}$$

An empirical model for saturation pressure as a pure function of temperature is given by the *Arden Buck equation*
http://en.wikipedia.org/wiki/Arden_Buck_equation

$$\boxed{E_s(T) = 6.1121 e^{\left\{\left(18.678 - \frac{T_c}{234.5}\right)\left(\frac{T_c}{T_c + 257.14}\right)\right\}}}$$

There are in fact many empirical formulae for vapour pressure.
A good selection are compared at
http://cires.colorado.edu/~voemel/vp.html

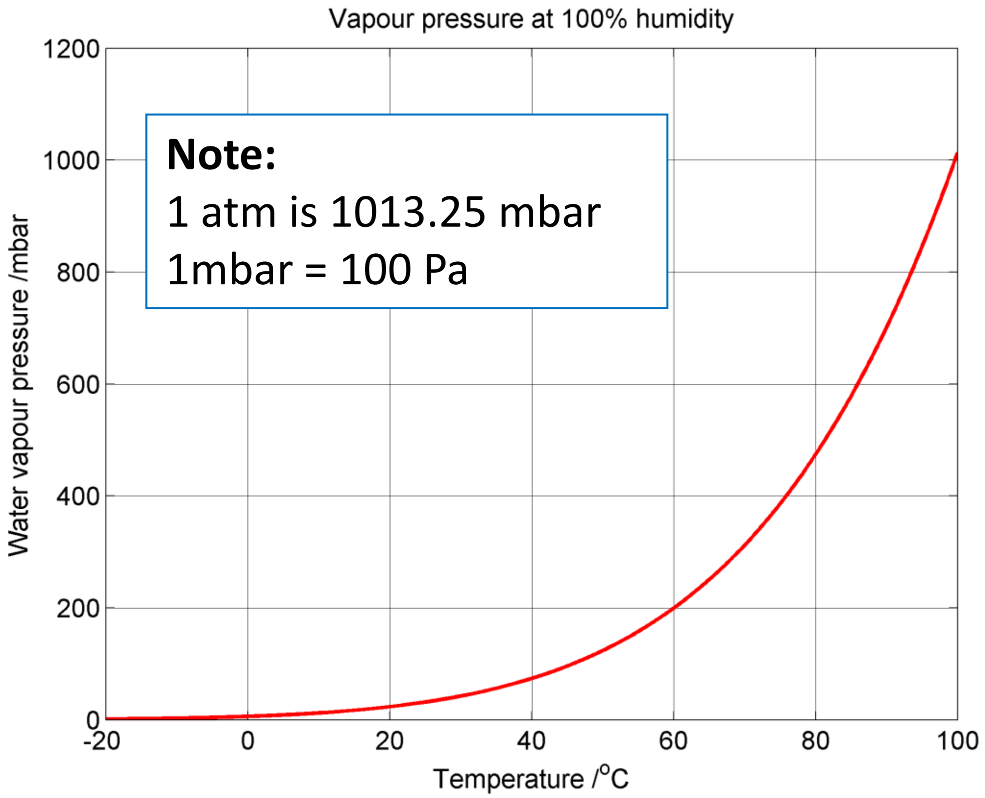
where the saturation vapour pressure E_s is given in mbar and temperature T_c in *degrees Celsius*

$$\boxed{T_c = T - 273}$$

Convert to Celsius from Kelvin temperatures T

Note the vapour pressure is *very small* compared to the pressure of dry air when temperatures are low. Therefore to a very good approximation we can ignore it in the pressure calculation.

However, the lapse rate *is significantly* affected by humidity.....


Vapour pressure at 100% humidity

Note:
1 atm is 1013.25 mbar
1mbar = 100 Pa

The overall density of the atmosphere is:

$$\rho = \frac{n_d M_d + n_V M_V}{V} = \frac{M_d}{V}\left(n_d + n_V \frac{M_V}{M_d} \right)$$

From the ideal gas equations:

$$n_V = \frac{V}{RT} P_V = \frac{V}{RT} U E_S$$

$$n_d = \frac{V}{RT} P_d = \frac{V}{RT}\left(P - P_V\right) = \frac{V}{RT}\left(P - U E_S\right)$$

Hence:

$$\boxed{\rho = \frac{M_d}{RT}\left(P - U\left(1 - \frac{M_V}{M_d}\right) E_s(T) \right)}$$

The pressure integral is now more complicated, but can be evaluated using a *numerical method*:

$$dP = -\rho g dh \qquad \rho = \frac{M_d}{RT}\left(P - U\left(1 - \frac{M_V}{M_d}\right)E_s(T)\right) \qquad T = T_0 - L(h - h_0)$$

$$\therefore \frac{dP}{dh} = -\frac{M_d g}{RT}\left(P - U\left(1 - \frac{M_V}{M_d}\right)E_s(T)\right)$$

A simple iterative numeric solution scheme might be to use a *finite* altitude change Δh, and start from a known temperature and pressure e.g. 15°C, 1013.25mbar.

