

# BPhO

## Computational Challenge

# 2022

# A Standard Atmosphere



The thin line of Earth's atmosphere and the setting sun are featured in this image photographed by the crew of the International Space Station while space shuttle Atlantis on the STS-129 mission was docked with the station.  
Image credit: [NASA](#)

**Instructions:** Welcome to the **British Physics Olympiad Computational Challenge 2022**. The goal is to build computer models based upon the instructions in this document. These can be achieved using a *spreadsheet* such as Microsoft Excel, although you are encouraged to use a *programming language* of your choice\* for the more sophisticated models.

The challenge runs from **Easter 2022 till August 2022**. To submit an entry you will need to fill in a web form. There may be a small administration charge of, payable online as per other BPhO competition entries.

The deliverable of the challenge is to produce a **screencast** of *maximum length two minutes* which describes your response to the challenge, i.e. the graphs and the code & spreadsheets and your explanation of these. The videos must be uploaded to **YouTube**, and we recommend you set these as *Unlisted with Comments disabled*. **Your entry will comprise a YouTube link.** *Instructions how to do this are at the end of this presentation.* To produce the screencast, we recommend the Google Chrome add-on [Screencastify](#).

You can enter the challenge **individually** or in **pairs**. If you opt for the latter, *both* of you must make equal contributions to the screencast.

**Gold**, **Silver** or **Bronze** e-certificates will be emailed to each complete entry, and the **top five** Golds will be invited to present their work at a special ceremony. You should receive a result by December 2022. Note no additional feedback will be provided, and the decision of the judges is final.

**Bronze:** Most of the initial spreadsheet-based challenge elements completed.

**Silver:** All the spreadsheet-based elements completed, and a commendable attempt at the programming-based elements.

**Gold:** All elements completed to a high standard, with possible extension work such as the construction of apps (i.e. programs with graphical user interfaces), significant development of the models, attempt at extension work, short research papers etc.

\*MATLAB or Python is recommended, although any system that can easily execute code in loops and plot graphs will do. e.g. Octave, Java, Javascript, C#, C++. Use what you can access and feel comfortable with. [Programming resources](#)

To quote Lloyd\* in *A Concise Guide to Weather*:

*“The atmosphere around our planet is critically important for our survival. It absorbs the energy we need, reflects the surplus back into space and provides a layer of protection against the harmful elements of incoming radiation. It contains gases, clouds, particles of dust and other particles called aerosols; the main gases being nitrogen and oxygen. Arranged in distinctive layers with no discernible outer limit, most of the atmospheric mass lies below an altitude of 100km. Compared to the Earth’s dimensions, the atmosphere is very thin: the equivalent of a layer of skin to an onion.”*

**Within the first 11 km of this onion skin is our fragile paradise, the Troposphere. Nearly all of life is contained, and the vast majority of weather effects occur here. The major reason for this is the rapid decay of air pressure with altitude. Since gases are highly compressible, this means a greater density of air molecules are present at lower altitudes.**

**We will develop what is known as the *International Standard Atmosphere* (ISA) model, with the goal of determining how average air pressure and temperature vary with altitude, and what effect humidity (i.e. the % saturation of water vapour) has on these variations.**

\*Lloyd, J. (2007) *A Concise Guide to Weather*, Paragon Books.

## TASK 1: PLOT THE ISA MODEL: Temperature vs altitude

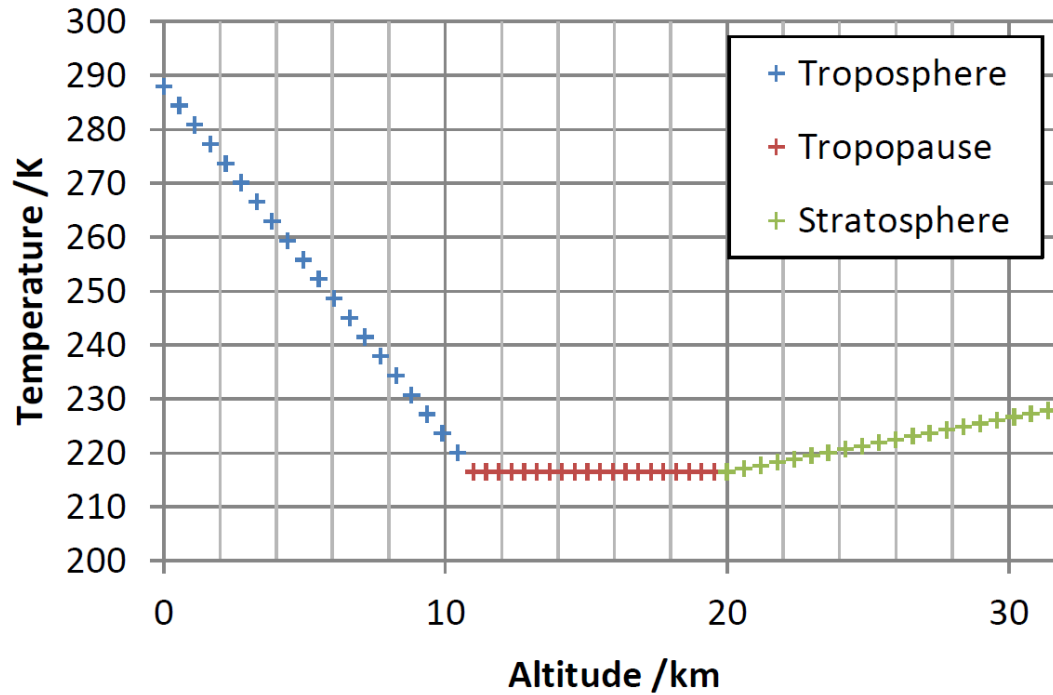
At the base of the *Troposphere*, altitude  $h = 0.0$  km, temperature  $T_0 = (273 + 15)$  K, pressure  $P_0 = 101,325$  Pa.

Level	Base $h$ (km)	Lapse rate $L$ (K/km)
Troposphere	0.0	-6.5
Tropopause	11.0	0.0
Stratosphere	20.0	1.0
Stratosphere	32.0	2.8
Stratopause	47.0	0.0
Mesosphere	51.0	-2.8
Mesosphere	71.0	-2.0

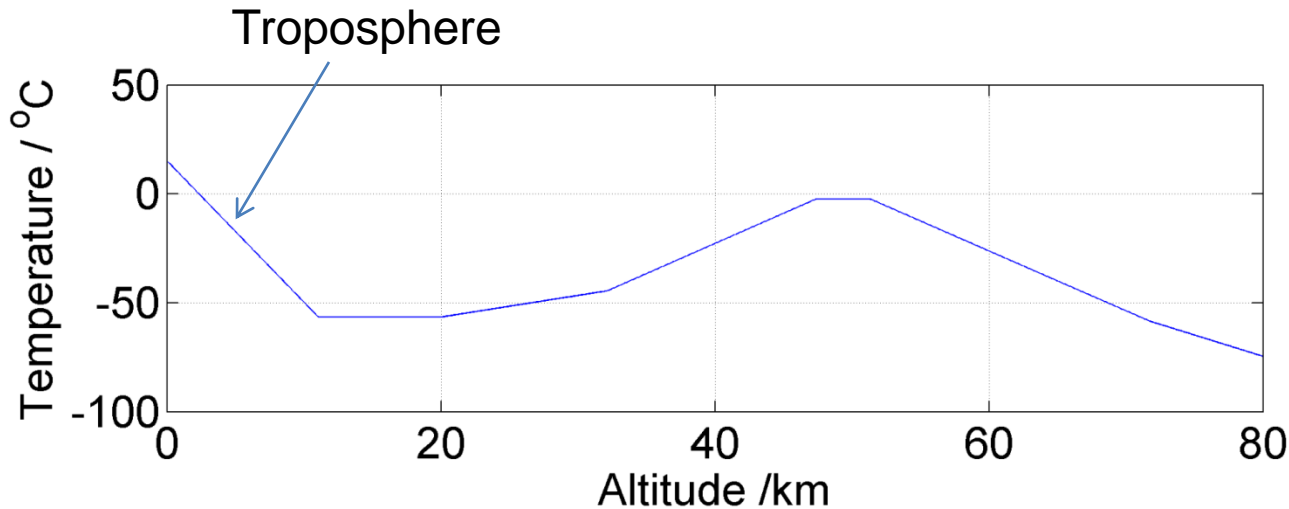
### Assumptions:

- The atmosphere comprises a number of fixed layers, each with a constant temperature gradient ('lapse rate') with altitude.
- The atmosphere consists of a single ideal 'air' gas, whose molecular mass takes into account the average compositions of different gases, e.g. nitrogen (78%), oxygen (21%), 0.9% argon, 0.04% carbon dioxide, etc.

# Atmosphere Temperature vs altitude



You could use  
a spreadsheet  
such as Excel ...

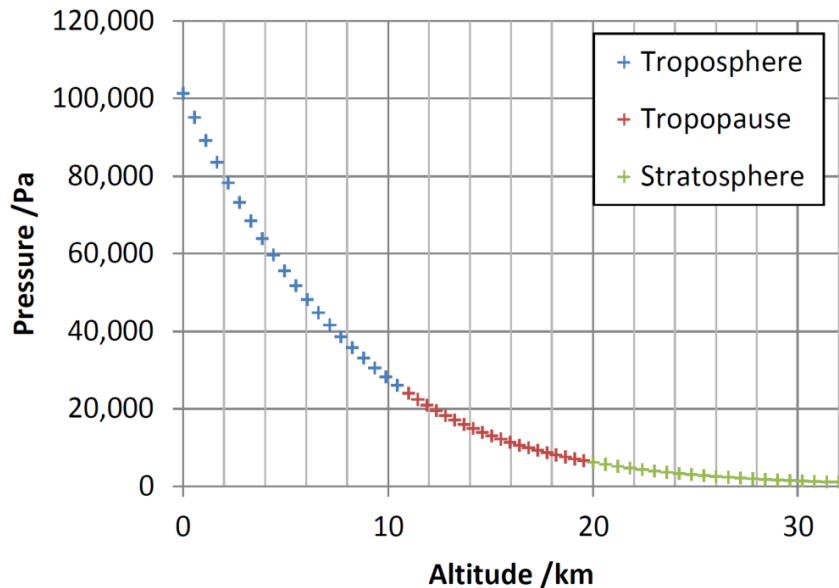


... Or a programming  
language like  
MATLAB or Python

## TASK 2: PLOT THE ISA MODEL: Pressure vs altitude, for dry air

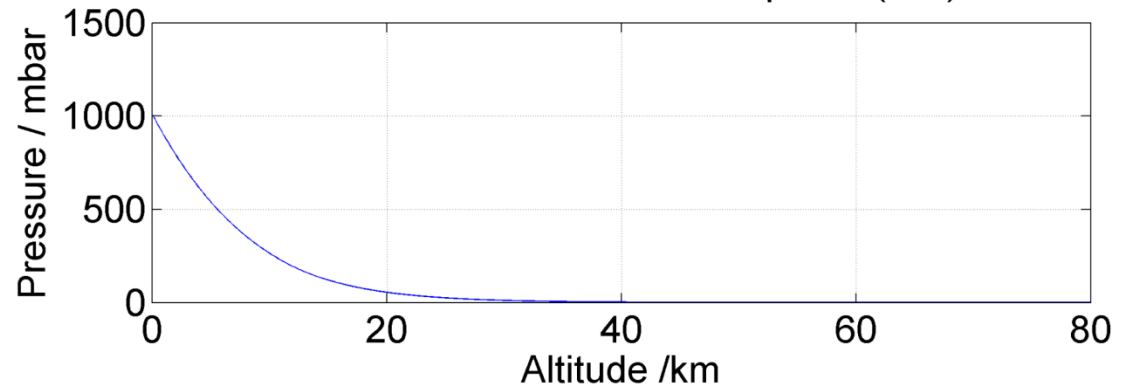
If we can ignore humidity (i.e. the contribution to air pressure from water vapour), air pressure is simply the *weight per unit area of a column of atmosphere*.

