BPhO Computational Challenge

Waves and optics

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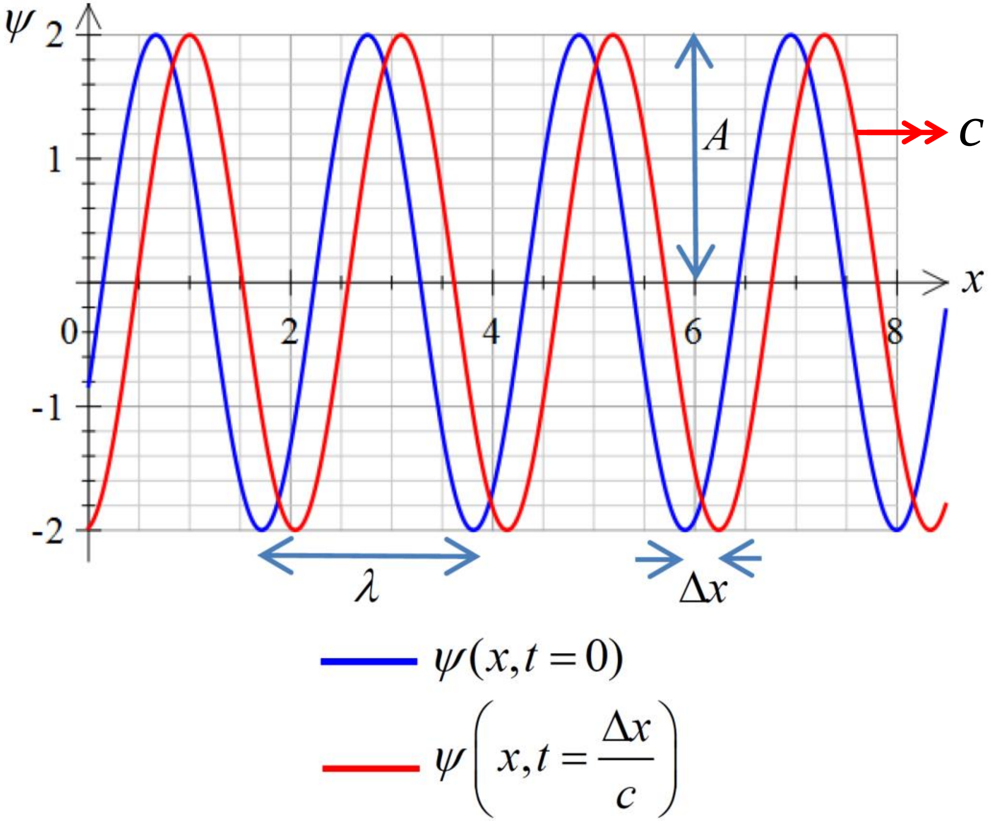
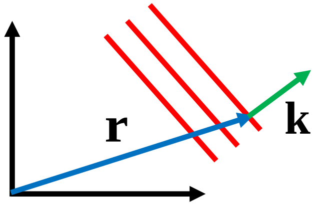
BPhO British Physics Olympiad

$$\psi(x,t) = Ae^{i((kx-\omega t-\phi)}$$

$$\psi(\mathbf{r},t) = Ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t-\phi)}$$





Wave speed $c = f\lambda$

Frequency = 1/period $f = \dfrac{1}{T}$

$$\omega = 2\pi f$$

Wavenumber $k = \dfrac{2\pi}{\lambda}$

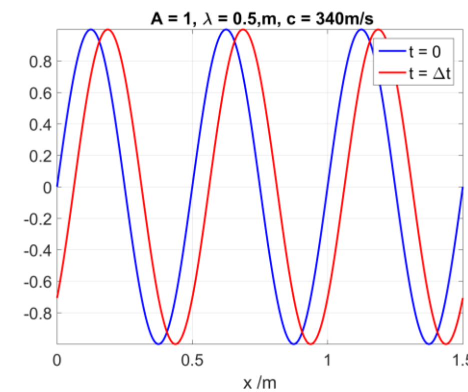
$$\omega = ck$$

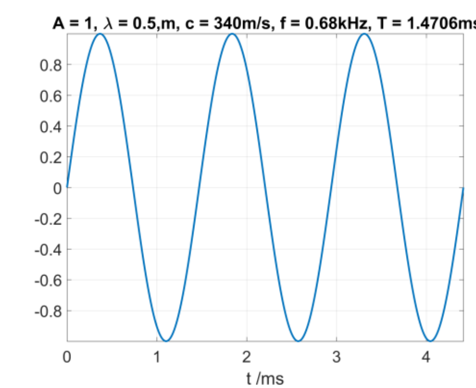
$$\boxed{\begin{aligned}\psi(x,t) &= A\cos\left(2\pi\frac{x-ct}{\lambda}-\phi\right)\\ \psi(x,t) &= A\cos(kx-\omega t-\phi)\end{aligned}}$$

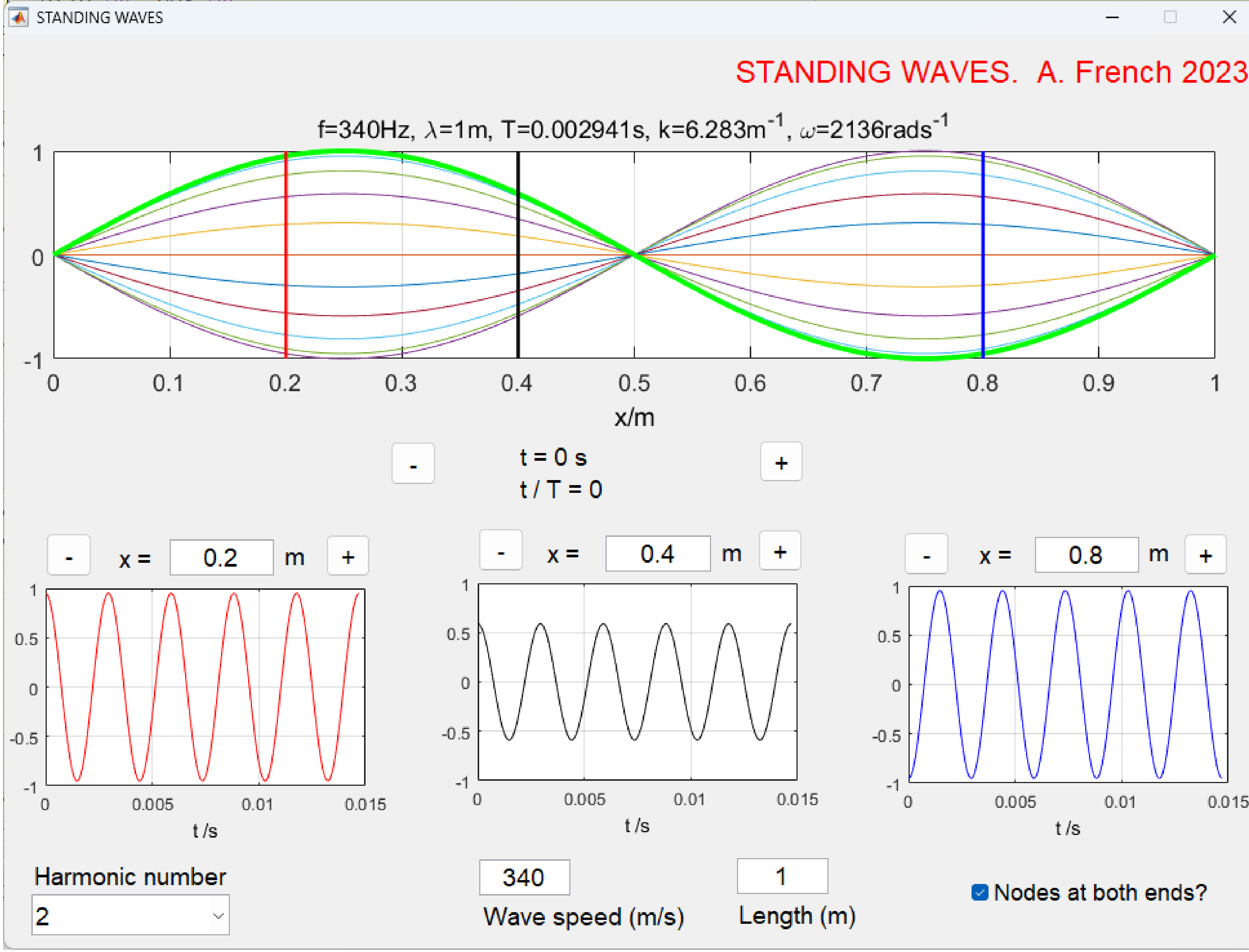
Wave *Fourier components* are **translated sinusoids**

```
A = 1;           %Amplitude
lambda = 0.5;    %Wavelength /m
c = 340;         %Wavespeed /ms^-1
f_unit = 'kHz';  %Frequency unit
t_unit = 'ms';   %Time unit

%Spatial sinusoid separation (fraction of
wavelength)
dx = lambda/8;

fsize = 18;     %Fontsize
N = 1000;       %Number of data points
Nwaves = 3;     %Number of waves to plot
linewidth = 2;  %Line width


%

%Determine period /s and frequency /Hz
f = c/lambda;
T = 1/f;

%Determine time delay associated with dx
dt = dx/c;

%Generate x,y and t,y data vectors
x = linspace( 0, Nwaves*lambda, N);
t = linspace( 0, Nwaves*T, N);

yx1 = A*sin( 2*pi*x/lambda );
yx2 = A*sin( 2*pi*( x- dx)/lambda );
yt = A*sin( 2*pi*t/T );
```

```
%Plot y vs x graph
figure('color',[1 1 1],'name','waves anatomy')
plot(x,yx1,'b-',x,yx2,'r-','linewidth',linewidth);
legend( {'t = 0','t = \Deltat'},'fontsize',fsize )
xlabel( 'x /m','fontsize',fsize )
title([' A = ',num2str(A),', \lambda = ',num2str(lambda),...
    ',m, c = ',num2str(c),'m/s'],'fontsize',fsize )
grid on;
set( gca,'fontsize',fsize )
axis tight
print( gcf,'wave y vs x.png','-dpng', '-r300' )
clf;

%Plot y vs t graph
plot(t/tuf( t_unit ),yt,'linewidth',linewidth);
xlabel( ['t /',t_unit],'fontsize',fsize )
title([' A = ',num2str(A),', \lambda = ',num2str(lambda),...
    ',m, c = ',num2str(c),'m/s, f = ',...
    num2str( f/fuf( f_unit),5),f_unit,', T = ',...
    num2str(T/tuf( t_unit ),5),t_unit],'fontsize',fsize )
grid on;
set( gca,'fontsize',fsize )
axis tight
print( gcf,'wave y vs t.png','-dpng', '-r300' )
close(gcf);

%%

%Frequency unit factor
function m = fuf( f_unit )
if strcmp( f_unit, 'THz' )==1
    m = 1e12;
elseif strcmp( f_unit, 'GHz' )==1
    m = 1e9;
elseif strcmp( f_unit, 'MHz' )==1
    m = 1e6;
elseif strcmp( f_unit, 'kHz' )==1
    m = 1e3;
else; m = 1; end

%

%Time unit factor
function m = tuf( t_unit )
if strcmp( t_unit, 'ps' )==1
    m = 1e-12;
elseif strcmp( t_unit, 'ns' )==1
    m = 1e-9;
elseif strcmp( t_unit, '\mus' )==1
    m = 1e-6;
elseif strcmp( t_unit, 'ms' )==1
    m = 1e-3;
else; m = 1; end
```

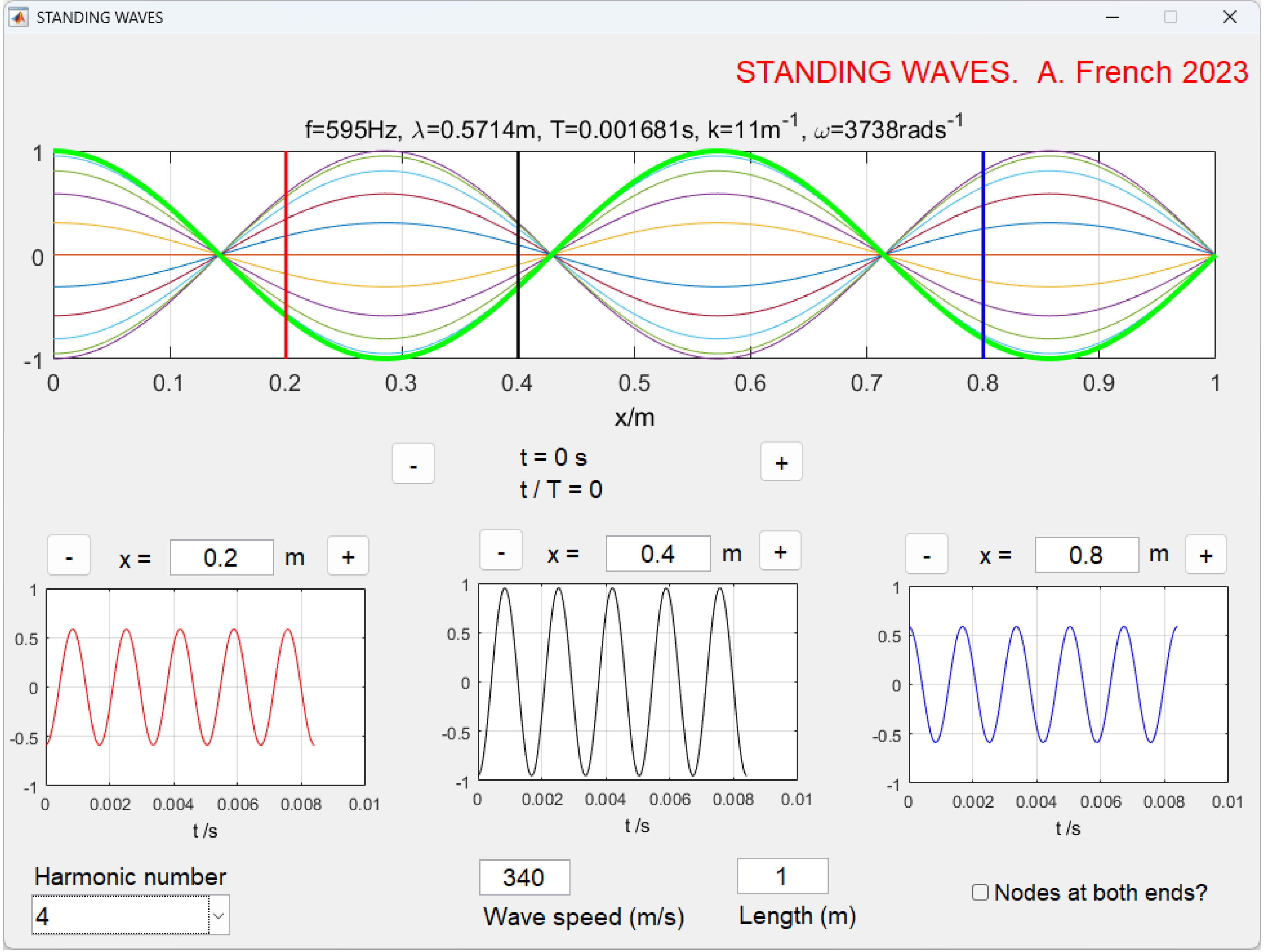






Length is a whole number of half wavelengths

$$L = n\tfrac{1}{2}\lambda$$
$$n = 1, 2, 3, 4\ldots$$

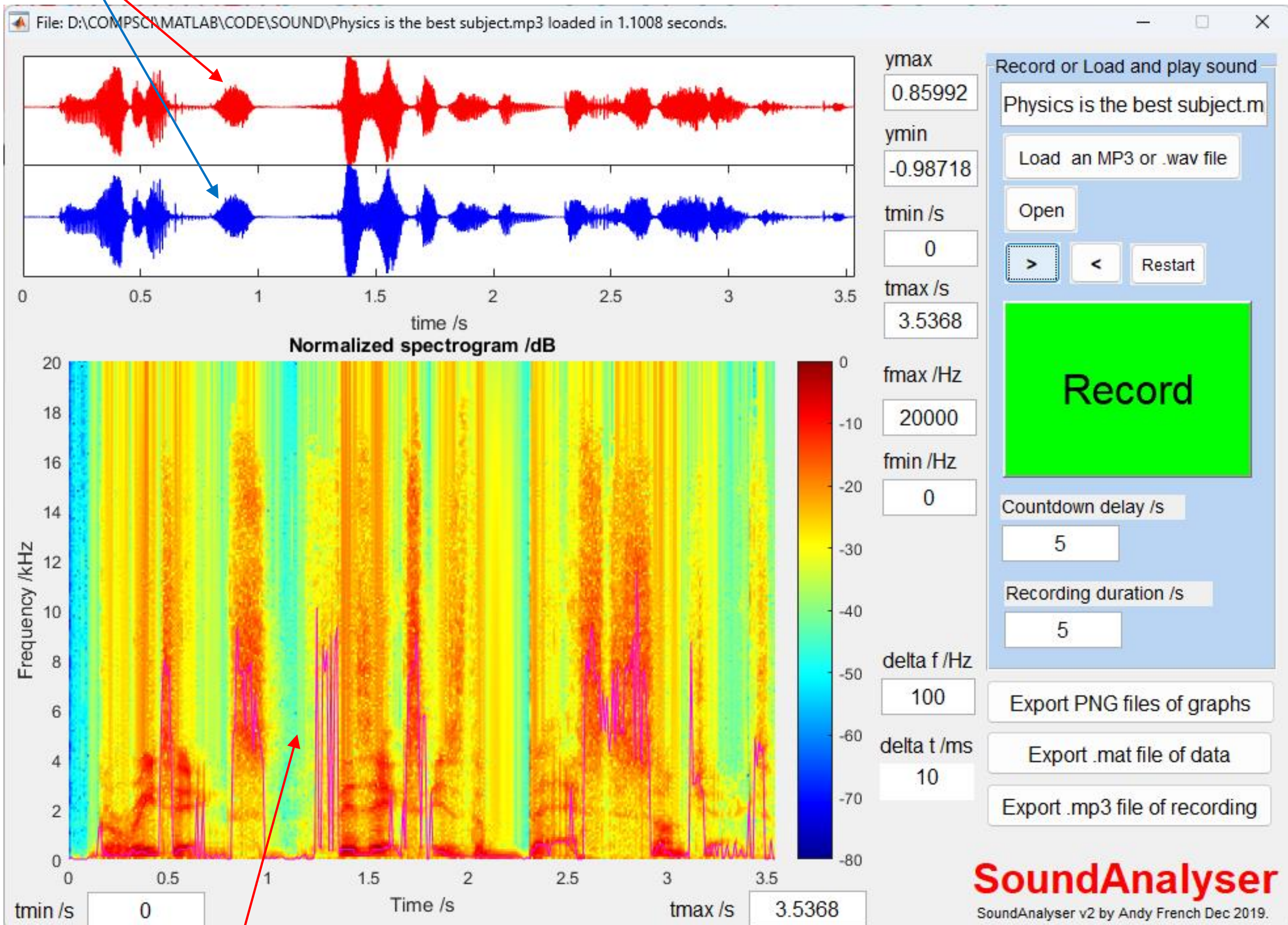


Length (plus *end correction*) is an odd number of quarter wavelengths

$$L+\tfrac{2}{3}r=\left(2n-1\right)\tfrac{1}{4}\lambda$$

$$n=1,2,3,4\ldots$$

Left and right *stereo* channels of a **sound wave recording**



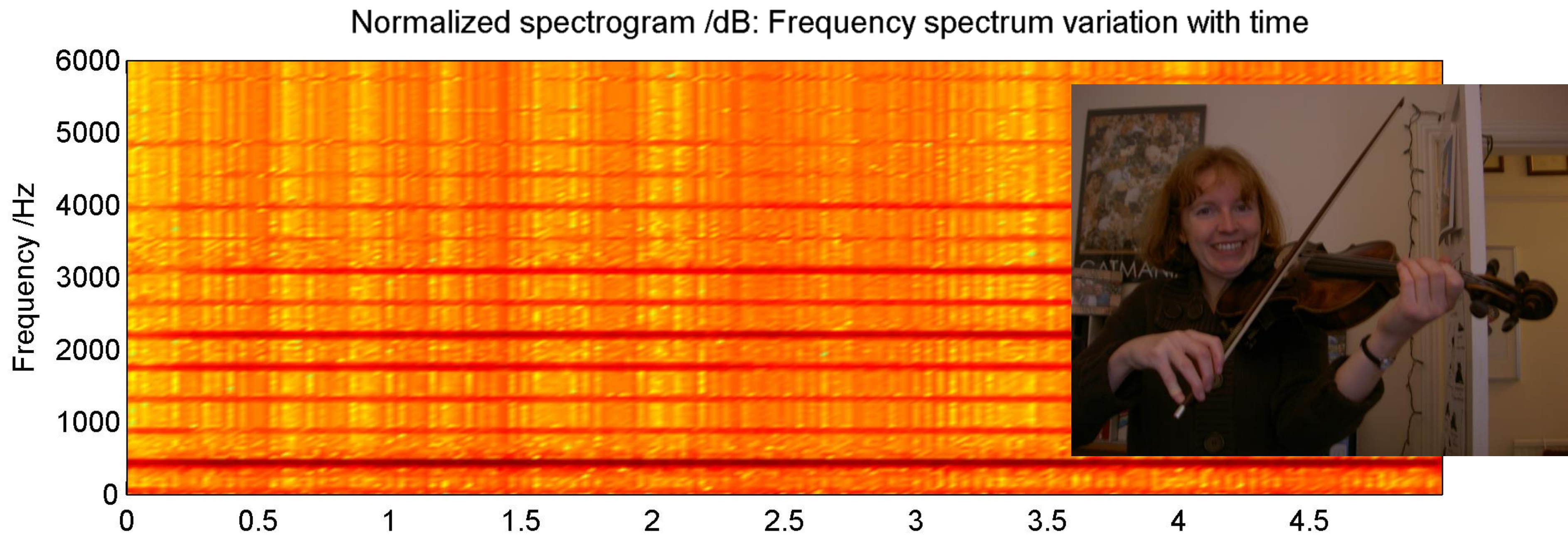
Plot of **frequency content** of waves vs time.
Colour scale is **decibels** (dB) with max power set at 0dB.

