

# BPhO

Computational  
Challenge

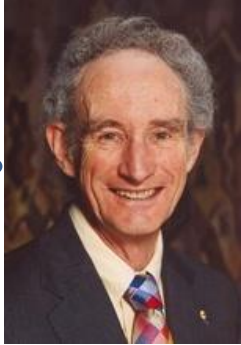
## Seminar 10: May's Chaotic Bunnies

Dr Andrew French.  
December 2021.

# The logistic map and population modelling



I published this model in 1976



Robert May  
1936-

Assume an ecosystem can support a maximum number of rabbits.  
Let  $x$  be the fraction of this maximum at year  $n$ .

To account for **reproduction**, next year's population is proportional to the previous.

To account for **starvation**, next year's population is *also proportional* to the fraction of the maximum population as yet unfilled.

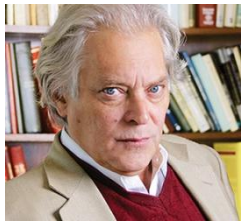


$$x_{n+1} = r x_n (1 - x_n)$$

Growth  
parameter

The population next year is predicted using this **iterative equation** called a **logistic map**

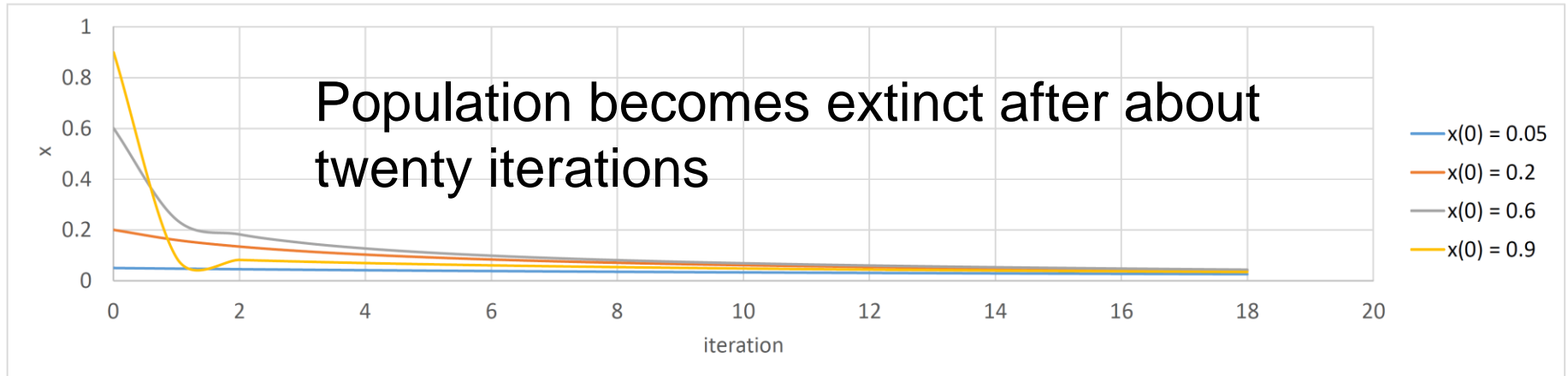
The pattern of  $x$  values with  $n$  is not always simple .....