$$h \to h + \Delta h$$

$$T \to T - L\Delta h \qquad T_c = T - 273$$

$$E_s = 6.1121 e^{\left\{\left(18.678 - \frac{T_c}{234.5}\right)\left(\frac{T_c}{T_c + 257.14}\right)\right\}}$$

$$\Delta P = -\frac{M_d g}{RT}\left(P - U\left(1 - \frac{M_V}{M_d}\right)E_s(T)\right)\Delta h$$

$$P \to P + \Delta P$$

i.e. you can now compute P vs h by running this **in a loop.**

$$M_d = 0.02896 \text{kgmol}^{-1}$$

$$M_v = 0.01802 \text{kgmol}^{-1}$$

$$R = 8.314 \text{JmolK}^{-1}$$

$$g = 9.81 \text{Nkg}^{-1}$$

Now the partial pressure of water vapour is typically very small compared to dry air. Therefore we *can* ignore it in pressure calculations. However, the **lapse rate** L is *significantly* affected by the presence of water vapour. A model for L is:

$$L = -\frac{dT}{dh}$$

Lapse rate in K per **metre**

$$\boxed{L = g\,\frac{1+\dfrac{r\Delta H_v}{R_{sd}T}}{c_{pd}+\dfrac{(\Delta H_v)^2 r}{R_{sw}T^2}}}$$

T is in Kelvin

$$\varepsilon = \frac{R_{sd}}{R_{sw}}$$

$$\boxed{r = \frac{\varepsilon U E_s}{P - U E_s}}$$

Essentially the mass fraction of water to dry air

$$M_d = 0.02896\,\mathrm{kg\,mol}^{-1}$$

$$M_v = 0.01802\,\mathrm{kg\,mol}^{-1}$$

$$g = 9.8076\,\mathrm{N\,kg}^{-1}$$

Specific latent heat of vaporization of water

ΔH_v = 2,501,000 J kg^{-1}

Specific heat of dry air at constant pressure is:

c_{pd} = 1003.5 Jkg^{-1}K^{-1}

Specific gas constant for dry air

R_{sd} = 287 J kg^{-1} K^{-1}

Specific gas constant for water vapour

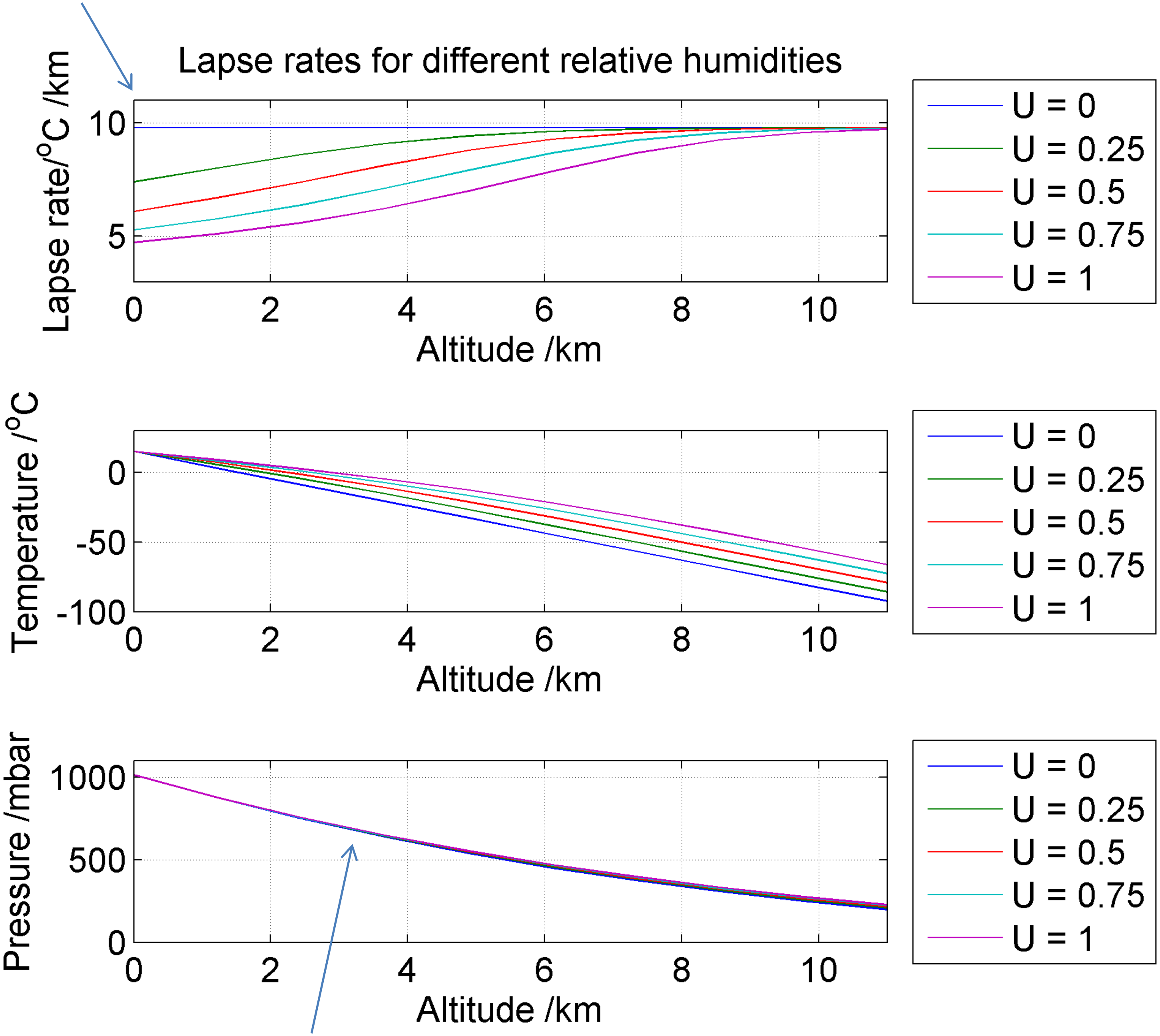
R_{sw} = 461.5 J kg^{-1} K^{-1}

Specific gas constant example

$$R_{sd} = \frac{R}{M_d} = \frac{8.314}{0.02896} \approx 287$$

http://en.wikipedia.org/wiki/Lapse_rate

Hence ‘rule of thumb’: *dry air lapse rate* is about **10° /km**, *wet air lapse rate* is about **5° /km**



So humidity only makes a significant difference at low pressures i.e. at high altitudes. At this point the assumption of a significant relative humidity is likely to be invalid anyway!

The **dew point** is the temperature to which a given parcel of air must be cooled, at constant barometric pressure, for water vapour to condense into water.

The *August-Roche Magnus approximation* defines the dew point to be defined by the following expression in terms of relative humidity U (with values 0…1) and *ambient* air temperature T. i.e. the air parcel being cooled will be *colder* than the ambient air.

Note in the formula below, T is defined in degrees Celsius.

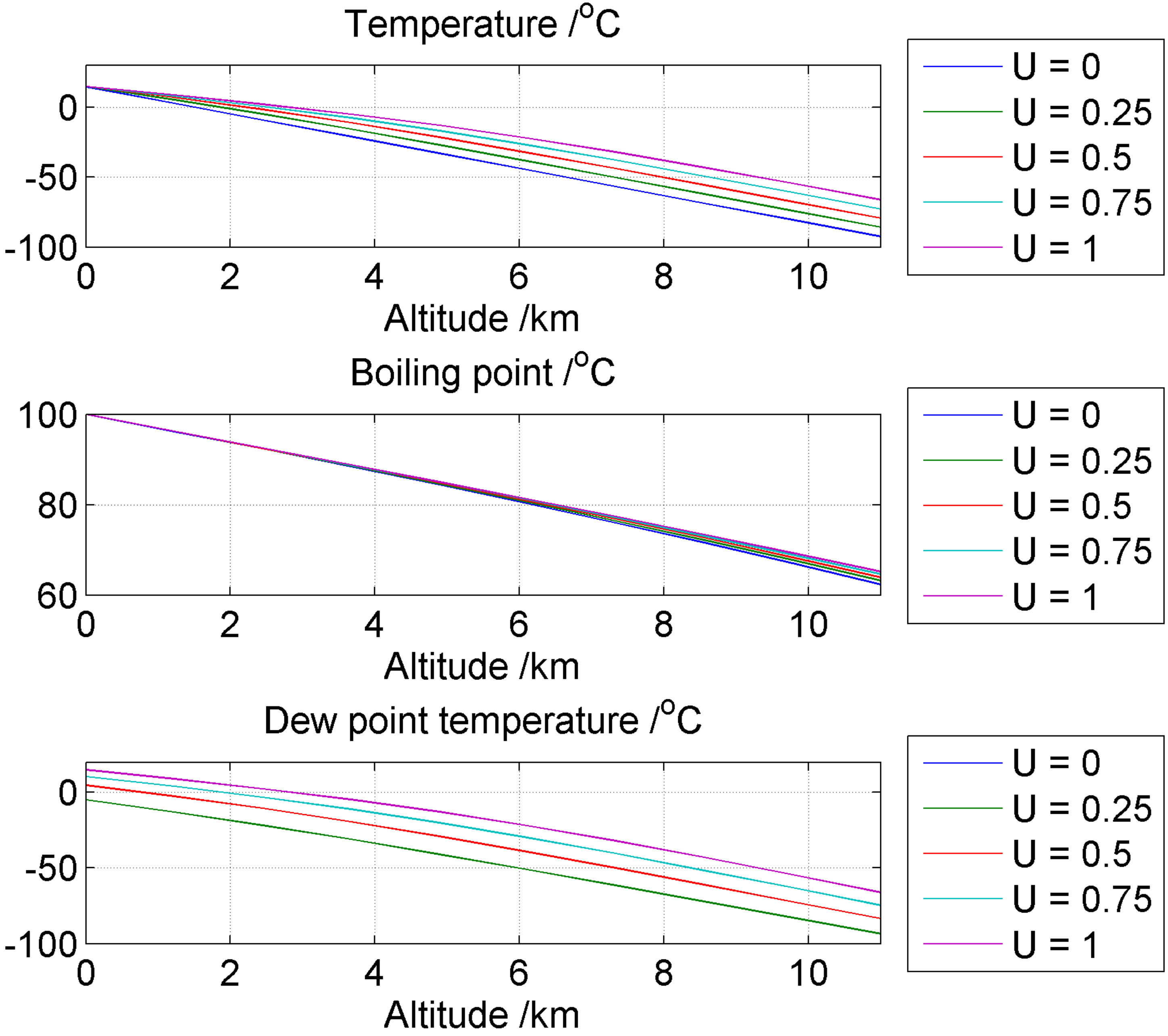
$$T_d = \frac{b\left(\ln U + \frac{aT}{b+T}\right)}{a - \ln U - \frac{aT}{b+T}}$$

