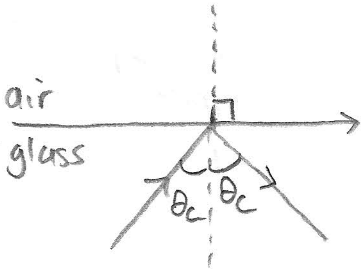
RAY OPTICS

Q1/ (i)



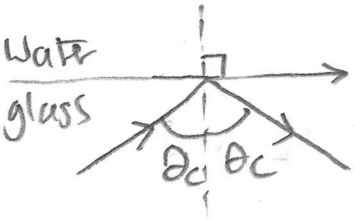
Snell's law:

$$n_g \sin\theta_c = n_a \sin 90^\circ$$

$$\therefore \theta_c = \sin^{-1}\left(\frac{n_a}{n_g}\right)$$

let $n_a = 1.00$, $n_g = 1.50$

$\therefore$ Critical angle is $\theta_c = \sin^{-1}\left(\frac{1.00}{1.50}\right) = \boxed{41.8^\circ}$



Snell's law: $n_g \sin\theta_c = n_w \sin 90^\circ$

$$\Rightarrow \theta_c = \sin^{-1}\left(n_w/n_g\right)$$

$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{1.34}{1.50}\right) = \boxed{63.3^\circ}$$

(ii) $\frac{c}{n} = f\lambda$ where c is the speed of light in a vacuum. Since f, c are the same across a boundary of refractive index

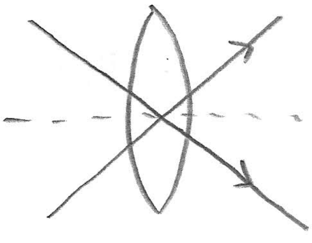
$$\Rightarrow \boxed{\lambda \propto \frac{1}{n}}$$

So $\lambda_{ice} = \lambda_{air}/n_{ice}$

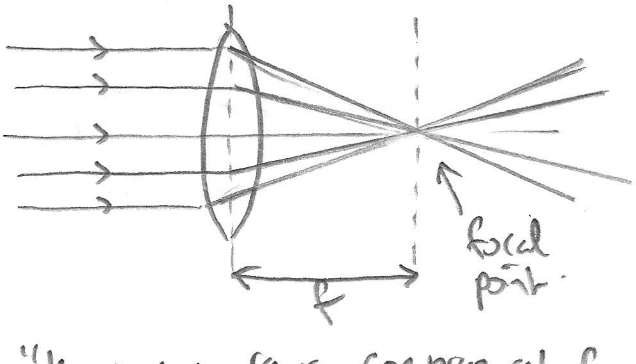
$$= 520\text{nm}/1.31 = \boxed{397\text{nm}}$$

So 5cm of ice is $\frac{5\times10^{-2}}{397\times10^{-9}} = \boxed{1.26\times10^{5} \text{ wavelengths}}$

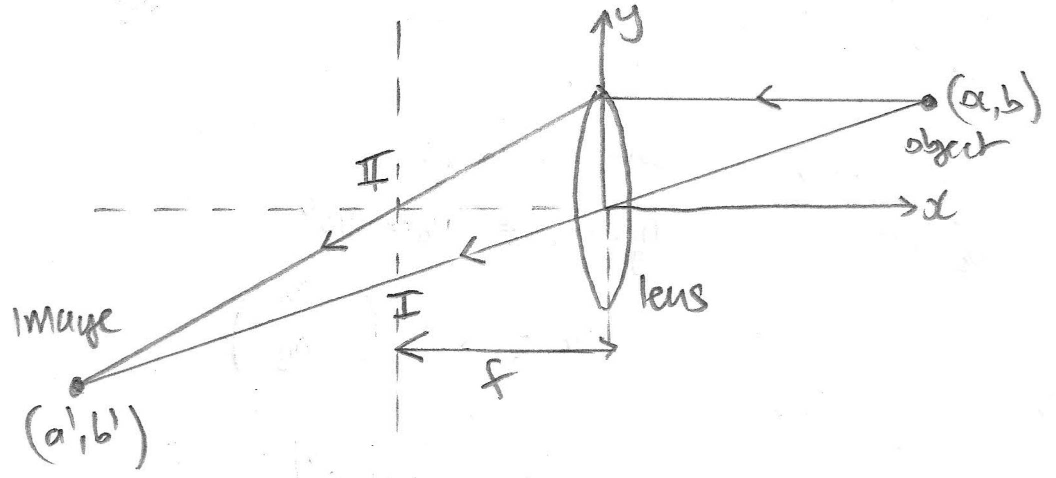
(iii) Ideal biconvex lens:



"Straight through"



"Horizontal rays converge at focus"



Note a

Cartesian equation of line I is: $\boxed{y_I = \frac{b}{a}x}$

" " " " II is: $\boxed{y_{II} = \frac{b}{f}x + b}$

The lines intersect at (a', b') ie the $\boxed{\text{real, inverted image}}$ position of an object at (a,b).

$\therefore y_I = y_{II} \Rightarrow \frac{b}{a}a' = \frac{b}{f}a' + b$

$\Rightarrow \frac{a'}{a} = \frac{a'}{f} + 1$

$\Rightarrow -1 = a'\left(\frac{1}{f} - \frac{1}{a}\right)$

if $a > f$ then this is +ve

$\Rightarrow \boxed{a' = -\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}}$ so a' is -ve

So since $y = \frac{b}{a}x$ passes through (a', b')

$\Rightarrow \boxed{b' = -\frac{b}{a}\left(\frac{1}{f} - \frac{1}{a}\right)^{-1}}$ as required.