Atmospheric pressure vs altitude



A mathematical recipe for these graphs will be derived in the next few slides...

International standard atmosphere (ISA)

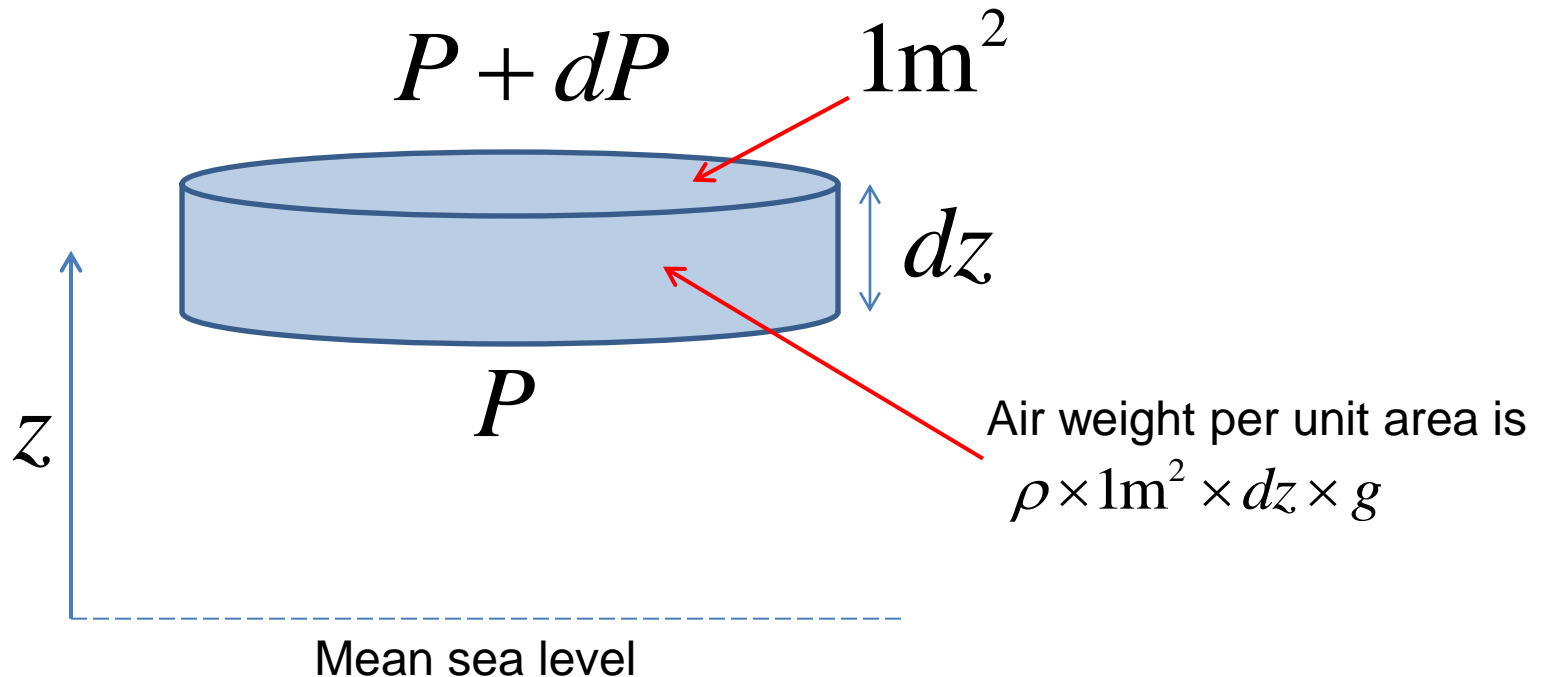


## Calculating air pressure

Consider a  $1\text{m}^2$  horizontal cross section parcel of air of density  $\rho$  at an altitude  $z$ , with vertical width  $dz$ . The atmospheric pressure change  $dP$  between altitudes  $z$  and  $z+dz$  resulting from the removal of the air parcel from the total weight of air above is:

$$dP = -\rho g dz$$

Strength of gravity  
 $g = 9.81\text{Nkg}^{-1}$



## Determining air pressure in the ISA

Let us assume that the air column is comprised of dry air with molar mass:

$$M = 0.02896 \text{ kg mol}^{-1}$$

Let us assume that the air column is an **ideal gas**. If  $n$  moles of gas occupies volume  $V$  at pressure  $P$  (Pascals) and (absolute) temperature  $T$  (Kelvin):

$$PV = nRT \quad \therefore \frac{P}{RT} = \frac{n}{V}$$

Ideal gas equation

Molar gas constant

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

The density of the air is the mass of  $n$  moles divided by the volume  $V$

$$\rho = \frac{nM}{V}$$

Hence:

$$\rho = \frac{MP}{RT}$$

Change in weight of air column

$$dP = -\rho g dz$$

Ideal gas

$$\rho = \frac{MP}{RT}$$

$$\frac{dP}{P} = -\frac{M}{R} \frac{g dz}{T}$$

$$\int_{P_0}^P \frac{1}{P} dP = -\frac{M}{R} \int_{z_0}^z \frac{g}{T} dz$$

$$\therefore \ln \left( \frac{P}{P_0} \right) = -\frac{M}{R} \int_{z_0}^z \frac{g}{T} dz$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{R} \int_{h_0}^h \frac{1}{T} dz$$

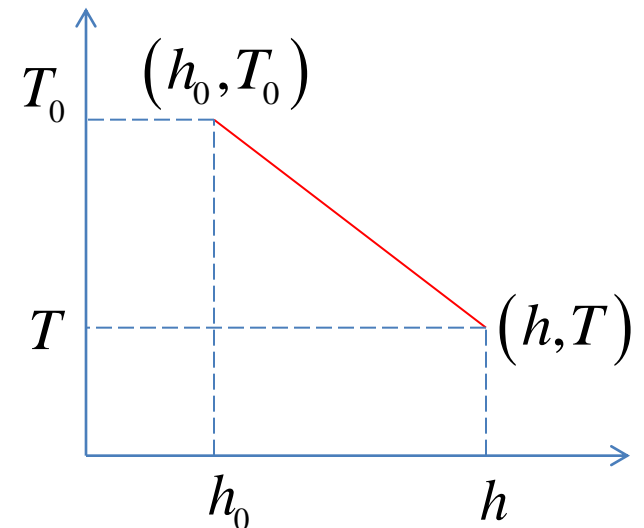
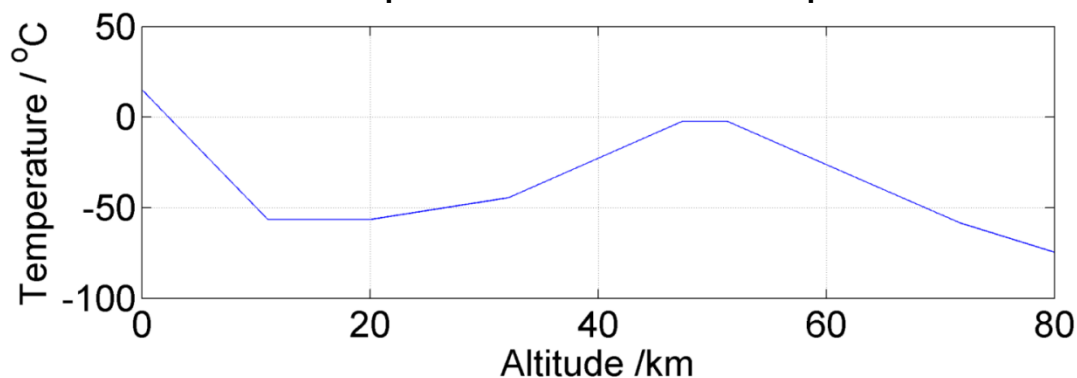
This assumes a **spherical Earth** and therefore *uniform gravitational field strength on the surface of the Earth*. This is quite a good approximation, but in reality the Earth is better modelled as an **ellipsoid** + topographic variations such as mountains.

Strength of gravity  $g = 9.81 \text{ N kg}^{-1}$

Now let us define the temperature  $T$  (in Kelvin) to be a linear function of altitude  $h$ . The (negative) gradient is defined to be the **lapse rate**  $L$ . Note this means a *negative* lapse rate implies a *rise* in temperature with height.

$$T = T_0 - L(h - h_0)$$

Effect of lapse rate on ISA temperature



**Case 1: Isothermal layer i.e.  $L = 0$**

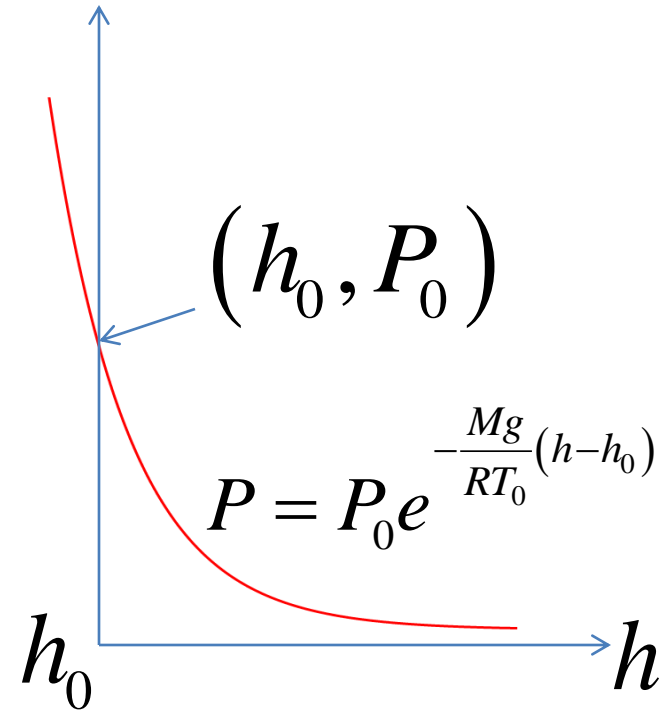
$$T = T_0$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{RT_0} \int_{h_0}^h dz$$

$$P = P_0 e^{-\frac{Mg}{RT_0}(h-h_0)}$$

i.e. an *exponential* decay of pressure with altitude.  
Notice the **Boltzmann factor** exponent.

Note to compute this  
in standard units you'll  
need altitude  $h$  in **metres**  
and temperature in K



Ludwig Boltzmann  
1844-1906

## Case 2: Constant but non-zero, lapse rate $L$

Note to compute this in standard units you'll need altitude  $h$  in **metres** and lapse rate  $L$  in **K per metre** and temperature in K

$$T = T_0 - L(h - h_0)$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{R} \int_{h_0}^h \frac{1}{T_0 - L(h - h_0)} dh$$

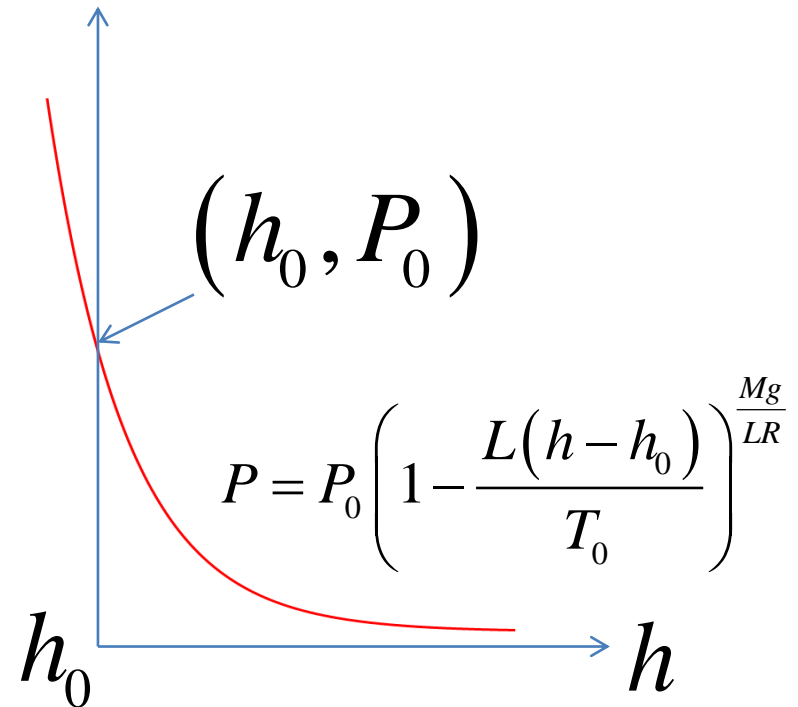
$$\ln\left(\frac{P}{P_0}\right) = -\frac{Mg}{(-L)R} \int_{h_0}^h \frac{-L}{T_0 + Lh_0 - Lh} dh$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{Mg}{LR} \left[ \ln(T_0 + Lh_0 - Lh) \right]_{h_0}^h$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{Mg}{LR} \ln\left(\frac{T_0 + Lh_0 - Lh}{T_0}\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \ln\left(\left(\frac{T_0 + Lh_0 - Lh}{T_0}\right)^{\frac{Mg}{LR}}\right)$$