$$\mathrm{dB} = 10\log_{10}\left(signal\ power\right)$$

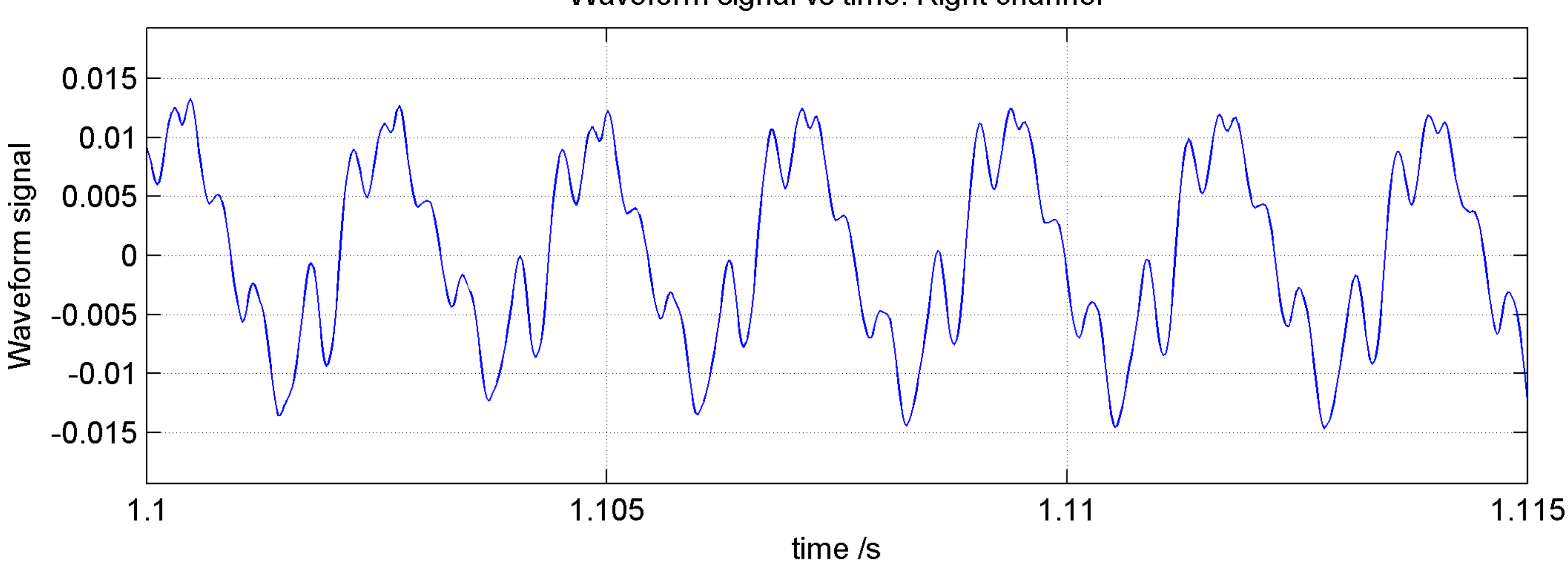
Musical harmony

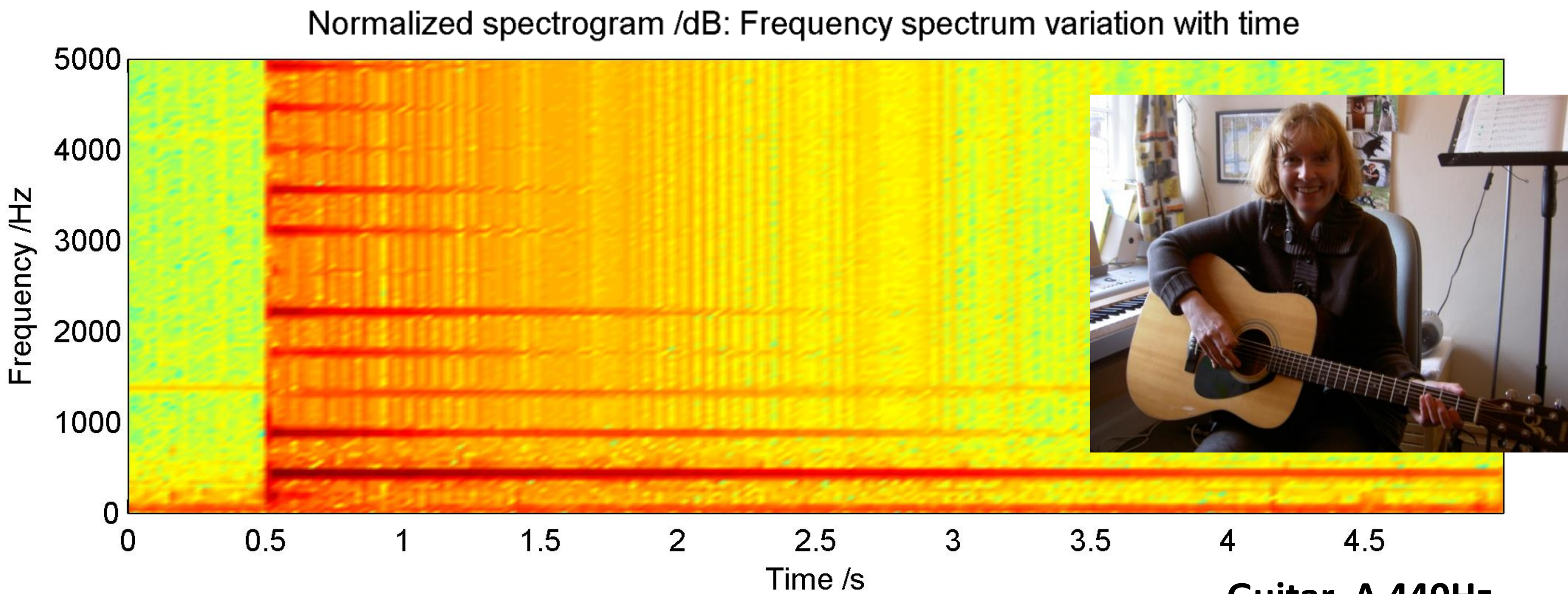
- The mathematics of music has been known since the time of Pythagoras, 2500 years ago
- Frequency intervals of simple fractions e.g. 3:2 (a fifth) yield 'harmonious' music
- An **octave** means a **frequency ratio of 2**. An octave above concert A (440Hz) is therefore 880Hz. An octave below is 220Hz.
- The modern 'equal-tempered scale' divides an octave (the frequency ratio 2) into twelve parts such that