For projector: $f = 20.0\,\text{mm}$, $|b'| = 1.6\,\text{M}$, $|a'| = 5.0\,\text{M}$

so $-\frac{1}{a'} = \frac{1}{a} - \frac{1}{f} \Rightarrow \boxed{a = \left(\frac{1}{a'} + \frac{1}{f}\right)^{-1}}$

$\therefore \; a = \left(-\frac{1}{5.0} + \frac{1}{20\times10^{-3}}\right)^{-1} = 2.01\times10^{-2}\,\text{M}$

$= \boxed{20.08\,\text{mm}} \rightarrow$ Calc memory

(i.e. just behind the back focus of the lens)

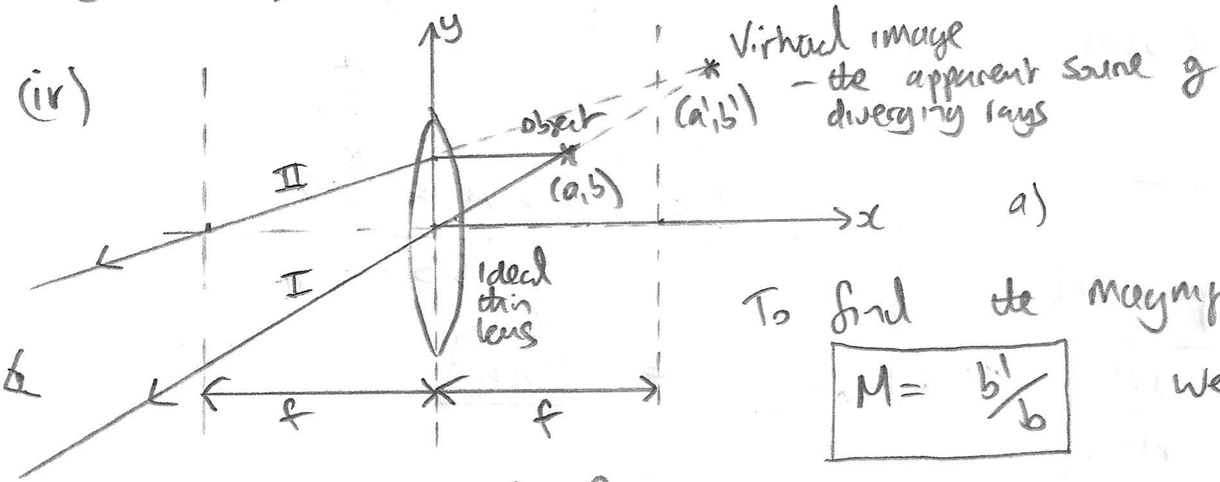
$\therefore \; \boxed{b = -ab'\left(\frac{1}{f} - \frac{1}{a}\right)}$

$b = -20.08 \times (-1.6)\left(\frac{1}{20\times10^{-3}} - \frac{1}{2.01\times10^{-2}}\right)$ (mm)

$\boxed{b = 6.43\,\text{mm}}$

{ Note signs are important in this geometry $b' = -1.6\,\text{M}$, $a' = -5.0\,\text{M}$ }

(iv)



a)

To find the magnification factor $\boxed{M = b'/b}$ we need to find a' and b' first.

Intersecting $y_I = \frac{b}{a}x$ and $y_{II} = \frac{b}{f}x + b$ as in Q1 (iii) at (a', b')

$\frac{b}{a}a' = \frac{b}{f}a' + b \Rightarrow a'\left(\frac{1}{a} - \frac{1}{f}\right) = 1$

Note this time $f > a$ so: $\boxed{a' = \left(\frac{1}{a} - \frac{1}{f}\right)^{-1}}$

and definitely +ve.

$$\therefore \quad b' = \frac{b}{a}\left(\frac{1}{a} - \frac{1}{f}\right)^{-1}$$

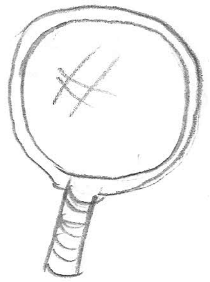
So
$$M = \frac{b'}{b} = \frac{1}{a}\left(\frac{1}{a} - \frac{1}{f}\right)^{-1}$$

$$= \left(\frac{f-a}{af}\right)^{-1}\frac{1}{a}$$

$$= \frac{af}{f-a}\,\frac{1}{a}$$

$$\Rightarrow \quad \boxed{M = \frac{f}{f-a}}$$

b) Sherlock Holmes' magnifying glass



$M = 5.0$
$a = 8.0\,\text{cm}$

$$(f-a)M = f$$

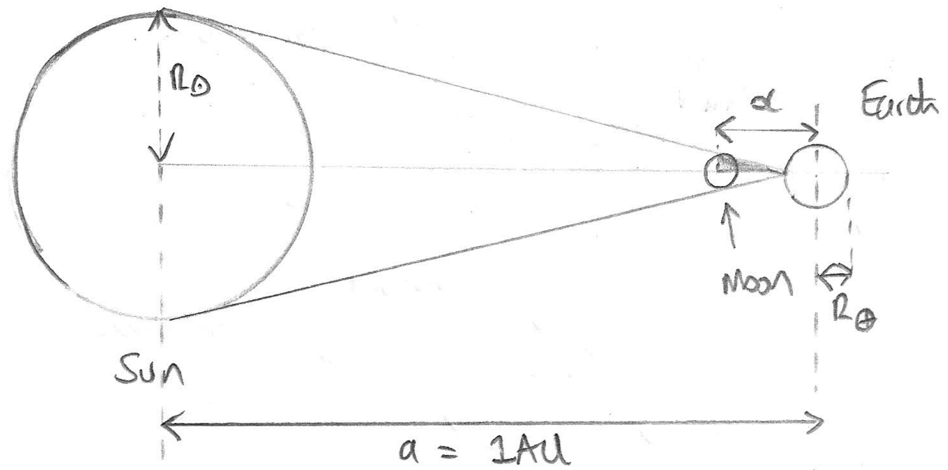
$$f(M-1) = aM$$

$$\boxed{f = \frac{aM}{M-1}}$$

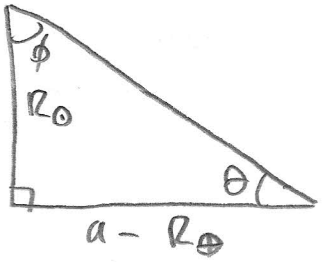
$$\therefore \quad f = \frac{8.0\,\text{cm} \times 5.0}{5.0 - 1}$$

$$= \boxed{10\,\text{cm}}$$

Check: $M = \frac{10}{10-8} = 5$ ✓



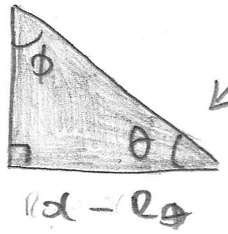
If a solar eclipse only occurs at a single point on the Earth:



Similar triangles

R_M

lunar radius



So: $\dfrac{R_M}{x - R_\oplus} = \dfrac{R_\odot}{a - R_\oplus}$

∴ Earth–Moon distance x (assume between the centres) is:

$$\boxed{\frac{R_M}{R_\odot}\left(a - R_\oplus\right) + R_\oplus = x}$$

∴ $x = \dfrac{1737.1}{696340}\left(1.496\times10^{11}/1000 - 6371\right) + 6371$ (km)

$$\boxed{x = 379{,}550 \text{ km}}$$

The distance between the surface of the Earth and the near surface of the moon is:

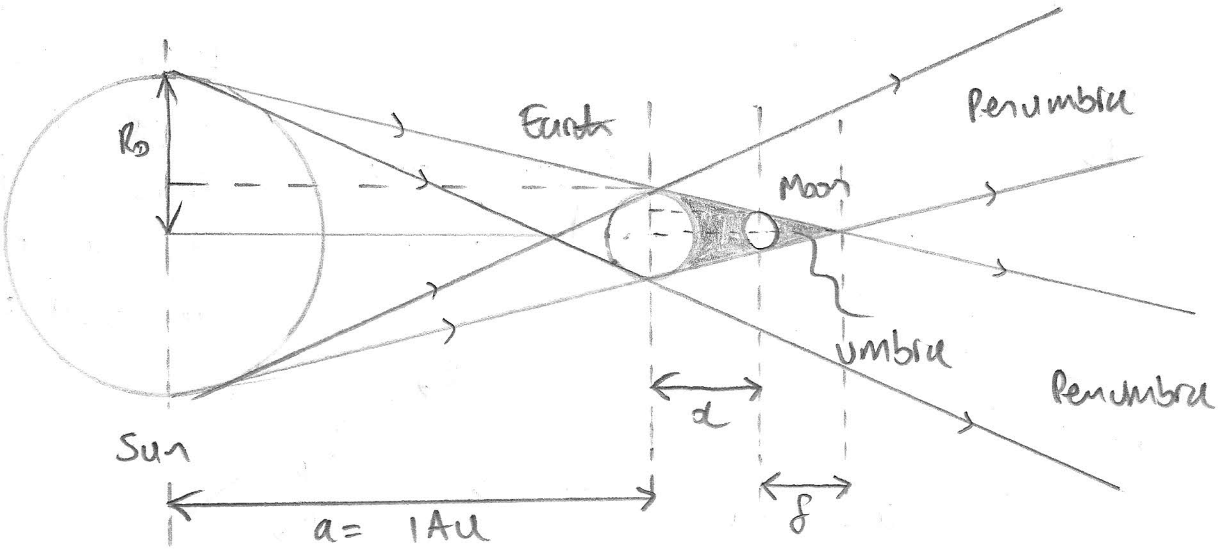
$$x - R_\oplus - R_M = \boxed{371{,}441 \text{ km}}$$

Now at perigee the x value is $\boxed{356{,}500\text{km.}}$

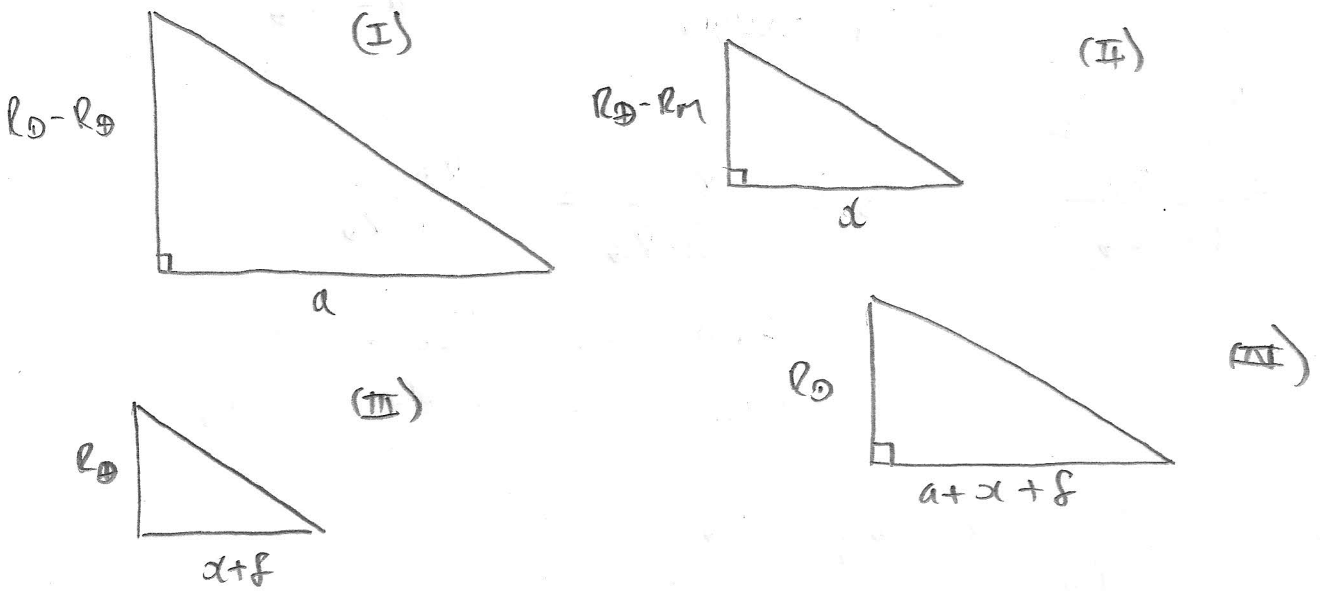
so # of years to get to 379,550 km is

$$\Delta t = \frac{(379{,}550 - 356{,}500)\times 10^5 \text{ cm}}{3.8 \text{ cm/yr}} = \boxed{607 \text{ million years}}$$