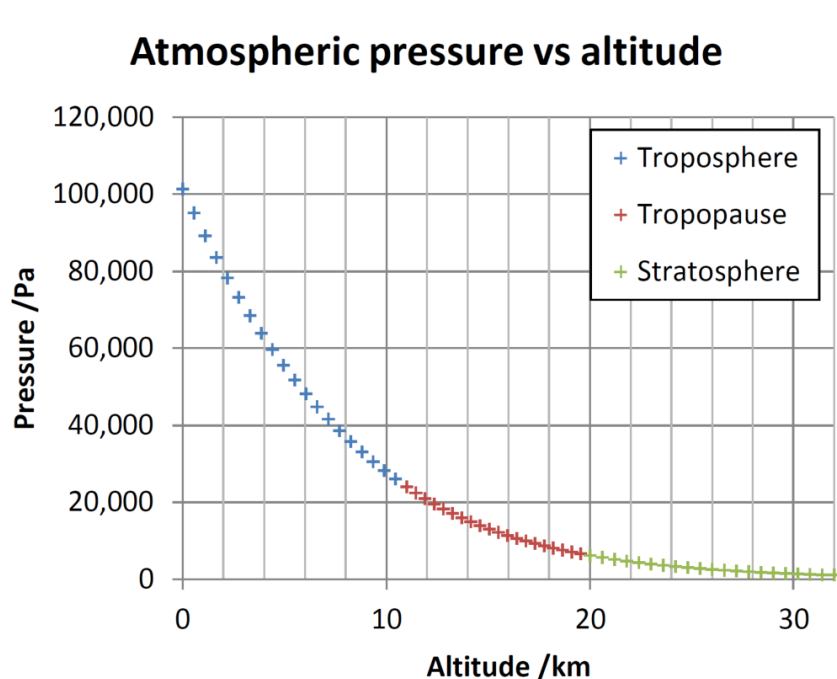
i.e. a **power-law** decay of pressure with altitude.



$$P = P_0 \left(1 - \frac{L(h - h_0)}{T_0}\right)^{\frac{Mg}{LR}}$$

## TASK 2 RECAP: PLOT THE ISA MODEL: Pressure vs altitude, for dry air

If we can ignore humidity (i.e. the contribution to air pressure from water vapour), air pressure is simply the weight per unit area of a column of atmosphere.



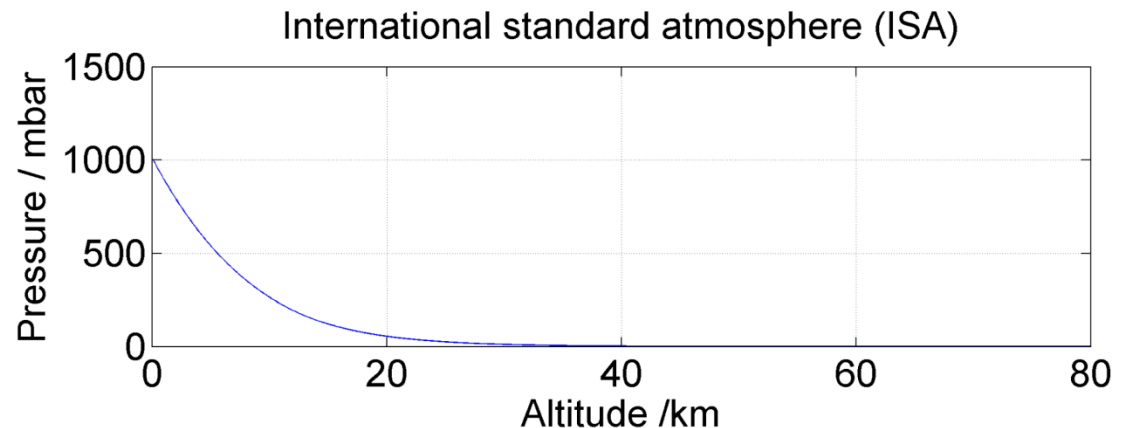
$$P = P_0 \left( 1 - \frac{L(h - h_0)}{T_0} \right)^{\frac{Mg}{LR}}$$

$$P = P_0 e^{-\frac{Mg}{RT_0}(h - h_0)}$$

↑  
If  $L = 0$

Note for these models you'll need altitude in **metres** if you use the other constants in standard units. This means you'll need to define the **Lapse rate in K per m**, which means dividing the **K/km** values by 1,000.

Note  $P_0$  and  $T_0$  correspond to the (Kelvin) temperatures *at the base of the layer*. So work upwards from the base of the Troposphere.



# Example spreadsheet for the first three ISA layers

Molar gas constant  $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$   
 Molar mass of air  $M = 0.02896 \text{ kg mol}^{-1}$   
 Strength of gravity  $g = 9.81 \text{ N kg}^{-1}$

Standard atmosphere model

	zstart /km	zfinish /km	Tstart /K	Tfinish /K	Lapse rate /K per km	pstart /Pa	pfinish /Pa
Troposphere	0	11	288	216.5	6.5	101,325	22,604
Tropopause	11	20	216.5	216.5	0.0	22,604	5,461
Stratosphere	20	32	216.5	228.5	-1.0	5,461	864

Mg/R
0.034171

Troposphere

	z /km	T /K	p /Pa
0	0	288	101,325
0.05	0.55	284.425	94,885
0.1	1.1	280.85	88,781
0.15	1.65	277.275	82,999
0.2	2.2	273.7	77,525
0.25	2.75	270.125	72,348
0.3	3.3	266.55	67,454
0.35	3.85	262.975	62,832
0.4	4.4	259.4	58,469
0.45	4.95	255.825	54,355
0.5	5.5	252.25	50,479
0.55	6.05	248.675	46,830
0.6	6.6	245.1	43,397
0.65	7.15	241.525	40,171
0.7	7.7	237.95	37,142
0.75	8.25	234.375	34,301
0.8	8.8	230.8	31,638
0.85	9.35	227.225	29,146
0.9	9.9	223.65	26,814
0.95	10.45	220.075	24,636
1	11	216.5	22,604

Tropopause

z /km	T /K	p /Pa
11	216.5	22,604
11.45	216.5	21,054
11.9	216.5	19,611
12.35	216.5	18,266
12.8	216.5	17,014
13.25	216.5	15,847
13.7	216.5	14,761
14.15	216.5	13,749
14.6	216.5	12,806
15.05	216.5	11,928
15.5	216.5	11,110
15.95	216.5	10,349
16.4	216.5	9,639
16.85	216.5	8,978
17.3	216.5	8,363
17.75	216.5	7,789
18.2	216.5	7,255
18.65	216.5	6,758
19.1	216.5	6,294
19.55	216.5	5,863
20	216.5	5,461

Stratosphere

z /km	T /K	p /Pa
20	216.5	5,461
20.6	217.1	4,968
21.2	217.7	4,521
21.8	218.3	4,115
22.4	218.9	3,747
23	219.5	3,412
23.6	220.1	3,108
24.2	220.7	2,832
24.8	221.3	2,581
25.4	221.9	2,353
26	222.5	2,146
26.6	223.1	1,957
27.2	223.7	1,786
27.8	224.3	1,629
28.4	224.9	1,487
29	225.5	1,358
29.6	226.1	1,240
30.2	226.7	1,133
30.8	227.3	1,035
31.4	227.9	946
32	228.5	864

Power law

Positive lapse rate

$$p(z) = p_0 \left( 1 - \frac{L(z - z_0)}{T_0} \right)^{\frac{Mg}{LR}}$$

Exponential

Isothermal

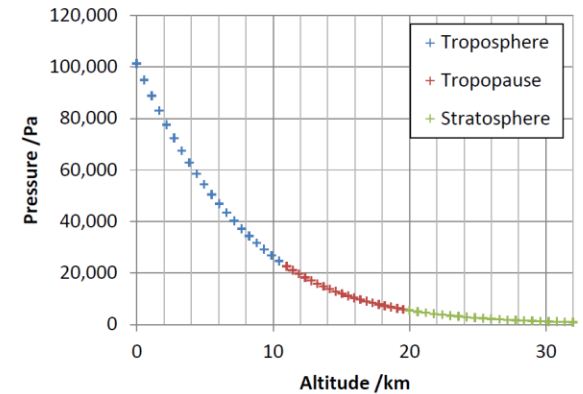
$$p(z) = p_0 e^{-\frac{Mg}{RT_0}(z - z_0)}$$

Power law

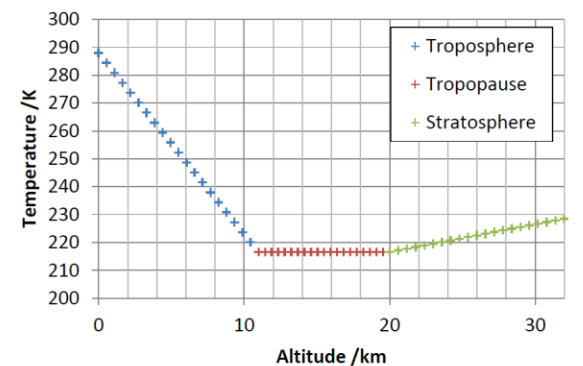
Negative lapse rate

$$p(z) = p_0 \left( 1 - \frac{L(z - z_0)}{T_0} \right)^{\frac{Mg}{LR}}$$

Atmospheric pressure vs altitude



Atmosphere Temperature vs altitude



### TASK3: What about humidity?

i.e. the average pressure, temperature structure of the air column

Any sensible description of *non-arid climatology* will need to take into account the impact of variable amounts of **water vapour** contained within the air column. In the *Troposphere* at least, the presence of water vapour can have a dramatic influence upon thermodynamic variables such as temperature and pressure, and is obviously a fundamental component of weather phenomena such as cloud and fog.