$$\boxed{f_n = 2^{n/12} = \sqrt[\frac{n}{12}]{2}}$$

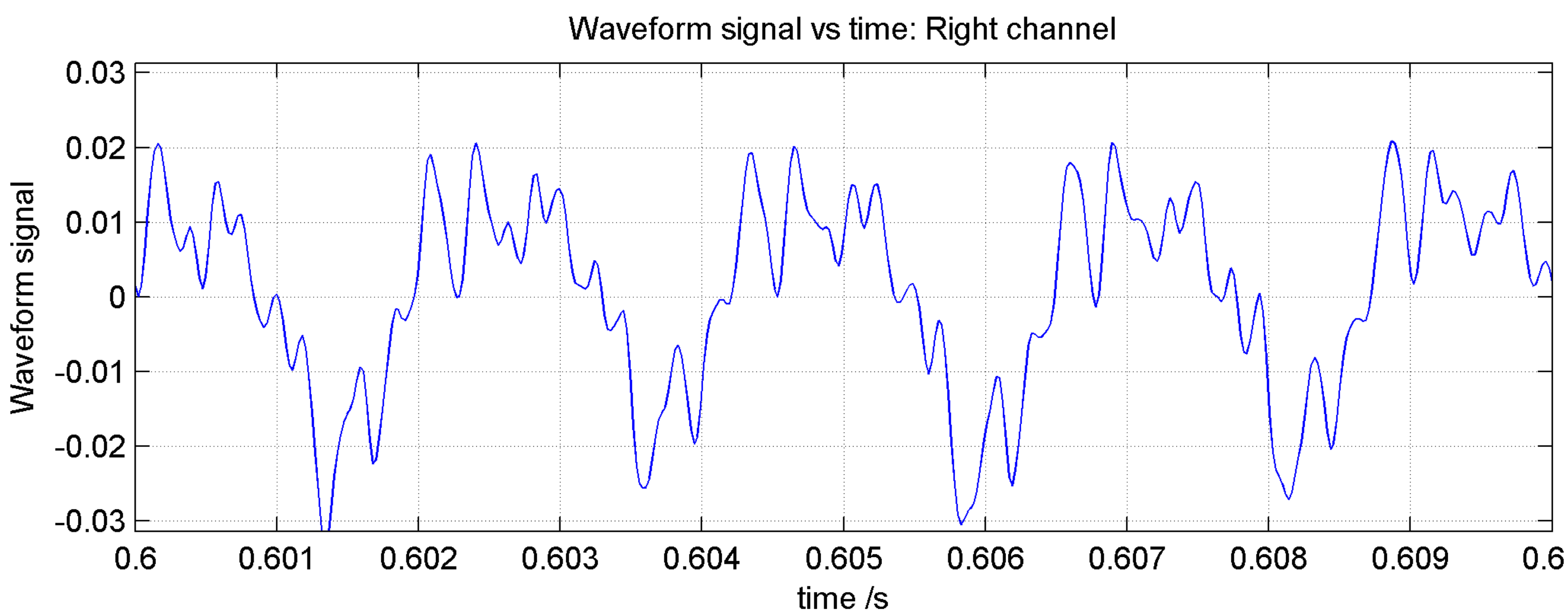


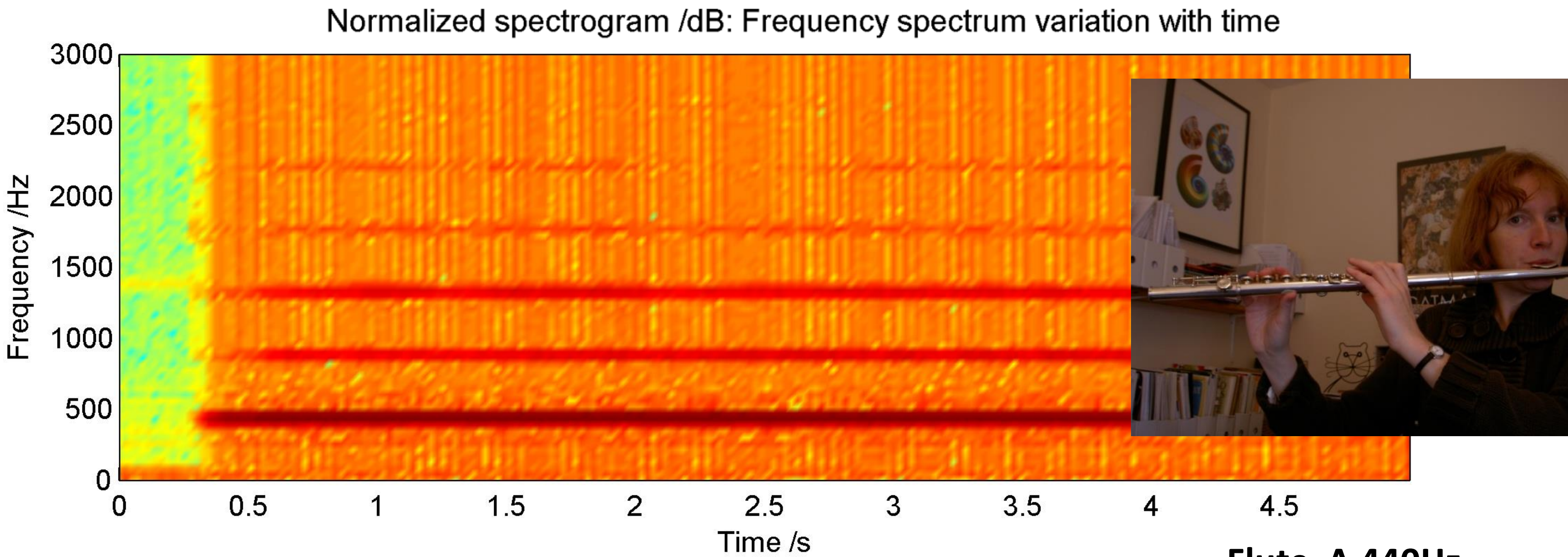
Violin A 440Hz



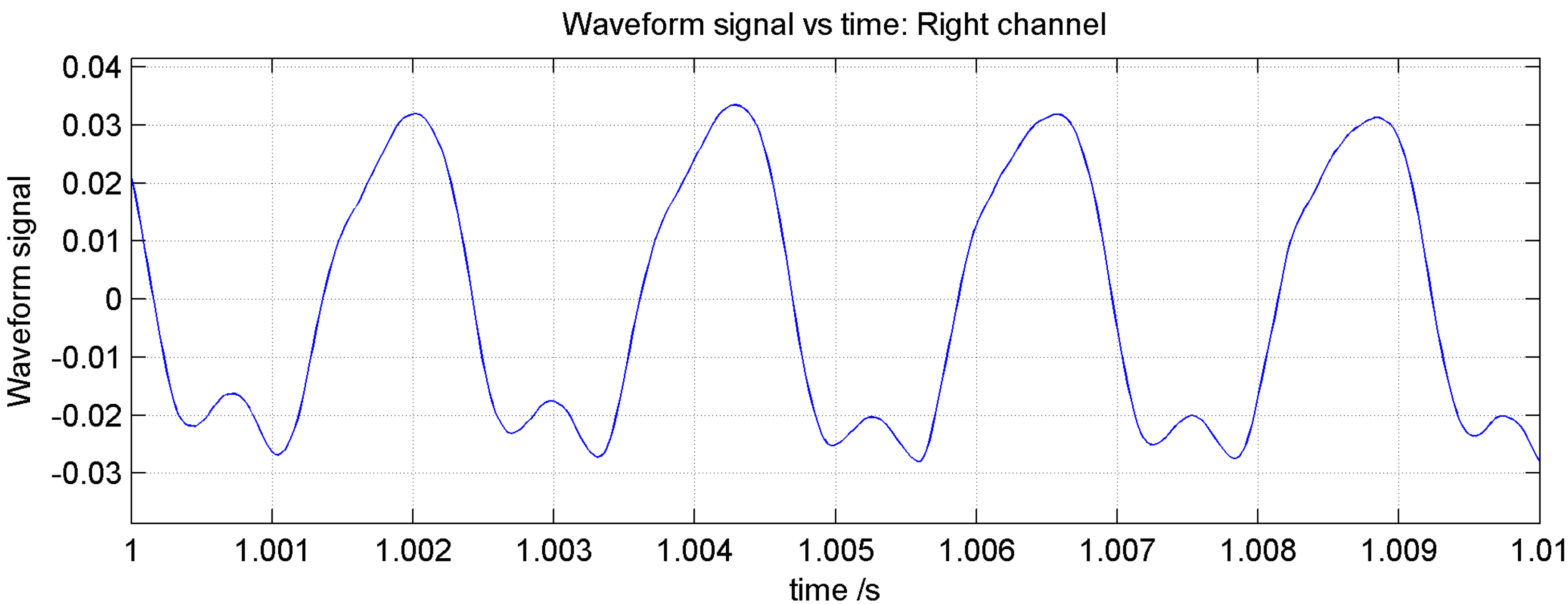


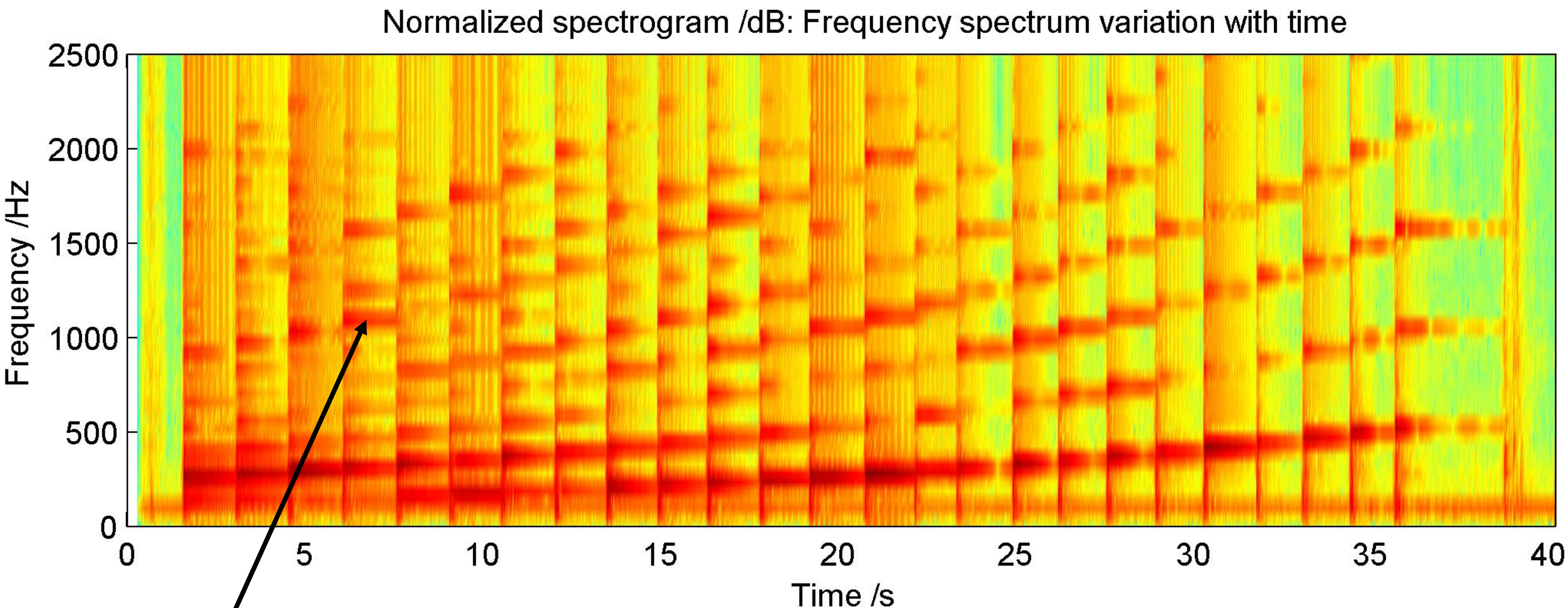
Guitar A 440Hz





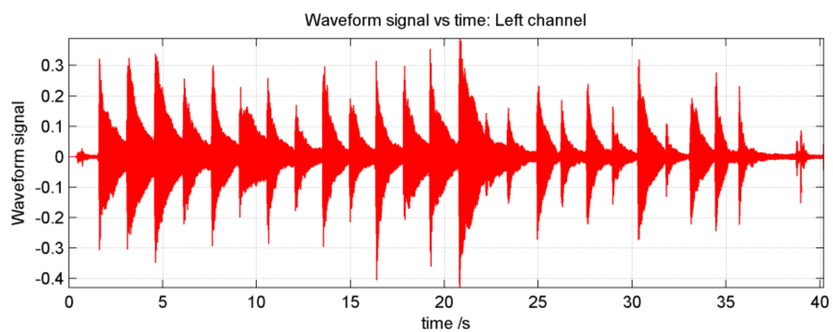
Flute A 440Hz

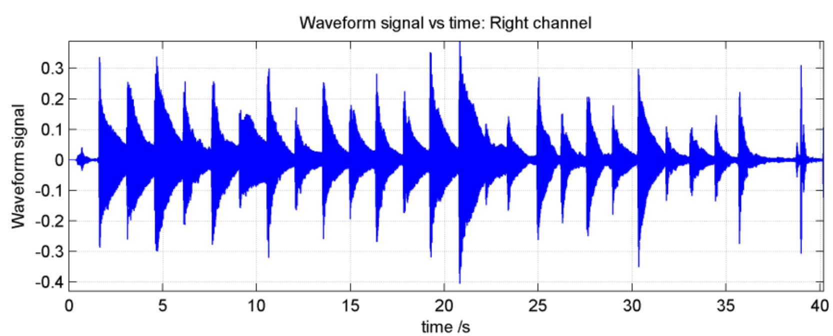




$$f_n = 2^{n/12}$$

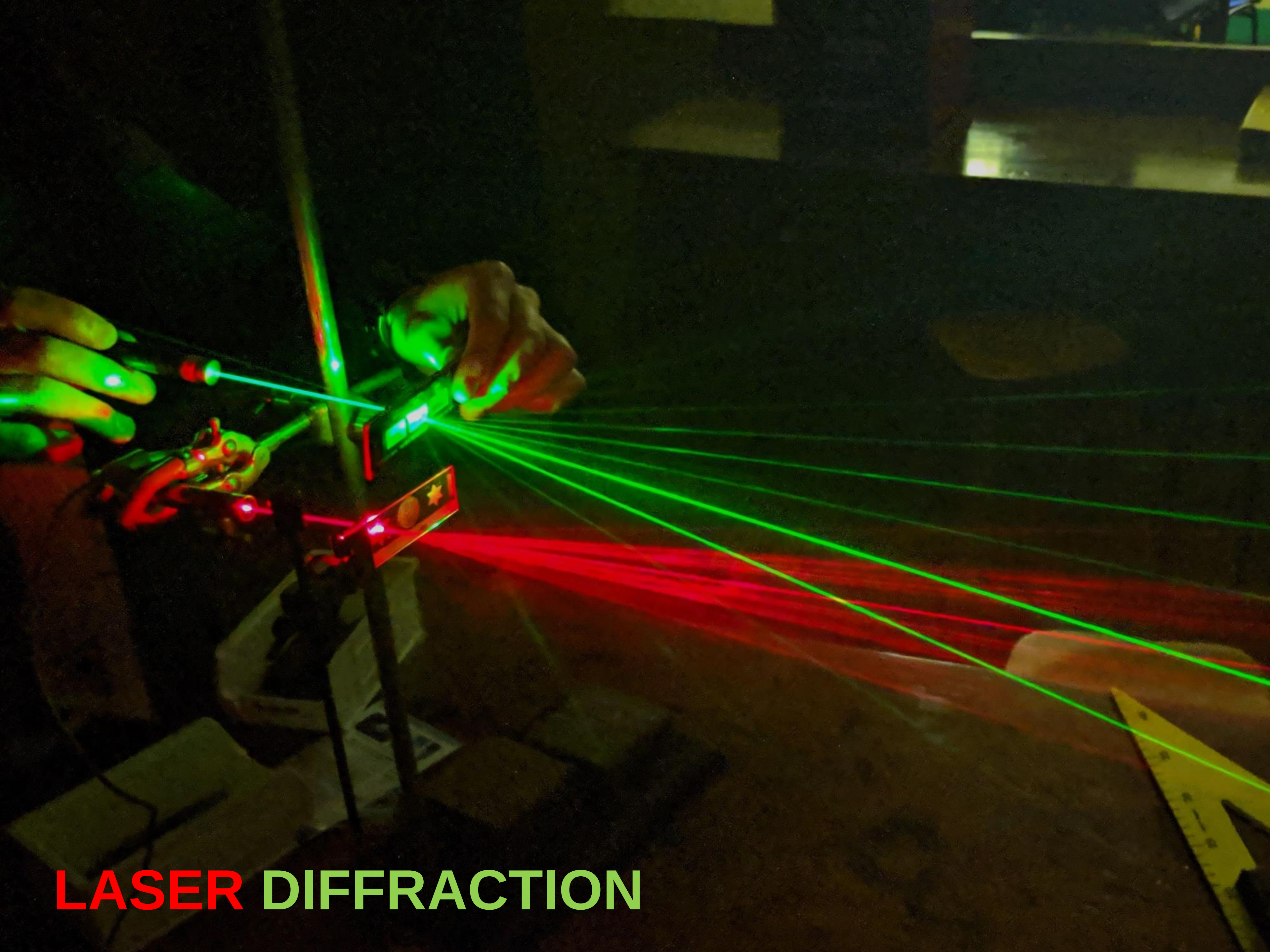
Notice *power law* of harmonics



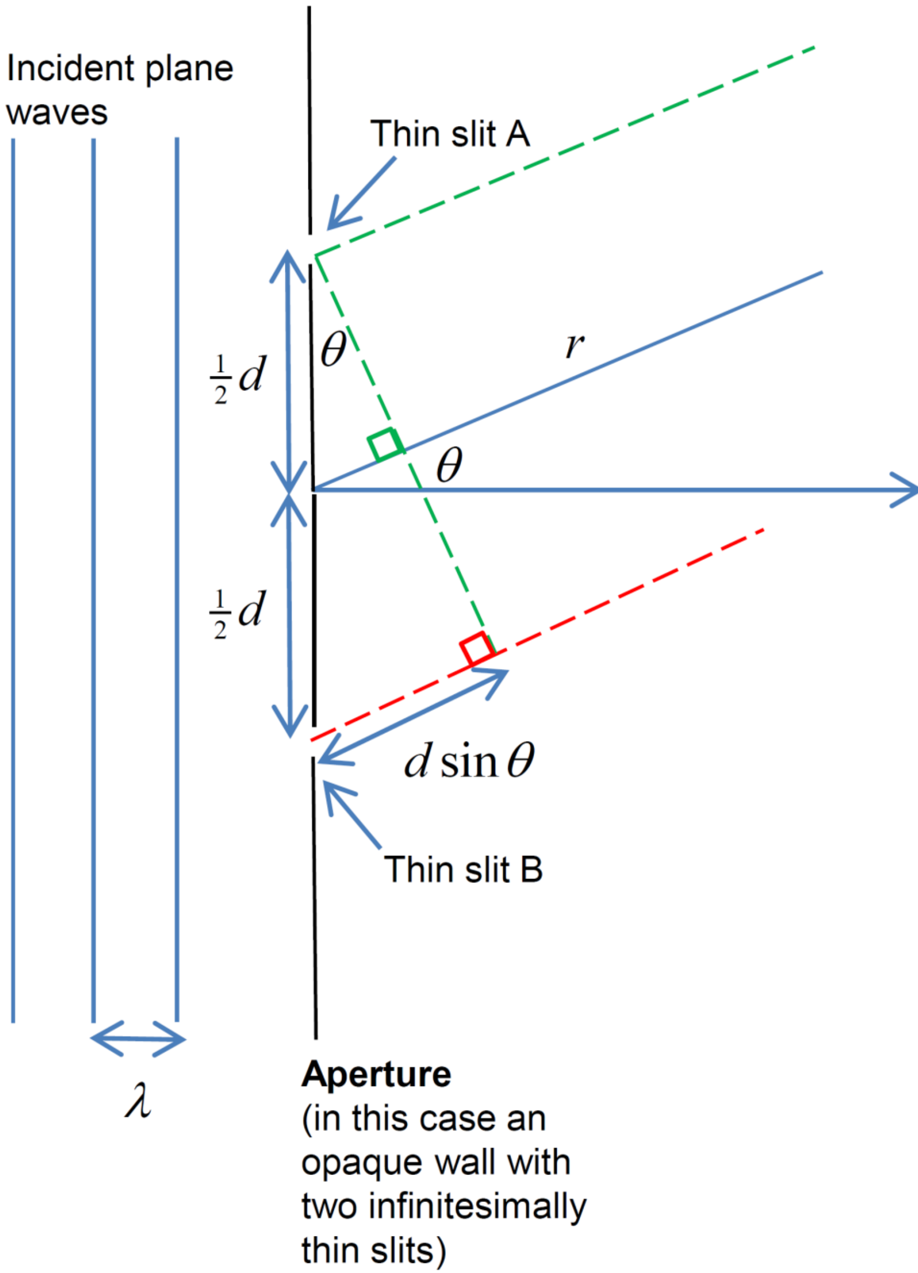


Chromatic scale (played on a piano) over two octaves

C,C#...Bb,B,
C,C#...Bb,B,C


LASER DIFFRACTION

Key geometrical idea from two infinitesimally thin slits ('Young's Slits')



Spherical waves will emanate from the slits, and interfere with each other.

For distances such that: $\boxed{r \gg \frac{d^2}{\lambda}}$
(we call this the **Far Field**)
we can assume waves from each slit are **plane waves**, for any given observational angle θ.

Constructive interference occurs when the *phase difference* between the waves from slits A and B is an integer multiple of 2π radians.

$$\frac{2\pi}{\lambda} d\sin\theta = 2\pi n$$

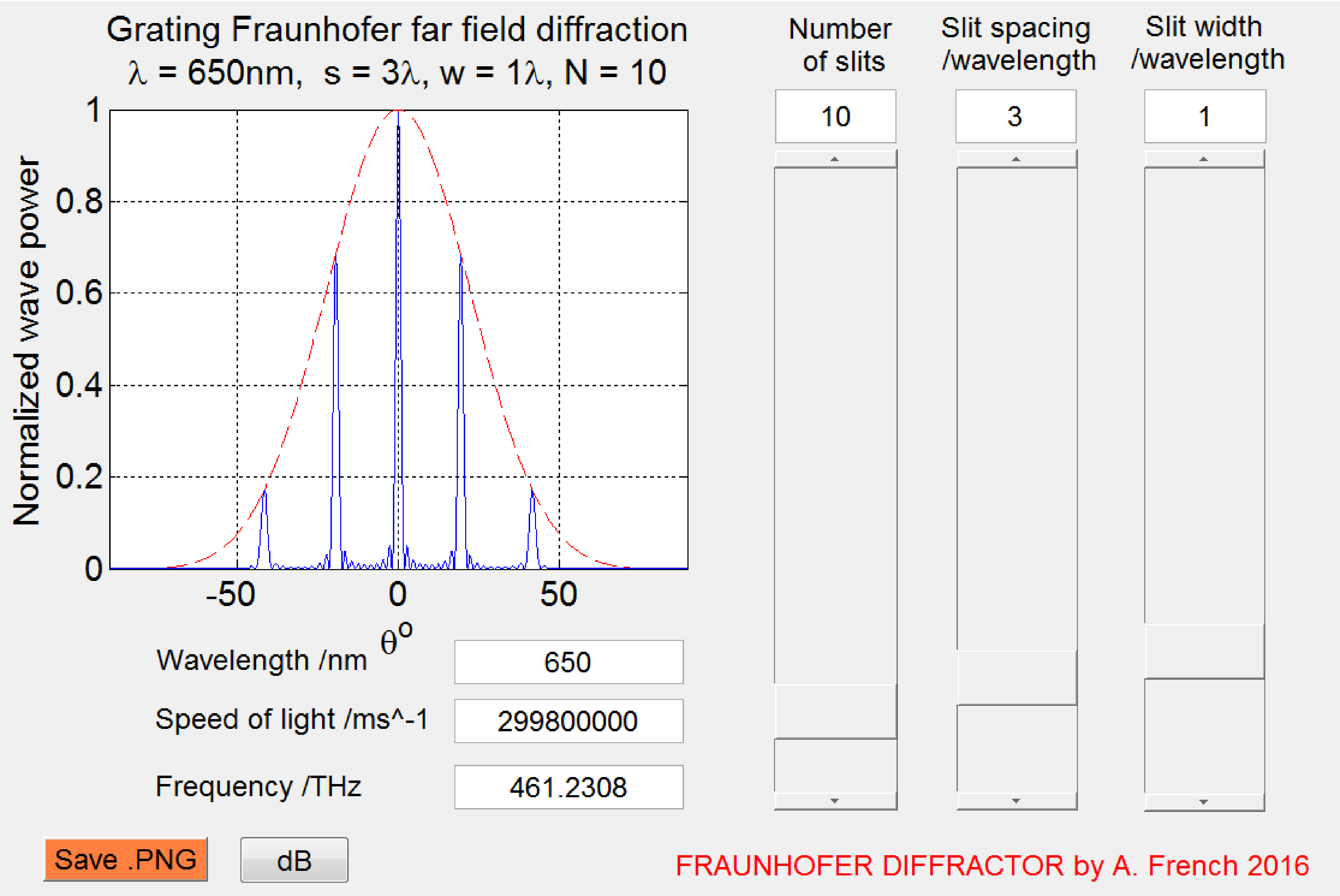
Wavenumber k

Path difference between waves from A and B

Integer n

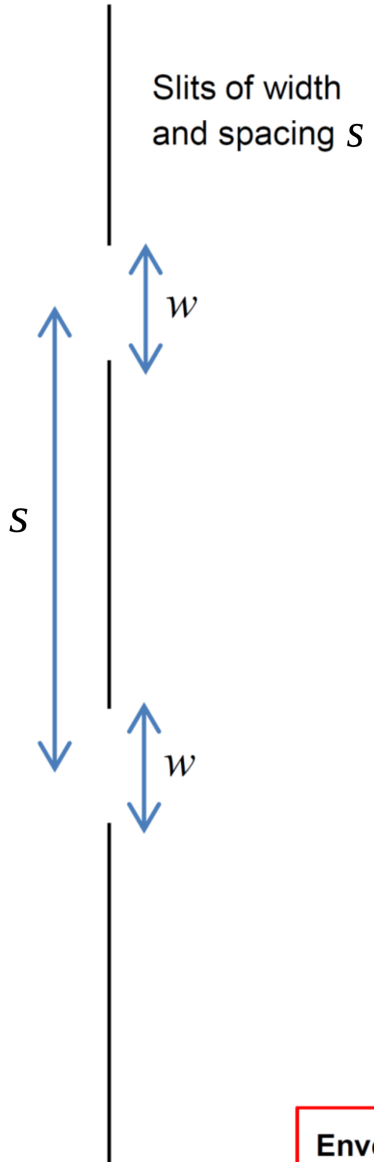
Hence expect **maxima** in the resulting **Far Field Diffraction pattern** (e.g. spots of a laser on a wall) at angles

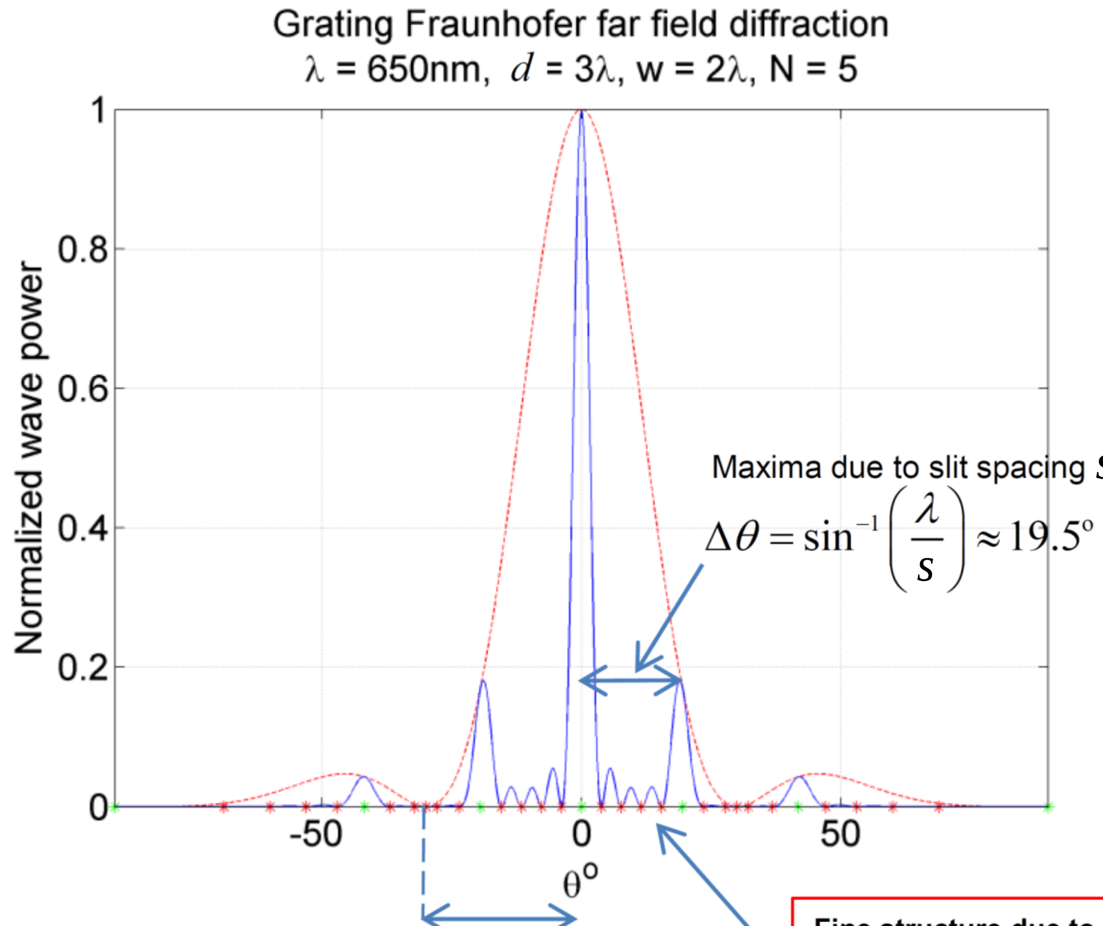
$$\boxed{\sin\theta = \frac{n\lambda}{d}}$$



Grating Fraunhofer far field diffraction
λ = 650nm, s = 3λ, w = 1λ, N = 10
Normalized wave power
1
0.8
0.6
0.4
0.2
0
-50
0
50
θ°
Number of slits
10
Slit spacing /wavelength
3
Slit width /wavelength
1
Wavelength /nm
650
Speed of light /ms^-1
299800000
Frequency /THz
461.2308
Save .PNG
dB
FRAUNHOFER DIFFRACTOR by A. French 2016

$$|\psi|^2 = \frac{A^2}{N^2 r^2}\left(\frac{\sin\left(\frac{\pi}{\lambda} w \sin\theta\right)}{\frac{\pi}{\lambda} w \sin\theta} \times \frac{\sin\left(\frac{\pi}{\lambda} N s \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} s \sin\theta\right)}\right)^2$$





Envelope due to finite slit width

Zeros at: $\theta = \sin^{-1}\left(\frac{n\lambda}{w}\right);\ n \neq 0$

Fine structure due to number of slits
(i.e. overall size of aperture)

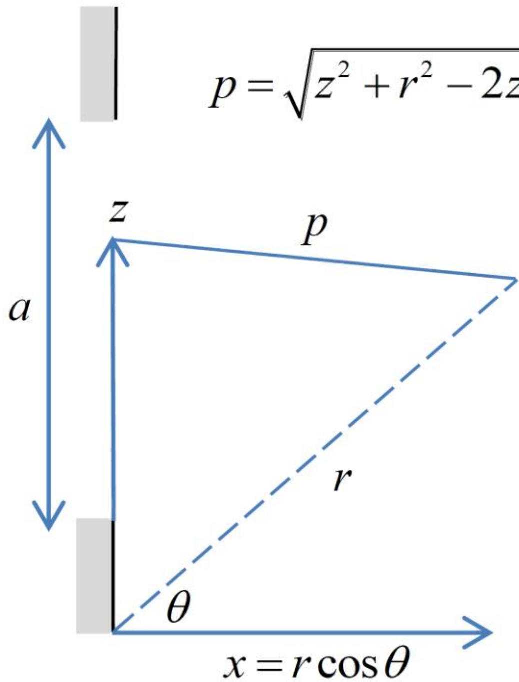
Zeros at: $\theta = \sin^{-1}\left(\frac{p\lambda}{Ns}\right)$

