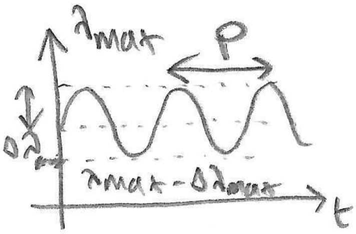
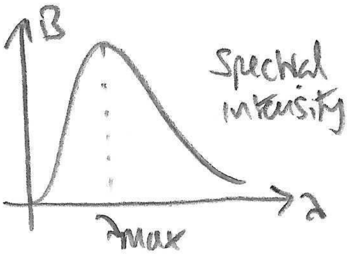
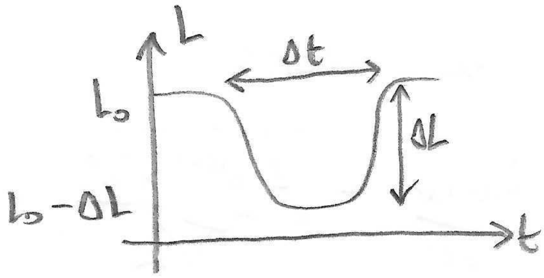
A SIMPLE MODEL OF EXOPLANET DETECTION AND CHARACTERIZATION

Assume: * Exoplanet of mass m orbits in a circular fashion about a Main sequence star of mass M. The star is observed to have a spectral peak at λ_{max}, which varies up to $\pm\Delta\lambda_{max}$ due to a doppler shift because of motion of the star about the barycentre of the star, planet system. The variation of λ_{max} with time yields the orbital period P.

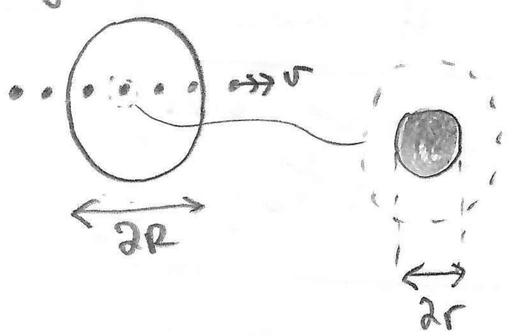




* The luminosity of the star vs time during a transit of the exoplanet is measured. The planet and star are deemed so far away that we can ignore divergence of the starlight on the scale of star to planet separation



(ΔL might be ≈ 5% of L_0)



If on a diameter

$$\boxed{2R = v\Delta t}$$

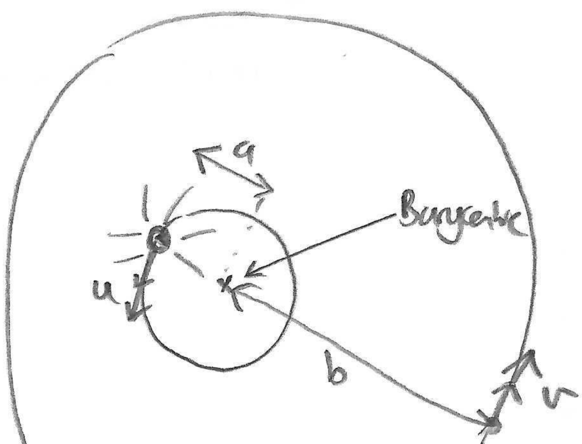
If r is the radius of the exoplanet and R is the radius of the star

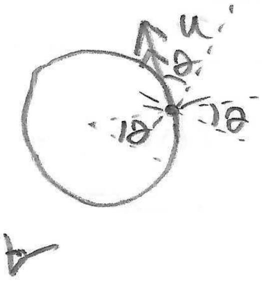
$$\boxed{\Delta L = \left(\frac{r}{R}\right)^2 L_0}$$

i.e. area ratio

$$\Delta L = \frac{\pi r^2}{\pi R^2} \times L_0$$

orbital period P





light from star will be doppler shifted

$$\boxed{\frac{\Delta\lambda}{\lambda} = \frac{u\cos\theta}{c}}$$

$$\boxed{\theta(t) = 2\pi t/P}$$

Newton II:

$$\frac{mv^2}{b} = \frac{GMm}{(a+b)^2} \qquad \frac{Mu^2}{a} = \frac{GMm}{(a+b)^2}$$

planet — star

$$\boxed{v = \frac{2\pi b}{P}} \qquad \boxed{u = \frac{2\pi a}{P}}$$

Planet — Star

$\Rightarrow$ Kepler III:

$$\boxed{P^2 = \frac{4\pi^2}{G(m+M)}(a+b)^3}$$

Now assume $M \gg m$ and $b \gg a$

$\therefore$

$$\boxed{P^2 \simeq \frac{4\pi^2}{GM} b^3}$$

Now if λ_{max} is known for the star:

$$\boxed{L/L_\odot \simeq \left(\frac{T}{T_\odot}\right)^{6.81}} \; * \qquad \text{and} \qquad \boxed{\frac{\lambda_{max}}{nm} = \frac{2.899 \times 10^6}{T/K}}$$

$L_\odot = 3.846 \times 10^{26}\ W/m^{-2}$ — $R_\odot = 696{,}340\,km$

$M_\odot = 1.99 \times 10^{30}\ kg$ — $T_\odot = 5780\,K$

Also:

$$\boxed{M/M_\odot \simeq \left(\frac{T}{T_\odot}\right)^{1.95}} \; *$$

* Assume Main Sequence.