To model the effect of humidity upon temperature and pressure, let us modify our original single ideal gas assumption to consider a *composite* of dry air and water vapour. The molar masses of dry air and water vapour are, respectively:

$$M_d = 0.02896 \text{ kg mol}^{-1} \quad M_v = 0.01802 \text{ kg mol}^{-1}$$

The respective ideal gas equations are, for  $n_d$  moles of dry air and  $n_v$  moles of water vapour:

$$P = P_v + P_d$$

i.e. the overall pressure  $P$  is the sum of the 'partial pressures' of the component gases

$$P_v = \frac{n_v RT}{V} \quad P_d = \frac{n_d RT}{V}$$

Molar gas constant

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

Let us define **relative humidity**  $U$  as the ratio of water vapour pressure to that at saturation  $E_s$

$$P_v = UE_s$$

An empirical model for saturation pressure as a pure function of temperature is given by the *Arden Buck equation*

[http://en.wikipedia.org/wiki/Arden\\_Buck\\_equation](http://en.wikipedia.org/wiki/Arden_Buck_equation)

$$E_s(T) = 6.1121e^{\left\{ \left( 18.678 - \frac{T_c}{234.5} \right) \left( \frac{T_c}{T_c + 257.14} \right) \right\}}$$

There are in fact many empirical formulae for vapour pressure. A good selection are compared at <http://cires.colorado.edu/~voemel/vp.html>

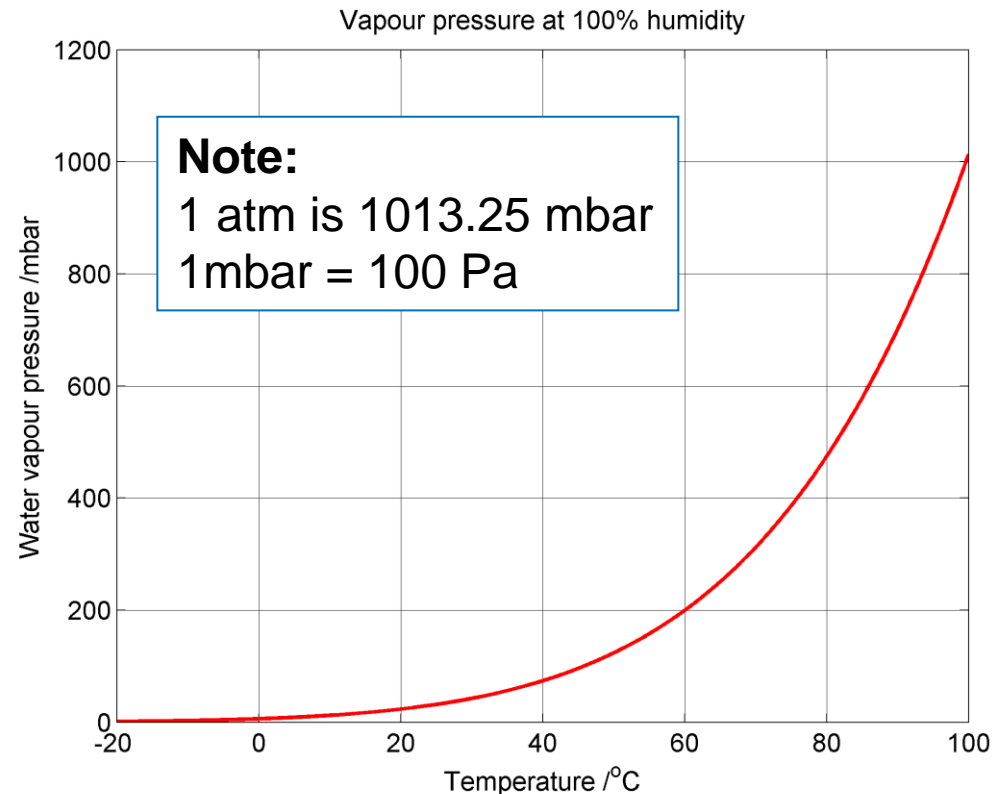
where the saturation vapour pressure  $E_s$  is given in mbar and temperature  $T_c$  in *degrees Celsius*

$$T_c = T - 273$$

Convert to Celsius from Kelvin temperatures  $T$

Note the vapour pressure is *very small* compared to the pressure of dry air when temperatures are low. Therefore to a very good approximation we can ignore it in the pressure calculation.

However, the lapse rate *is significantly* affected by humidity.....



The overall density of the atmosphere is:

$$\rho = \frac{n_d M_d + n_v M_v}{V} = \frac{M_d}{V} \left( n_d + n_v \frac{M_v}{M_d} \right)$$

From the ideal gas equations:

$$n_v = \frac{V}{RT} P_v = \frac{V}{RT} U E_s$$

$$n_d = \frac{V}{RT} P_d = \frac{V}{RT} (P - P_v) = \frac{V}{RT} (P - U E_s)$$

Hence:

$$\rho = \frac{M_d}{RT} \left( P - U \left( 1 - \frac{M_v}{M_d} \right) E_s(T) \right)$$

The pressure integral is now more complicated, but can be evaluated using a *numerical method*:

$$dP = -\rho g dh \quad \rho = \frac{M_d}{RT} \left( P - U \left( 1 - \frac{M_v}{M_d} \right) E_s(T) \right) \quad T = T_0 - L(h - h_0)$$

$$\therefore \frac{dP}{dh} = -\frac{M_d g}{RT} \left( P - U \left( 1 - \frac{M_v}{M_d} \right) E_s(T) \right)$$

A simple iterative numeric solution scheme might be to use a *finite* altitude change  $\Delta h$ , and start from a known temperature and pressure e.g. 15°C, 1013.25mbar.

$$h \rightarrow h + \Delta h$$

$$T \rightarrow T - L\Delta h \quad T_c = T - 273$$

$$E_s = 6.1121e^{\left\{ \left( 18.678 - \frac{T_c}{234.5} \right) \left( \frac{T_c}{T_c + 257.14} \right) \right\}}$$

$$\Delta P = -\frac{M_d g}{RT} \left( P - U \left( 1 - \frac{M_v}{M_d} \right) E_s(T) \right) \Delta h$$

$$P \rightarrow P + \Delta P$$

i.e. you can now  
compute  $P$  vs  $h$  by running  
this **in a loop**.

$$M_d = 0.02896 \text{ kg mol}^{-1}$$

$$M_v = 0.01802 \text{ kg mol}^{-1}$$

$$R = 8.314 \text{ J mol K}^{-1}$$

$$g = 9.81 \text{ N kg}^{-1}$$

Now the partial pressure of water vapour is typically very small compared to dry air. Therefore we *can* ignore it in pressure calculations. However, the **lapse rate**  $L$  is *significantly* affected by the presence of water vapour. A model for  $L$  is:

$$L = -\frac{dT}{dh}$$

Lapse rate in K per metre

$$L = g \frac{1 + \frac{r\Delta H_v}{R_{sd}T}}{c_{pd} + \frac{(\Delta H_v)^2 r}{R_{sw}T^2}}$$

T is in Kelvin

$$M_d = 0.02896 \text{ kg mol}^{-1}$$

$$M_v = 0.01802 \text{ kg mol}^{-1}$$

$$g = 9.8076 \text{ N kg}^{-1}$$

$$r = \frac{\varepsilon U E_s}{P - U E_s}$$

Essentially the mass fraction of water to dry air

**Specific latent heat of vaporization of water**

$$\Delta H_v = 2,501,000 \text{ J kg}^{-1}$$

**Specific heat of dry air at constant pressure is:**

$$c_{pd} = 1003.5 \text{ J kg}^{-1} \text{ K}^{-1}$$

**Specific gas constant for dry air**

$$R_{sd} = 287 \text{ J kg}^{-1} \text{ K}^{-1}$$

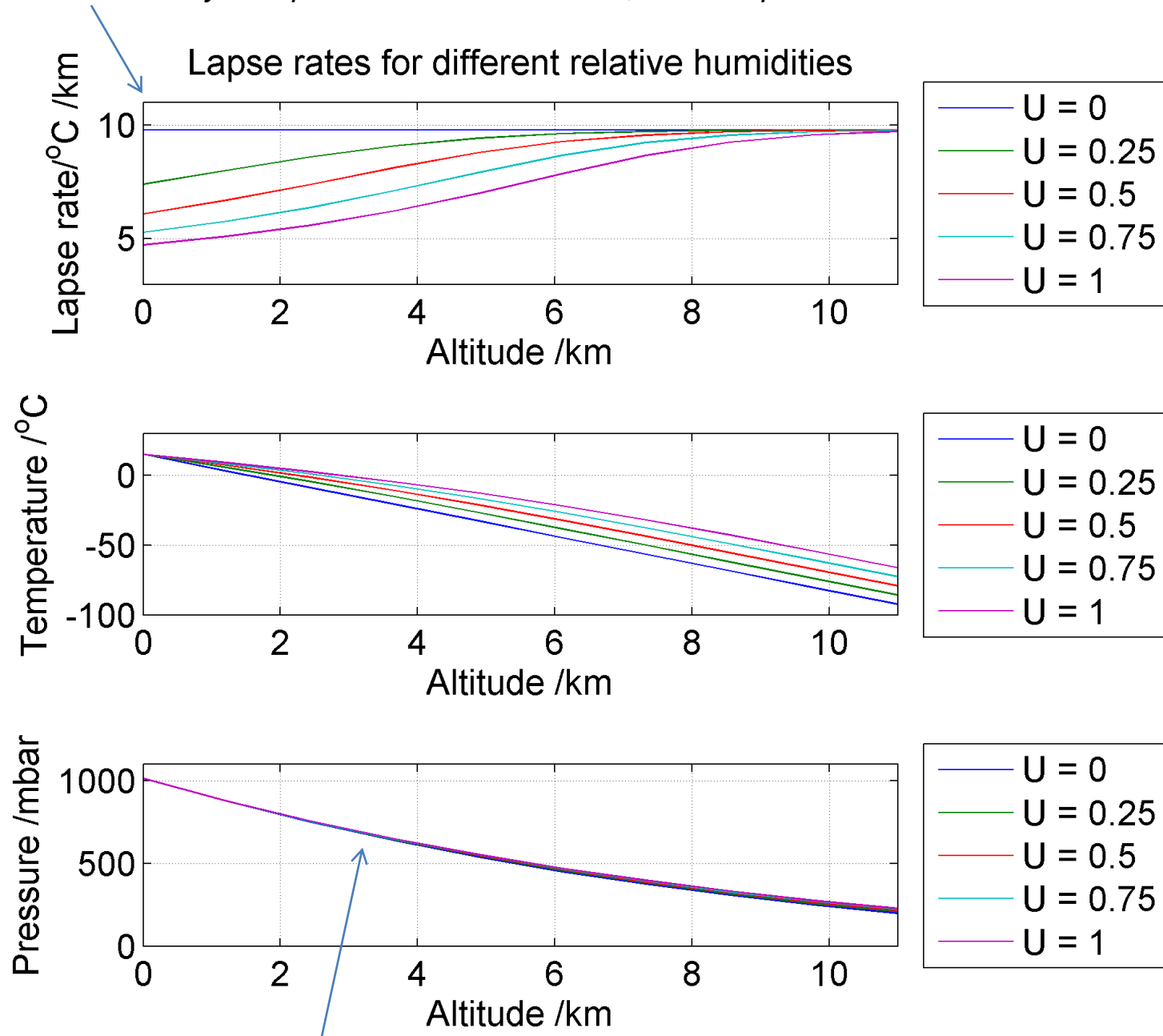
**Specific gas constant for water vapour**

$$R_{sw} = 461.5 \text{ J kg}^{-1} \text{ K}^{-1}$$

Specific gas constant example

$$R_{sd} = \frac{R}{M_d} = \frac{8.314}{0.02896} \approx 287$$

Hence 'rule of thumb': *dry air lapse rate* is about **10° /km**, *wet air lapse rate* is about **5° /km**



So humidity only makes a significant difference at low pressures i.e. at high altitudes. At this point the assumption of a significant relative humidity is likely to be invalid anyway!

The **dew point** is the temperature to which a given parcel of air must be cooled, at constant barometric pressure, for water vapour to condense into water.

The *August-Roche Magnus approximation* defines the dew point to be defined by the following expression in terms of relative humidity  $U$  (with values 0...1) and *ambient* air temperature  $T$ . i.e. the air parcel being cooled will be *colder* than the ambient air.