$$r = 1 \quad x_{n+1} = rx_n (1 - x_n)$$

iteration number n

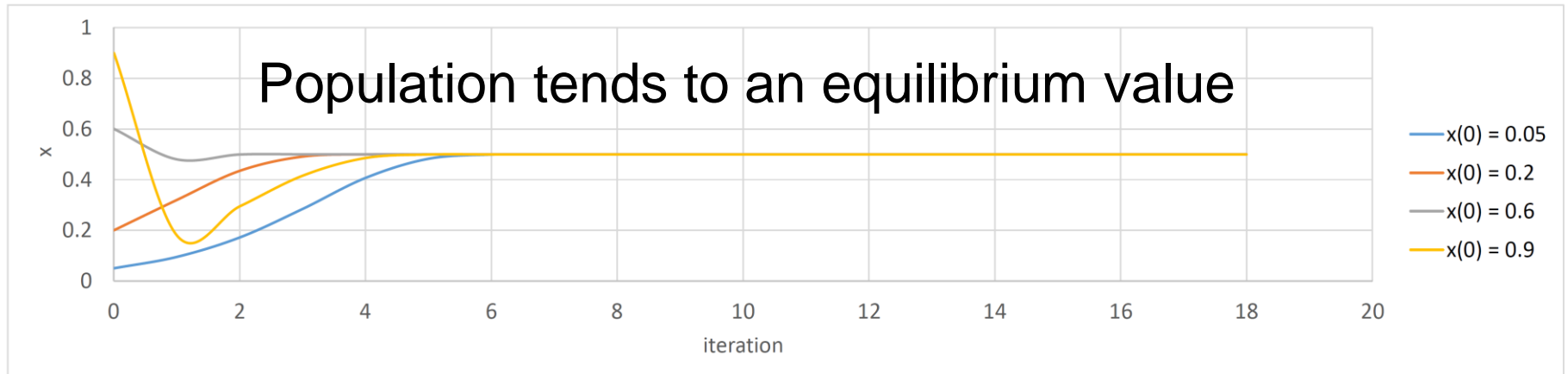
x(n)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858	0.025185
0.1	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359	0.033180
0.15	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321	0.036870
0.2	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496	0.038858
0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804	0.040084
0.3	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628	0.040838
0.35	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154	0.041304
0.4	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348	0.041604
0.45	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658	0.041764
0.5	0.25	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804
0.55	0.2475	0.186244	0.151557	0.128587	0.112053	0.099497	0.089597	0.08157	0.074916	0.069304	0.064501	0.06034	0.056699	0.053485	0.050624	0.048061	0.045751	0.043658	0.041764
0.6	0.24	0.1824	0.14913	0.12689	0.110789	0.098515	0.08881	0.080923	0.074374	0.068843	0.064103	0.059994	0.056395	0.053214	0.050383	0.047844	0.045555	0.04348	0.041604
0.65	0.2275	0.175744	0.144858	0.123874	0.108529	0.096751	0.08739	0.079753	0.073392	0.068006	0.063381	0.059364	0.05584	0.052722	0.049942	0.047448	0.045197	0.043154	0.041304
0.7	0.21	0.1659	0.138377	0.119229	0.105013	0.093986	0.085152	0.077901	0.071833	0.066673	0.062228	0.058355	0.05495	0.05193	0.049234	0.04681	0.044619	0.042628	0.040838
0.75	0.1875	0.152344	0.129135	0.112459	0.099812	0.08985	0.081777	0.075089	0.069451	0.064627	0.060451	0.056796	0.053571	0.050701	0.04813	0.045814	0.043715	0.041804	0.040084
0.8	0.16	0.1344	0.116337	0.102802	0.092234	0.083727	0.076717	0.070831	0.065814	0.061483	0.057703	0.054373	0.051417	0.048773	0.046394	0.044242	0.042284	0.040496	0.038858
0.85	0.1275	0.111244	0.098869	0.089094	0.081156	0.07457	0.069009	0.064247	0.060119	0.056505	0.053312	0.05047	0.047923	0.045626	0.043544	0.041648	0.039914	0.038321	0.036870
0.9	0.09	0.0819	0.075192	0.069538	0.064703	0.060516	0.056854	0.053622	0.050746	0.048171	0.045851	0.043749	0.041835	0.040084	0.038478	0.036997	0.035628	0.034359	0.033180
0.95	0.0475	0.045244	0.043197	0.041331	0.039623	0.038053	0.036605	0.035265	0.034021	0.032864	0.031784	0.030773	0.029826	0.028937	0.028099	0.02731	0.026564	0.025858	0.025185
1	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16	-2.2E-16