(vi)



Similar triangles:



comparing (I) and (II):

$$\frac{d}{R_\oplus - R_m} = \frac{a}{R_\odot - R_\oplus}$$

$$\therefore \boxed{d = \frac{a\,(R_\oplus - R_m)}{R_\odot - R_\oplus}}$$

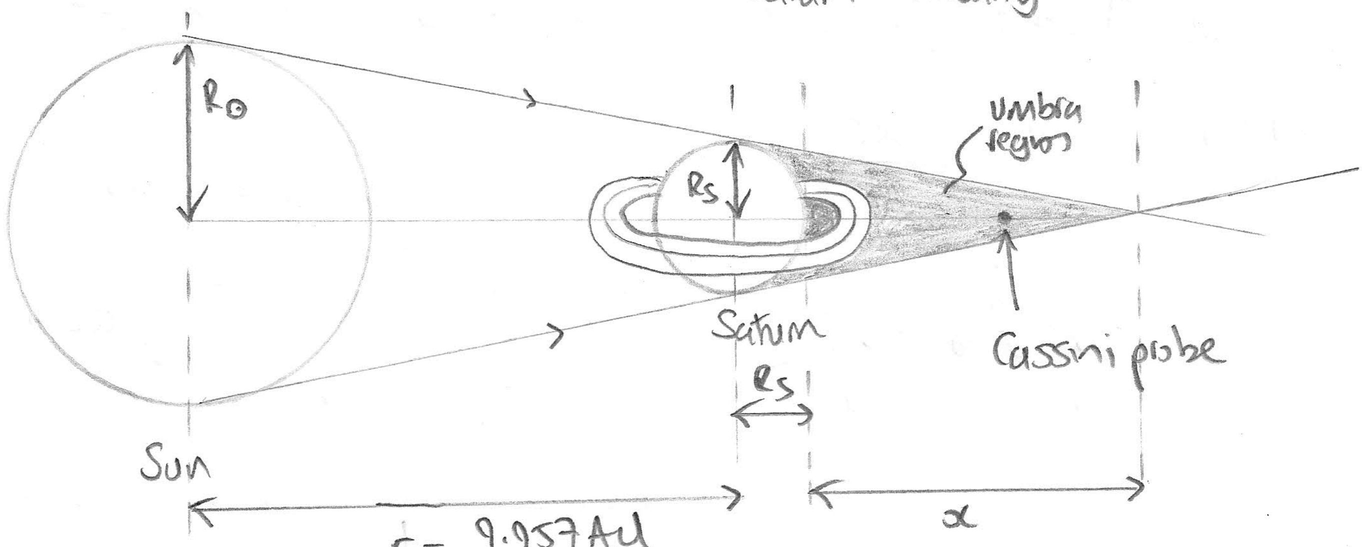
[Note could use (II) and (III) to find f given d]

$$\therefore d = \frac{1.496\times10^8 \times (6371 - 1737.1)}{696\,340 - 6371} \text{ km}$$

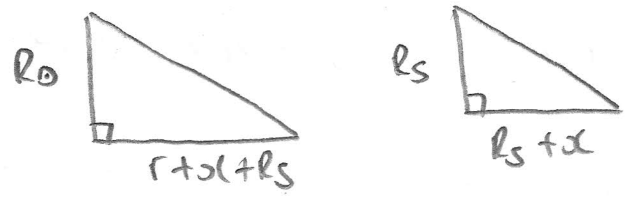
$$\boxed{d = 1{,}004{,}728 \text{ km}}$$

(So you will get total lunar eclipses long after the Solar eclipse becomes an ANTUMBRA ◉) as the moon moves away from the Earth.

(vii) Saturn "occulting" the sun.



Similar triangles :



$$\therefore \quad \frac{r+x+R_S}{R_\odot} = \frac{R_S+x}{R_S}$$

$$\therefore \quad x\left(\frac{1}{R_S} - \frac{1}{R_\odot}\right) = \frac{r+R_S}{R_\odot} - 1$$

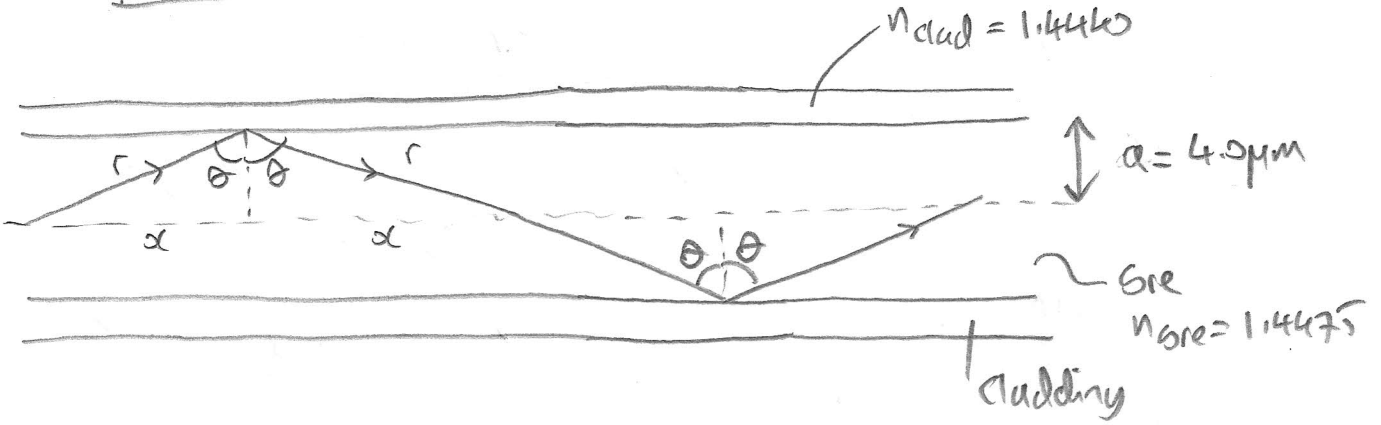
$$\therefore \quad \boxed{x = \frac{\frac{r+R_S}{R_\odot} - 1}{1/R_S - 1/R_\odot}}$$