Note in the formula below,  $T$  is defined in degrees Celsius.

$$T_d = \frac{b \left( \ln U + \frac{aT}{b+T} \right)}{a - \ln U - \frac{aT}{b+T}}$$

$$a = 17.625$$

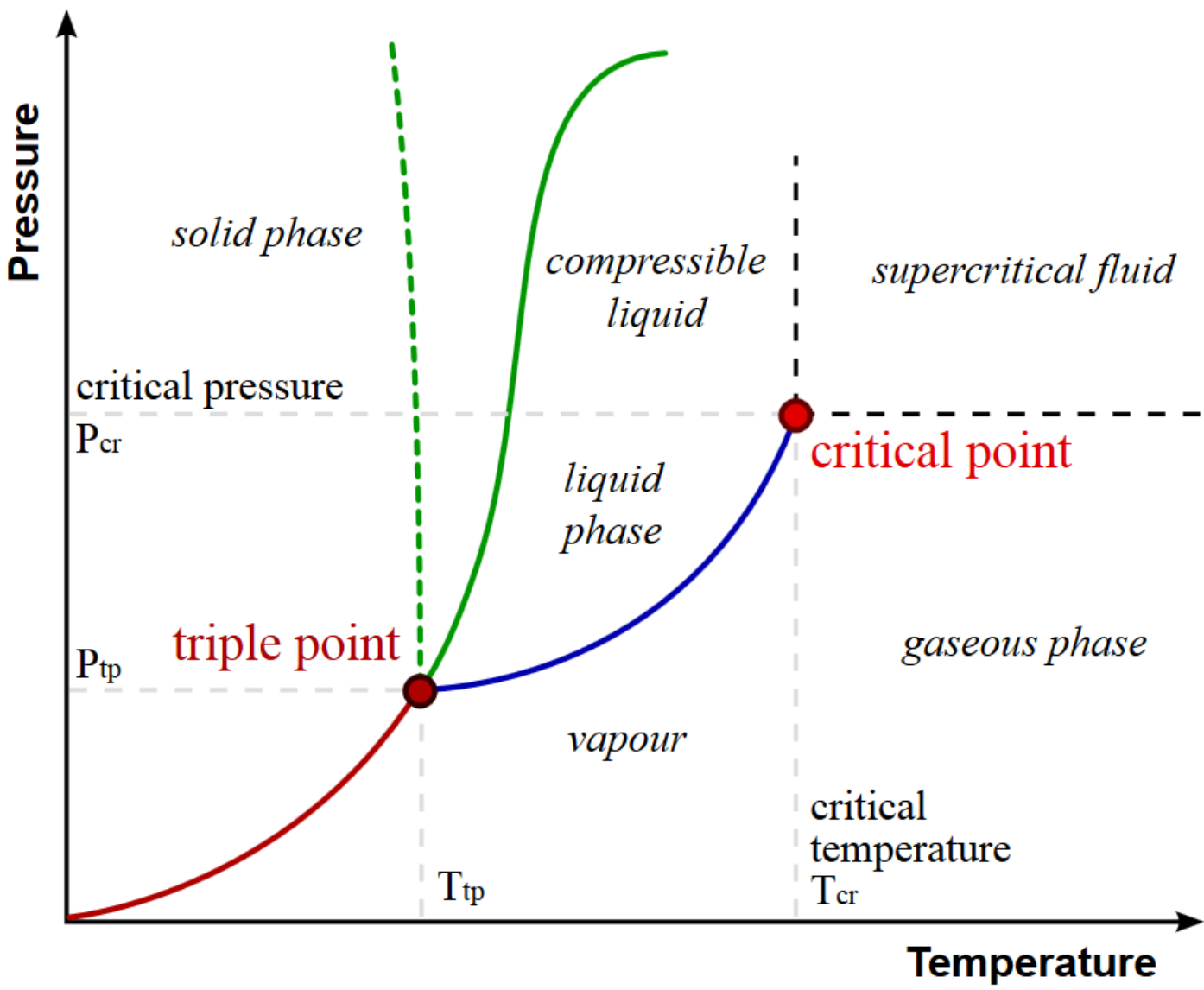
$$b = 243.04$$



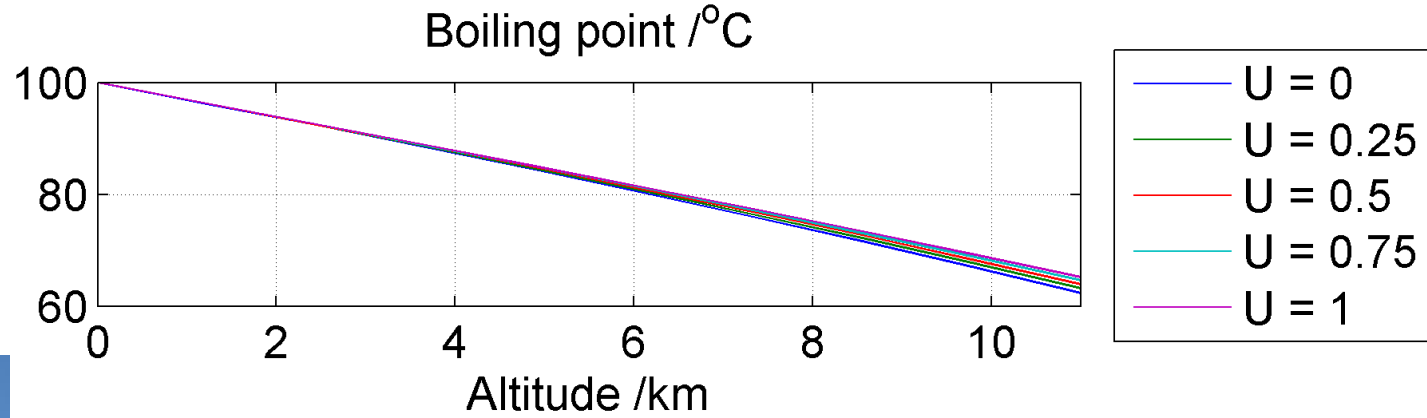
“The dew point is the temperature at which the water vapour in a sample of air at constant barometric pressure condenses into liquid water at the same rate at which it evaporates”

[http://en.wikipedia.org/wiki/Dew\\_point](http://en.wikipedia.org/wiki/Dew_point)

**What about boiling point?** To boil water, it must undergo a *phase transition* from liquid to gas. This requires a certain amount of heat, *the latent heat of vaporization*, to break the inter-molecular bonds inherent in the water.



The boiling point of water *decreases* as atmospheric pressure *decreases*.  
You can observe this on a mountaineering expedition as you ascend in altitude.



K2. August 2018

The gradient of a phase transition line in the  $P,T$  diagram is given by the *Clausius-Clapeyron equation*:

Pressure in  
Pascals

Latent heat /Jmol<sup>-1</sup>

$$\frac{dP}{dT} = \frac{\Delta H}{T \Delta V}$$

Temperature in  
Kelvin

Volume change of 1 mole of  
substance during the phase  
transition

In a liquid to gas transition we can assume the volume change is sufficiently large as to ignore the original fluid volume. If one assumes the resulting gas is ideal:

$$\Delta V = \frac{RT}{P}$$

Hence:

$$\frac{dP}{dT} = \frac{\Delta H}{RT^2} P$$

We can use this relationship to determine the liquid-to-gas line in the  $P,T$  diagram if the latent heat of vaporization is assumed to be temperature independent. (In reality this is not the case, but is less of an issue at low temperatures).

$$\frac{dP}{dT} = \frac{\Delta H}{RT^2} P$$

$$\int_{P_*}^P \frac{1}{P} dP = \frac{\Delta H}{R} \int_{T_*}^T \frac{1}{T^2} dT$$

$$\ln\left(\frac{P}{P_*}\right) = -\frac{\Delta H}{R} \left(\frac{1}{T} - \frac{1}{T_*}\right)$$

$$P = P_* e^{-\frac{\Delta H}{R} \left(\frac{1}{T} - \frac{1}{T_*}\right)}$$

$$T = \left( \frac{1}{T_*} - \frac{R}{\Delta H} \ln\left(\frac{P}{P_*}\right) \right)^{-1}$$

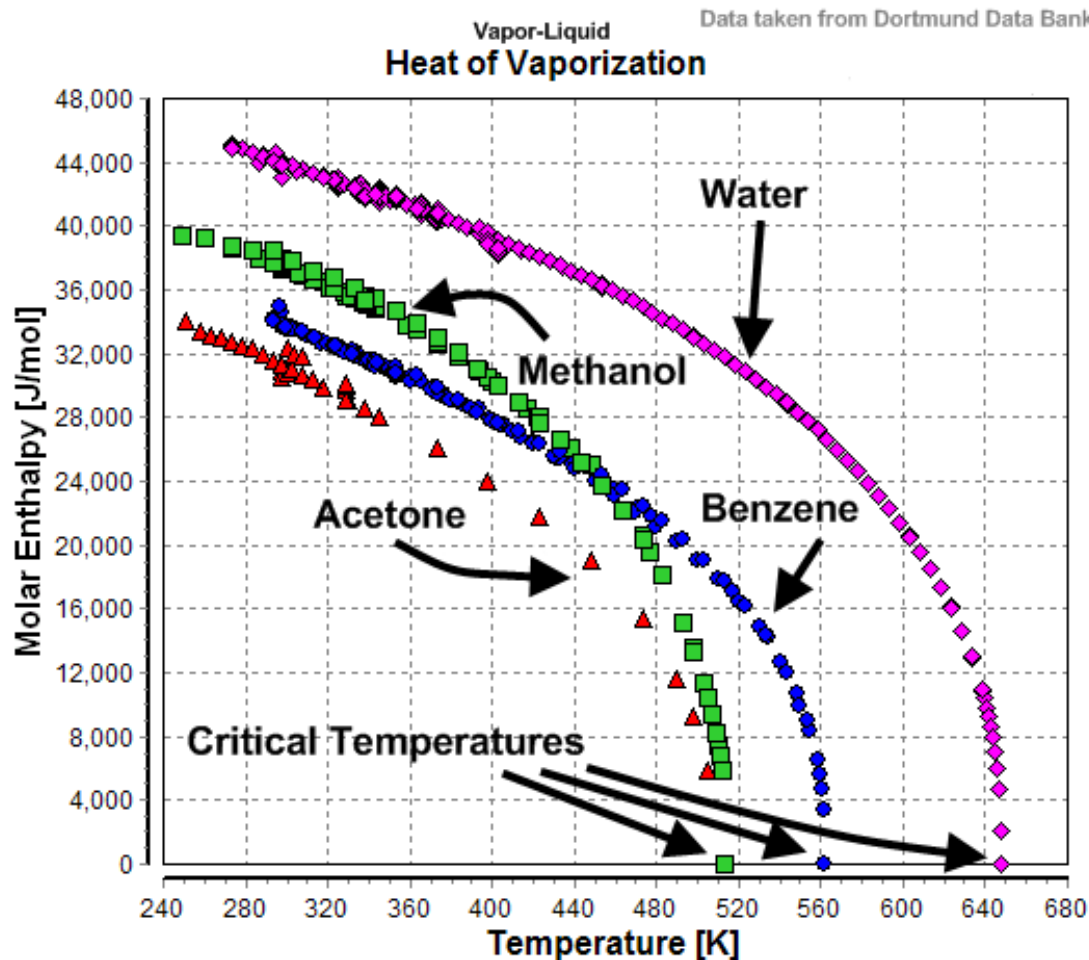
Since the temperature corresponds to the liquid-to-gas transition line, we can therefore derive an expression for the **boiling point** of a liquid relative to ambient temperature  $T$  and pressure  $P$ , as long as one fixed boiling point  $T_*$  and corresponding pressure  $P_*$  is known.

$$T_{boil} = \left( \frac{1}{T_*} - \frac{R}{\Delta H} \ln \left( \frac{P}{P_*} \right) \right)^{-1}$$

Boiling point of water at different atmospheric pressures /mbar.