$$a = 17.625$$

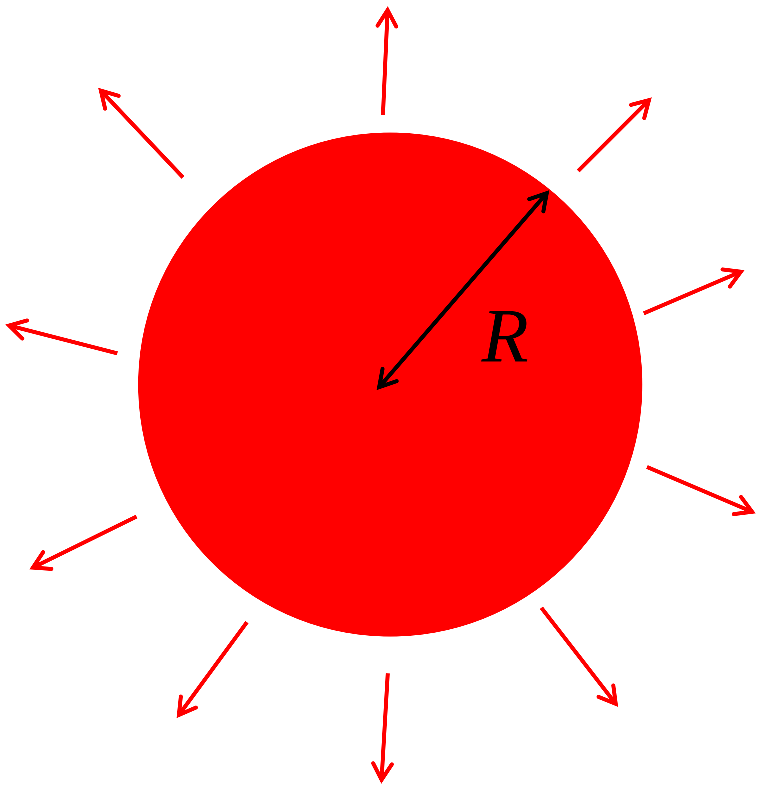
$$b = 243.04$$

"The dew point is the temperature at which the water vapour in a sample of air at constant barometric pressure condenses into liquid water at the same rate at which it evaporates"
http://en.wikipedia.org/wiki/Dew_point

http://andrew.rsmas.miami.edu/bmcnoldy/Humidity.html


Temperature /°C
0
-50
-100
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Boiling point /°C
100
80
60
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Dew point temperature /°C
0
-50
-100
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1

5b. Minimizing heat loss



Radiation power from a sphere of radius R is:

$$\frac{dQ}{dt} = 4\pi R^2 \varepsilon \sigma T^4$$

Surface temperature /K

emissivity is about 0.97 for ice

Stefan Boltzmann constant

$$\sigma = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$$

The heat contained within a body is the specific heat capacity times the density times the volume:

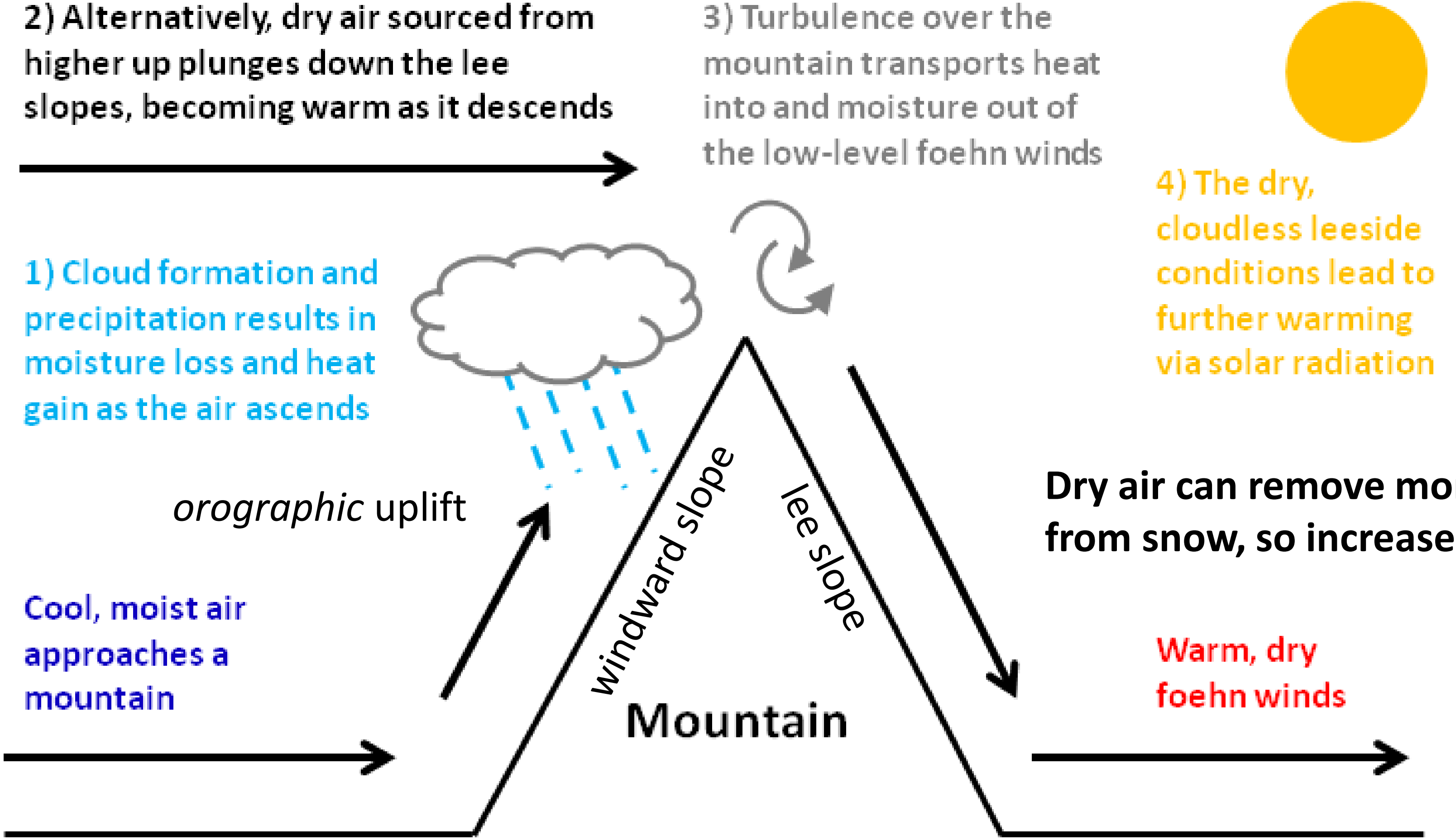
$$Q = c\rho \tfrac{4}{3}\pi R^3$$

Hence to minimize the rate of heat loss, the ratio of radiated power to total heat contained in the body should be as *small* as possible. This means making R as large as possible.

$$\frac{1}{Q}\frac{dQ}{dt} \propto \frac{1}{R}$$

So small children will lose heat faster than large adults, and a huddle (which increases R) can reduce the rate of heat loss. Penguins survive Antarctic winters this way.

5c. Mountain breeze and Föhn (or 'chinook')



2) Alternatively, dry air sourced from higher up plunges down the lee slopes, becoming warm as it descends

3) Turbulence over the mountain transports heat into and moisture out of the low-level foehn winds

4) The dry, cloudless leeside conditions lead to further warming via solar radiation

1) Cloud formation and precipitation results in moisture loss and heat gain as the air ascends

