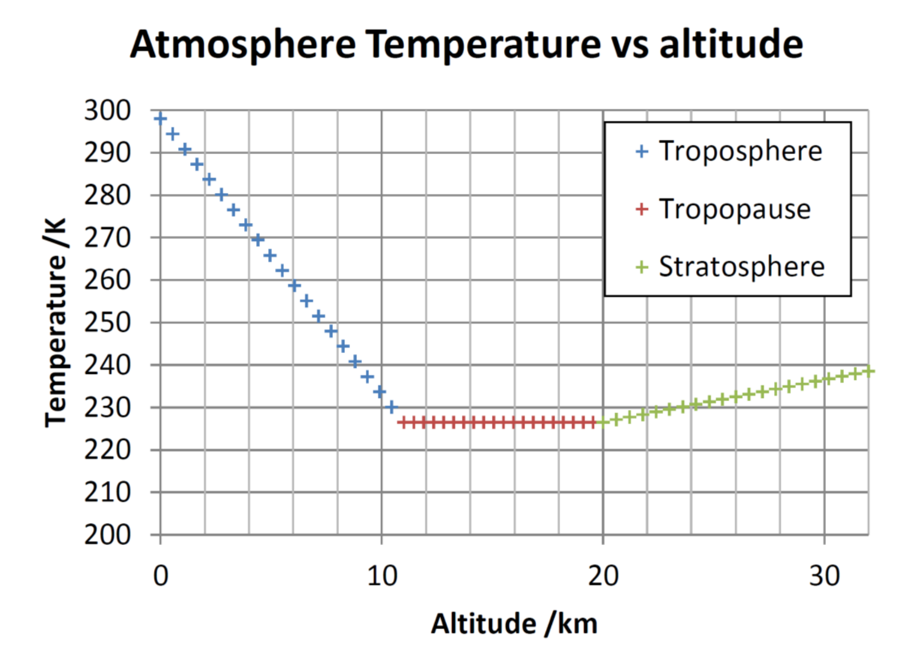
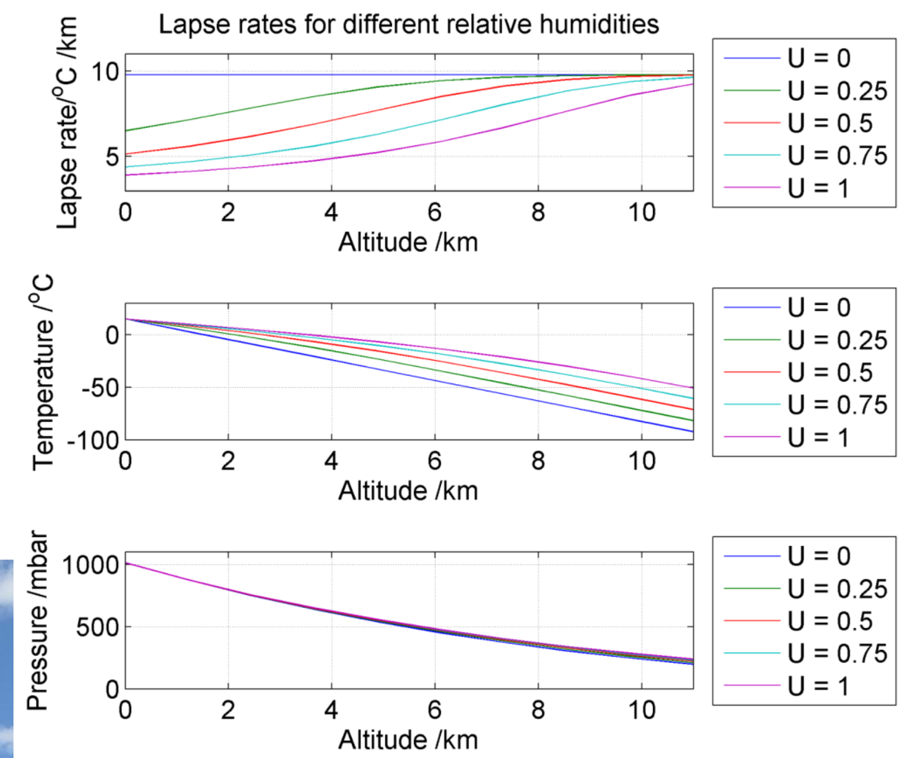