$$r = 2 \quad x_{n+1} = rx_n (1 - x_n)$$

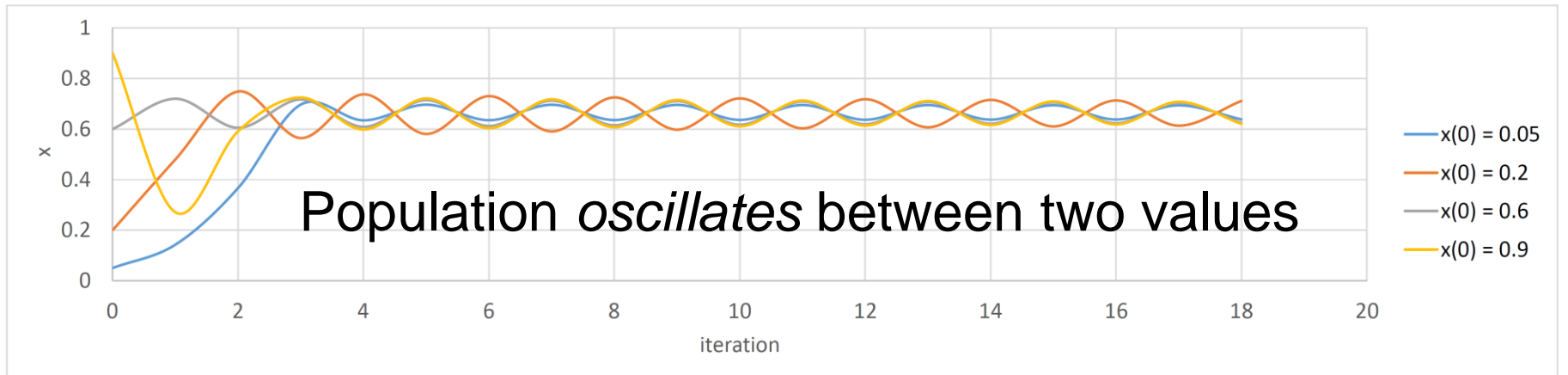
iteration number n

x(n)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.1	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.15	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.2	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.25	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.3	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.35	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.4	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.45	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.55	0.495	0.49995	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.6	0.48	0.4992	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.65	0.455	0.49595	0.499967	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.7	0.42	0.4872	0.499672	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.75	0.375	0.46875	0.498047	0.499992	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.8	0.32	0.4352	0.491602	0.499859	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.85	0.255	0.37995	0.471176	0.498338	0.499994	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.9	0.18	0.2952	0.416114	0.485926	0.499604	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.95	0.095	0.17195	0.284766	0.407349	0.482832	0.49941	0.499999	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
1	-4.4E-16	-8.9E-16	-1.8E-15	-3.6E-15	-7.1E-15	-1.4E-14	-2.8E-14	-5.7E-14	-1.1E-13	-2.3E-13	-4.5E-13	-9.1E-13	-1.8E-12	-3.6E-12	-7.3E-12	-1.5E-11	-2.9E-11	-5.8E-11	



$$r = 3 \quad x_{n+1} = rx_n (1 - x_n)$$

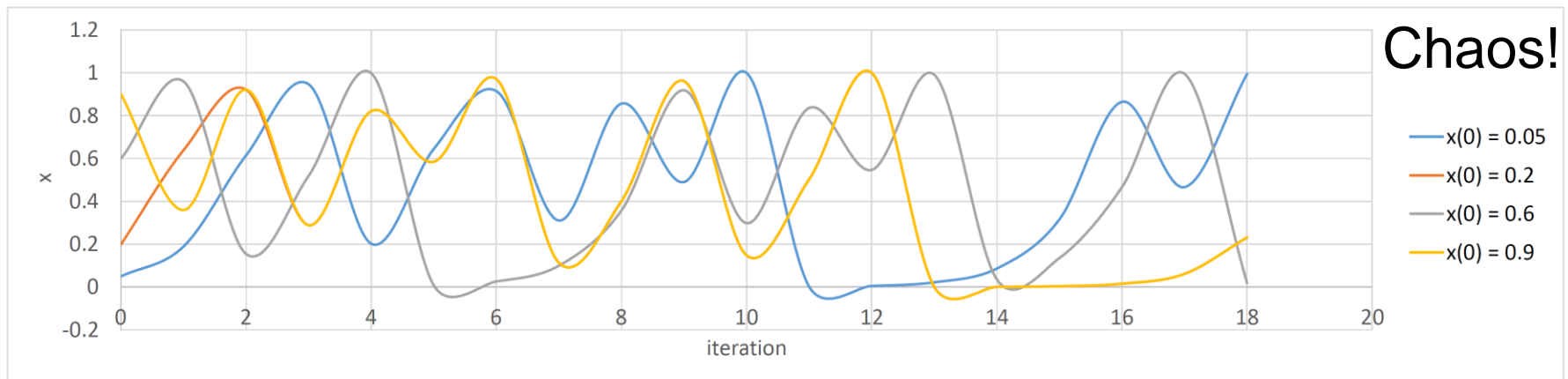
iteration number n		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x(n)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.05	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632	
	0.1	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219	
	0.15	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253	
	0.2	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343	
	0.25	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757	
	0.3	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969	
	0.35	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691	
	0.4	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387	
	0.45	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107	
	0.5	0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	
	0.55	0.7425	0.573581	0.733757	0.586072	0.727775	0.594356	0.723291	0.600424	0.719745	0.605136	0.716839	0.608942	0.714395	0.612105	0.712298	0.614789	0.71047	0.617107	
	0.6	0.72	0.6048	0.717051	0.608667	0.714575	0.611873	0.712453	0.614591	0.710607	0.616934	0.708979	0.618983	0.707529	0.620795	0.706226	0.622413	0.705045	0.62387	
	0.65	0.6825	0.650081	0.682427	0.650161	0.682355	0.65024	0.682284	0.650318	0.682213	0.650395	0.682144	0.65047	0.682076	0.650545	0.682009	0.650619	0.681942	0.650691	
	0.7	0.63	0.6993	0.630839	0.698644	0.631622	0.698027	0.632356	0.697446	0.633046	0.696897	0.633695	0.696377	0.634308	0.695884	0.634889	0.695415	0.635439	0.694969	
	0.75	0.5625	0.738281	0.579666	0.73096	0.589973	0.725715	0.597158	0.721681	0.602573	0.718436	0.606857	0.715745	0.610362	0.71346	0.613304	0.711487	0.61582	0.709757	
	0.8	0.48	0.7488	0.564296	0.737598	0.580641	0.730491	0.590622	0.725363	0.597634	0.721403	0.602943	0.718208	0.607155	0.715553	0.61061	0.713296	0.613514	0.711343	
	0.85	0.3825	0.708581	0.619482	0.707172	0.621239	0.705904	0.622811	0.704752	0.62423	0.703701	0.625518	0.702736	0.626694	0.701846	0.627775	0.701021	0.628772	0.700253	
	0.9	0.27	0.5913	0.724993	0.598135	0.721109	0.603333	0.717967	0.607471	0.71535	0.610873	0.713121	0.613738	0.711191	0.616195	0.709496	0.618334	0.707991	0.620219	
	0.95	0.1425	0.366581	0.696598	0.634047	0.696094	0.634641	0.695615	0.635204	0.695159	0.635738	0.694725	0.636246	0.694311	0.63673	0.693915	0.637191	0.693536	0.637632	
	1	-6.7E-16	-2E-15	-6E-15	-1.8E-14	-5.4E-14	-1.6E-13	-4.9E-13	-1.5E-12	-4.4E-12	-1.3E-11	-3.9E-11	-1.2E-10	-3.5E-10	-1.1E-09	-3.2E-09	-9.6E-09	-2.9E-08	-8.6E-08	