So

$$\boxed{b \simeq \left(\frac{GM}{4\pi^2}\right)^{\frac{1}{3}} P^{\frac{2}{3}}}$$

From Stefan's law: $L = 4\pi R^2 \sigma T^4$

$$\therefore \quad L/L_\odot = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

$$\therefore \quad \boxed{R/R_\odot = \left(\frac{L}{L_\odot}\right)^{\frac{1}{2}} \left(\frac{T}{T_\odot}\right)^{-2}}$$

$$\Rightarrow \quad \boxed{R/R_\odot \simeq \left(\frac{T}{T_\odot}\right)^{\frac{6.81}{2} - 2}}$$

From transit luminosity dip $\Delta L = \left(\frac{r}{R}\right)^2 L_\odot$

$$\Rightarrow \quad \boxed{r/R_\odot = \left(\frac{\Delta L}{L_\odot}\right)^{\frac{1}{2}} \left(\frac{R}{R_\odot}\right)}$$

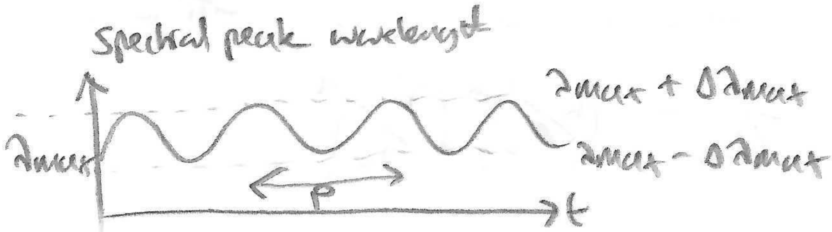
NII: $\quad \frac{u^2}{a} \simeq \frac{GM}{b^2} \qquad (M \gg m, \; b \gg a)$

$u = \frac{\Delta \lambda_{max}}{\lambda_{max}} c \qquad u = \frac{2\pi a}{P} \qquad \therefore \; a = \frac{uP}{2\pi}$

$$\therefore \quad \frac{u^2}{uP/2\pi} = \frac{GM}{b^2}$$

$$\Rightarrow \quad \boxed{M = \frac{2\pi b^2}{G} \frac{u}{P}} \qquad \left\{ \text{so } u = \frac{GPM}{2\pi b^2} \right\}$$

In Summary:



Measure:

$$\boxed{\begin{array}{c}\lambda_{max} \\ \Delta\lambda_{max} \\ P\end{array}}$$

from time variation of spectral peak wavelength

From main sequence star correlations from Hertzsprung-Russell diagram, calculate star parameters

$$\boxed{\begin{aligned} T/T_\odot &= \left(\frac{\lambda_{max}}{502\,nm}\right)^{-1} \\ M/M_\odot &= \left(T/T_\odot\right)^{1.95} \\ R/R_\odot &= \left(T/T_\odot\right)^{\frac{6.81}{2}-2} \\ L/L_\odot &= \left(T/T_\odot\right)^{6.81} \end{aligned}}$$

Find exoplanet orbital radius b from $\boxed{b = \left(\frac{GM}{4\pi^2}\right)^{\frac{1}{3}} P^{\frac{2}{3}}}$

Note $AU = \left(\frac{GM_\odot}{4\pi^2}\right)^{\frac{1}{3}} Yr^{\frac{2}{3}}$

So $$\boxed{b/AU = \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} \left(\frac{P}{Yr}\right)^{\frac{2}{3}}}$$

Find $u = \frac{\Delta\lambda_{max}}{\lambda_{max}} c$ and $\boxed{\frac{m}{M_\oplus} = \frac{2\pi b^2}{GM_\oplus} \frac{u}{P}}$

(4) Earth mass $M_\oplus = 5.97 \times 10^{24}$ kg.