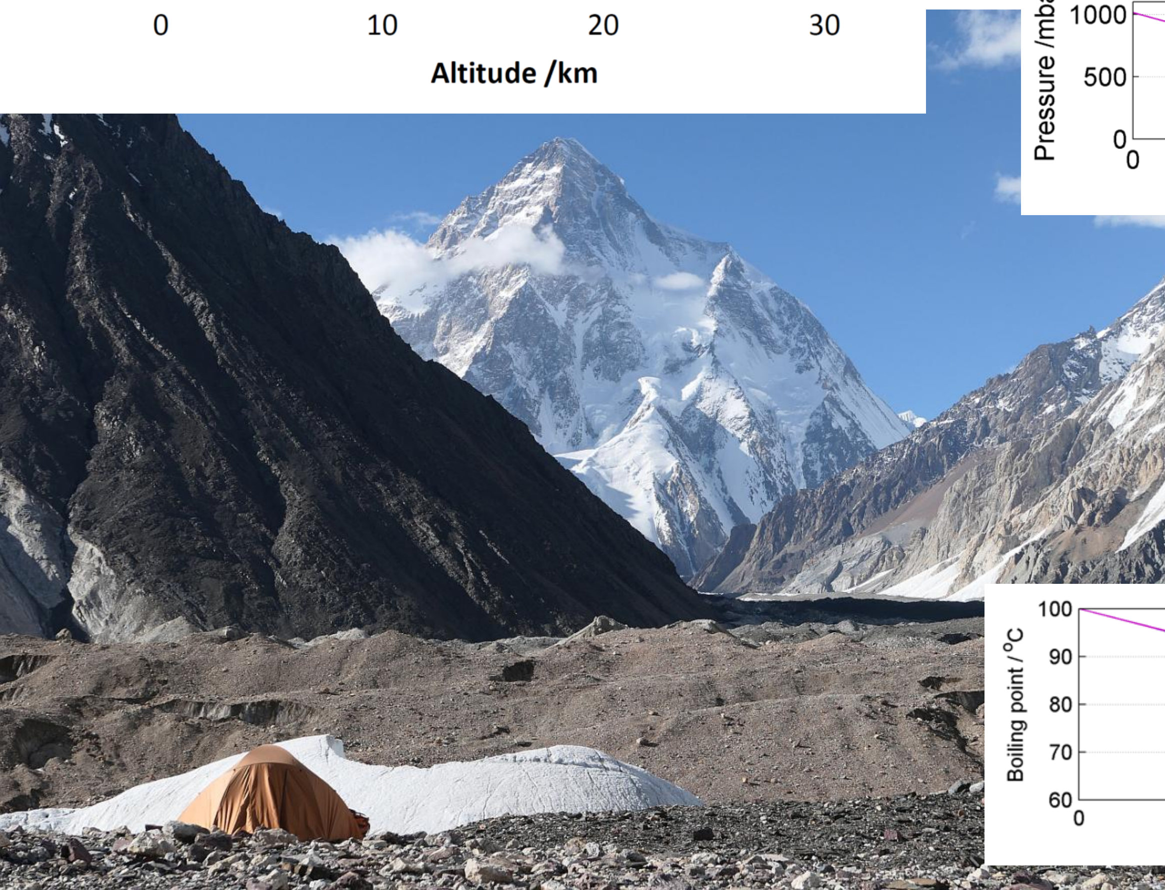
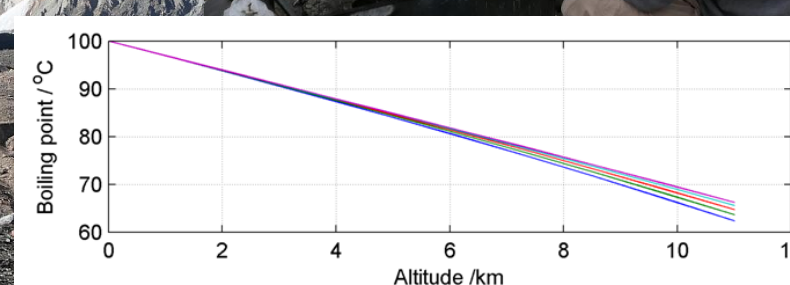