But *maxima* when $\frac{p}{N}$ integer m

Modelling general diffraction effects from a finite width slit

We can use a computer to evaluate the wavefield in the vicinity of a finite width slit which is uniformly illuminated. We are therefore not restricted to the limitations of the Fraunhofer and Fresnel regimes

$$p = \sqrt{z^2 + r^2 - 2zr\cos\left(\tfrac{1}{2}\pi - \theta\right)} = \sqrt{z^2 + r^2 - 2zr\sin\theta}$$

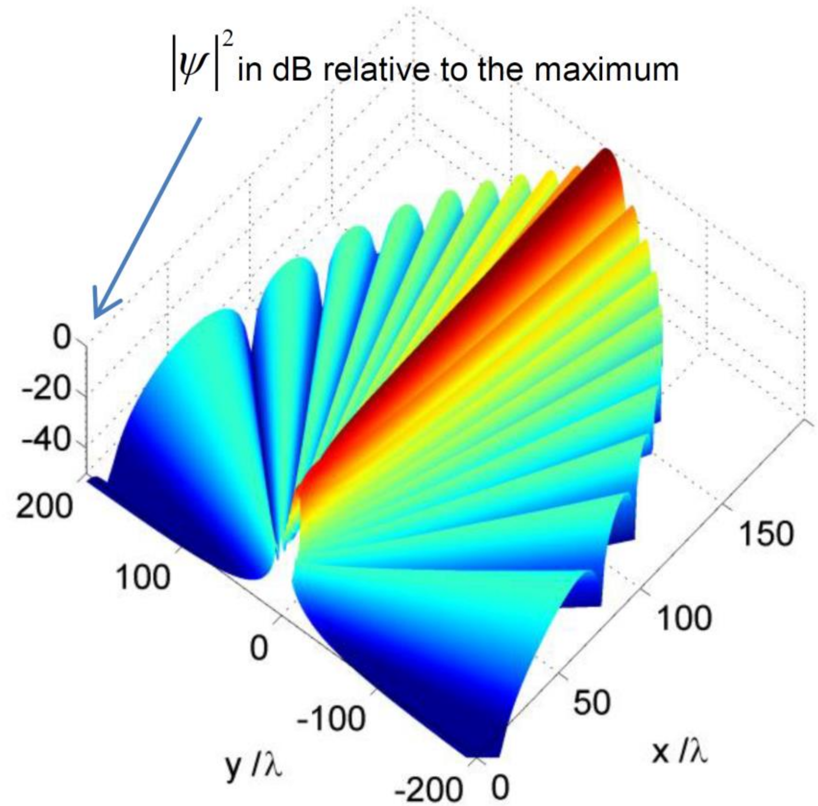


$$\psi(r,t) = \frac{A}{a} e^{-i\omega t} \int \frac{e^{ikp}}{p} dz$$

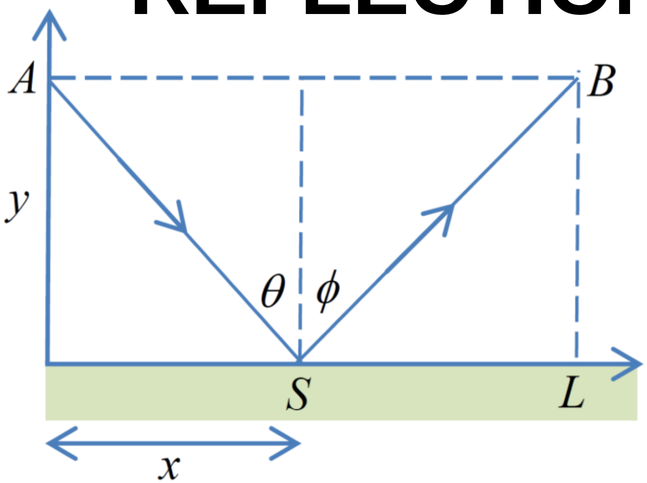
The idea is to evaluate the wavefield power at various r and θ values. The integral is approximated by a *sum* based on a large number of finite dz values

In the examples below :

$$dz = \tfrac{1}{500} a$$



REFLECTION



$$x = y\tan\theta$$

$$L - x = Y\tan\phi$$

$$\boxed{t = \frac{\sqrt{x^2+y^2}}{c/n} + \frac{\sqrt{(L-x)^2+y^2}}{c/n}}$$

$$\frac{\partial t}{\partial x} = \frac{n}{c}\left(\frac{\frac{1}{2}2x}{\sqrt{x^2+y^2}} + \frac{\frac{1}{2}2(L-x)(-1)}{\sqrt{(L-x)^2+y^2}}\right)$$

$$\frac{1}{\cos^2\theta} = 1+\tan^2\theta$$

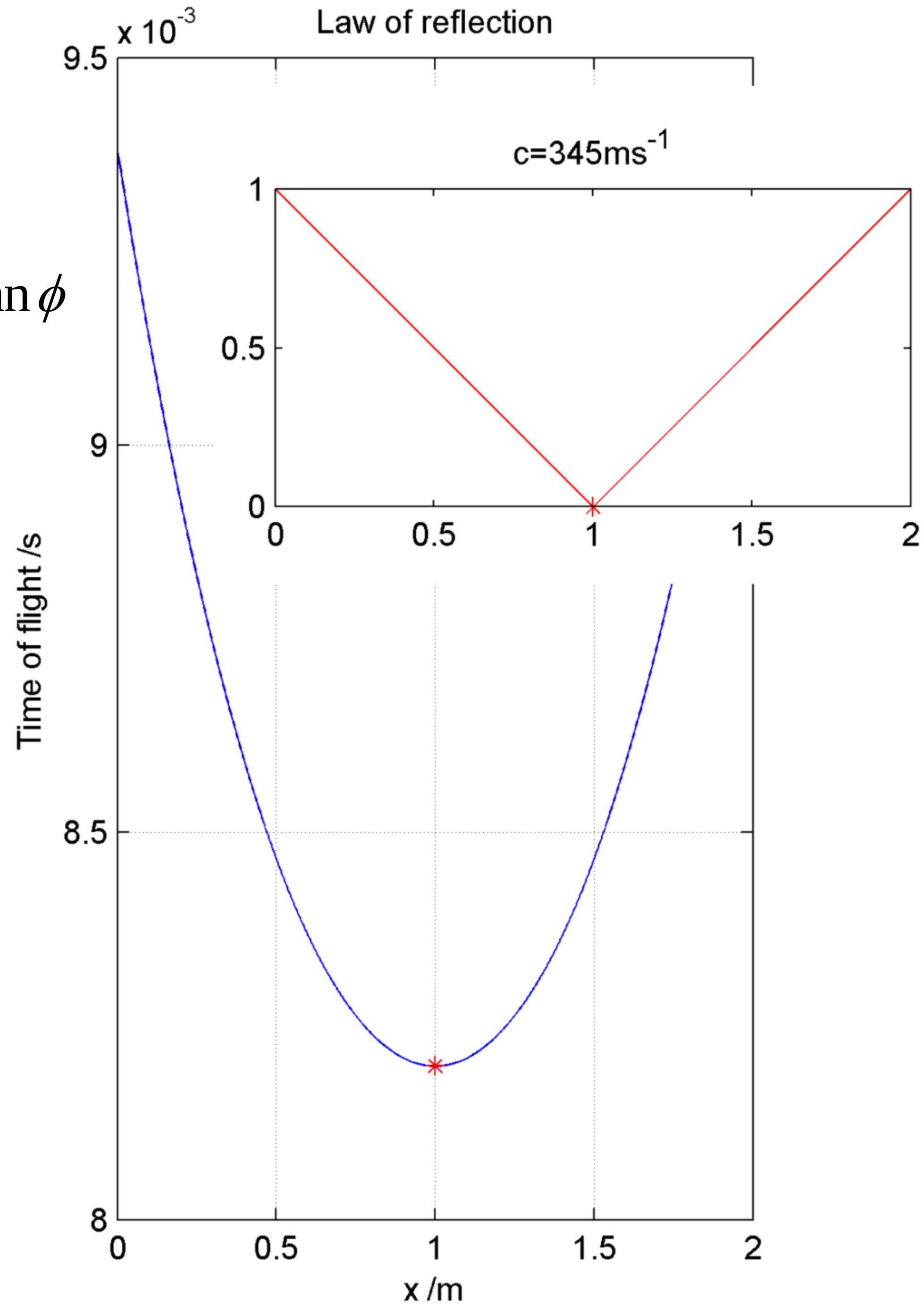
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \frac{\partial t}{\partial x} = \frac{n}{c}\left(\frac{y\tan\theta}{\sqrt{y^2\tan^2\theta + y^2}} - \frac{y\tan\phi}{\sqrt{y^2\tan^2\phi + y^2}}\right)$$

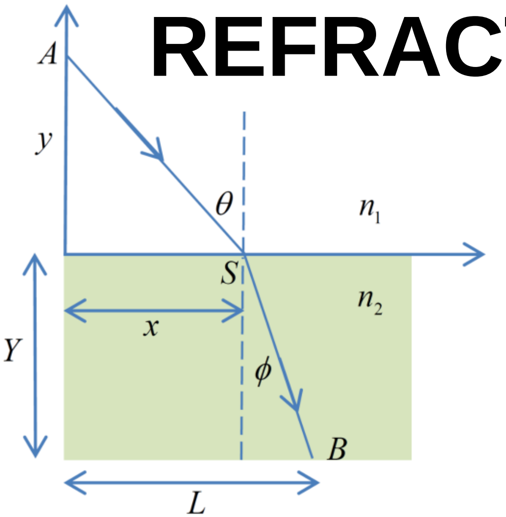
$$\therefore \frac{\partial t}{\partial x} = \frac{n}{c}(\sin\theta - \sin\phi)$$

The travel time is minimized when $\frac{\partial t}{\partial x} = 0$

$$\boxed{\begin{aligned}&\therefore \sin\theta = \sin\phi\\ &\therefore \theta = \phi\end{aligned}}$$



REFRACTION



$$x = y\tan\theta$$
$$L - x = Y\tan\phi$$

$$\boxed{t = \frac{\sqrt{x^2 + y^2}}{c/n_1} + \frac{\sqrt{(L-x)^2 + Y^2}}{c/n_2}}$$

$$\frac{\partial t}{\partial x} = \frac{1}{c}\left(\frac{\frac{1}{2}2xn_1}{\sqrt{x^2+y^2}} + \frac{\frac{1}{2}2(L-x)(-1)n_2}{\sqrt{(L-x)^2+Y^2}}\right)$$

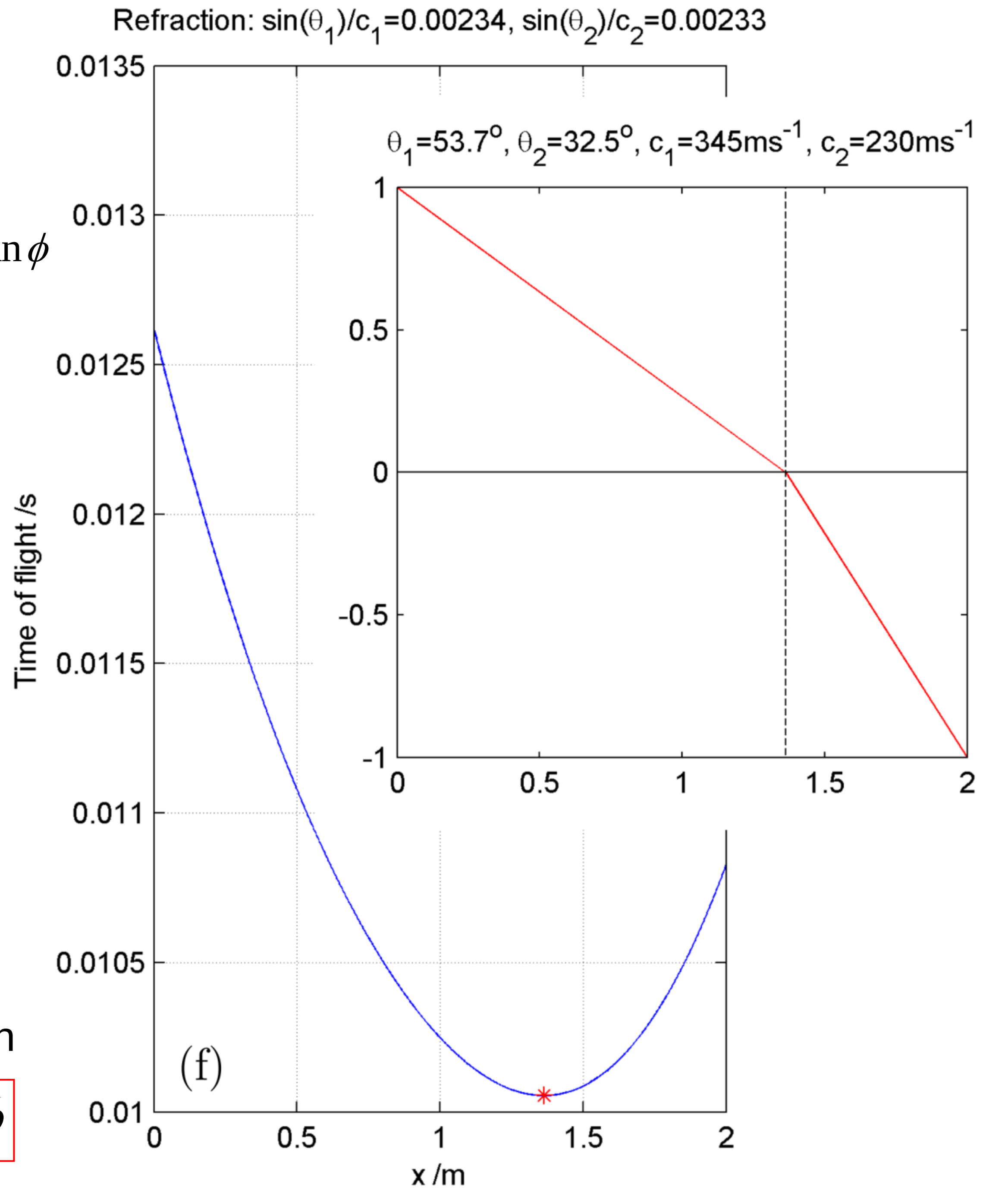
$$\frac{\partial t}{\partial x} = \frac{1}{c}\left(\frac{y\tan\theta n_1}{y\sqrt{\tan^2\theta+1}} + \frac{-Y\tan\phi n_2}{Y\sqrt{\tan^2\phi+1}}\right)$$

$$\frac{\partial t}{\partial x} = \frac{1}{c}(\cos\theta\tan\theta n_1 - \cos\phi\tan\phi n_2)$$

$$\frac{\partial t}{\partial x} = \frac{1}{c}(n_1\sin\theta - n_2\sin\phi)$$

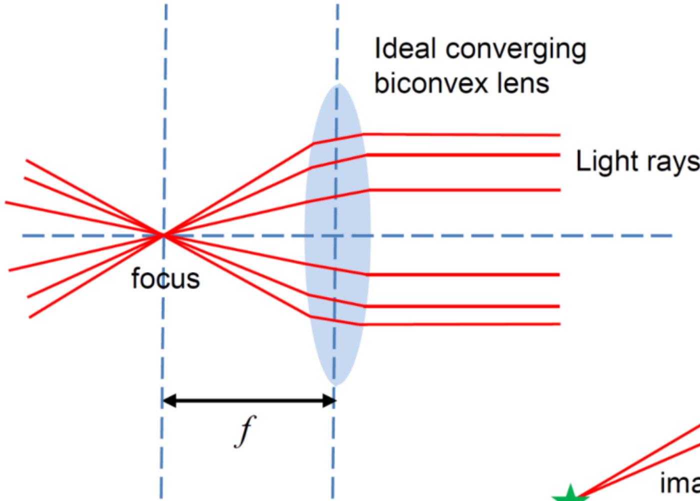
The travel time is minimized when

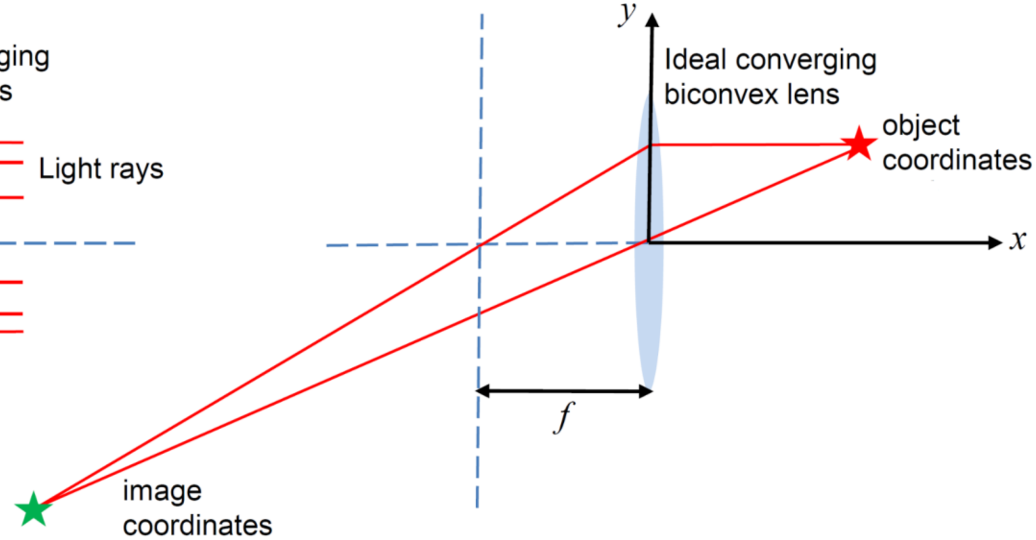
$$\frac{\partial t}{\partial x} = 0 \qquad \boxed{\therefore n_1\sin\theta = n_2\sin\phi}$$

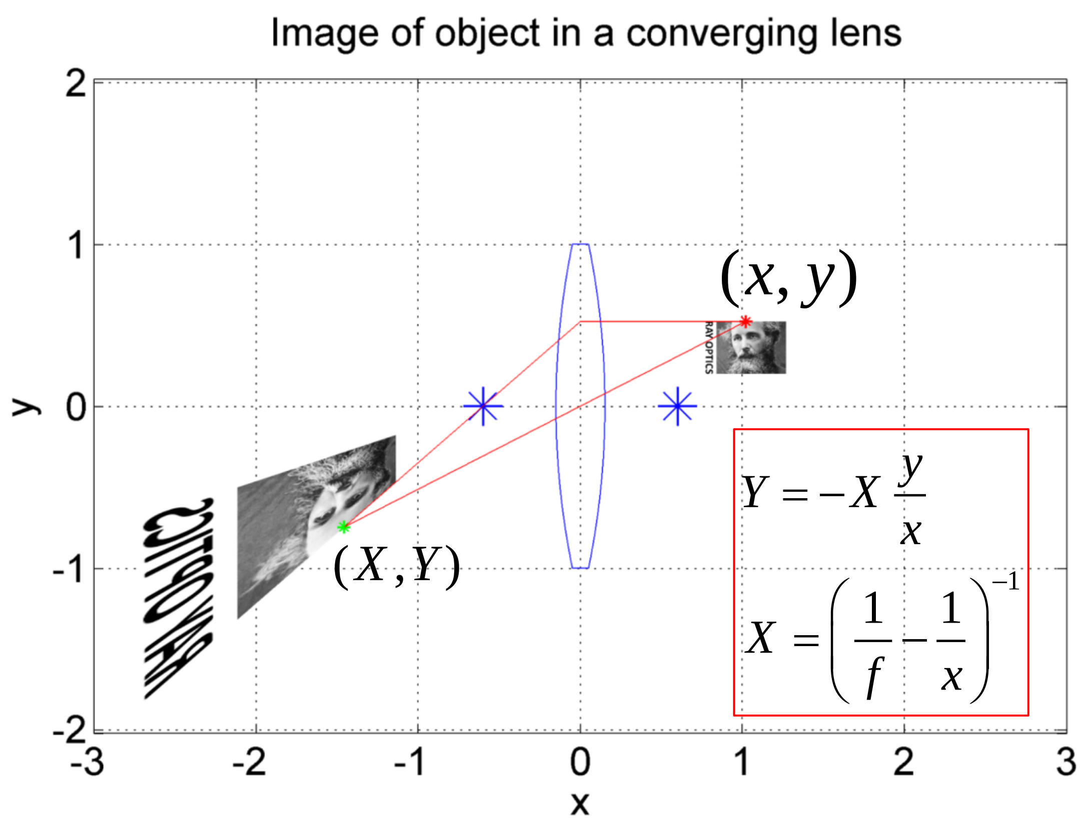


RAY OPTICS:

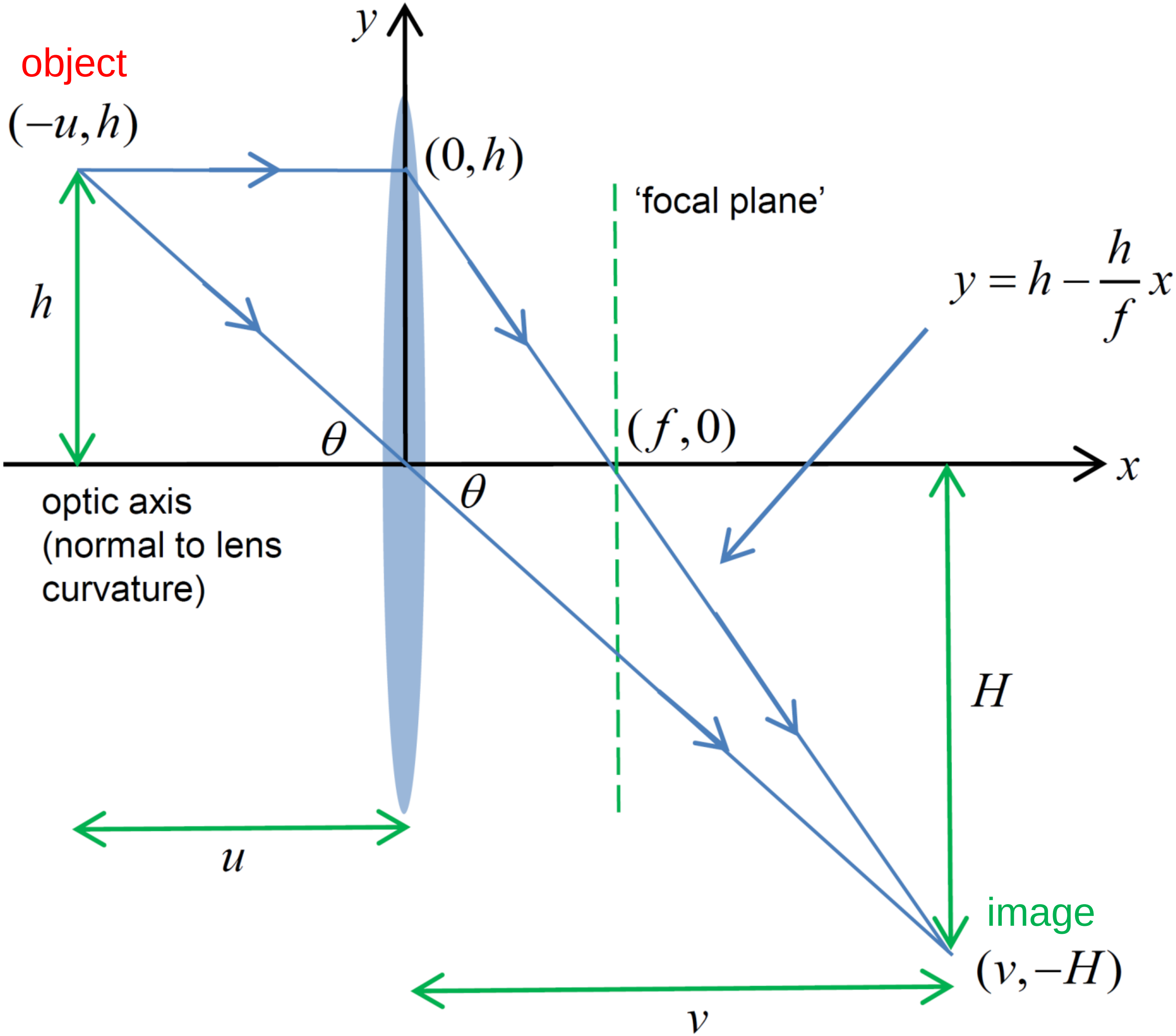
THIN BICONVEX LENS







THIN LENS EQUATION



$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

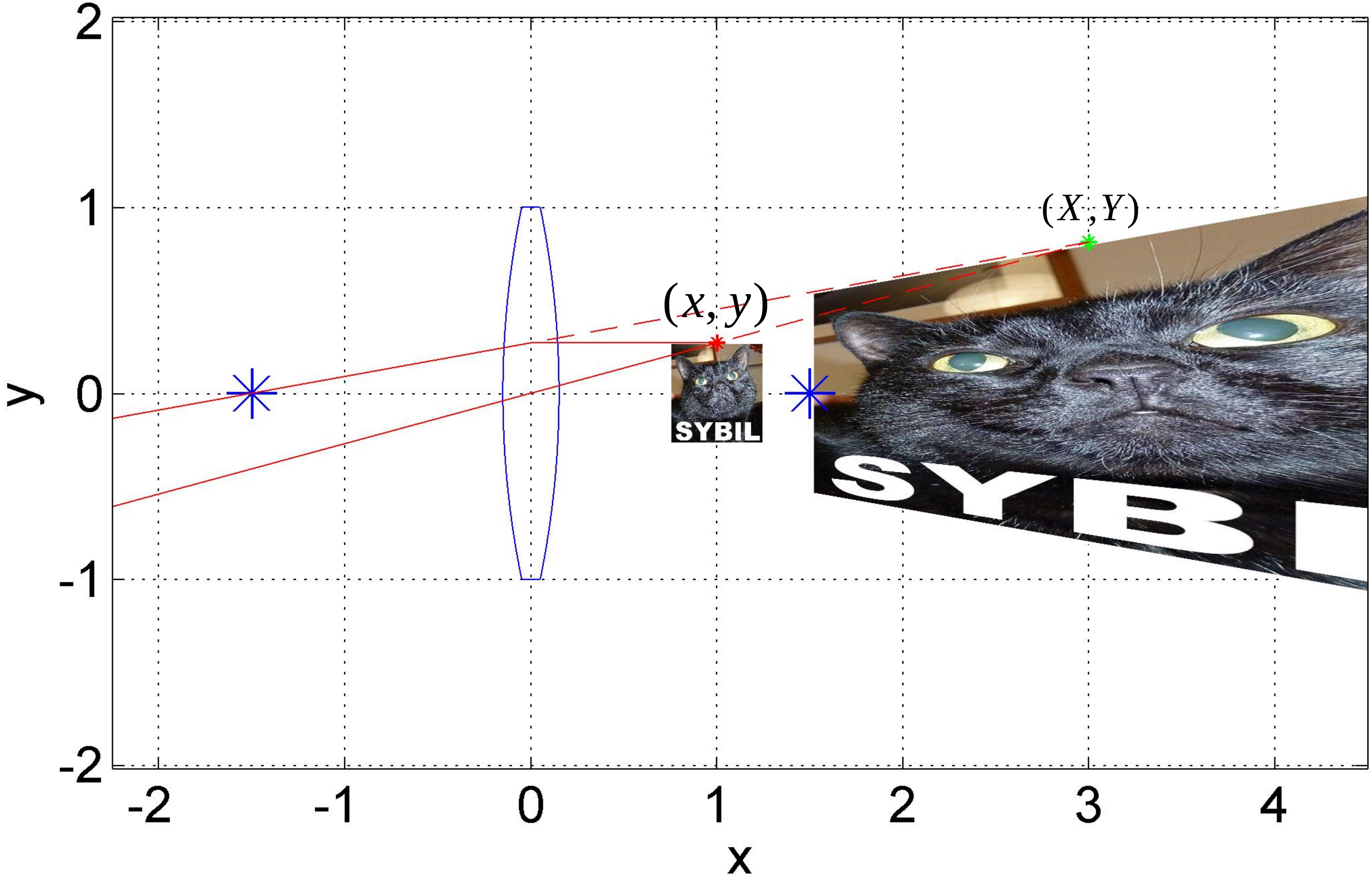
$$\frac{H}{v} = \frac{h}{u}$$

$$v = \left(\frac{1}{f} - \frac{1}{u}\right)^{-1}$$

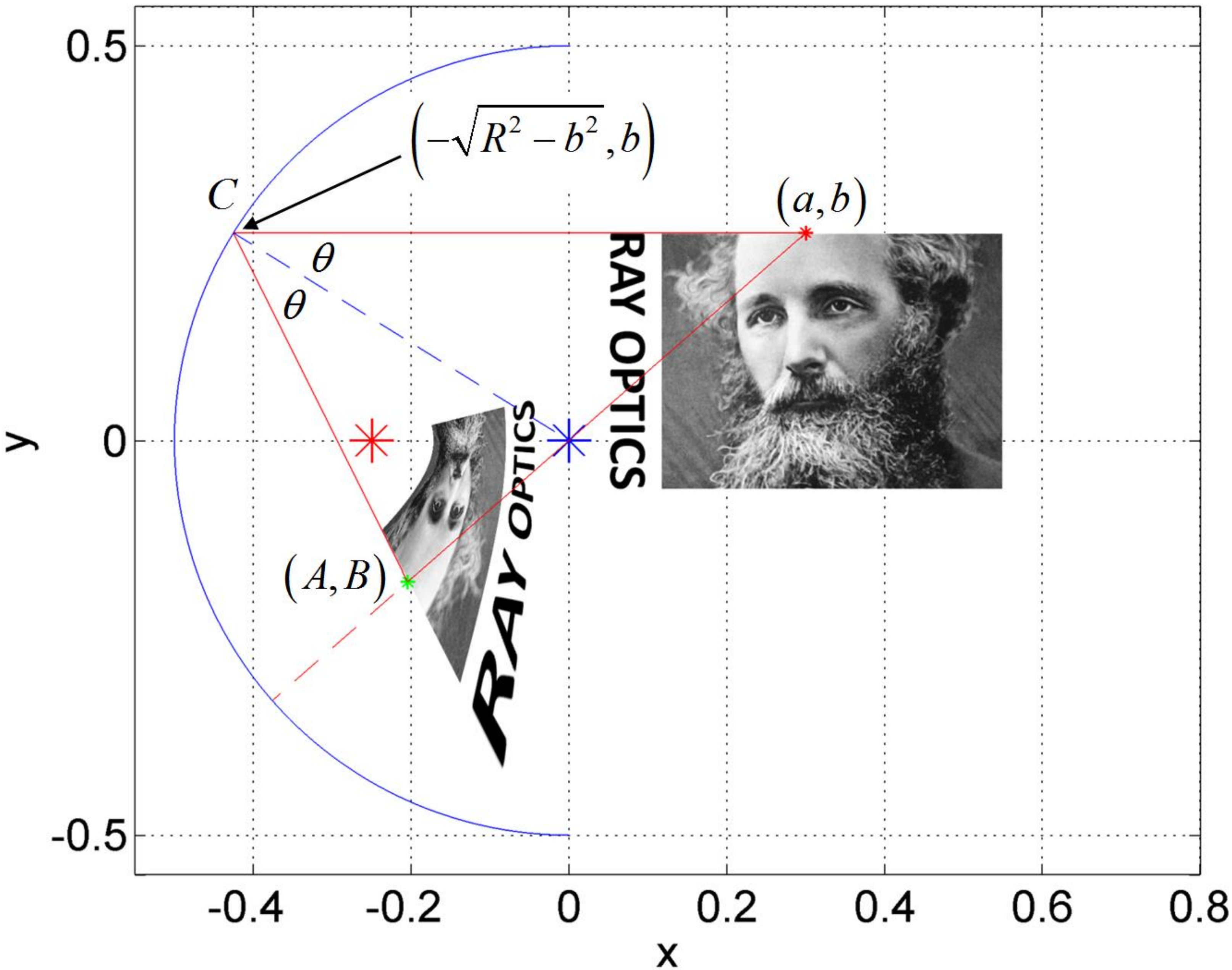
$$\boxed{\begin{aligned} Y &= -X\,\frac{y}{x} \\ X &= \left(\frac{1}{f} - \frac{1}{x}\right)^{-1} \end{aligned}}$$

Image is a *coordinate transformation*

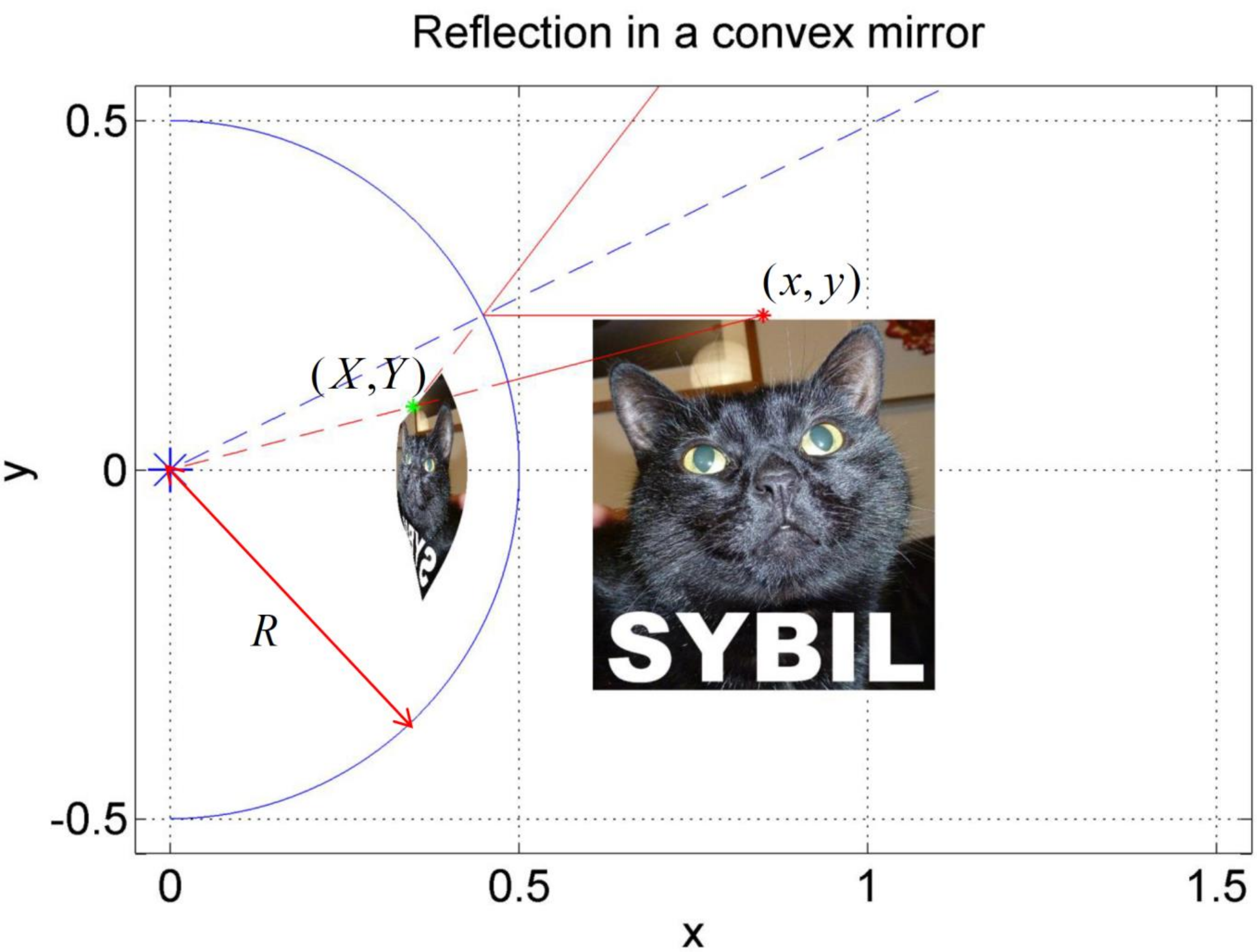
Virtual image of object in a magnifying lens



Reflection in a concave mirror




(a)
optics
smartLine
(b)



$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{x}\right)$$

$$k = \frac{x}{\cos(2\alpha)}$$

$$Y = \frac{k\sin\alpha}{\tfrac{k}{R} - \cos\alpha + \tfrac{x}{y}\sin\alpha}$$

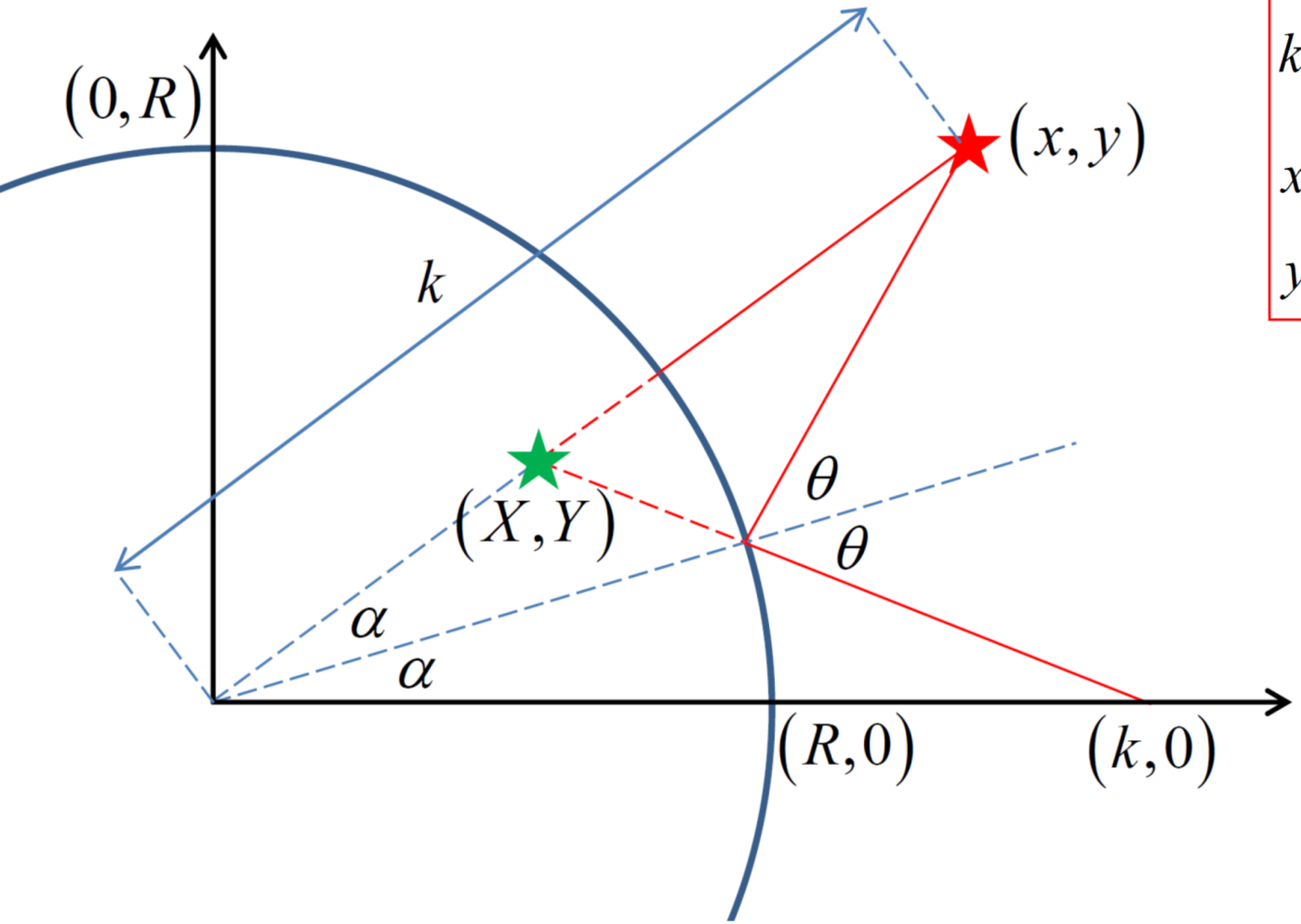
$$X = x\frac{Y}{y}$$

Virtual image from object coordinates

We see an upright, distorted *virtual image* in a cylindrical mirror.