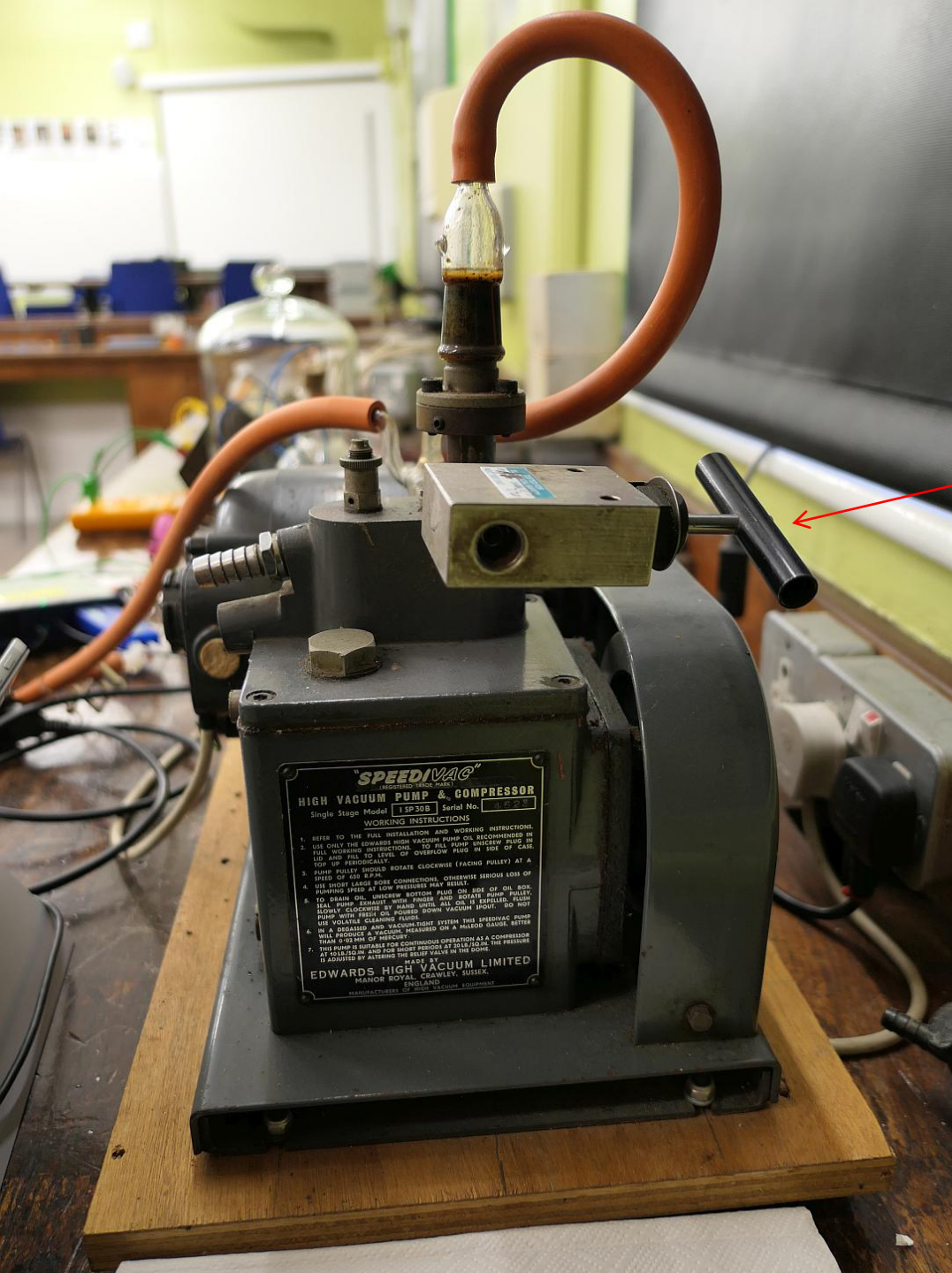
Latent heat of vaporization of water is  $\Delta H = 40.7 \text{ kJ mol}^{-1}$  at  $100^\circ\text{C}$  and 1013.25 mbar ambient air pressure.

At 'typical' troposphere temperatures, a higher value of  $45.07 \text{ kJ mol}^{-1}$  is used in some calculations, such as the lapse rate model quoted earlier.



## References:

[http://en.wikipedia.org/wiki/Lapse\\_rate](http://en.wikipedia.org/wiki/Lapse_rate)  
[http://en.wikipedia.org/wiki/Enthalpy\\_of\\_vaporization](http://en.wikipedia.org/wiki/Enthalpy_of_vaporization)  
[http://en.citizendium.org/wiki/Heat\\_of\\_vaporization](http://en.citizendium.org/wiki/Heat_of_vaporization)



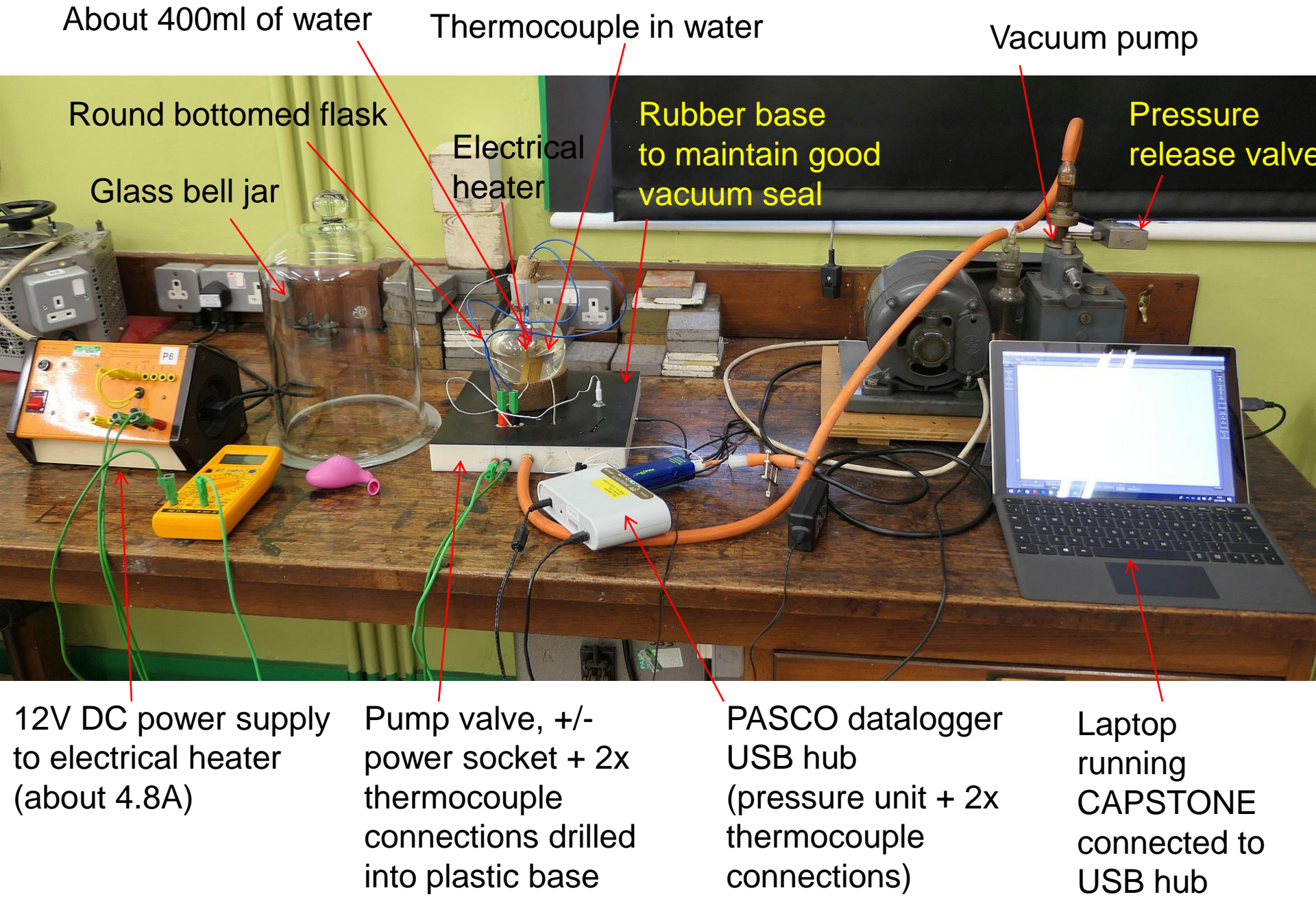
If you have access to a vacuum pump then you can experience the variation of water boiling point with air pressure yourself.

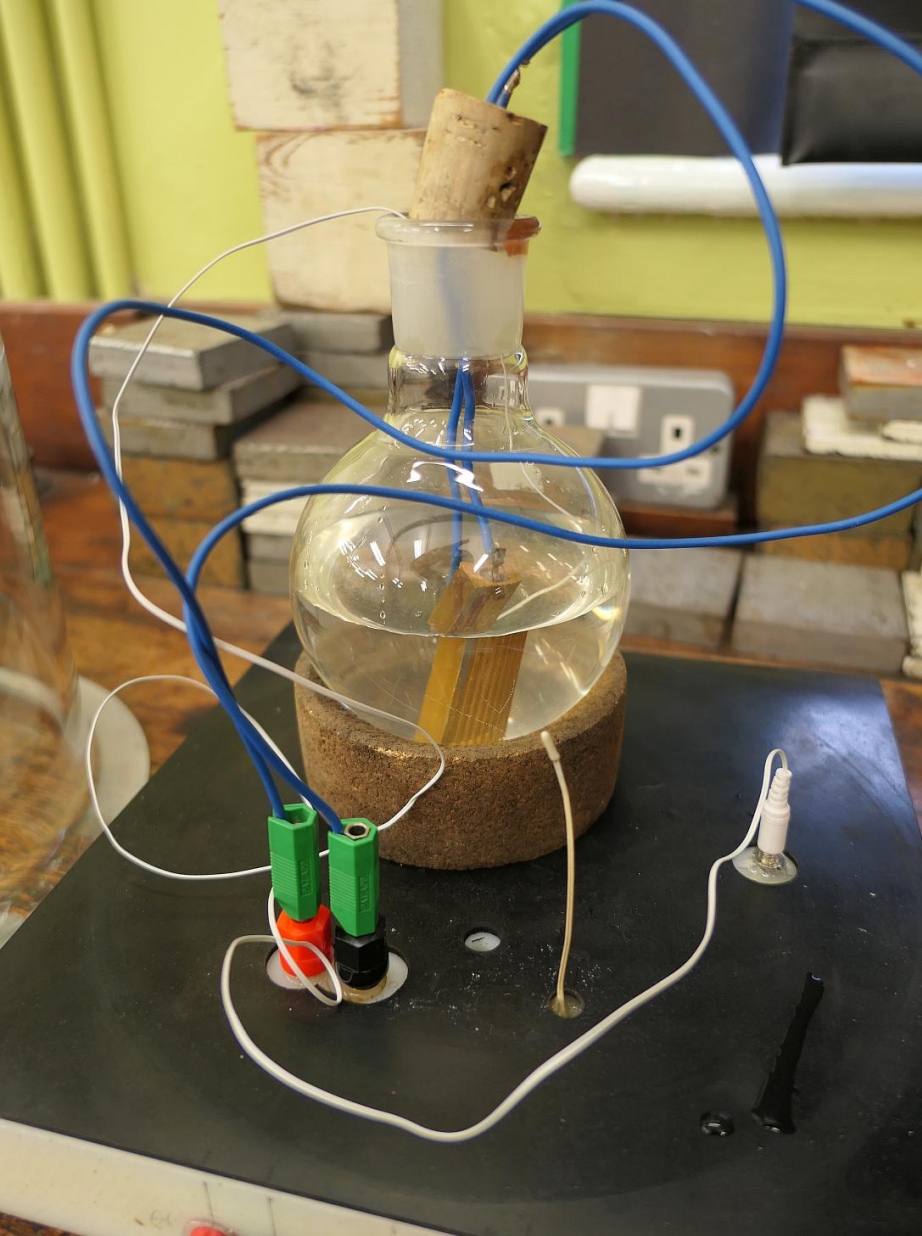
Vacuum pump, connected to a bell jar.

About 3% of atmospheric pressure is possible.