$$\therefore \quad x = \frac{\frac{9.957 \times 1.496 \times 10^8 + 58232}{696340} - 1}{1/58232 - 1/696340} \quad \text{(km)}$$

$$\boxed{x = 1{,}245{,}486 \text{ km}} \qquad (1{,}245{,}000 \text{ km to 4 s.f.}).$$

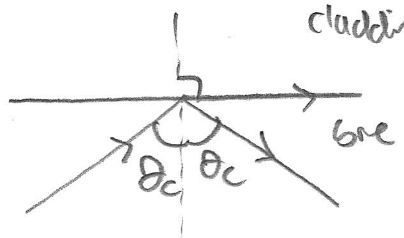
If Cassini is < this distance from Saturn, Saturn will completely block the Sun. For the dramatic photo one assumes (given the slight ring around Saturn) that the distance from the surface was close to 1,245,000 km.

(viii) $\boxed{\text{optical fiber:}}$



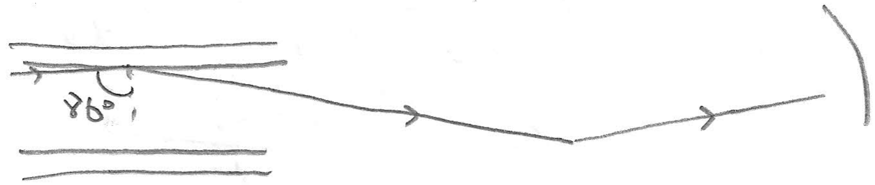
a) Snell's law : $n_{core} \sin\theta_c = n_{clad} \sin 90°$

$$\therefore \theta_c = \sin^{-1}\left(\frac{n_{clad}}{n_{core}}\right)$$

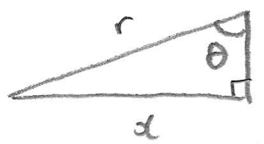


$$= \sin^{-1}\left(\frac{1.4440}{1.4475}\right)$$

$$= \boxed{86.0°}$$

(so more like 

)

b) 

$x = r\sin\theta$ $\quad\therefore\quad \boxed{r = x/\sin\theta}$

c) So if cable length is $2\pi R_\oplus$, then light travels $2\pi R_\oplus / \sin\theta$. Hence time to travel round the earth is

$$\Delta t_\oplus = \frac{2\pi R_\oplus}{\sin\theta} \Big/ \frac{c}{n_{core}}$$

$$\boxed{\Delta t_\oplus = \frac{2\pi R_\oplus n_{core}}{c \sin\theta}} = \frac{2\pi \times 6371 \times 10^3 \times 1.4475}{2.998 \times 10^8 \times \sin 86.0°} \text{ (s)}$$

$$= \boxed{0.194 \text{ s}}$$

d) Hence Δt for London to Sydney is:

$$\Delta t = \frac{16983 \times 10^3 \times 1.4475}{2.998 \times 10^8 \times \sin 86^\circ}$$

$$\boxed{\Delta t = 0.082\,s}$$

[Note at $\theta = \theta_c$, $\sin\theta_c = n_{clad}/n_{core}$

$$\therefore \quad \Delta t = \frac{x}{\sin\theta_c}\frac{1}{c/n_{core}} = \frac{x\, n_{core}}{c \times n_{clad}/n_{core}}$$

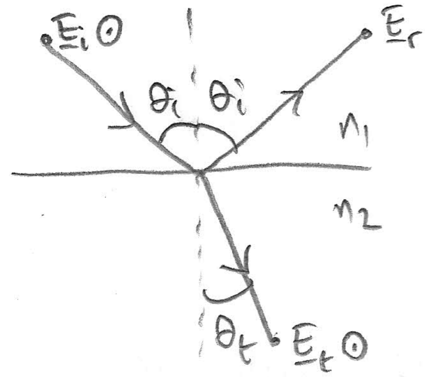
$$\therefore \quad \boxed{\Delta t = \frac{x}{c} \times \frac{n_{core}^2}{n_{clad}}}$$

So London to Sydney in vacuo in a straight line is

$$\frac{16983 \times 10^3}{2.998 \times 10^8} = \boxed{0.0575} \quad \therefore \quad \frac{n_{core}^2}{n_{clad}} = \frac{1.4475^2}{1.440} = 1.455.$$

$0.057 \times 1.455 = 0.082(9)$ ✓ So correct, subject to the rounding error in (a)].