i.e. the *apparent source* of (diverging) light rays from the mirror

Convex mirror object to virtual image transformation (and reverse transformation)



$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{Y}{X}\right)$$

$$k = \frac{R\left(Y\cos\alpha - X\sin\alpha\right)}{Y - R\sin\alpha}$$

$$x = k\cos\left(2\alpha\right)$$

$$y = k\sin\left(2\alpha\right)$$

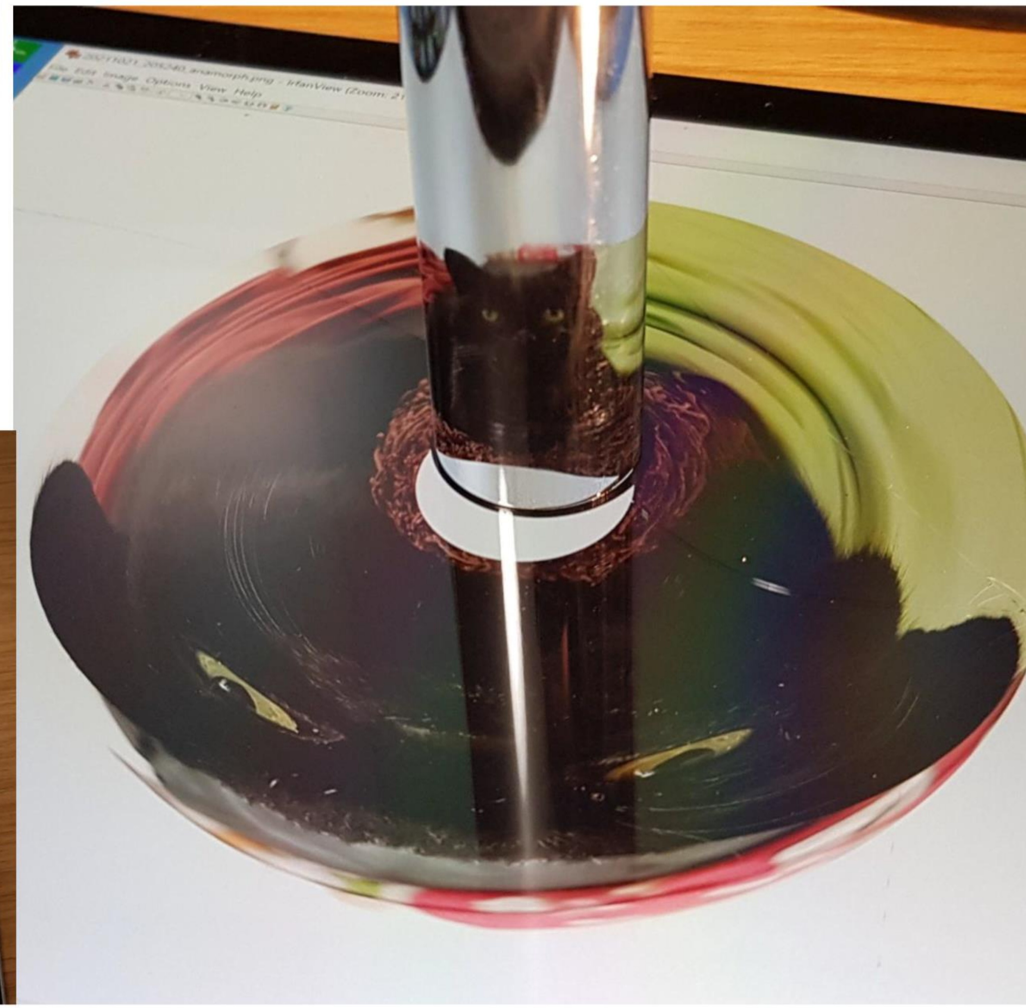
$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{x}\right)$$

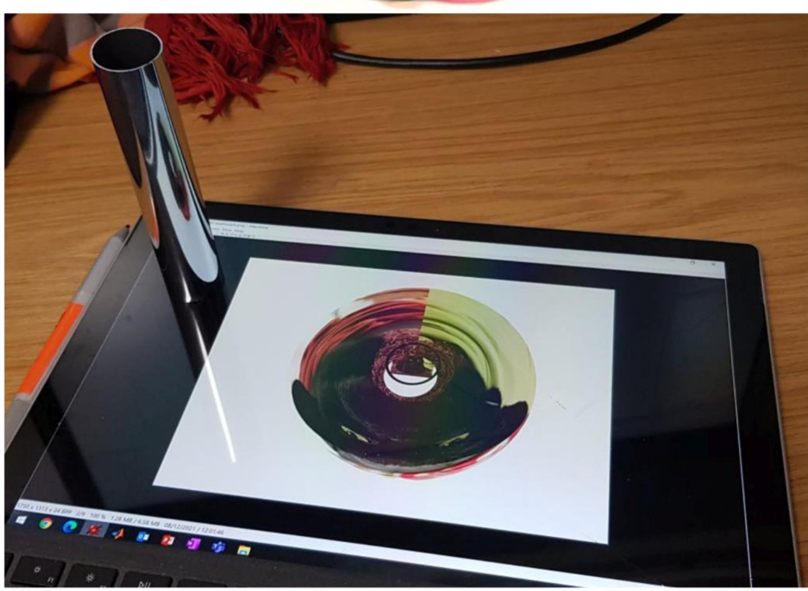
$$k = \frac{x}{\cos\left(2\alpha\right)}$$

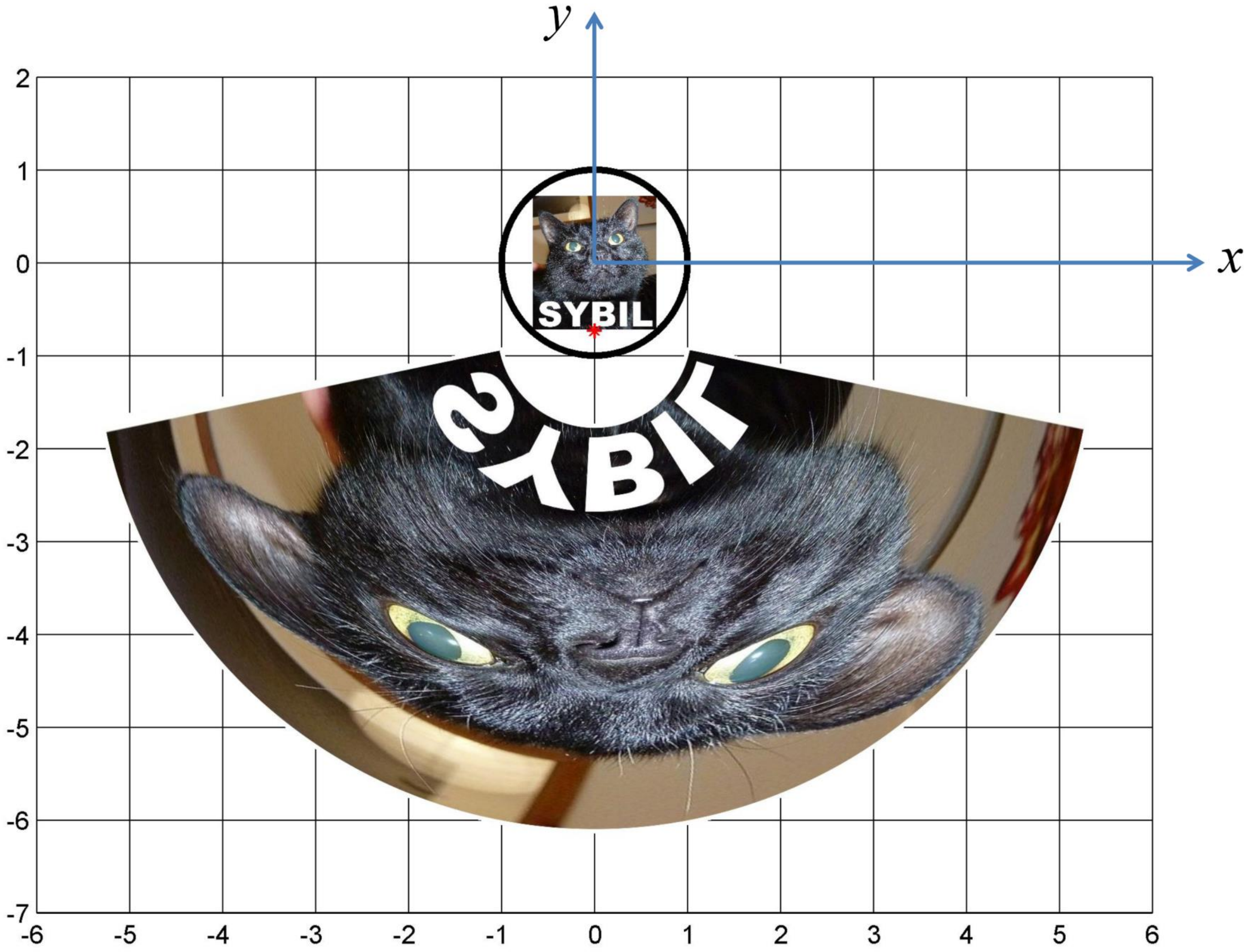
$$Y = \frac{k\sin\alpha}{\frac{k}{R} - \cos\alpha + \frac{x}{y}\sin\alpha}$$

$$X = x\frac{Y}{y}$$

Sybil the cat was unperturbed by this anamorphic transformation.

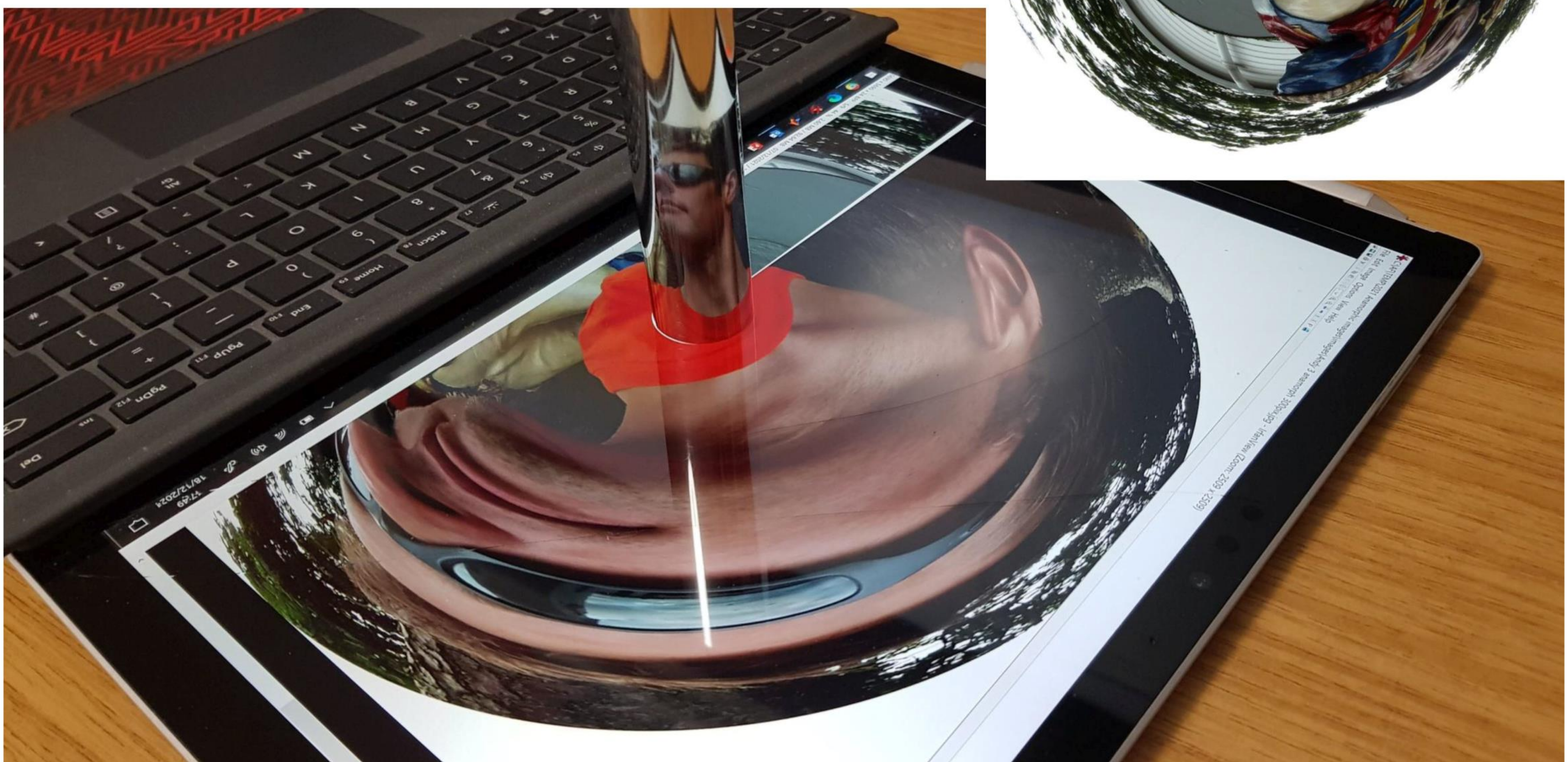






arc_deg = 160   Rf = 3

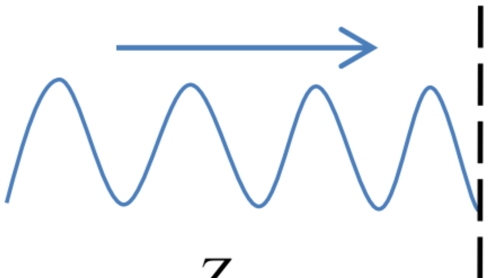
‘Fear and loathing’ in Portmeirion ...



Reflection and transmission of waves on boundaries

$$Z = \rho c$$ acoustic impedance

$$\boxed{\frac{A_R}{A_I} = \frac{Z_1 - Z_2}{Z_2 + Z_1}} \qquad \boxed{\frac{A_T}{A_I} = \frac{2Z_1}{Z_2 + Z_1}}$$



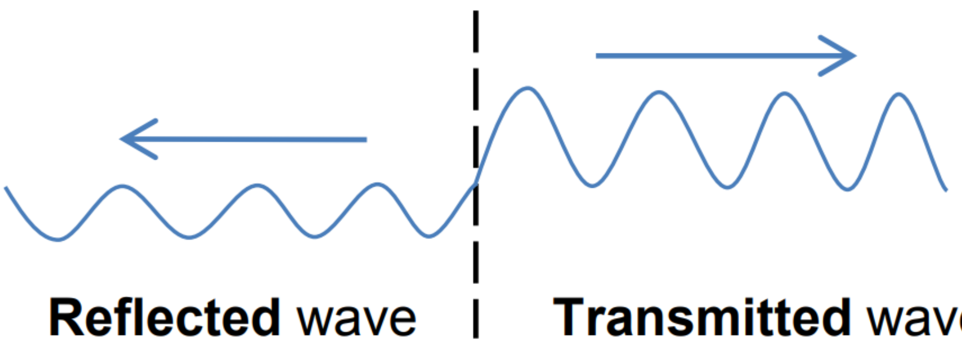
Z_1 $\qquad$ Z_2

$$\boxed{P = \tfrac{1}{2} Z A^2 \omega^2}$$ Wave power

$$\boxed{\frac{P_R}{P_I} = \left| \frac{Z_1 - Z_2}{Z_1 + Z_2} \right|^2} \qquad \boxed{\frac{P_T}{P_I} = 1 - \left| \frac{Z_1 - Z_2}{Z_1 + Z_2} \right|^2}$$

Incident wave

$$\psi_I = A_I e^{i(k_1 x - \omega t)}$$



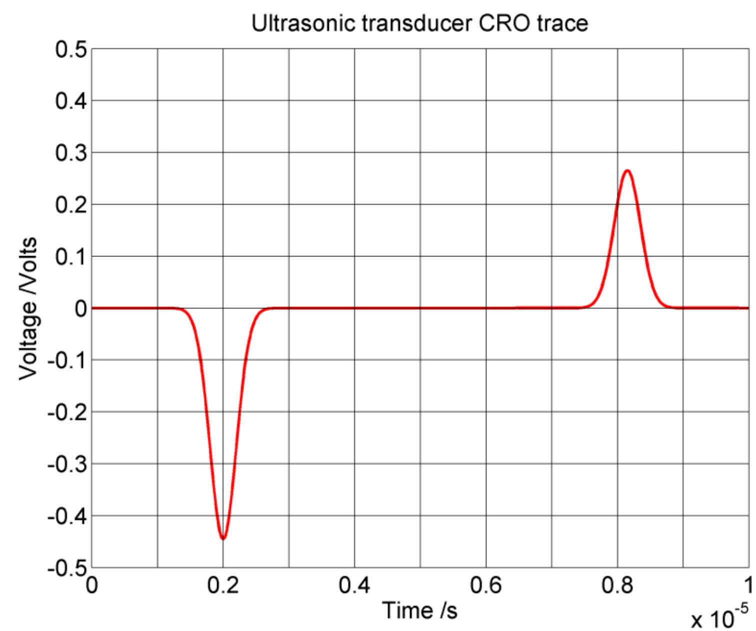
Reflected wave

$$\psi_R = A_R e^{i(-k_1 x - \omega t)}$$

Transmitted wave

$$\psi_T = A_T e^{i(k_2 x - \omega t)}$$

Z_1 $\qquad$ Z_2



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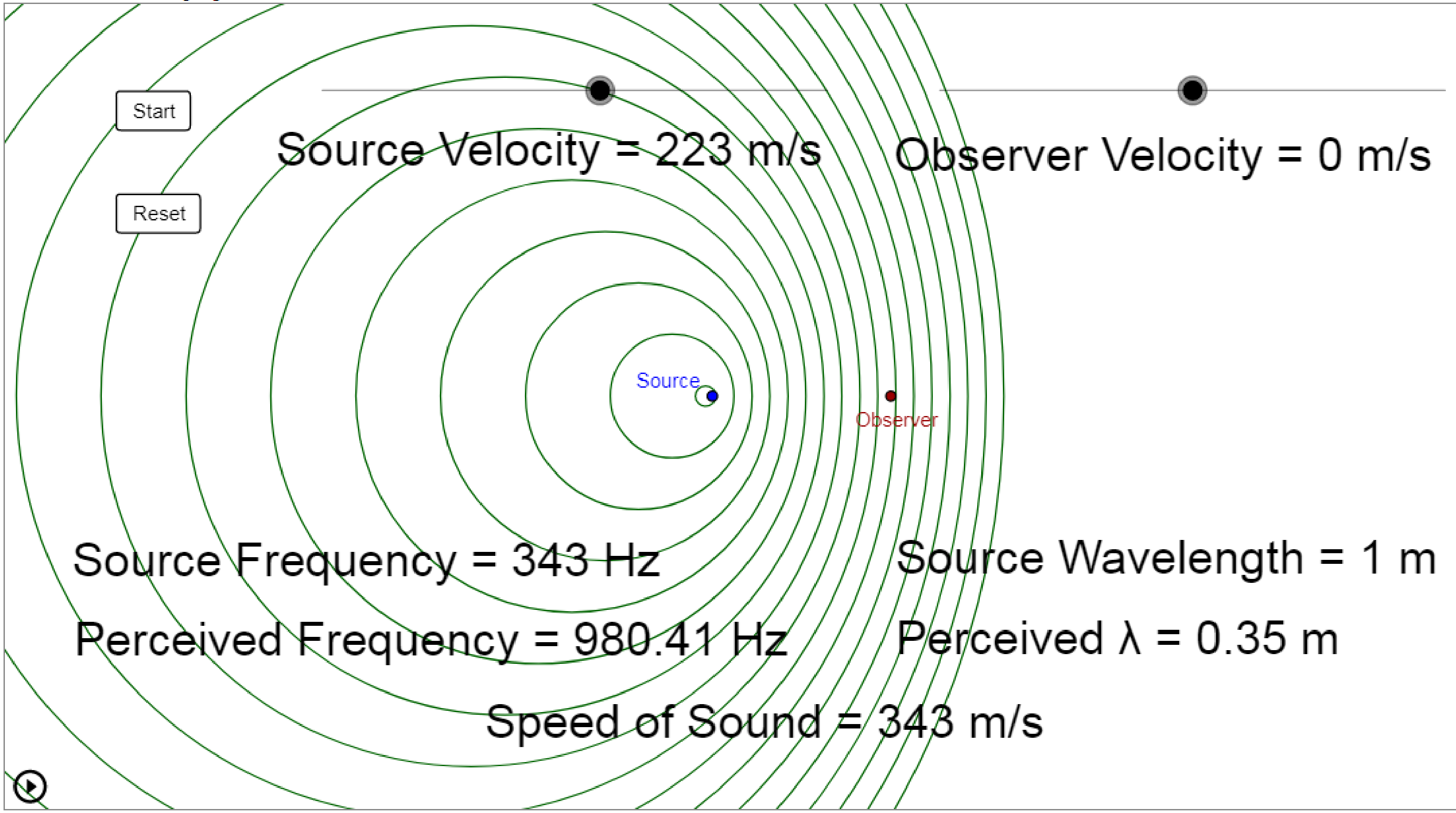
The Doppler Effect & Sonic Boom