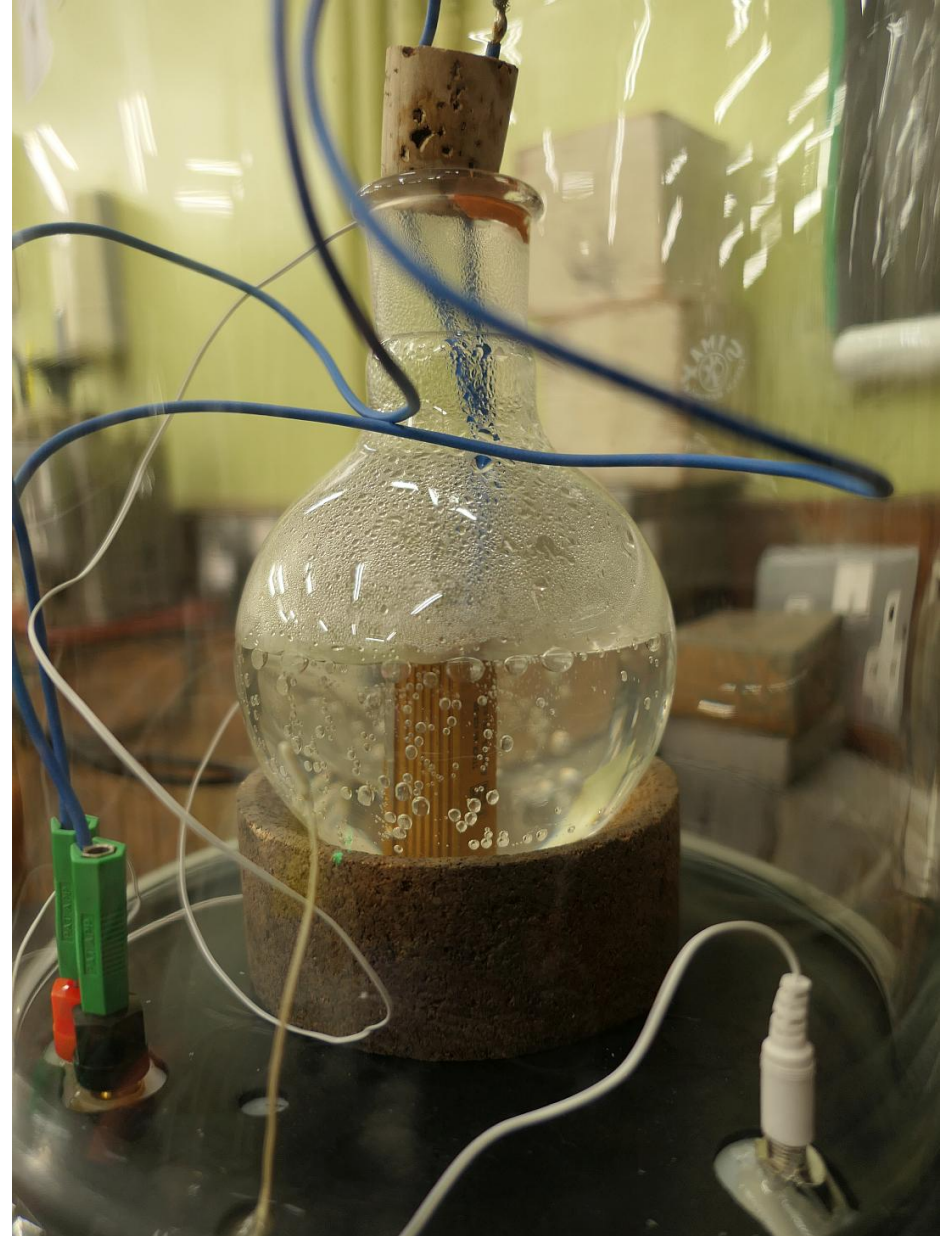
Turn pressure valve *slowly* to return air to bell jar between experiments.

# Equipment for boiling point of water vs air pressure experiment



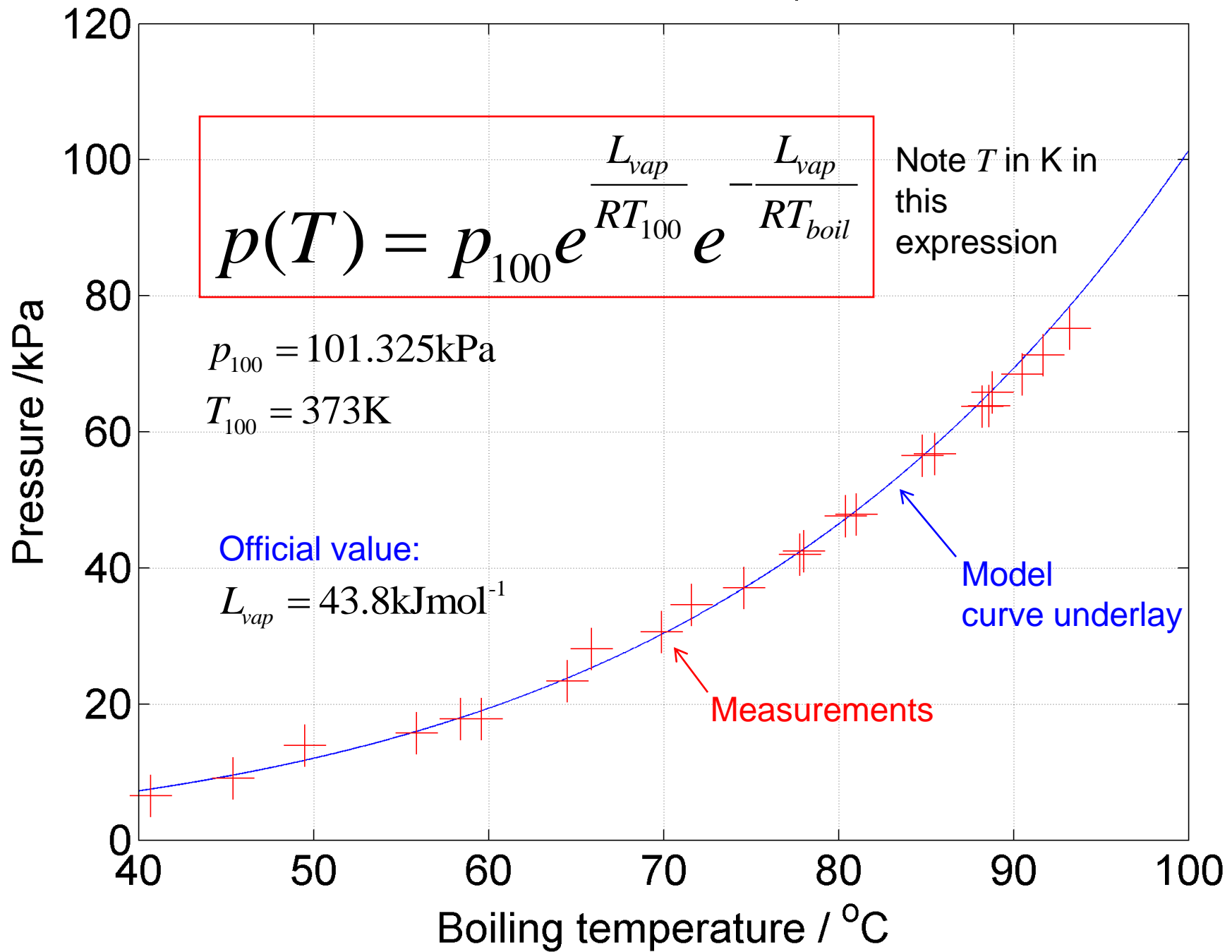


Heat water up to a desired temperature using electrical heater. (It is pretty slow, i.e about 10s per deg C, so you can leave it on)



Then pump out air till water spontaneously boils. At this point turn off the pump and record pressure and temperature using the datalogger display on the PC

Pressure vs boiling temperature.  $L_{vap} = (42.7 \pm 0.8) \text{ kJ/mol}$



In **summary**, we can model the variation of pressure, temperature, lapse rate boiling point and dew point with altitude using the following iterative scheme:

$$T_0 = 15^\circ\text{C} \quad T_* = 100^\circ\text{C} \quad P_0 = P_* = 1013.25\text{mbar} \quad \Delta h = 0.01\text{km}$$

$$h \rightarrow h + \Delta h \quad T \rightarrow T - L\Delta h$$

$T$  in degrees Celsius

$T_K = T + 273$

$T$  in Kelvin

$$E_s = 6.1121e^{\left\{ \left( 18.678 - \frac{T}{234.5} \right) \left( \frac{T}{T+257.14} \right) \right\}}$$

$$r = \frac{R_{sd}}{R_{sw}} \frac{UE_s}{P - UE_s}$$

$$L = g \frac{1 + \frac{r\Delta H_v}{R_{sd}T_K}}{c_{pd} + \frac{(\Delta H_v)^2 r}{R_{sw}T_K^2}}$$

$$\Delta P = -\frac{M_d g}{RT_K} \left( P - U \left( 1 - \frac{M_v}{M_d} \right) E_s(T) \right) \Delta h$$

$$P \rightarrow P + \Delta P$$

$$T_{boil} = \left( \frac{1}{T_*} - \frac{R}{\Delta H} \ln \left( \frac{P}{P_*} \right) \right)^{-1}$$

$T$  in Kelvin

$$T_{dew} = \frac{b \left( \ln U + \frac{aT}{b+T} \right)}{a - \ln U - \frac{aT}{b+T}}$$

