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$$(R_1-r)^2=e^2+f^2$$
$$(R_2+r)^2=(e-a)^2+(f-b)^2$$
$$(R_3+r)^2=(e-c)^2+(f-d)^2$$

Goal is to find the centre (e,f) coordinates and radius r of a circle which touches the inside of a large circle (origin $(0,0)$)and also two other circles which also touch the inside of the larger circle

$$R_1^2-2R_1r+r^2=e^2+f^2 \qquad \textbf{(1)}$$
$$R_2^2+2R_2r+r^2=e^2-2ea+a^2+f^2-2fb+b^2 \qquad \textbf{(2)}$$
$$R_3^2+2R_3r+r^2=e^2-2ec+c^2+f^2-2fd+d^2 \qquad \textbf{(3)}$$

$$R_2^2+2R_2r+r^2-\left(R_1^2-2R_1r+r^2\right)=e^2-2ea+a^2+f^2-2fb+b^2-\left(e^2+f^2\right)$$
$$R_2^2-R_1^2+2\left(R_2+R_1\right)r=-2ea+a^2-2fb+b^2 \qquad \textbf{(4) = (2) – (1)}$$

$$R_3^2+2R_3r+r^2-\left(R_1^2-2R_1r+r^2\right)=e^2-2ec+c^2+f^2-2fd+d^2-\left(e^2+f^2\right)$$
$$R_3^2-R_1^2+2\left(R_3+R_1\right)r=-2ec+c^2-2fd+d^2 \qquad \textbf{(5) = (3) – (1)}$$

$$R_2^2d-R_1^2d+2\left(R_2+R_1\right)dr=-2ead+a^2d-2fbd+b^2d \qquad \textbf{(6)} = d\textbf{(4)}$$
$$R_3^2b-R_1^2b+2\left(R_3+R_1\right)br=-2ecb+c^2b-2fdb+d^2b \qquad \textbf{(7)} = b\textbf{(5)}$$

$$\left(R_2^2-R_1^2\right)d-\left(R_3^2-R_1^2\right)b+2r\left\{\left(R_2+R_1\right)d-\left(R_3+R_1\right)b\right\}=a^2d-c^2b+b^2d-d^2b+2e\left(cb-ad\right)$$
$$\therefore e=A+Br \qquad \textbf{(8) = (6) – (7)}$$

$$A=\frac{\left(R_2^2-R_1^2\right)d-\left(R_3^2-R_1^2\right)b+c^2b+d^2b-a^2d-b^2d}{2\left(cb-ad\right)}$$
$$B=\frac{\left(R_2+R_1\right)d-\left(R_3+R_1\right)b}{cb-ad}$$

$$R_2^2c-R_1^2c+2\left(R_2+R_1\right)cr=-2eca+a^2c-2fbc+b^2c \qquad \textbf{(9)} = c\textbf{(4)}$$
$$R_3^2a-R_1^2a+2\left(R_3+R_1\right)ar=-2eca+c^2a-2fda+d^2a \qquad \textbf{(10)} = a\textbf{(5)}$$

$$\left(R_2^2-R_1^2\right)c-\left(R_3^2-R_1^2\right)a+2r\left\{\left(R_2+R_1\right)c-\left(R_3+R_1\right)a\right\}=a^2c-c^2a+b^2c-d^2a+2f\left(da-bc\right)$$
$$\therefore f=C+Dr \qquad \textbf{(11) = (9) – (10)}$$

$$C=\frac{\left(R_2^2-R_1^2\right)c-\left(R_3^2-R_1^2\right)a+c^2a+d^2a-a^2c-b^2c}{2\left(da-bc\right)}$$
$$D=\frac{\left(R_2+R_1\right)c-\left(R_3+R_1\right)a}{da-bc}$$

$$R_1^2-2R_1r+r^2=e^2+f^2 \qquad \textbf{(1)}$$
$$R_1^2-2R_1r+r^2=(A+Br)^2+(C+Dr)^2$$ Substitute for $e=A+Br$ and $f=C+Dr$
$$R_1^2-2R_1r+r^2=A^2+2ABr+B^2r^2+C^2+2CDr+D^2r^2$$
$$\left(1-B^2-D^2\right)r^2+\left(-2R_1-2AB-2CD\right)r+R_1^2-A^2-C^2=0$$
$$\therefore \alpha r^2+\beta r+\gamma=0$$

$$\alpha=1-B^2-D^2$$
$$\beta=-2R_1-2AB-2CD$$
$$\gamma=R_1^2-A^2-C^2$$
$$\therefore r=\frac{-\beta\pm\sqrt{\beta^2-4\alpha\gamma}}{2\alpha}$$

$$e=A+Br$$
$$f=C+Dr$$

Check of three circle intersection code