Atmosphere Temperature vs altitude
Temperature /K
300
290
280
270
260
250
240
230
220
210
200
0
10
20
30
Altitude /km
Troposphere
Tropopause
Stratosphere


Lapse rates for different relative humidities
Lapse rate/°C /km
10
5
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Temperature /°C
0
-50
-100
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1
Pressure /mbar
1000
500
0
0
2
4
6
8
10
Altitude /km
U = 0
U = 0.25
U = 0.5
U = 0.75
U = 1




Boiling point / °C
100
90
80
70
60
0
2
4
6
8
10
12
Altitude /km

BPhO Computational Challenge

2022

A Standard Atmosphere

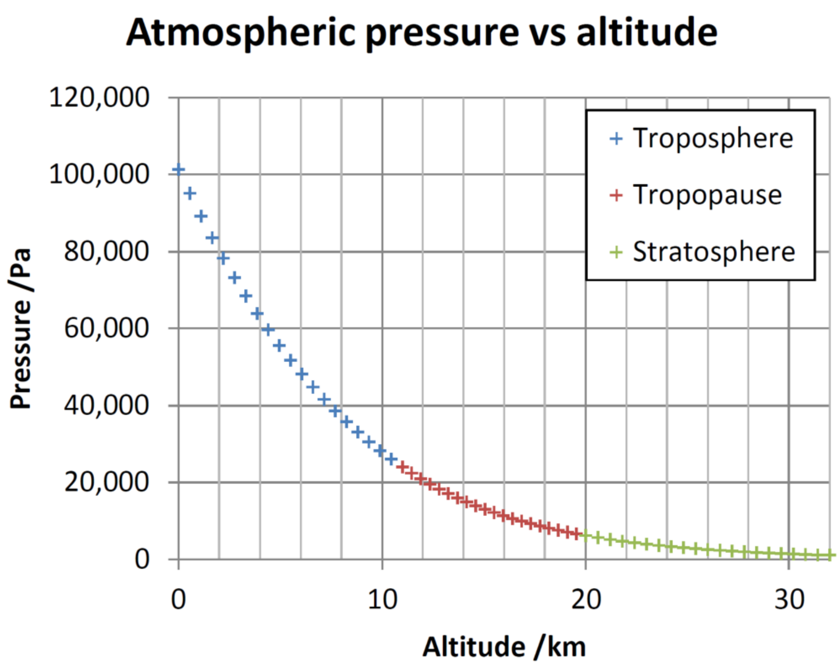
The thin line of Earth's atmosphere and the setting sun are featured in this image photographed by the crew of the International Space Station while space shuttle Atlantis on the STS-129 mission was docked with the station. Image credit: NASA

BPhO British Physics Olympiad

v2 April 2022

TASK 2 RECAP: PLOT THE ISA MODEL: Pressure vs altitude, for dry air

If we can ignore humidity (i.e. the contribution to air pressure from water vapour), air pressure is simply the weight per unit area of a column of atmosphere.



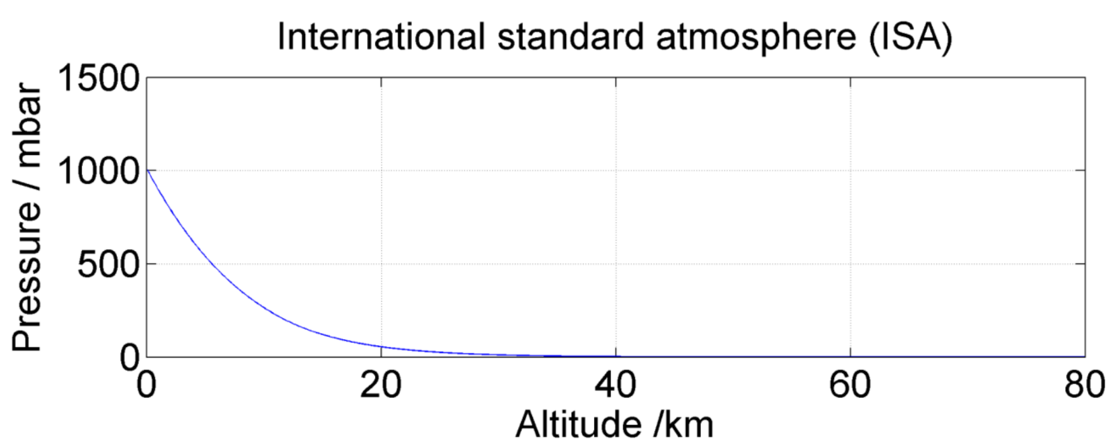
$$P = P_0\left(1 - \frac{L(h-h_0)}{T_0}\right)^{\frac{Mg}{LR}}$$