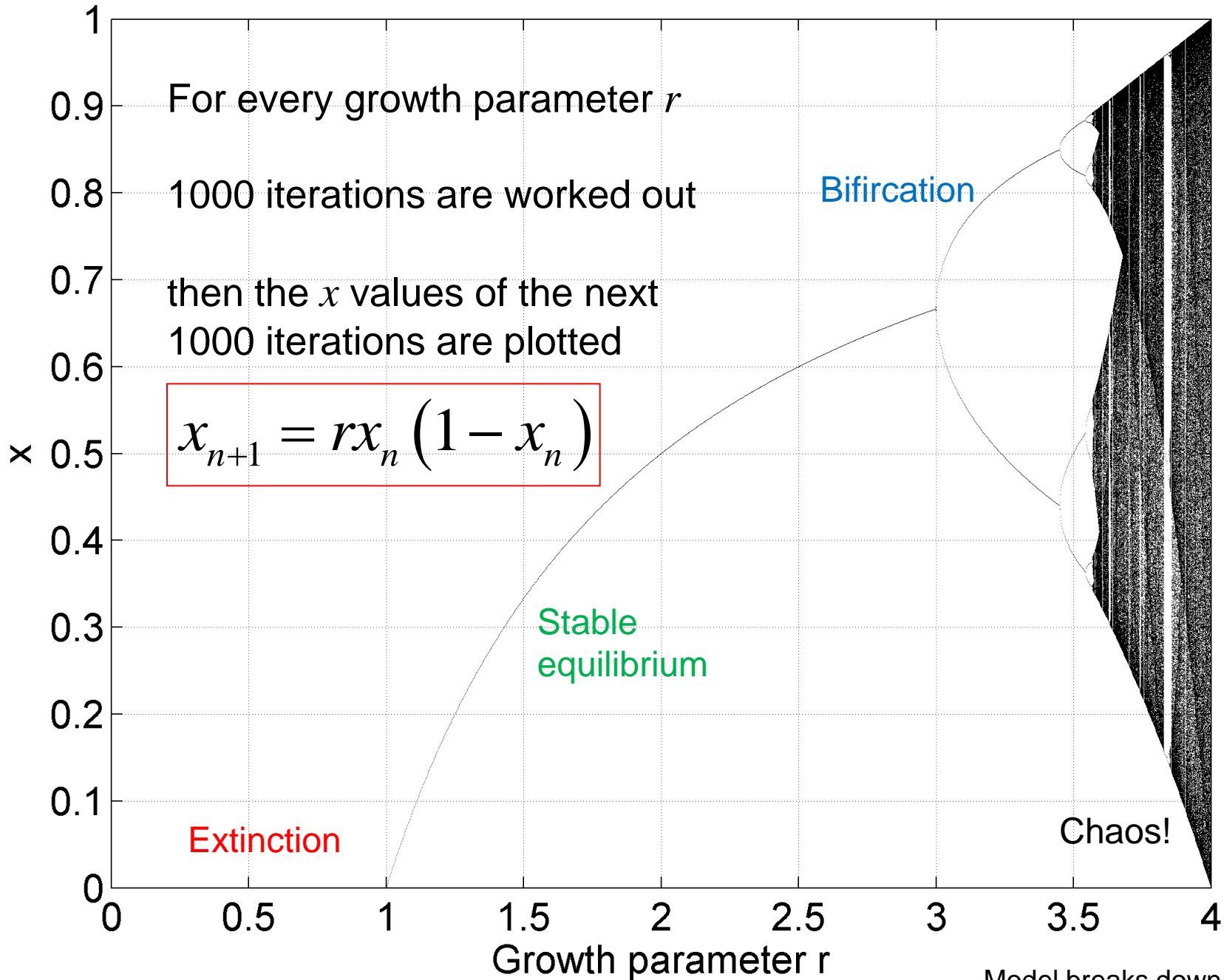
$$r = 4 \quad x_{n+1} = rx_n (1 - x_n)$$