(ix)



S-polarized light
⊥ to plane ⊙

power coefficient (well fraction) for reflection is:

$$|r_\perp|^2 = \left|\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right|^2$$

$$\text{So } |r_\perp|^2 = \left|\frac{1.00\cos 42^\circ - 1.5\cos\theta_t}{1.00\cos 42 + 1.5\cos\theta_t}\right|^2$$

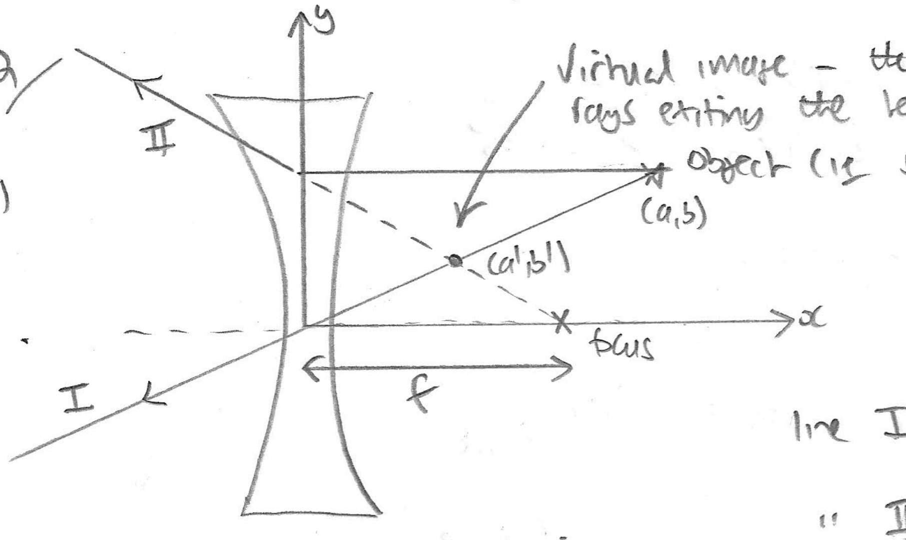
$= \boxed{0.083}$ (i.e. 8.3% reflected power).

where $\theta_t = \sin^{-1}\left(\frac{\sin 42^\circ \times 1.00}{1.5}\right) = \boxed{26.5^\circ}$

So fraction transmitted $|t_\perp|^2 = 1 - |r_\perp|^2$ is $\boxed{91.7\%}$

Q2

(i)



Diverging biconcave lens

line I: $y_I = \frac{b}{a}x$

" II: $y_{II} = -\frac{b}{f}x + b$

$y_I = y_{II}$ at (a',b') : $\therefore$ $\frac{b}{a}a' = -\frac{b}{f}a' + b$

$\therefore$ $a'\left(\frac{1}{a} + \frac{1}{f}\right) = 1$

$\therefore$ $\boxed{a' = \left(\frac{1}{a} + \frac{1}{f}\right)^{-1}}$

$\therefore$ using $y_I = \frac{b}{a}x \Rightarrow \boxed{b' = \frac{b}{a}a'}$

(ii) Magnification factor $M = b'/b$

$M = a'/a$

$M = \left(\frac{f+a}{fa}\right)^{-1}\frac{1}{a}$

$M = \frac{fa}{f+a}\frac{1}{a}$

$\boxed{M = \frac{f}{f+a}}$

Since $f, a > 0 \Rightarrow M < 1$ as expected from ray diagram. ie virtual image is both upright and $\boxed{\text{demagnified}}$

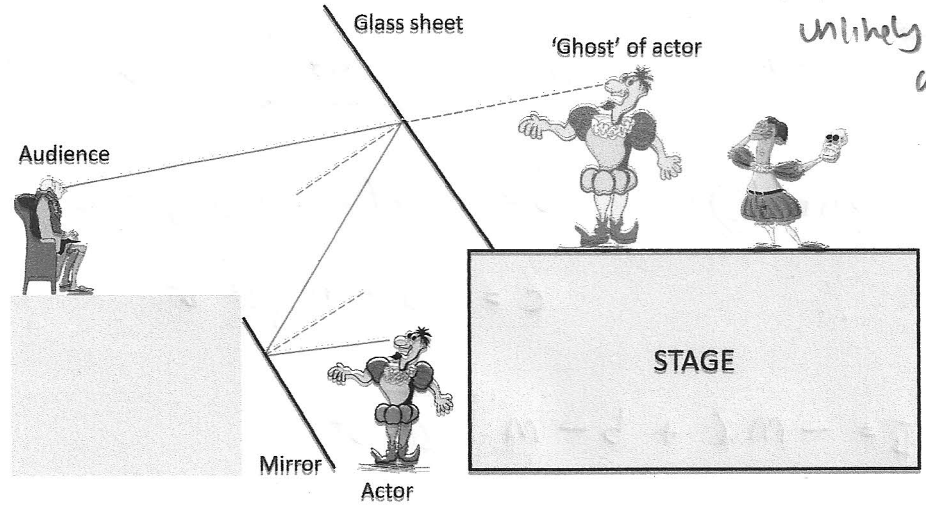
↑

b' has same sign as b

Q3/

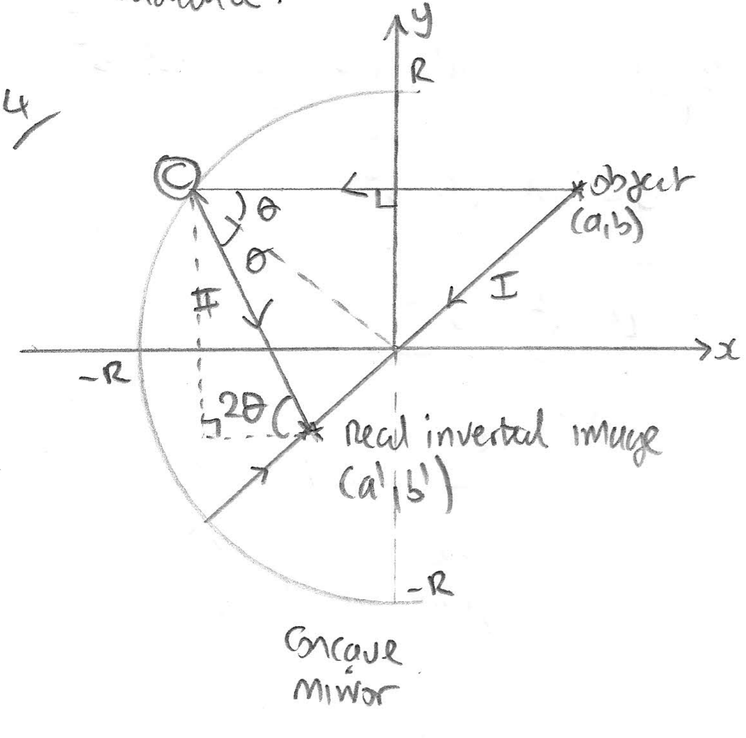
PEPPER'S GHOST

The theatrical effect is all the more effective since the angled glass sheet means the audience are unlikely to observe any reflected light originating from the audience.