$$P = P_0 e^{-\frac{Mg}{RT_0}(h-h_0)}$$

If $L = 0$

Note for these models you'll need altitude in **metres** if you use the other constants in standard units. This means you'll need to define the **Lapse rate in K per m, which means dividing the K/km values by 1,000.**

Note P_0 and T_0 correspond to the (Kelvin) temperatures *at the base of the layer*. So work upwards from the base of the Troposphere.



Example spreadsheet for the first three ISA layers

Molar gas constant $R = 8.314\,\mathrm{J\,mol^{-1}K^{-1}}$

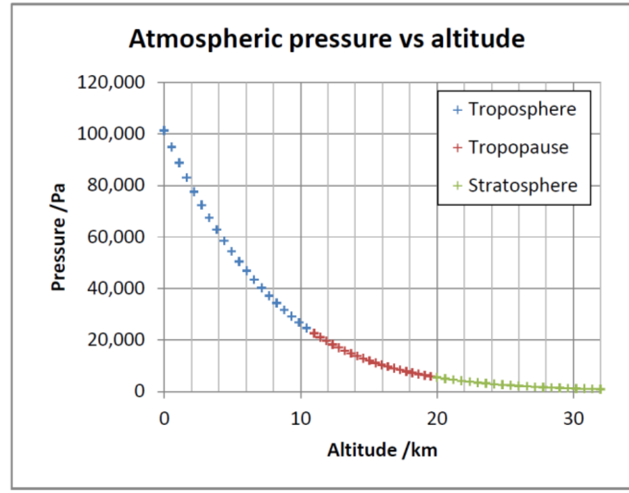
Molar mass of air $M = 0.02896\,\mathrm{kg\,mol^{-1}}$

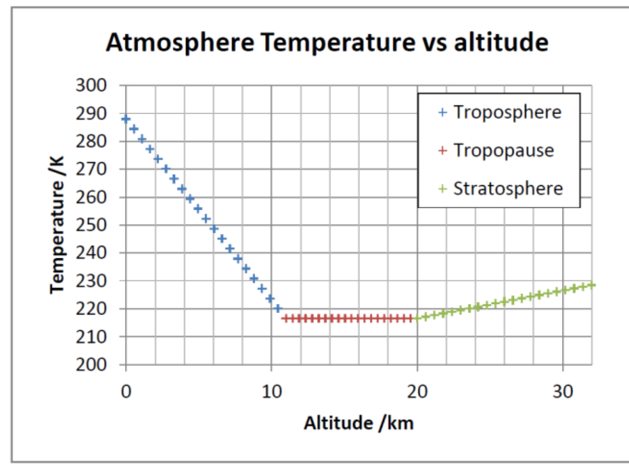
Strength of gravity $g = 9.81\,\mathrm{N\,kg^{-1}}$

Standard atmosphere model

	zstart /km	zfinish /km	Tstart /K	Tfinish /K	Lapse rate /K per km	pstart /Pa	pfinish /Pa
Troposphere	0	11	288	216.5	6.5	101,325	22,604
Tropopause	11	20	216.5	216.5	0.0	22,604	5,461
Stratosphere	20	32	216.5	228.5	-1.0	5,461	864

Mg/R
0.034171





Troposphere

	z /km	T /K	p /Pa
0	0	288	101,325
0.05	0.55	284.425	94,885
0.1	1.1	280.85	88,781
0.15	1.65	277.275	82,999
0.2	2.2	273.7	77,525
0.25	2.75	270.125	72,348
0.3	3.3	266.55	67,454
0.35	3.85	262.975	62,832
0.4	4.4	259.4	58,469
0.45	4.95	255.825	54,355
0.5	5.5	252.25	50,479
0.55	6.05	248.675	46,830
0.6	6.6	245.1	43,397
0.65	7.15	241.525	40,171
0.7	7.7	237.95	37,142
0.75	8.25	234.375	34,301
0.8	8.8	230.8	31,638
0.85	9.35	227.225	29,146
0.9	9.9	223.65	26,814
0.95	10.45	220.075	24,636
1	11	216.5	22,604