orographic uplift

windward slope

lee slope

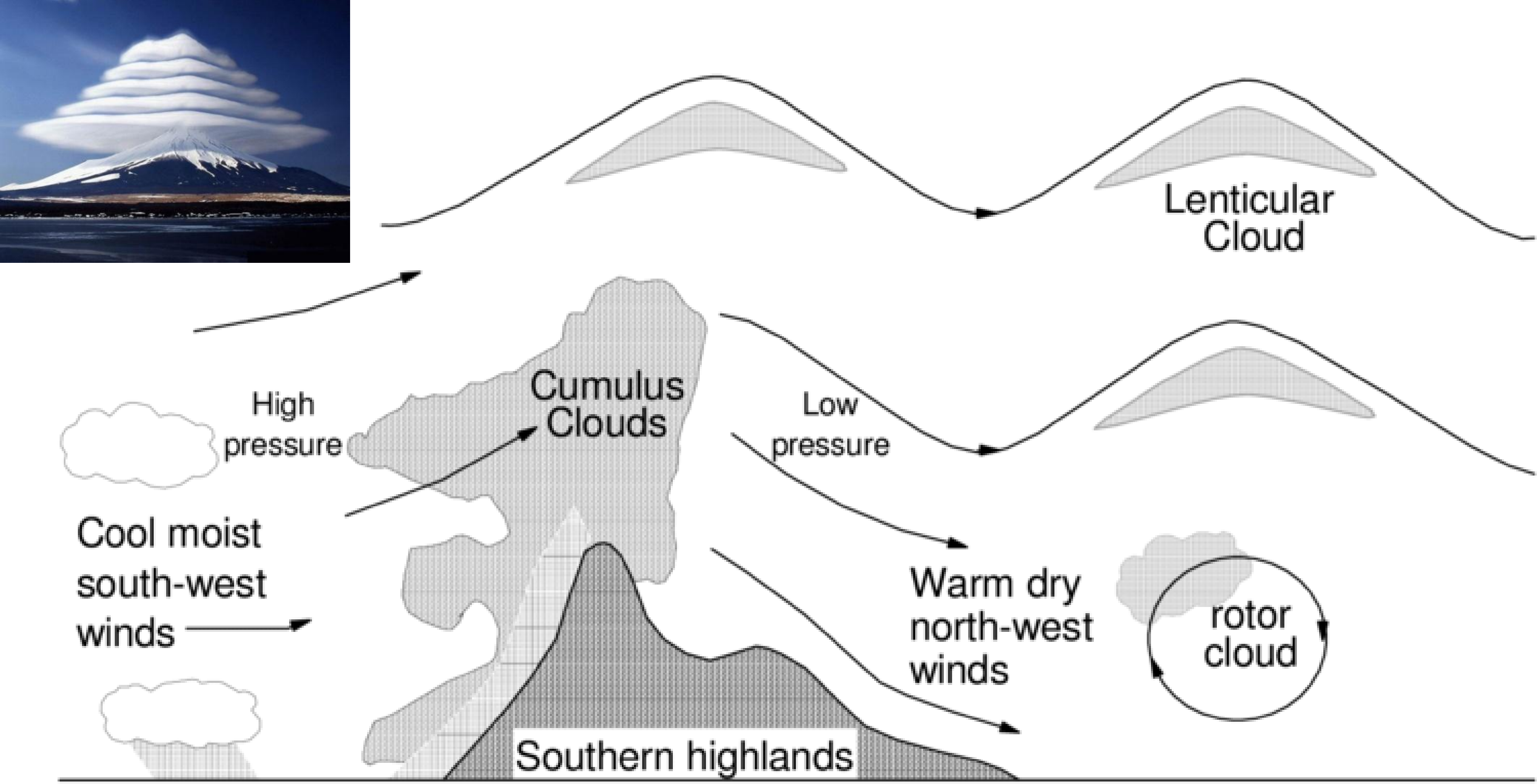
Mountain

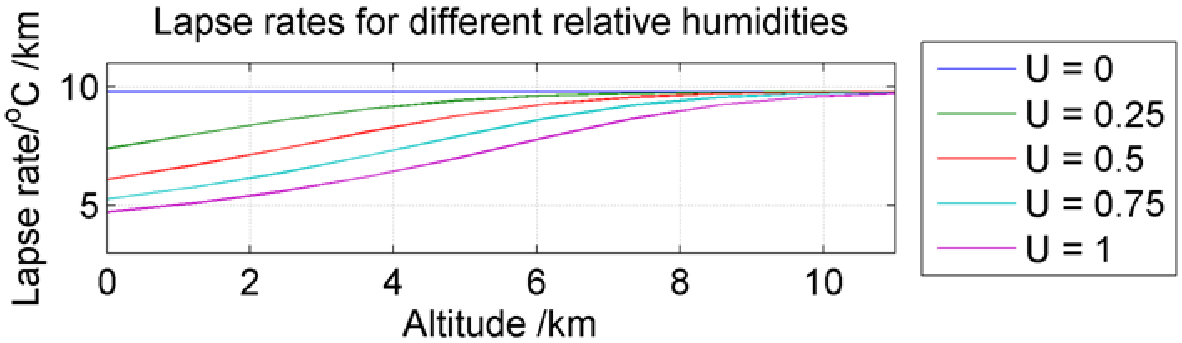
Cool, moist air approaches a mountain

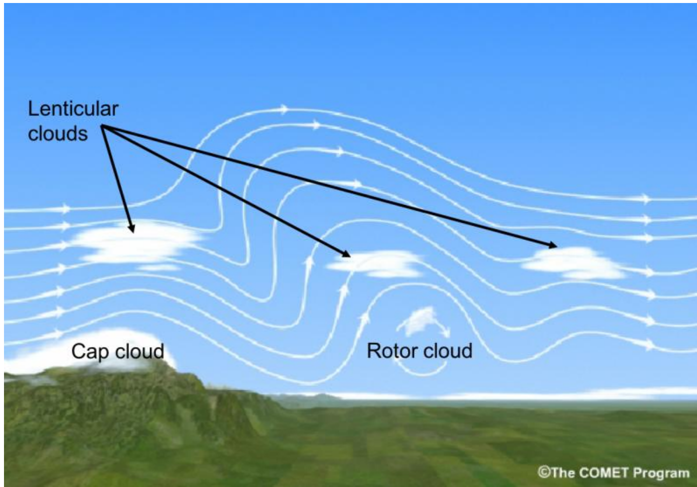
Dry air can remove more moisture from snow, so increases ablation

Warm, dry foehn winds

Lapse rate for dry air is about 10°C per km of altitude, double that for humid air. So *descending* dry air is a warmer than the moist air approaching the mountain

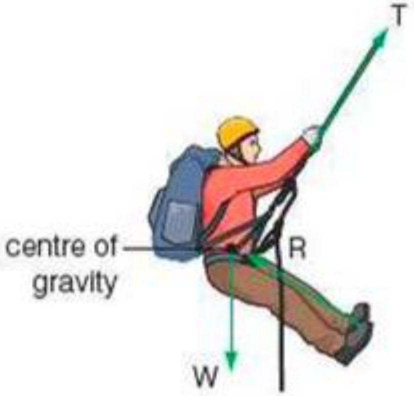

Lenticular Cloud
High pressure
Cumulus Clouds
Low pressure
Cool moist south-west winds
Warm dry north-west winds
rotor cloud
Southern highlands

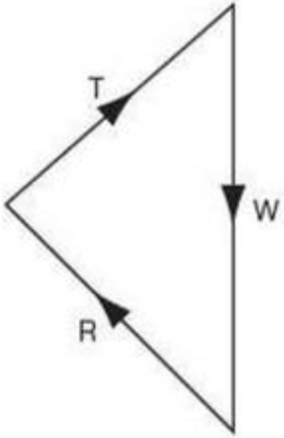

Lapse rates for different relative humidities
Lapse rate/°C /km
10
5
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1