The "Ghost" is the $\boxed{\text{virtual image}}$ of the actor below the stage, i.e. the apparent source of rays reaching the audience.

Q4/



(i) Cartesian equation of circle is:

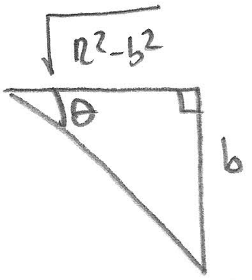
$R^2 = x^2+y^2$

so when $y=b$

$x = -\sqrt{R^2-b^2}$

$\therefore$ coordinates of reflection point at © is:

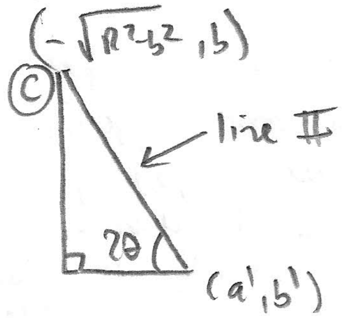
$$\boxed{\left(-\sqrt{R^2-b^2}\,,\,b\right)}$$



$\therefore \tan\theta = \dfrac{b}{\sqrt{R^2-b^2}}$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{b}{\sqrt{R^2-b^2}}\right)}$$

(ii)



line II has Cartesian equation:

$$y_{II} = -Mx + C$$

$\boxed{M = \tan 2\theta}$ is - the gradient

Using part C: $b = -M(-\sqrt{R^2-b^2}) + C$

$\therefore$ $C = b - M\sqrt{R^2-b^2}$

$$\therefore \quad y_{II} = -Mx + b - M\sqrt{R^2-b^2}$$

at (a', b'), this is the intersection of lines I and II.

$$y_I = \frac{b}{a}x$$

$\therefore$ when $y_I = y_{II}$: $\frac{b}{a}a' = -Ma' + b - M\sqrt{R^2-b^2}$

$$\therefore \quad a'\left(\frac{b}{a} + M\right) = b - M\sqrt{R^2-b^2}$$

$$\therefore \boxed{a' = -\frac{M\sqrt{R^2-b^2} - b}{M + b/a}}$$

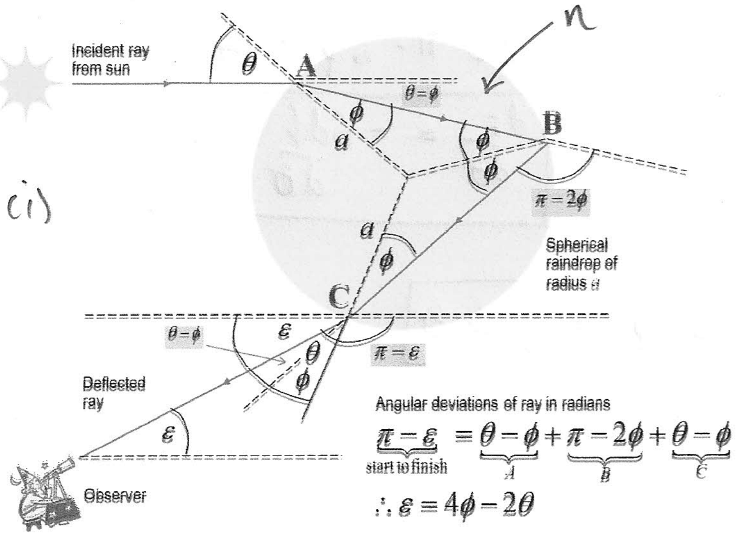
(Note a' is -ve from the diagram)

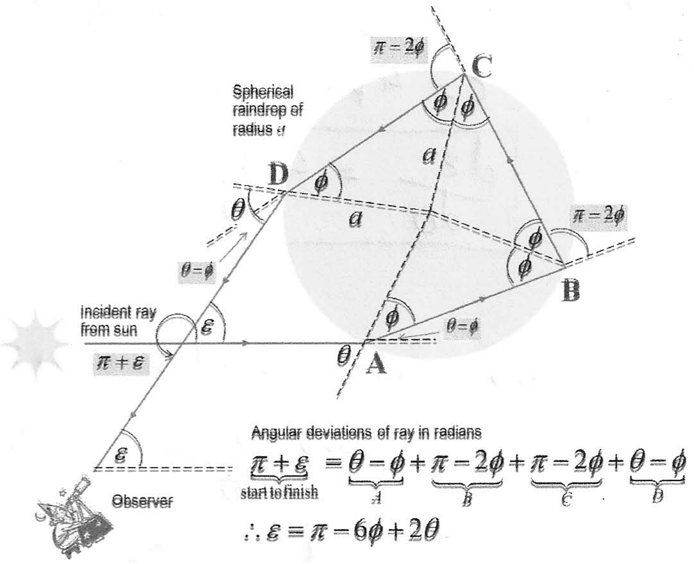
and, using $y_I = \frac{b}{a}x$

$$\Rightarrow \boxed{b' = \frac{b}{a}a'}$$

Q5/

(i)





Single internal reflection $\Rightarrow$ "primary rainbow"

Double internal reflection $\Rightarrow$ "secondary rainbow"