$$a = 17.625$$

$$b = 243.04$$

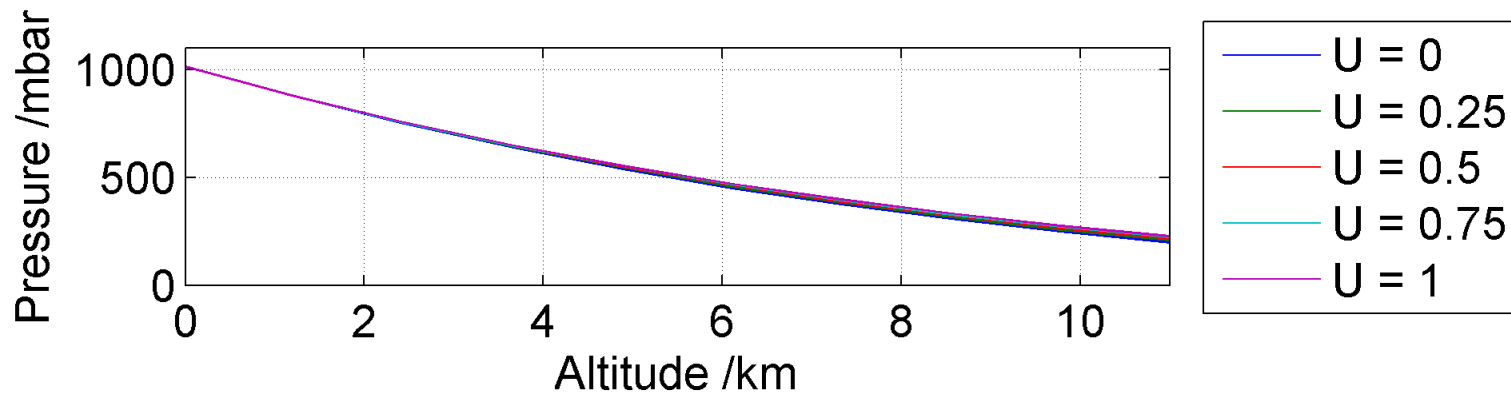
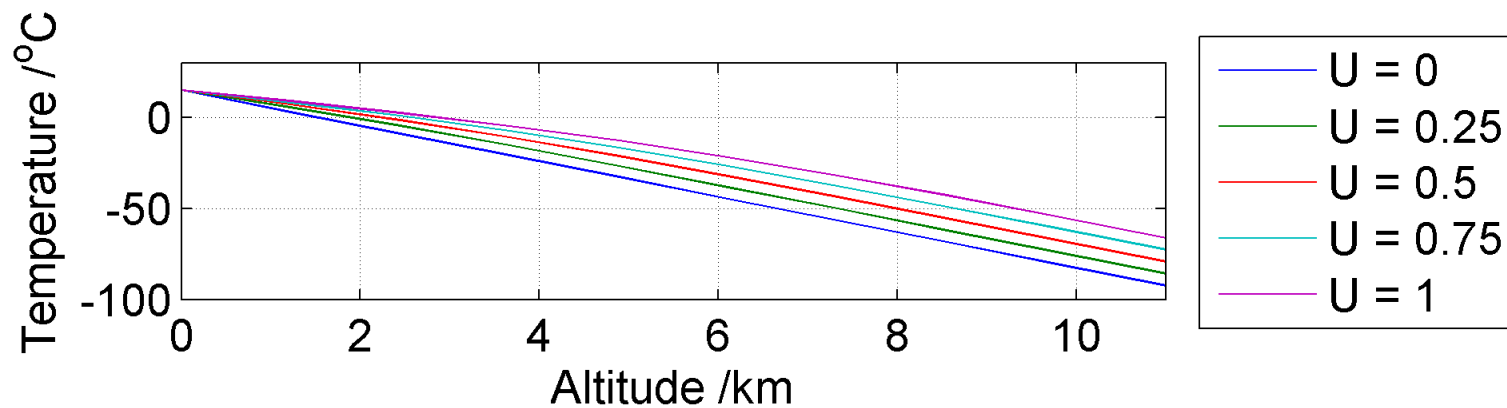
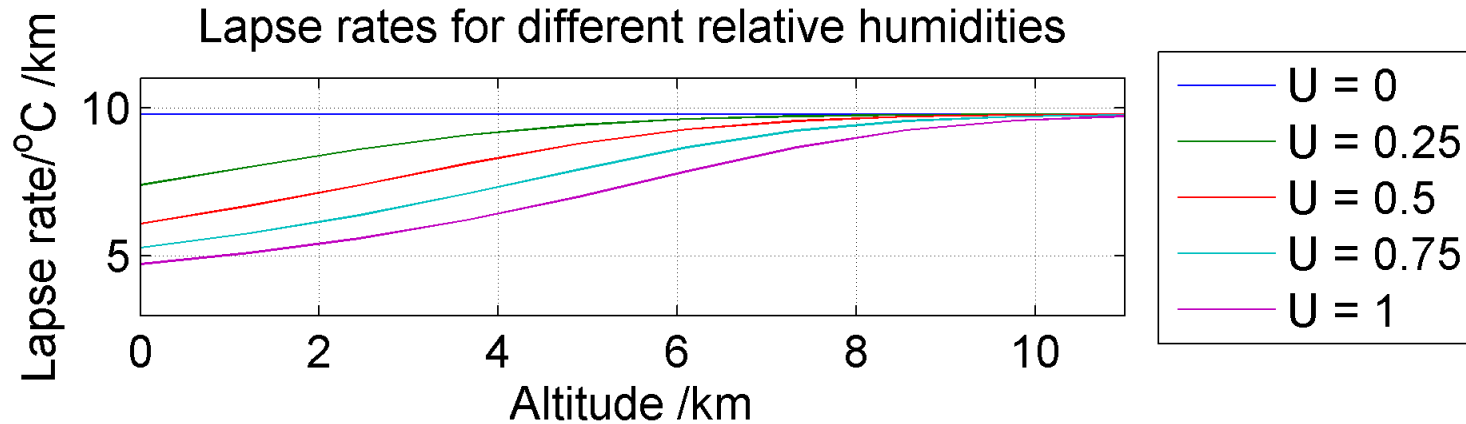
$T$  in degrees Celsius

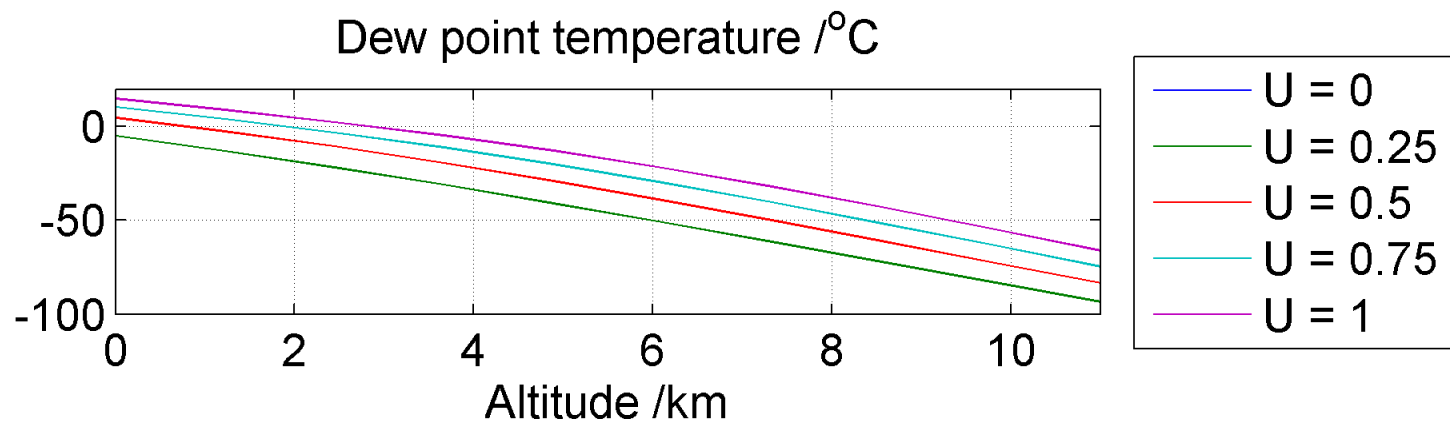
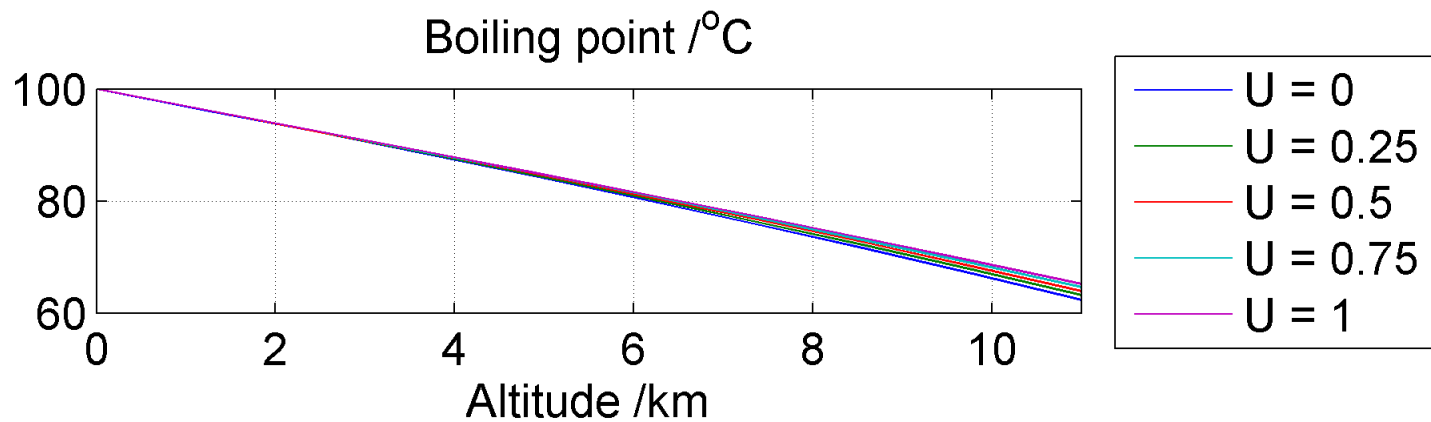
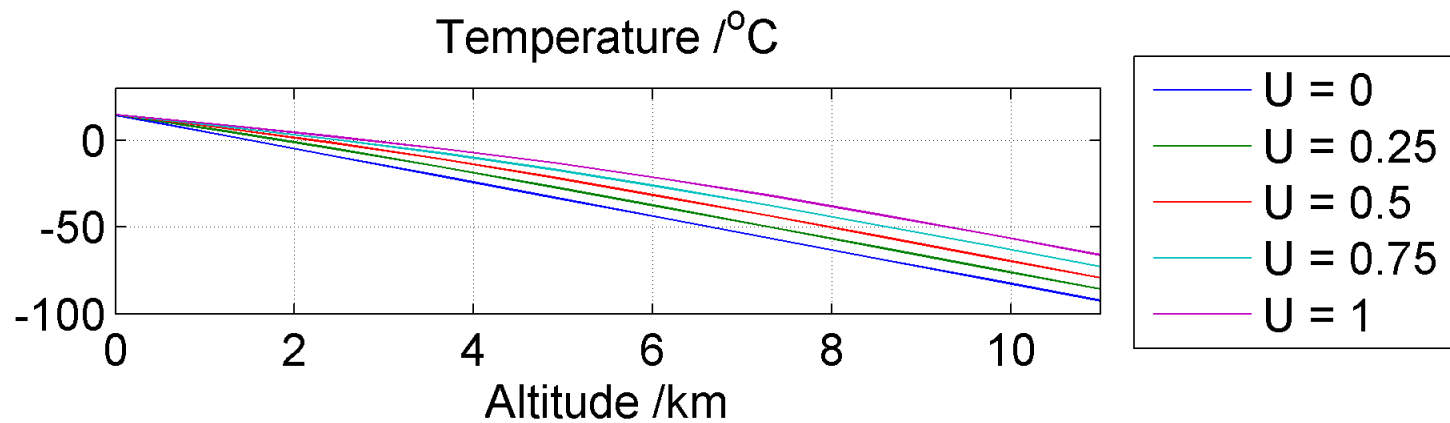
$$\begin{aligned} M_d &= 0.02896 \text{kgmol}^{-1} & \Delta H_v &= 2,501,000 \text{ J kg}^{-1} \\ M_v &= 0.01802 \text{kgmol}^{-1} & c_{pd} &= 1003.5 \text{ J kg}^{-1} \text{K}^{-1} \\ R &= 8.314 \text{JmolK}^{-1} & R_{sd} &= 287 \text{ J kg}^{-1} \text{K}^{-1} \\ g &= 9.81 \text{Nkg}^{-1} & R_{sw} &= 461.5 \text{ J kg}^{-1} \text{K}^{-1} \end{aligned}$$

**TASK3: Write a computer program or spreadsheet to evaluate  $P$  and  $T$  vs altitude for the whole range of the ISA.**

**Plot  $L$ , boiling point and dew point vs altitude for different  $U$  values.**

Lapse rates for different relative humidities





## Extension opportunities:

- Compare the ISA for Earth with the atmospheres for *other planets in the Solar System*. Note these may have a very different gas mixture to the Earth's atmosphere.
- Write a **graphical user interface** (GUI) for the atmosphere model and encode this as an 'app'. Coding up an iOS/Android smartphone app will particularly impress the judges! Inputs such as humidity  $U$  could be fixed numbers, or perhaps variables that could be dynamically varied via a slider or perhaps a swipe gesture on a smartphone or tablet.
- Write up your model as a **short paper**. (Aim for about 10 sides of A4, two columns). If you have never written a paper before, download a few from the *Physics Education* journal. *The Epidemiology of Eyam* might be a good start...

Don't forget to include any extension projects in your video, as this is the only way you will gain credit for your work in the BPhO Computational Challenge. I'm afraid we cannot accept any other files. **Submit only the YouTube link to your two-minute screencast.**

## How to make a screencast using **Screencastify** and upload this to Youtube

1. Download the [Google Chrome web browser](#)
2. Download the [Screencastify](#) add-on to Chrome. The free educational version will allow up to 5 minutes of video.
3. When you are ready to make your video (have all the program windows open in advance, and prepare what you are going to say), click on the Screencastify arrow in the corner of your browser. Follow the instructions to record a screen, and a three second countdown will begin.
4. Record your video!
5. Export your video to a .webm or .mp4 file. There is also a direct to YouTube upload option.
6. Upload your video to [YouTube](#) (you will need to set up an account first and establish a Channel).
7. Navigate to your video and copy to the clipboard the YouTube weblink. Submit this link in your submission form in the BPhO website.
8. It is recommended that (i) you *don't* have a presenter image in your video (you can turn off this in Screencastify) , i.e. **only have a voice-over**. Also *turn off Comments* in YouTube and make the video *Unlisted*. This means nobody can leave comments, and only those with the link will find your video.

**Here is a short video of how to do this!**