Power law

Positive lapse rate

$$p(z) = p_0\left(1 - \frac{L(z-z_0)}{T_0}\right)^{\frac{Mg}{LR}}$$

Tropopause

z /km	T /K	p /Pa
11	216.5	22,604
11.45	216.5	21,054
11.9	216.5	19,611
12.35	216.5	18,266
12.8	216.5	17,014
13.25	216.5	15,847
13.7	216.5	14,761
14.15	216.5	13,749
14.6	216.5	12,806
15.05	216.5	11,928
15.5	216.5	11,110
15.95	216.5	10,349
16.4	216.5	9,639
16.85	216.5	8,978
17.3	216.5	8,363
17.75	216.5	7,789
18.2	216.5	7,255
18.65	216.5	6,758
19.1	216.5	6,294
19.55	216.5	5,863
20	216.5	5,461

Exponential

Isothermal

$$p(z) = p_0 e^{-\frac{Mg}{RT_0}(z-z_0)}$$

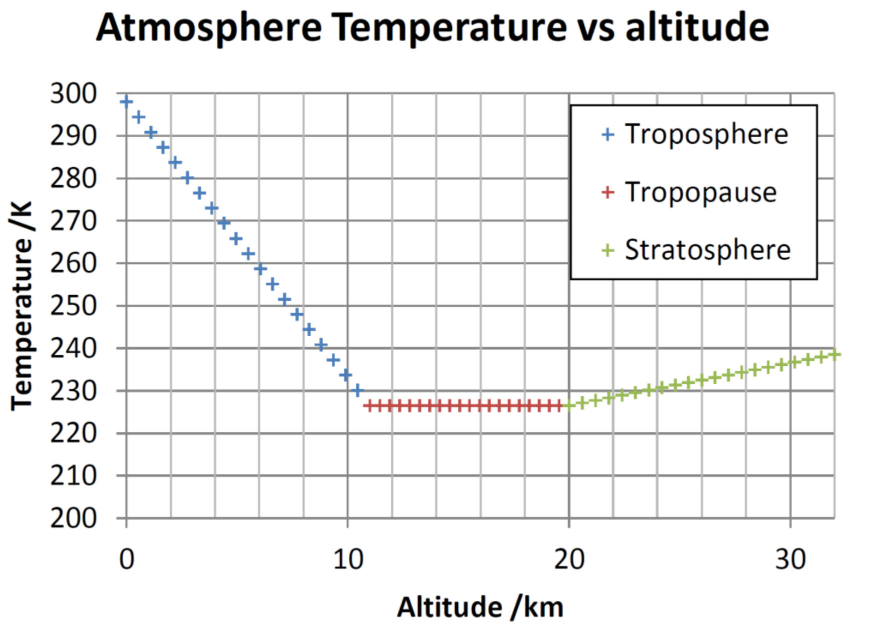
Stratosphere

z /km	T /K	p /Pa
20	216.5	5,461
20.6	217.1	4,968
21.2	217.7	4,521
21.8	218.3	4,115
22.4	218.9	3,747
23	219.5	3,412
23.6	220.1	3,108
24.2	220.7	2,832
24.8	221.3	2,581
25.4	221.9	2,353
26	222.5	2,146
26.6	223.1	1,957
27.2	223.7	1,786
27.8	224.3	1,629
28.4	224.9	1,487
29	225.5	1,358
29.6	226.1	1,240
30.2	226.7	1,133
30.8	227.3	1,035
31.4	227.9	946
32	228.5	864