iteration number n

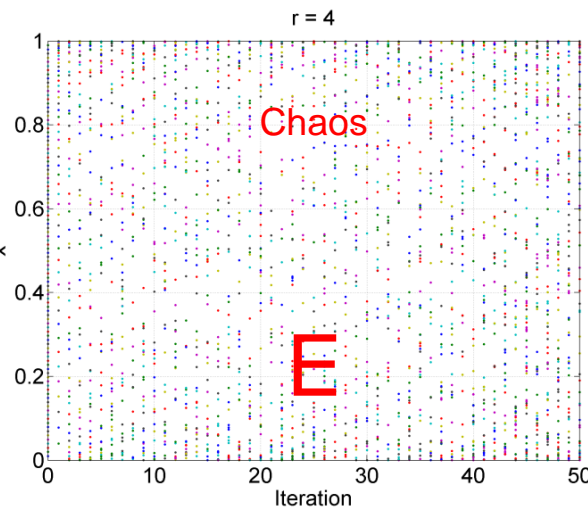
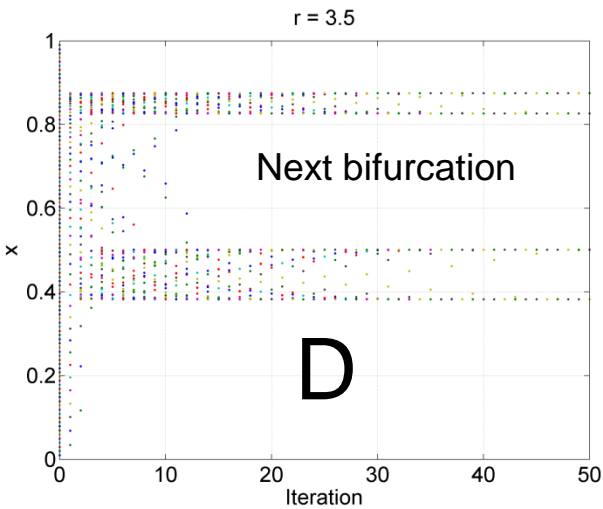
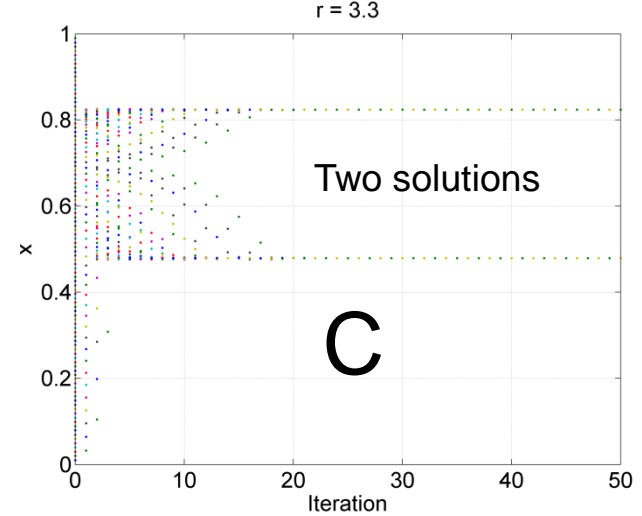
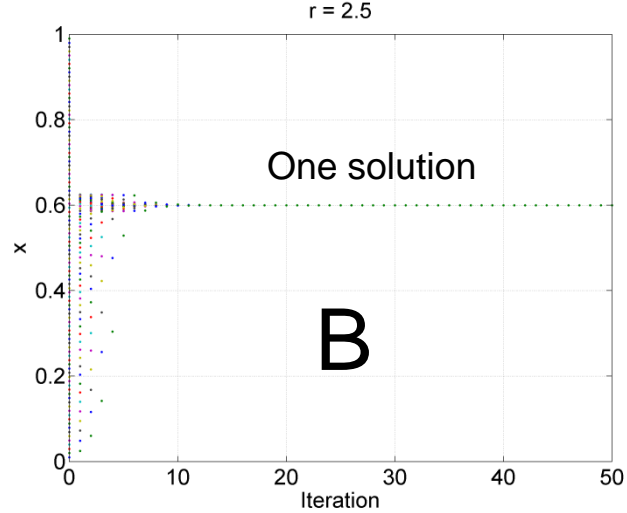
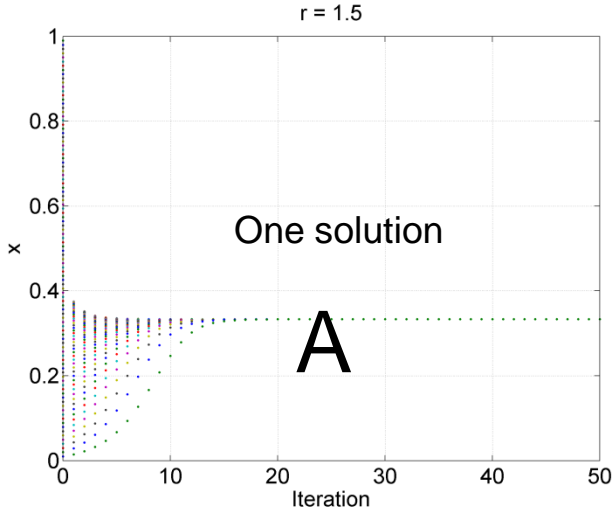
x(n)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582	
0.1	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173	
0.15	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787	
0.2	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173	
0.25	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
0.3	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097	
0.35	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367	
0.4	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978	
0.45	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666	
0.5	1	4.44E-16	1.78E-15	7.11E-15	2.84E-14	1.14E-13	4.55E-13	1.82E-12	7.28E-12	2.91E-11	1.16E-10	4.66E-10	1.86E-09	7.45E-09	2.98E-08	1.19E-07	4.77E-07	1.91E-06	
0.55	0.99	0.0396	0.152127	0.515939	0.998984	0.00406	0.016176	0.063657	0.238418	0.7263	0.795154	0.651537	0.908147	0.333665	0.889331	0.393686	0.954789	0.172666	
0.6	0.96	0.1536	0.520028	0.998395	0.006408	0.025467	0.099273	0.35767	0.918969	0.29786	0.836557	0.546917	0.991195	0.034909	0.134761	0.466403	0.995485	0.017978	
0.65	0.91	0.3276	0.881113	0.419012	0.973764	0.102192	0.366996	0.92924	0.263011	0.775345	0.69674	0.845174	0.523421	0.997806	0.008757	0.034722	0.134065	0.464367	
0.7	0.84	0.5376	0.994345	0.022492	0.087945	0.320844	0.871612	0.447617	0.989024	0.043422	0.166146	0.554165	0.988265	0.046391	0.176954	0.582565	0.972732	0.106097	
0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
0.8	0.64	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173	
0.85	0.51	0.9996	0.001599	0.006387	0.025386	0.098965	0.356683	0.917841	0.301635	0.842605	0.530488	0.996282	0.014817	0.058389	0.219918	0.686217	0.861293	0.47787	
0.9	0.36	0.9216	0.289014	0.821939	0.585421	0.970813	0.113339	0.401974	0.961563	0.147837	0.503924	0.999938	0.000246	0.000985	0.003936	0.015682	0.061745	0.23173	
0.95	0.19	0.6156	0.946547	0.202385	0.6457	0.915085	0.310816	0.856838	0.490667	0.999652	0.001393	0.005565	0.022137	0.086589	0.316366	0.865114	0.466766	0.995582	
1	-8.9E-16	-3.6E-15	-1.4E-14	-5.7E-14	-2.3E-13	-9.1E-13	-3.6E-12	-1.5E-11	-5.8E-11	-2.3E-10	-9.3E-10	-3.7E-09	-1.5E-08	-6E-08	-2.4E-07	-9.5E-07	-3.8E-06	-1.5E-05	