Lastly, from transit luminosity dip ΔL

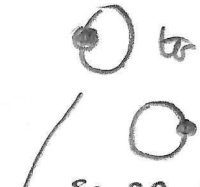
$$\frac{r}{R_\odot} = \left(\frac{\Delta L}{L_\odot}\right)^{\frac{1}{2}} \left(\frac{R}{R_\odot}\right)$$

and

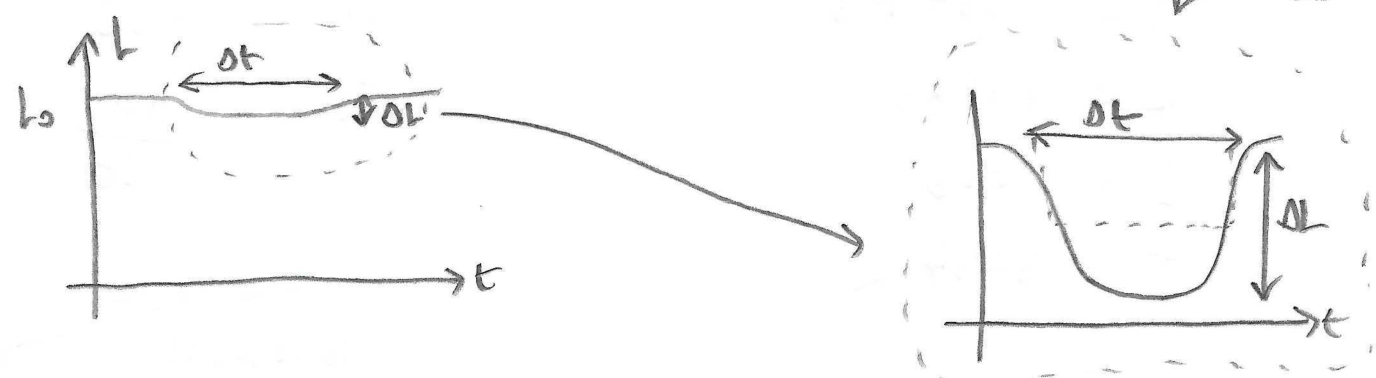
$$\frac{r}{R_\oplus} = \frac{r}{R_\odot} \times \frac{R_\odot}{R_\oplus}$$

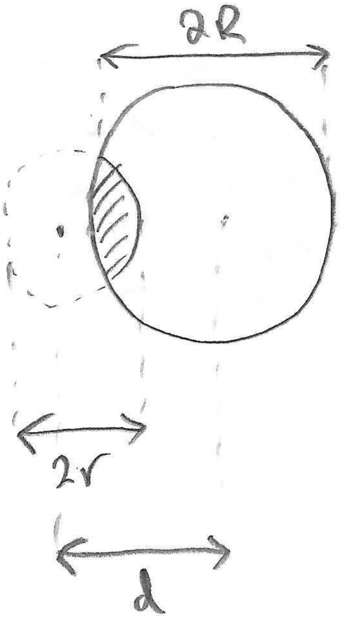
$$\boxed{\frac{r}{R_\oplus} = \left(\frac{\Delta L}{L_\odot}\right)^{\frac{1}{2}} \left(\frac{R}{R_\odot}\right) \frac{696340}{6371}}$$

Also $\Delta t = \dfrac{2R}{v}$ and $v = \dfrac{2\pi b}{P}$



so $2R = v\Delta t$





Really we should consider a smooth transition from $L_\odot$ to $L_\odot - \Delta L$.

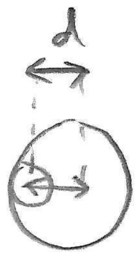
Intersection area is:

$$A = r^2 \cos^{-1}\left(\frac{d^2 + r^2 - R^2}{2dr}\right) + R^2 \cos^{-1}\left(\frac{d^2 + R^2 - r^2}{2dR}\right)$$

$$- \frac{1}{2}\sqrt{(-d+r+R)(d+r-R)(d-r+R)(d+r+R)}$$

(Mathworld). i.e. zero when $d \geq R + r$

πr^2 when $d \leq R - r$



(Assume $R > r$).

Example exoplanet:

"Earth 2.0"

Kepler 452b

$M = (5 \pm 2)\, M_\oplus$

$b = 1.046$ AU

$P = 384.84$ days

$r = 1.5\, R_\oplus$

Kepler 452 star

- G-type main sequence
- 1402 ly away, in Cygnus constellation

parallax 1.7838 mas

ie $1.7838 \times \frac{\pi}{180} \div \frac{1}{1000} \div \frac{1}{3600}$

radians $= \Delta\theta$

$\therefore$ distance $= \frac{AU}{\Delta\theta}$

$M = 1.037\, M_\odot$

$R = 1.11\, R_\odot$

$L = 1.2\, L_\odot$

$T = 5757$ K

Question:

→ How is M calculated?

The simple M.S. Correlations yield $M/M_\odot = 0.992$

see spreadsheet model:

$\Rightarrow$ $\Delta L / L = 0.0153\ \%$

$\Delta t = 14.49$ hours

$\Delta \lambda_{max} = 7.23 \times 10^{-7}$ nm (!)

$v = 29.57$ km/s

$u = 0.430$ m/s

In other words, changes to luminosity and λ_{max} are VERY small. Impressive detective work.