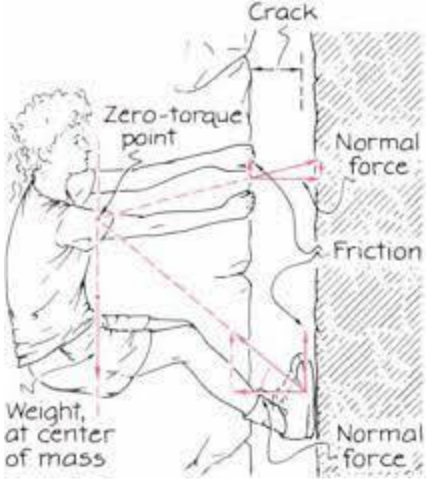

Lenticular clouds
Cap cloud
Rotor cloud
©The COMET Program

1. Mountains high or low, hard or soft
2. Gravity rules
3. Water, snow and ice
4. Glacier puzzles
5. Heat, cold and air
6. Rock climbing
7. Miscellaneous

Mechanics of climbing







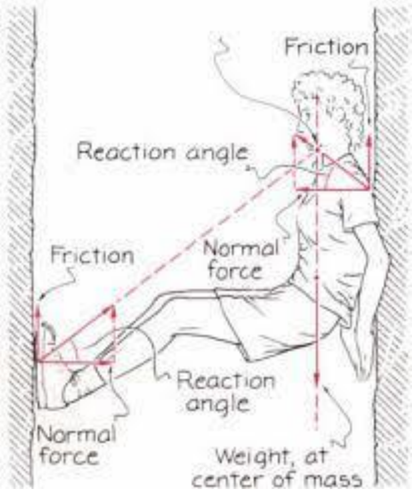
Friction force balances weight down the slope

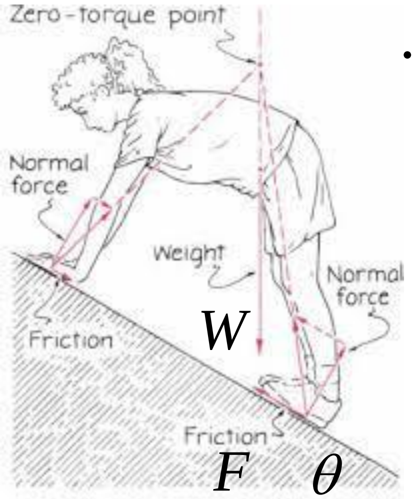
$$W \sin\theta = F$$

$$F \le \mu W \cos\theta$$

$$\therefore \tan\theta \le \mu$$

Friction force is less than or equal to coefficient of friction times normal contact force for no sliding





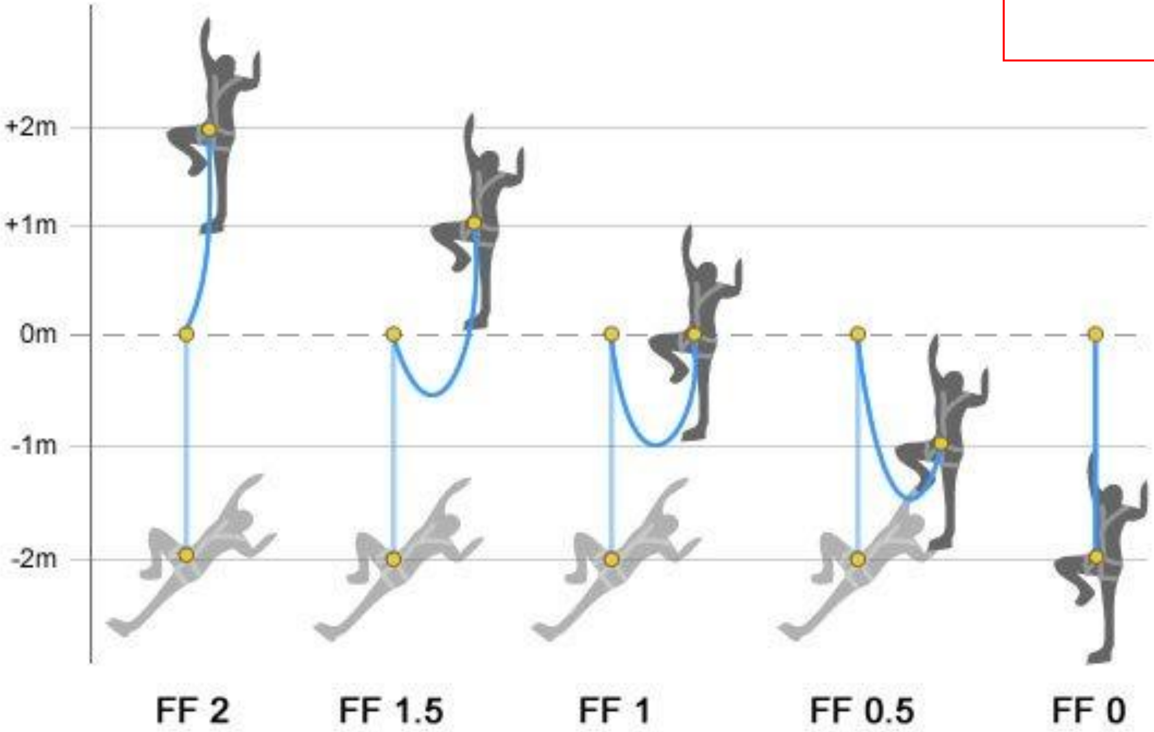
	Static friction coefficient μ_s	Dynamic friction coefficient μ_d
Isoprene rubber (TypeA)	1.528±0.137	1.539±0.133
Polyester non-woven fabric (TypeB)	0.248±0.004	0.222±0.004