Power law

Negative lapse rate

$$p(z) = p_0\left(1 - \frac{L(z-z_0)}{T_0}\right)^{\frac{Mg}{LR}}$$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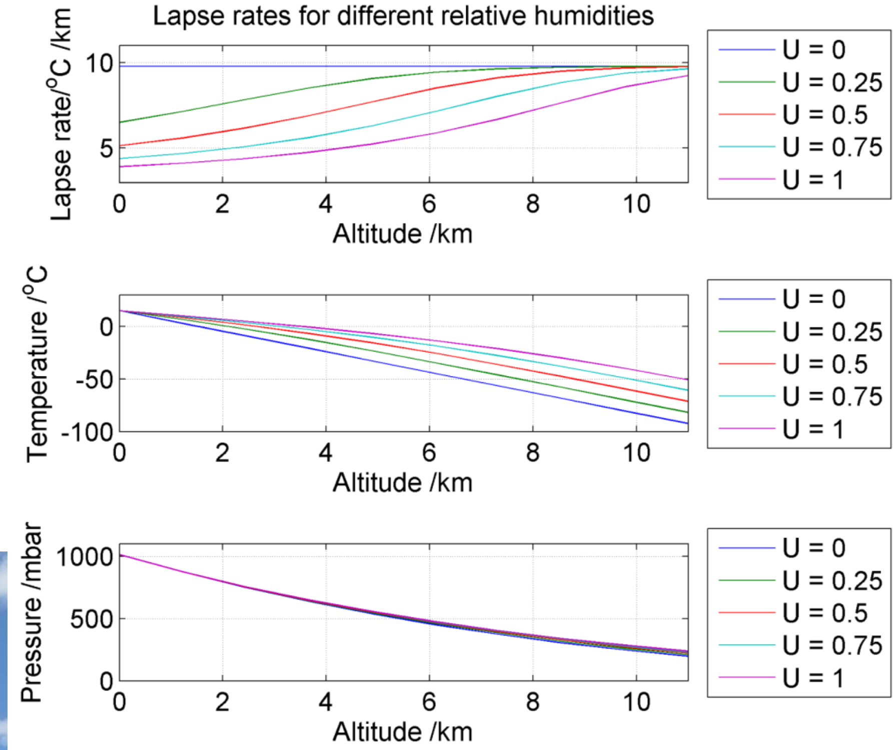
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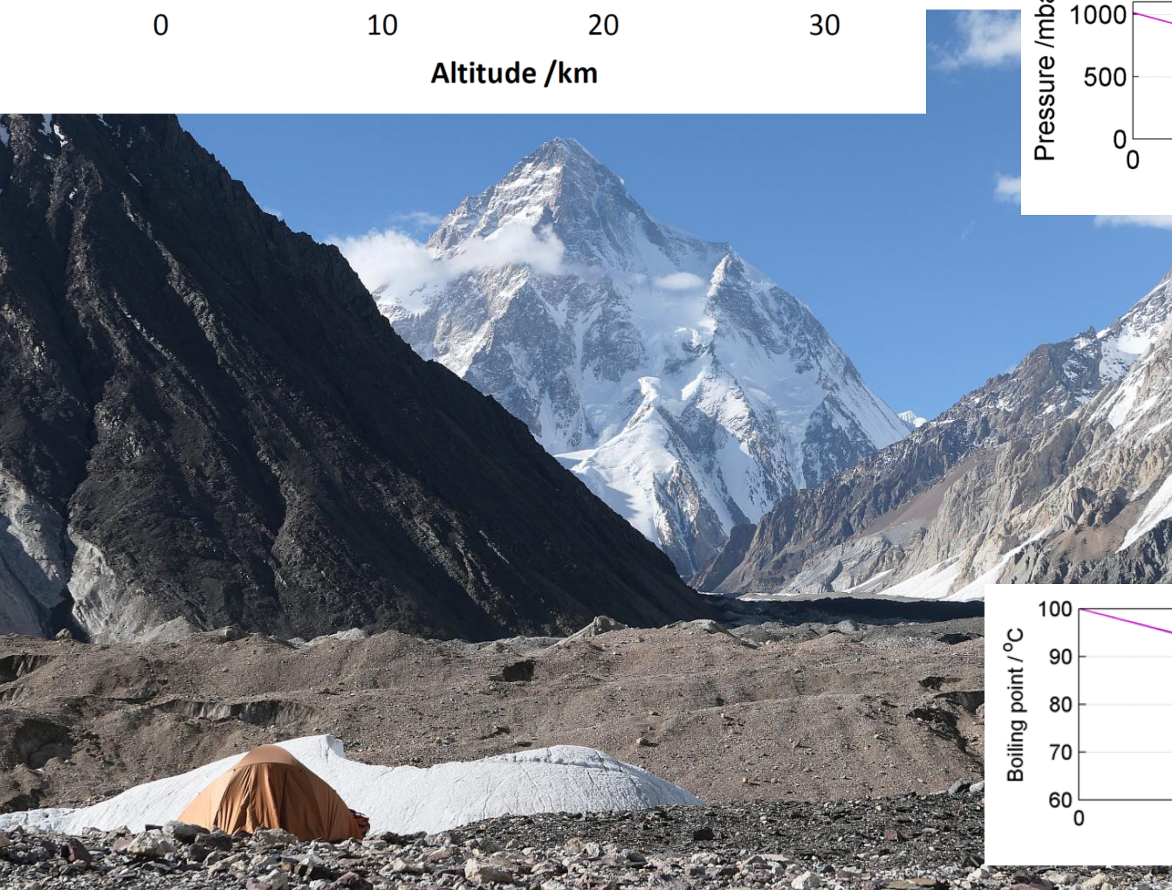