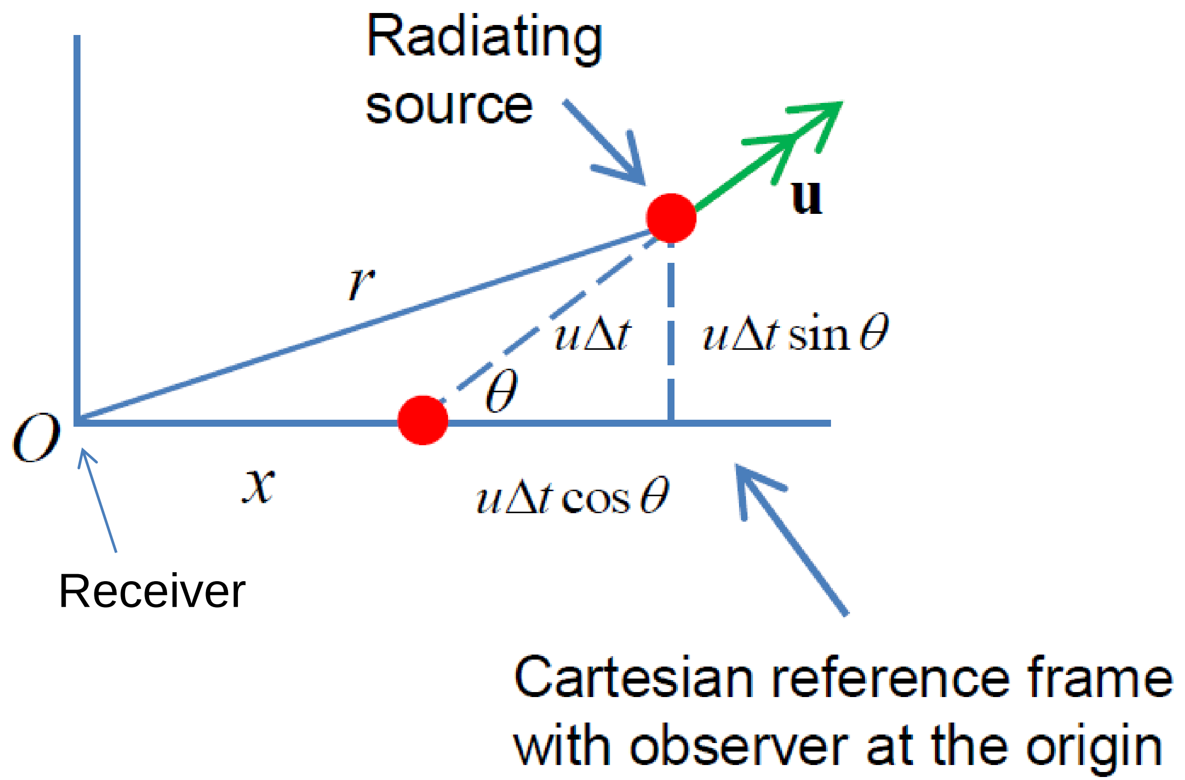


Christian Doppler
1803-1853

https://ophysics.com/w11.html

DOPPLER EFFECT

Consider a **receding** wave source of frequency f. It crosses the x axis of a Cartesian reference frame at angle θ with speed u. The receiver of the waves is stationary at the origin of the Cartesian frame.



The speed of waves, relative to the observer, is w.

Depending on the velocity of the wave source relative to the observer, the observer will experience a *frequency shift* from f. If the source *recedes*, the frequency *diminishes* and the *wavelength increases* ('redshift'). If the source is *approaching*, the observed *frequency will increase* and the *wavelength will decrease* ('blueshift').

Excel model of Doppler Shift

- Plot the circular wavefront(s) when the source has got to position 8.
- i.e. you see a wavefront radius which *increases* as the *initial position decreases*, and also a *shift of wavefront centre.* The combination of these two effects causes the bunching up of wavefronts

SIMULATION OF DOPPLER EFFECT OR MACH CONE

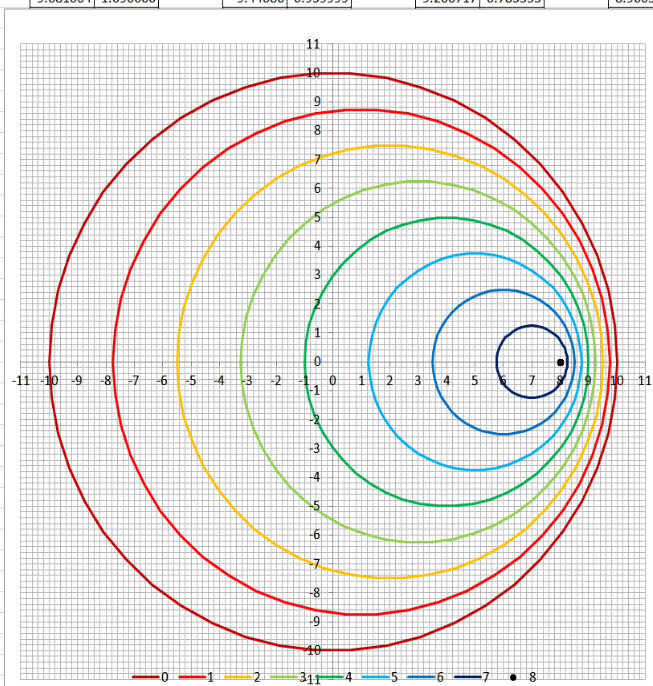
A. French Dec 2021

Mach number = speed of wave source / speed of waves

M	0.8

Wave source position:	0	1	2	3	4	5	6	7	8
Wavefront radius:	10.00	8.75	7.50	6.25	5.00	3.75	2.50	1.25	0.00

		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle		Wavefront circle	
	Angle /rad	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**
0	0	10	0	9.75	0	9.5	0	9.25	0	9	0	8.75	0	8.5	0	8.25	0	8	0
.02	0.125664	9.921147	1.253332	9.681004	1.096666	9.44086	0.939999	9.200717	0.783333	8.960574	0.626666	8.72043	0.47	8.480287	0.313333	8.240143	0.156667	8	0
.04	0.251327	9.685832	2.486899								1.243449	8.632187	0.932587	8.421458	0.621725	8.210729	0.310862	8	0
.06	0.376991	9.297765	3.681246								1.840623	8.486662	1.380467	8.324441	0.920311	8.162221	0.460156	8	0
.08	0.502655	8.763067	4.817537								2.408768	8.28615	1.806576	8.190767	1.204384	8.095383	0.602192	8	0
0.1	0.628319	8.09017	5.877853								2.938926	8.033814	2.204195	8.022542	1.469463	8.011271	0.734732	8	0
.12	0.753982	7.289686	6.845471								3.422736	7.733632	2.567052	7.822422	1.711368	7.911211	0.855684	8	0
.14	0.879646	6.37424	7.705132								3.852566	7.39034	2.889425	7.59356	1.926283	7.79678	0.963142	8	0
.16	1.00531	5.358268	8.443279								4.22164	7.00935	3.16623	7.339567	2.11082	7.669783	1.05541	8	0
.18	1.130973	4.257793	9.048271								4.524135	6.596672	3.393101	7.064448	2.262068	7.532224	1.131034	8	0
0.2	1.256637	3.09017	9.510565								4.755283	6.158814	3.566462	6.772542	2.377641	7.386271	1.188821	8	0
.22	1.382301	1.873813	9.822873								4.911436	5.70268	3.683577	6.468453	2.455718	7.234227	1.227859	8	0
.24	1.507964	0.627905	9.980267								4.990134	5.235464	3.7426	6.156976	2.495067	7.078488	1.247533	8	0
.26	1.633628	-0.62791	9.980267								4.990134	4.764536	3.7426	5.843024	2.495067	6.921512	1.247533	8	0
.28	1.759292	-1.87381	9.822873								4.911436	4.29732	3.683577	5.531547	2.455718	6.765773	1.227859	8	0
0.3	1.884956	-3.09017	9.510565								4.755283	3.841186	3.566462	5.227458	2.377641	6.613729	1.188821	8	0
.32	2.010619	-4.25779	9.048271								4.524135	3.403328	3.393101	4.935552	2.262068	6.467776	1.131034	8	0
.34	2.136283	-5.35827	8.443279								4.22164	2.99065	3.16623	4.660433	2.11082	6.330217	1.05541	8	0
.36	2.261947	-6.37424	7.705132								3.852566	2.60966	2.889425	4.40644	1.926283	6.20322	0.963142	8	0
.38	2.38761	-7.28969	6.845471								3.422736	2.266368	2.567052	4.177578	1.711368	6.088789	0.855684	8	0
0.4	2.513274	-8.09017	5.877853								2.938926	1.966186	2.204195	3.977458	1.469463	5.988729	0.734732	8	0
.42	2.638938	-8.76307	4.817537								2.408768	1.71385	1.806576	3.809233	1.204384	5.904617	0.602192	8	0
.44	2.764602	-9.29776	3.681246								1.840623	1.513338	1.380467	3.675559	0.920311	5.837779	0.460156	8	0
.46	2.890265	-9.68583	2.486899								1.243449	1.367813	0.932587	3.578542	0.621725	5.789271	0.310862	8	0
.48	3.015929	-9.92115	1.253332								0.626666	1.27957	0.47	3.519713	0.313333	5.759857	0.156667	8	0
0.5	3.141593	-10	-7.7E-15								-3.8E-15	1.25	-2.9E-15	3.5	-1.9E-15	5.75	-9.6E-16	8	0
.52	3.267256	-9.92115	-1.25333								-0.62667	1.27957	-0.47	3.519713	-0.31333	5.759857	-0.15667	8	0
.54	3.39292	-9.68583	-2.4869								-1.24345	1.367813	-0.93259	3.578542	-0.62172	5.789271	-0.31086	8	0
.56	3.518584	-9.29776	-3.68125								-1.84062	1.513338	-1.38047	3.675559	-0.92031	5.837779	-0.46016	8	0
.58	3.644247	-8.76307	-4.81754								-2.40877	1.71385	-1.80658	3.809233	-1.20438	5.904617	-0.60219	8	0
0.6	3.769911	-8.09017	-5.87785								-2.93893	1.966186	-2.20419	3.977458	-1.46946	5.988729	-0.73473	8	0
.62	3.895575	-7.28969	-6.84547								-3.42274	2.266368	-2.56705	4.177578	-1.71137	6.088789	-0.85568	8	0
.64	4.021239	-6.37424	-7.70513								-3.85257	2.60966	-2.88942	4.40644	-1.92628	6.20322	-0.96314	8	0
.66	4.146902	-5.35827	-8.44328								-4.22164	2.99065	-3.16623	4.660433	-2.11082	6.330217	-1.05541	8	0
.68	4.272566	-4.25779	-9.04827								-4.52414	3.403328	-3.3931	4.935552	-2.26207	6.467776	-1.13103	8	0
0.7	4.39823	-3.09017	-9.51057	-1.7039	-8.32174	-0.31763	-7.13292	1.068644	-5.9441	2.454915	-4.75528	3.841186	-3.56646	5.227458	-2.37764	6.613729	-1.18882	8	0
.72	4.523893	-1.87381	-9.82287	-0.63959	-8.59501	0.59464	-7.36715	1.828867	-6.1393	3.063093	-4.91144	4.29732	-3.68358	5.531547	-2.45572	6.765773	-1.22786	8	0



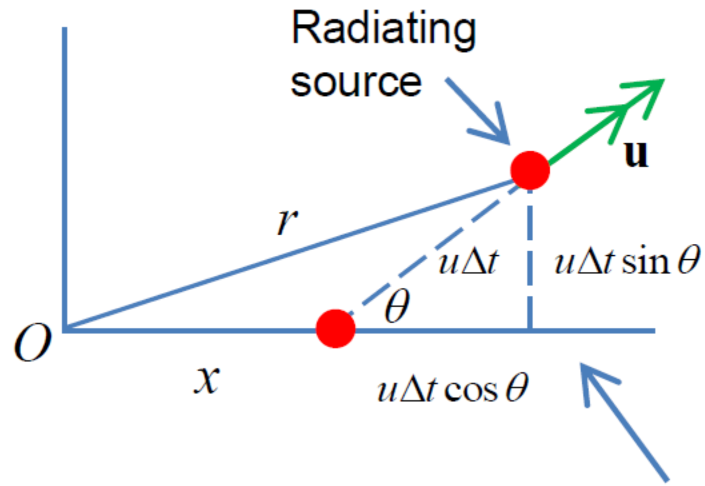
The period T of waves received by an observer (in the x direction) at the frame origin O is:

$$\boxed{T = \Delta t + \frac{r - x}{w}}$$

extra distance travelled by source between wave crests

time between wave crests *at source*

wave speed



Cartesian reference frame with observer at the origin

From geometry:

$$r = \sqrt{(x + u\Delta t\cos\theta)^2 + u^2\Delta t^2\sin^2\theta}$$

$$r = \sqrt{x^2 + u^2\Delta t^2\cos^2\theta + 2ux\Delta t\cos\theta + u^2\Delta t^2\sin^2\theta}$$

$$r = \sqrt{x^2 + u^2\Delta t^2 + 2ux\Delta t\cos\theta}$$

$$r = x\sqrt{1 + 2\cos\theta\frac{u\Delta t}{x} + \left(\frac{u\Delta t}{x}\right)^2}$$

If $u\Delta t \ll x$ $\quad r \approx x\sqrt{1 + 2\cos\theta\frac{u\Delta t}{x}} \approx x\left(1 + \cos\theta\frac{u\Delta t}{x}\right) = x + u\Delta t\cos\theta$

$$\therefore \boxed{r - x \approx u\Delta t\cos\theta}$$

Hence frequency of radiation received at O is $F = 1/T$ where:

$$\frac{1}{F} = \Delta t + \frac{u\Delta t\cos\theta}{w} = \Delta t\left(1 + \frac{u\cos\theta}{w}\right)$$

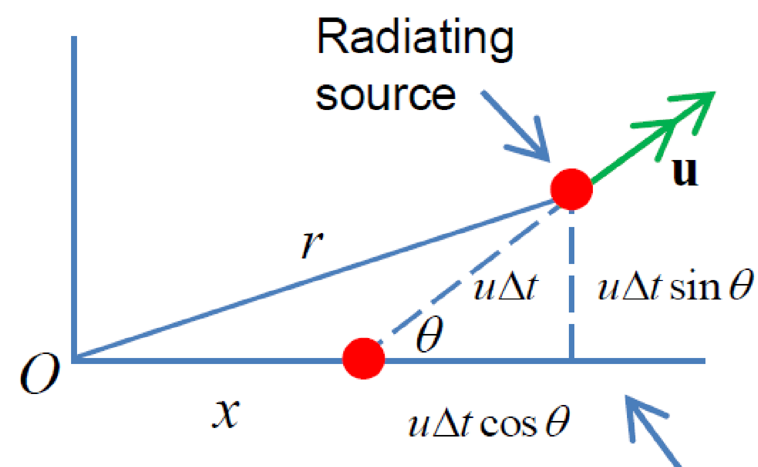
In a *Classical* scenario, where u, w are much less than the speed of light:

$$\boxed{f = 1/\Delta t}$$

Received frequency

$$\boxed{\therefore F = \frac{1}{1 + \frac{u}{w}\cos\theta}\,f}$$

Emitted frequency



If $w \gg u,\ \theta \approx 0$

$$\boxed{F \approx f - \frac{u}{w}f}$$

i.e. a **Doppler Shift** of: $\boxed{\Delta f = \frac{u}{w}f}$

Cartesian reference frame with observer at the origin

Recession velocity from observer

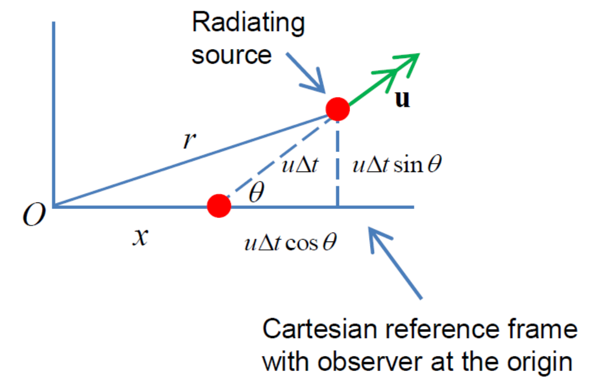
Observed wavelength

$$\boxed{\frac{\lambda_o}{\lambda_e} = 1 + \frac{u}{w}\cos\theta}$$

Emitted wavelength

wave speed

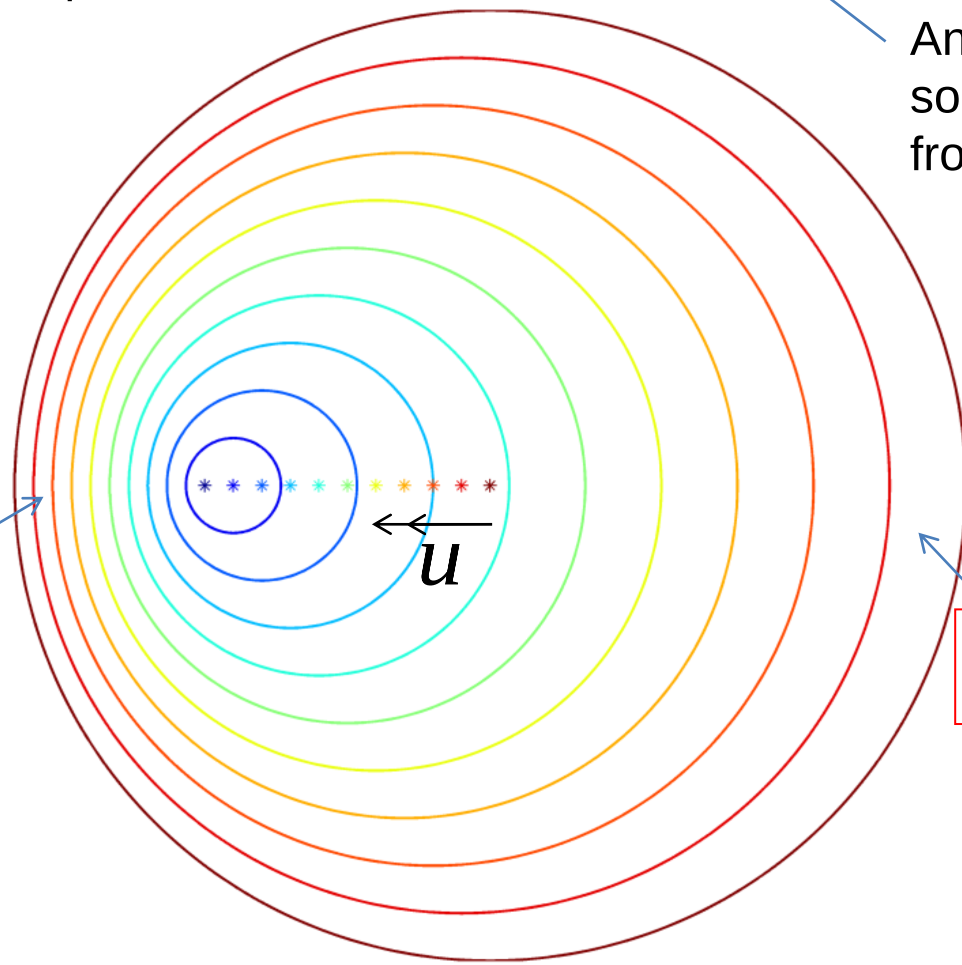
Angle of source velocity from horizontal