$$\tan^{-1} 1.53 = 56.8^{\mathrm{o}}$$

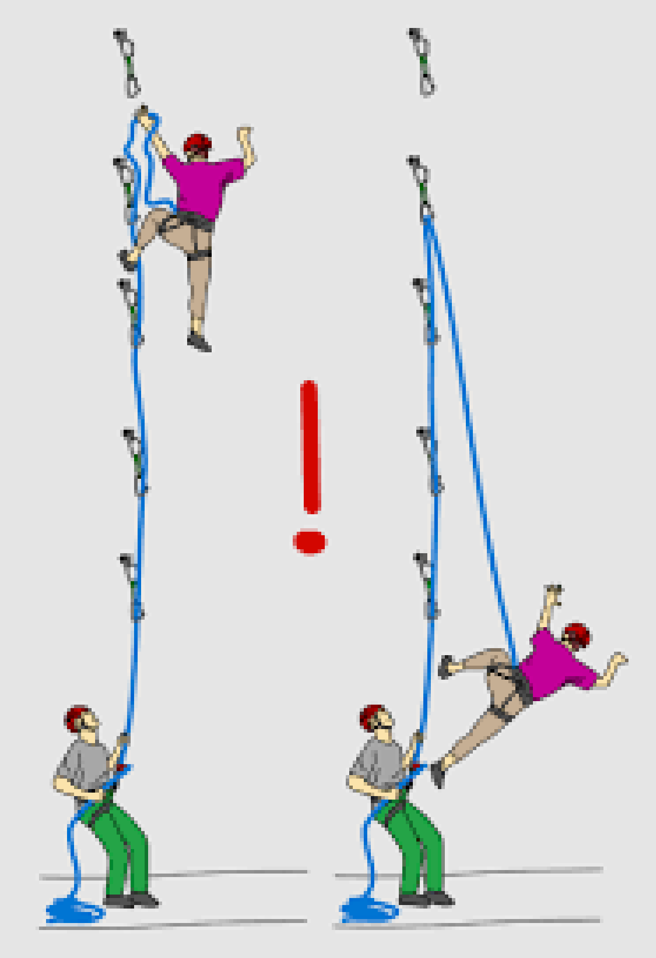
Rock shoes can be very grippy!

Lead climbing and Fall factor

$$\text{fall factor } F = \frac{\text{length of fall}}{\text{length of rope out}}$$



Don’t forget rope stretch!



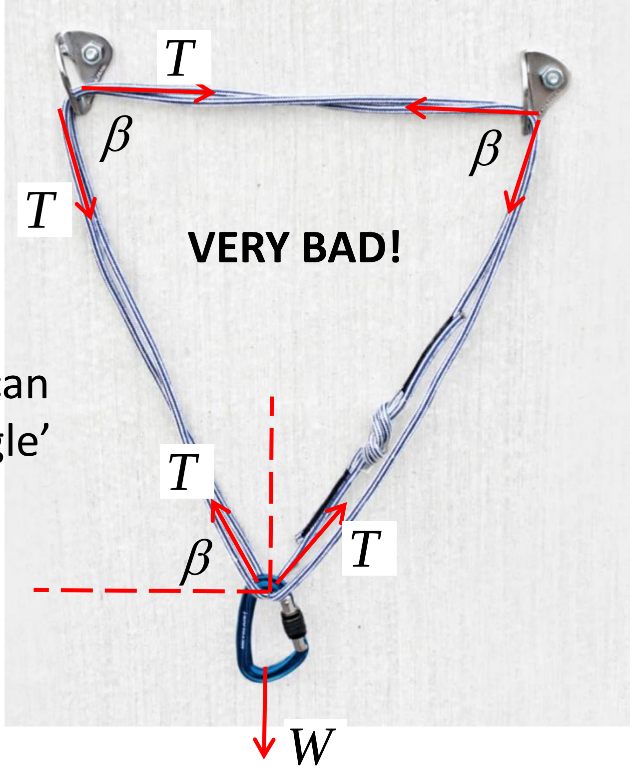
DMM *Belay master,* rated to 25kN through major axis, 10kN through minor axis and 8kN with screw gate open.

Setting up a belay anchor

http://howtoclimbharder.com/basic-safety-in-rock-climbing/basic-climbing-safety-basic-belays/equalising-two-anchors/

Equalize anchors

‘The American death triangle’



$$W = 2T\sin\beta$$

Tension in sling: $\therefore T = \dfrac{W}{2\sin\beta}$

Force on each anchor point

$$F_{net} = \sqrt{(T + T\cos\beta)^2 + T^2\sin^2\beta}$$

$$= \sqrt{T^2 + 2T^2\cos\beta + T^2\cos^2\beta + T^2\sin^2\beta}$$

$$= T\sqrt{2}\sqrt{1+\cos\beta}$$

unnecessary tension multiplier!

1. Mountains high or low, hard or soft
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Lichens and fairy rings

Rhizocarpon geographicum on quartz;
Habitat: Stein am Mandl, Styria, Austria