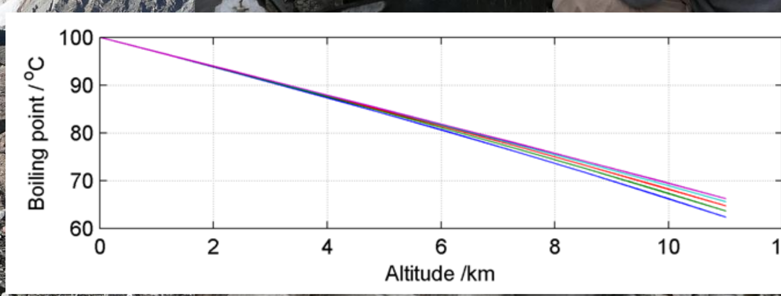
Boiling point / °C
100
90
80
70
60
0
2
4
6
8
10
12
Altitude /km

In **summary**, we can model the variation of pressure, temperature, lapse rate boiling point and dew point with altitude using the following iterative scheme:

$$T_0 = 15^{o}\mathrm{C} \qquad T_* = 100^{o}\mathrm{C} \qquad P_0 = P_* = 1013.25\mathrm{mbar} \qquad \Delta h = 0.01\mathrm{km}$$

T in degrees Celsius

$$T_K = T + 273$$ T in Kelvin

$$h \rightarrow h + \Delta h \qquad T \rightarrow T - L\Delta h$$

$$E_s = 6.1121 e^{\left\{\left(18.678 - \frac{T}{234.5}\right)\left(\frac{T}{T+257.14}\right)\right\}}$$

$$r = \frac{R_{sd}}{R_{sw}} \frac{UE_s}{P - UE_s}$$

$$L = g \frac{1 + \frac{r\Delta H_v}{R_{sd} T_K}}{c_{pd} + \frac{(\Delta H_v)^2 r}{R_{sw} T_K{}^2}}$$

$$\Delta P = -\frac{M_d g}{RT_K}\left(P - U\left(1 - \frac{M_V}{M_d}\right) E_s(T)\right)\Delta h$$

$$P \rightarrow P + \Delta P$$

$$T_{boil} = \left(\frac{1}{T_*} - \frac{R}{\Delta H}\ln\left(\frac{P}{P_*}\right)\right)^{-1}$$

T in Kelvin

$$T_{dew} = \frac{b\left(\ln U + \frac{aT}{b+T}\right)}{a - \ln U - \frac{aT}{b+T}}$$

$$a = 17.625$$

$$b = 243.04$$

T in degrees Celsius

$M_d = 0.02896 \mathrm{kgmol}^{-1}$

$M_v = 0.01802 \mathrm{kgmol}^{-1}$

$R = 8.314 \mathrm{JmolK}^{-1}$

$g = 9.81 \mathrm{Nkg}^{-1}$

ΔH_v = 2,501,000 J kg^{-1}

c_{pd} = 1003.5 Jkg^{-1}K^{-1}

R_{sd} = 287 J kg^{-1} K^{-1}

R_{sw} = 461.5 J kg^{-1} K^{-1}

TASK3: Write a computer program or spreadsheet to evaluate P and T vs altitude for the whole range of the ISA.

Plot L, boiling point and dew point vs altitude for different U values.