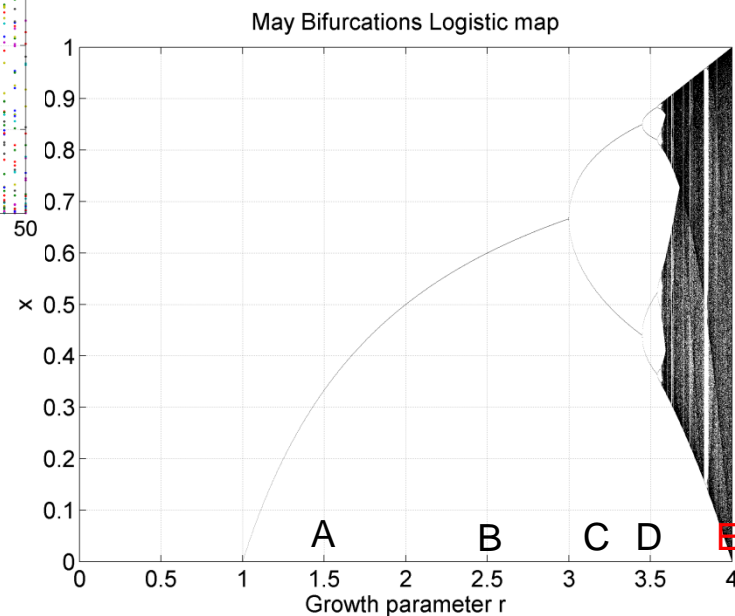
# May Bifurcations Logistic map



Model breaks down for  $r > 4$



$$x_{n+1} = rx_n(1 - x_n)$$



Tracking the bifurcations maps the 'road to chaos'. The ratio of successive bifurcation intervals is a **universal constant!**  
4.669201609...