In this example:

$$\theta = 0$$

$$u = 0.6w$$



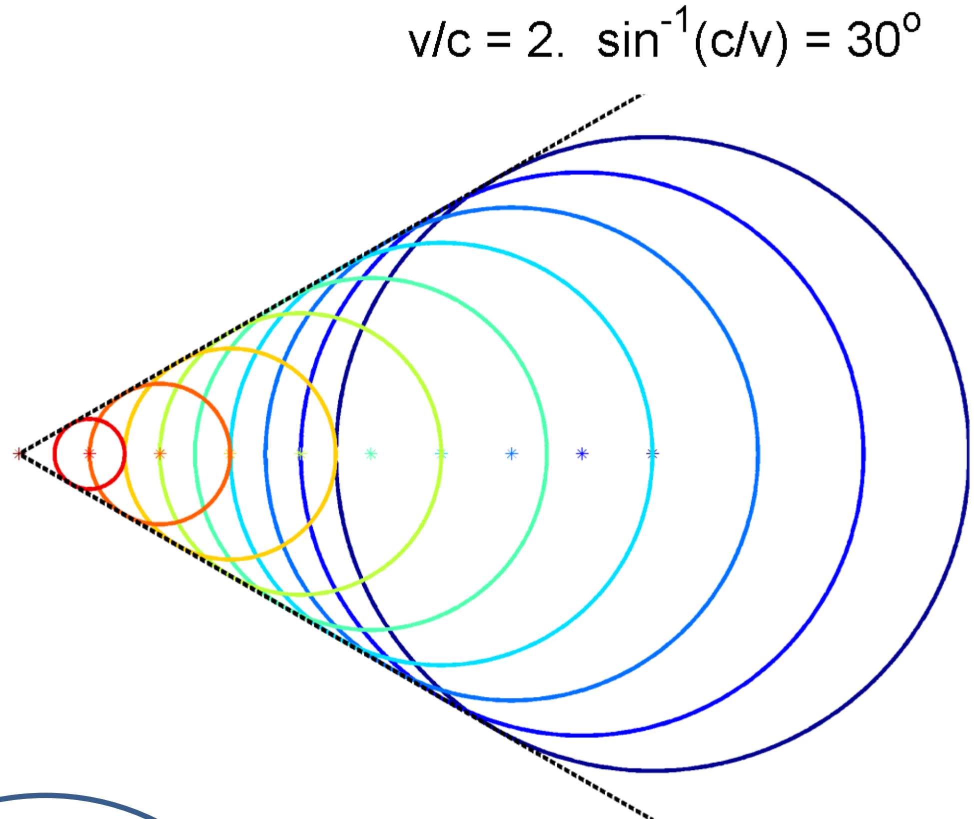
$$\boxed{\lambda_o = \lambda_e(1-0.6)}$$

$$\boxed{\lambda_o = \lambda_e(1+0.6)}$$

Mach's construction

c is the wave speed

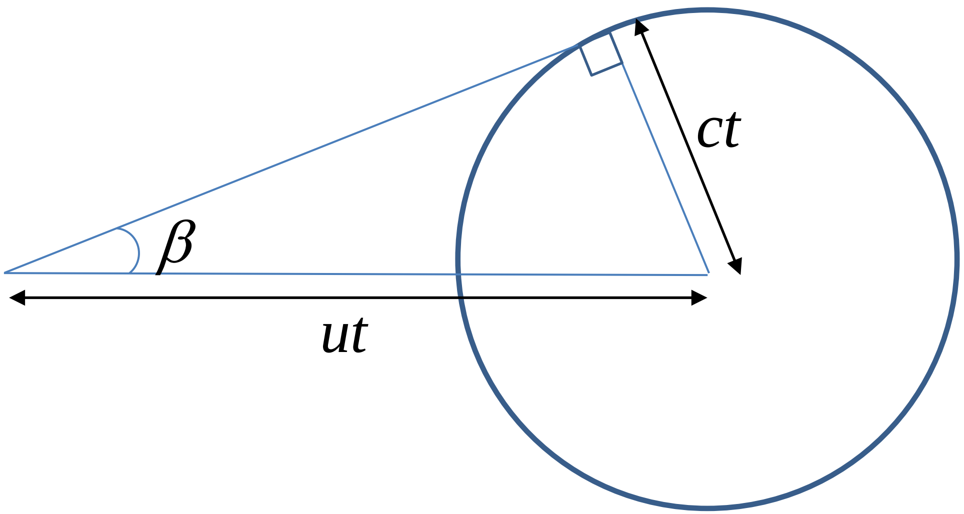
u is the velocity of the object making the waves



Ernst Mach 1838-1916

Mach number

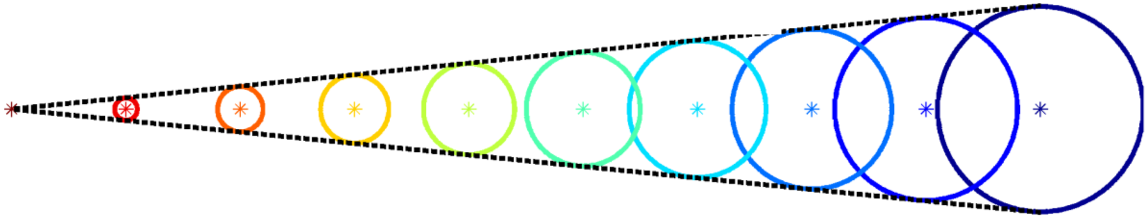
$$\boxed{M = \frac{u}{c}}$$

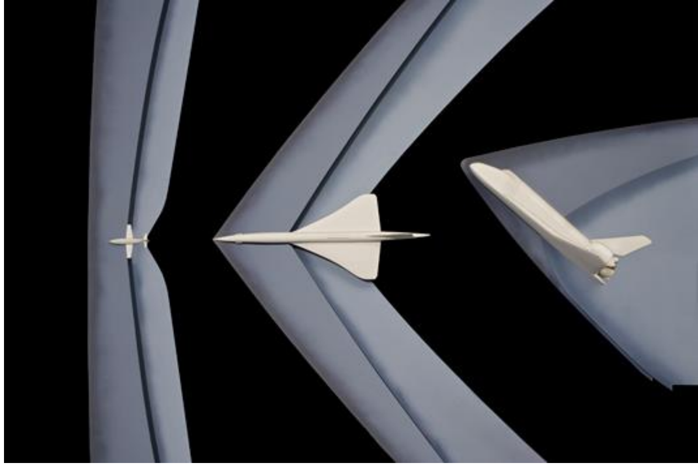


$$ut\sin\beta = ct$$

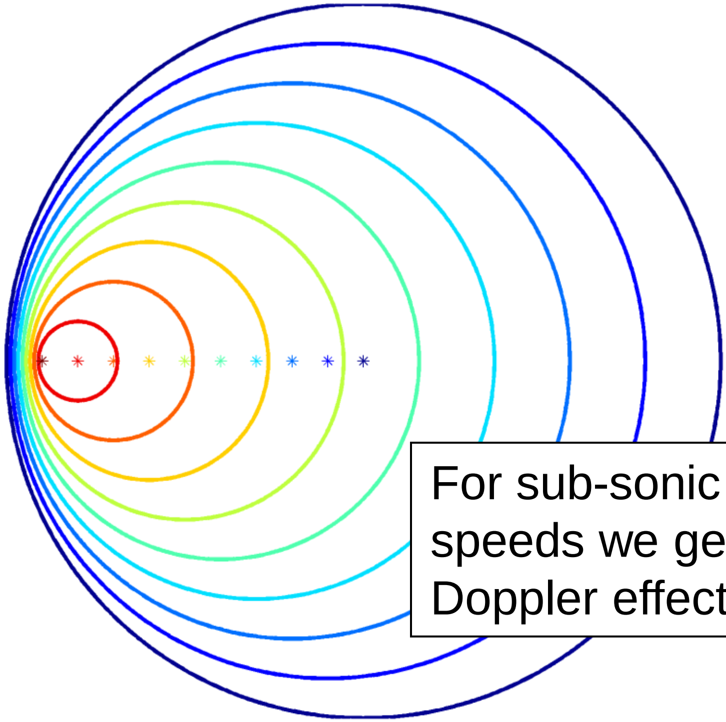
$$\therefore \boxed{\beta = \sin^{-1}\left(\frac{c}{u}\right) = \sin^{-1}\frac{1}{M}}$$

v/c = 10. $\sin^{-1}(c/v) = 5.7392^o$



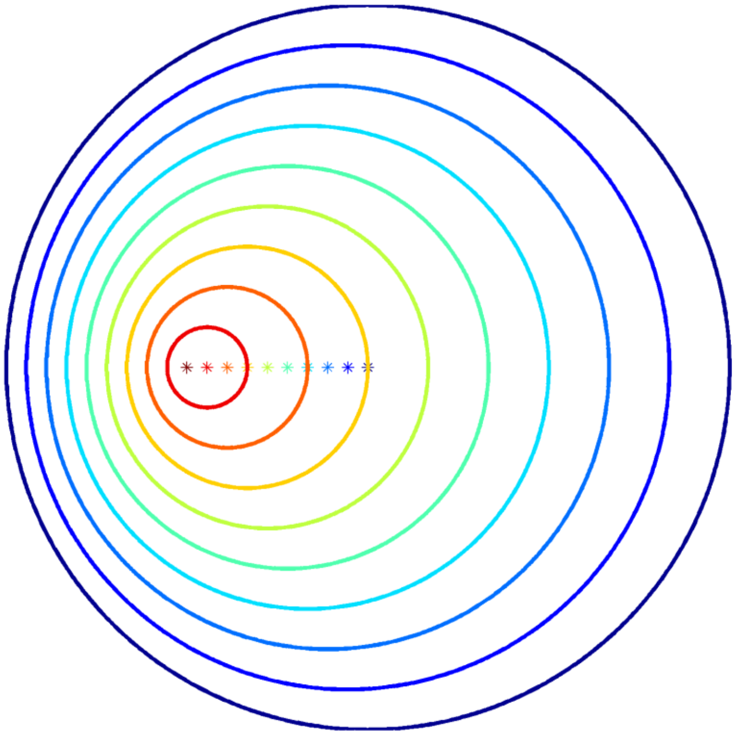


v/c = 0.9. $\sin^{-1}(c/v) = \text{NaN}^o$



For sub-sonic speeds we get the Doppler effect!

v/c = 0.5. $\sin^{-1}(c/v) = \text{NaN}^o$

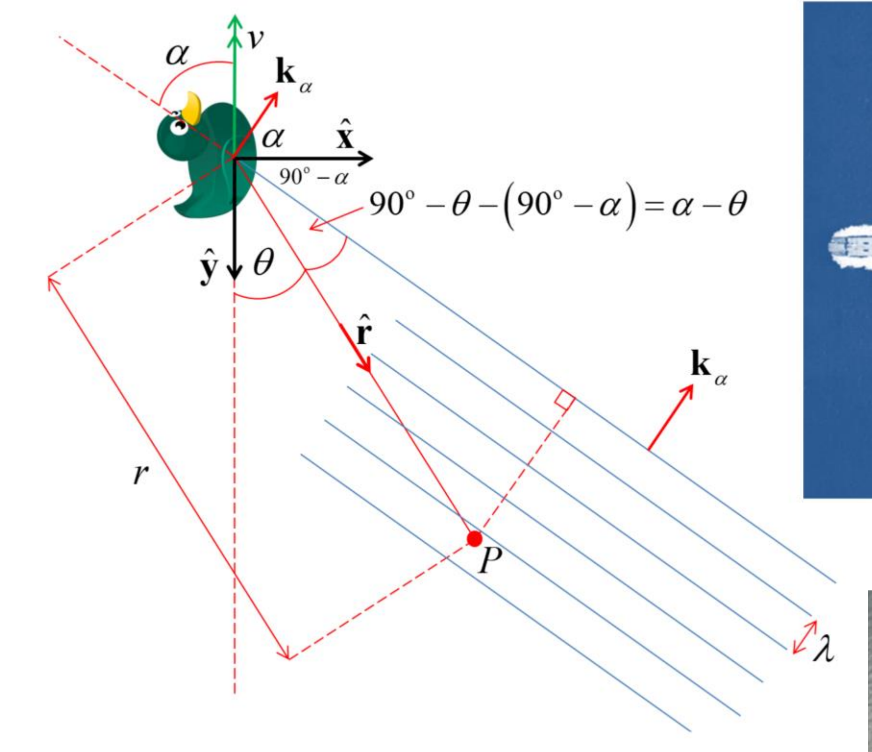


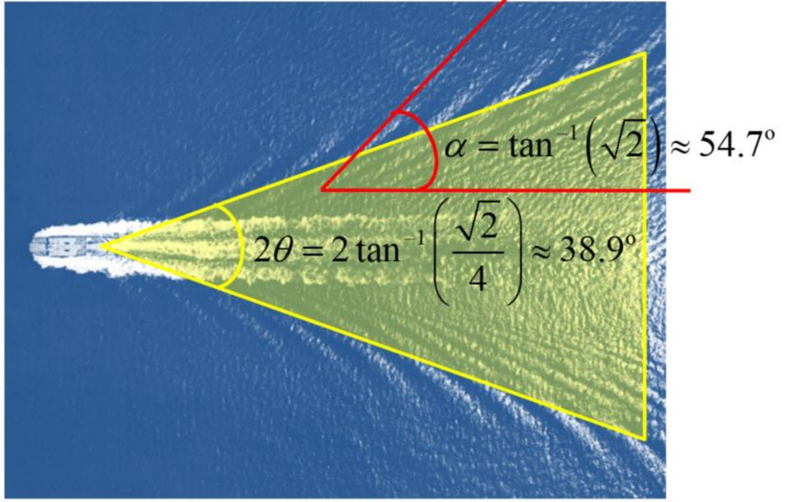


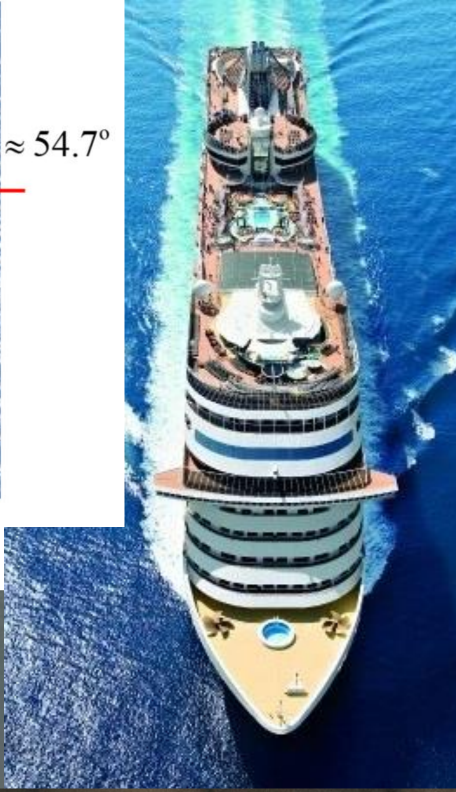

A wake is an *interference pattern* of waves formed by the motion of a body through a fluid. Intriguingly, the angular width of the wake produced by ships (and ducks!) in deep water is the same (about 38.9^{o}). A mathematical explanation for this phenomenon was first proposed by Lord Kelvin (1824-1907). The triangular envelope of the wake pattern has since been known as

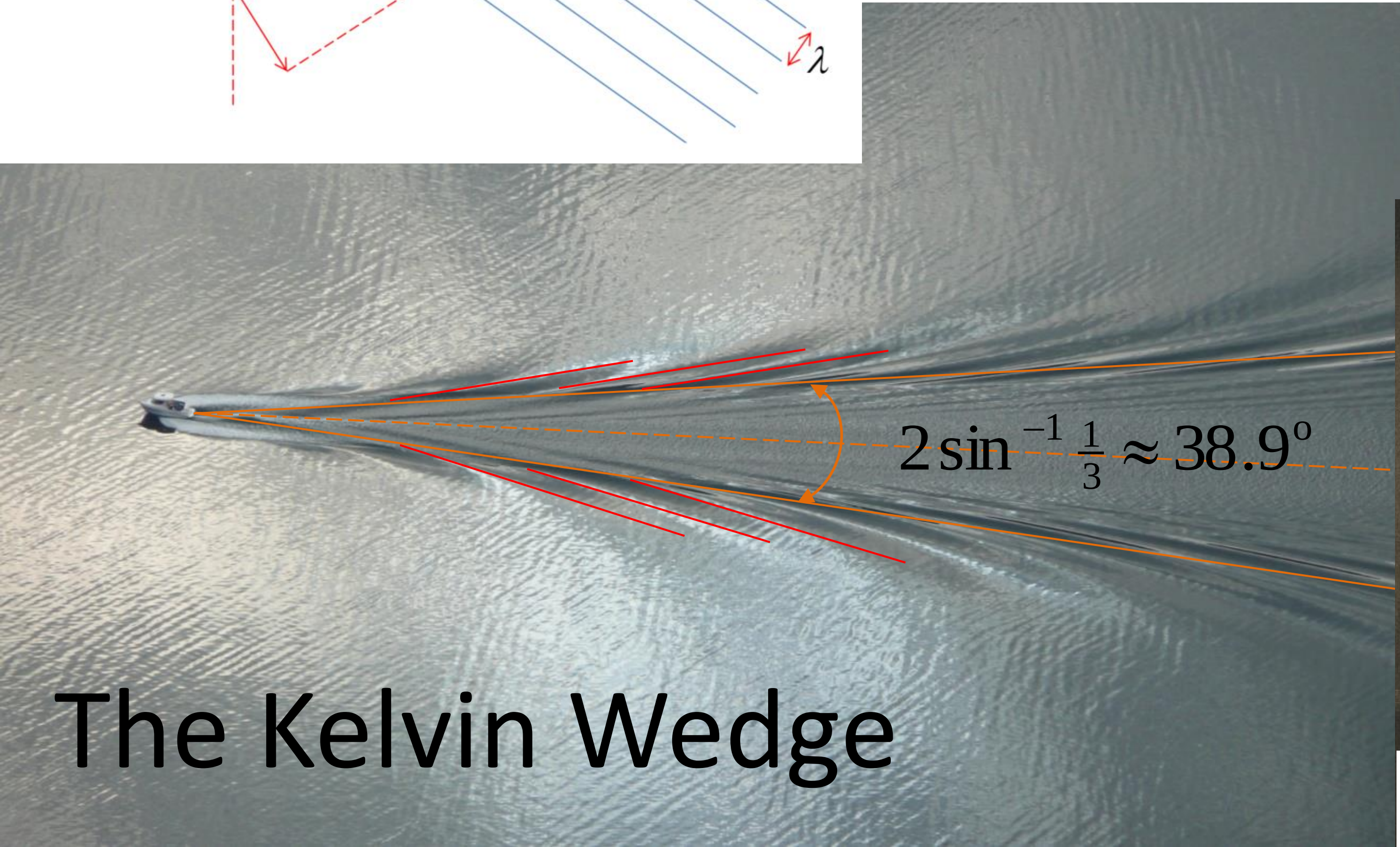
http://en.wikipedia.org/wiki/Wake

the Kelvin Wedge

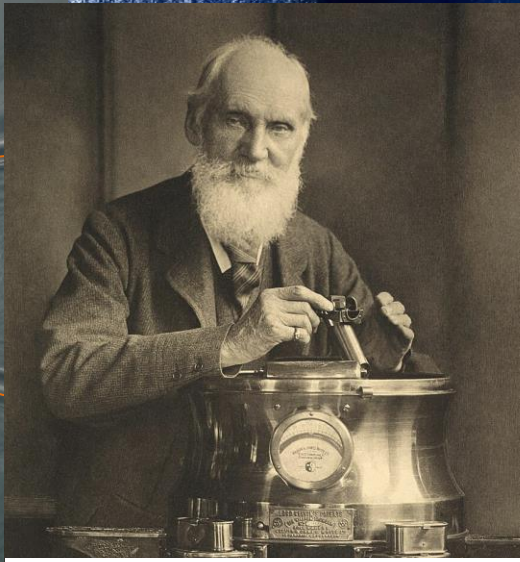








The Kelvin Wedge



William Thomson, 1st Baron Kelvin
1824-1907