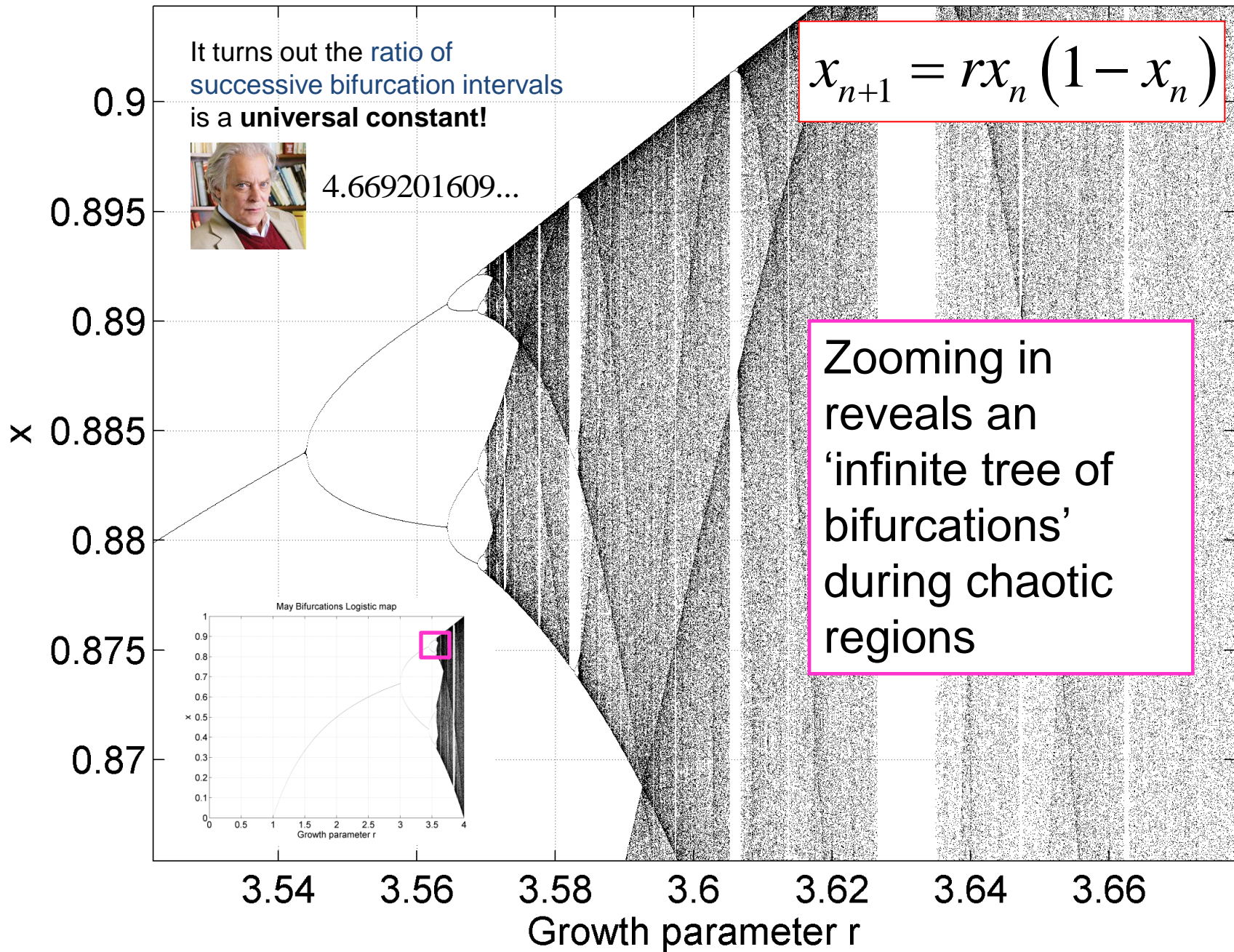
# May Bifurcations Logistic map

It turns out the ratio of successive bifurcation intervals is a **universal constant!**

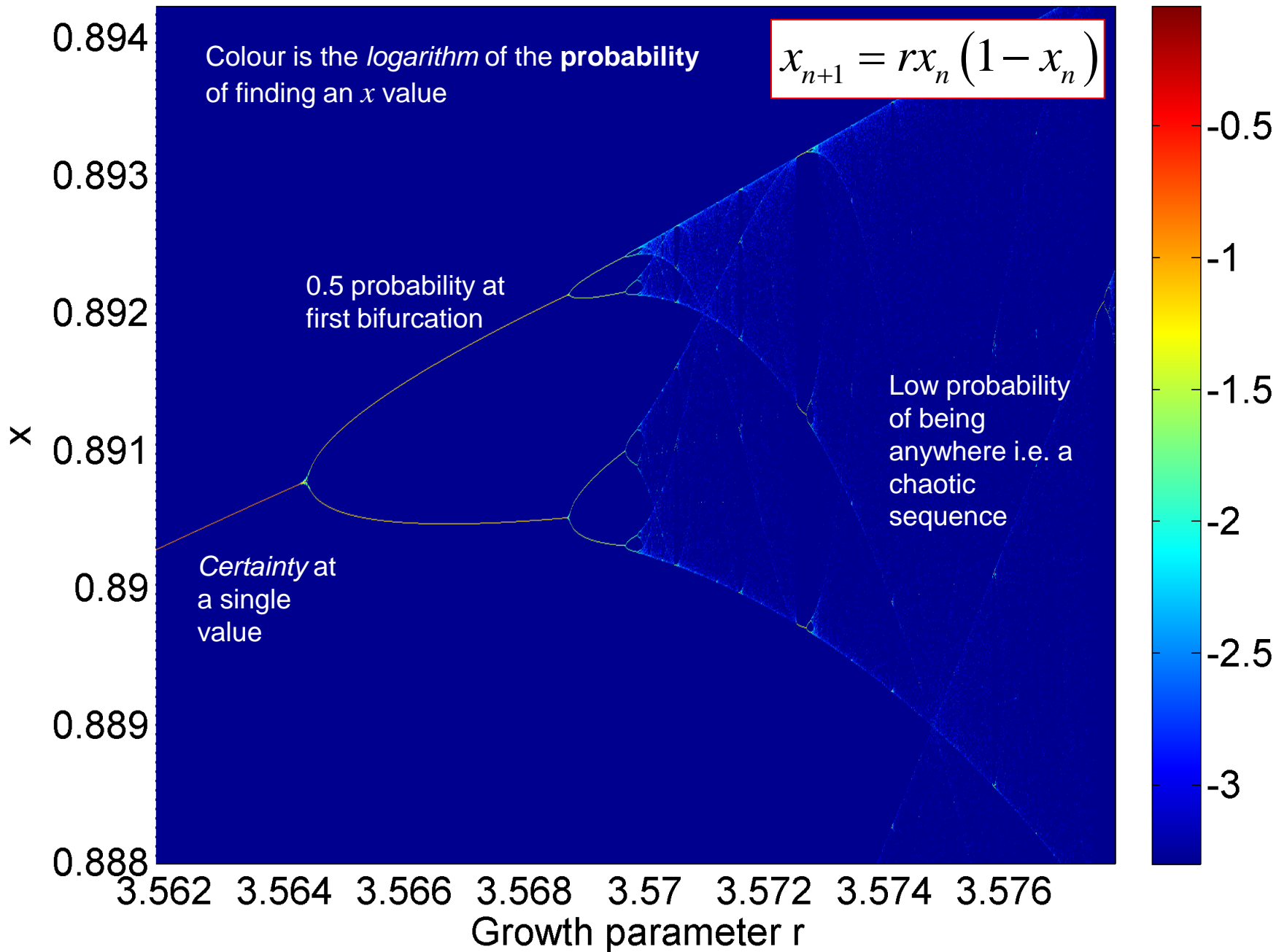


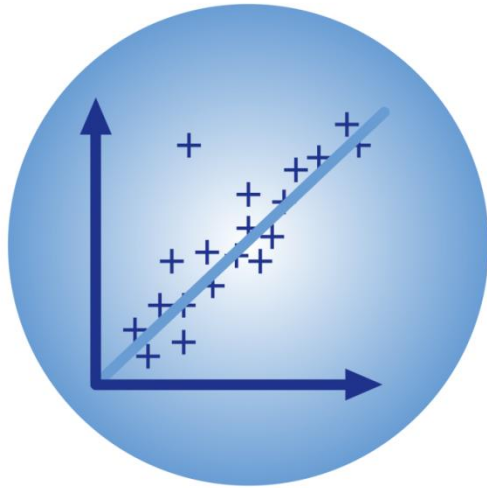
4.669201609...

$$x_{n+1} = rx_n(1 - x_n)$$



# May Bifurcations Logistic map probability





# BPhO

## Computational Challenge

- Suggested homework
- Q&A