Snell's law at A

$1.00 \sin\theta = n \sin\phi$

$\therefore \boxed{\phi = \sin^{-1}\left(\frac{\sin\theta}{n}\right)}$

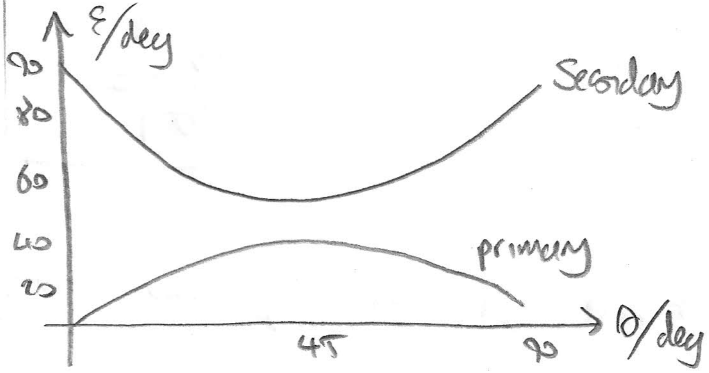
So since elevation $\varepsilon = 4\phi - 2\theta$

$\Rightarrow \boxed{\varepsilon = 4\sin^{-1}\left(\frac{\sin\theta}{n}\right) - 2\theta}$

$\varepsilon = \pi - 6\phi + 2\theta$

$\therefore \boxed{\varepsilon = \pi - 6\sin^{-1}\left(\frac{\sin\theta}{n}\right) + 2\theta}$

(ii) If you plot ε vs θ:



for a given value of n, which varies with light frequency.

But this variation is slight, so over the visible light spectral range 405–790 THz the $\varepsilon(\theta)$ curves only change subtley

Both curves have a stationary point i.e. where $d\varepsilon/d\theta = 0$.

This means a single ε value for a range of angles of incidence θ.

In practical terms this implies a focussing of light, so we expect to see an enhanced light intensity at ε angles when $d\varepsilon/d\theta = 0$

ie THESE ARE THE ELEVATION ANGLES THAT WE OBSERVE A RAINBOW

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(iii) Primary bow $d\varepsilon/d\theta = 0$

$\varepsilon = 4\phi - 2\theta$

$$\therefore \boxed{\frac{d\varepsilon}{d\theta} = 4\frac{d\phi}{d\theta} - 2}$$

Secondary bow $d\varepsilon/d\theta = 0$

$\varepsilon = \pi - 6\phi + 2\theta$

$$\boxed{\frac{d\varepsilon}{d\theta} = -6\frac{d\phi}{d\theta} + 2}$$

Now from Snell's law: $\boxed{\sin\phi = \frac{1}{n}\sin\theta}$

$$\therefore \cos\phi \frac{d\phi}{d\theta} = \frac{1}{n}\cos\theta$$

$$\cos^2\phi\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{n^2}\cos^2\theta$$

$$(1-\sin^2\phi)\left(\frac{d\phi}{d\theta}\right)^2 = \frac{\cos^2\theta}{n^2}$$

$$\left(1 - \frac{\sin^2\theta}{n^2}\right)\left(\frac{d\phi}{d\theta}\right)^2 = \frac{\cos^2\theta}{n^2}$$

$$(n^2 - \sin^2\theta)\left(\frac{d\phi}{d\theta}\right)^2 = 1 - \sin^2\theta$$

$$\therefore \boxed{\left(\frac{d\phi}{d\theta}\right)^2 = \frac{1-\sin^2\theta}{n^2-\sin^2\theta}}$$

Hence for primary bow: $d\varepsilon/d\theta = 0$

$$\Rightarrow 0 = 4\frac{d\phi}{d\theta} - 2 \quad \Rightarrow \left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{4}$$

$$\therefore \frac{1-\sin^2\theta}{n^2-\sin^2\theta} = \frac{1}{4} \quad \Rightarrow 4 - 4\sin^2\theta = n^2 - \sin^2\theta$$

$$\Rightarrow 4 - n^2 = 3\sin^2\theta$$

$$\Rightarrow \boxed{\theta = \sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right)}$$

Hence for secondary bow: $\frac{d\varepsilon}{d\theta}$

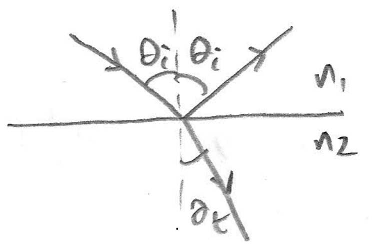
$$\Rightarrow \quad 0 = -6\frac{d\phi}{d\theta} + 2 \quad \Rightarrow \quad \frac{d\phi}{d\theta} = \frac{1}{3} \quad \Rightarrow \quad \left(\frac{d\phi}{d\theta}\right)^2 = \frac{1}{9}$$

$$\therefore \quad \frac{1-\sin^2\theta}{n^2-\sin^2\theta} = \frac{1}{9} \quad \Rightarrow \quad 9 - 9\sin^2\theta = n^2 - \sin^2\theta$$

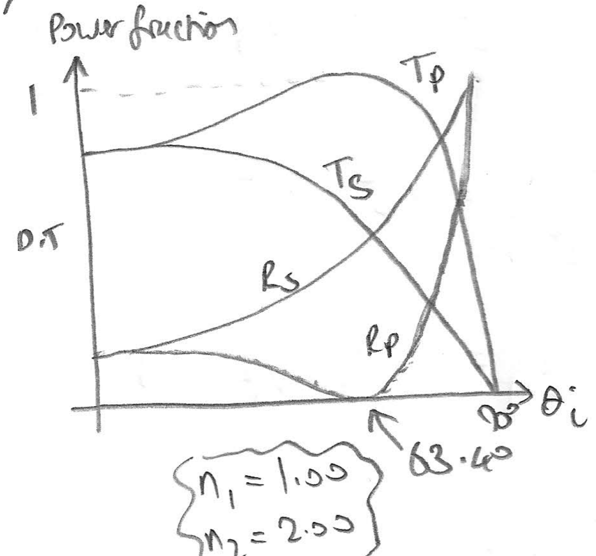
$$9 - n^2 = 8\sin^2\theta$$

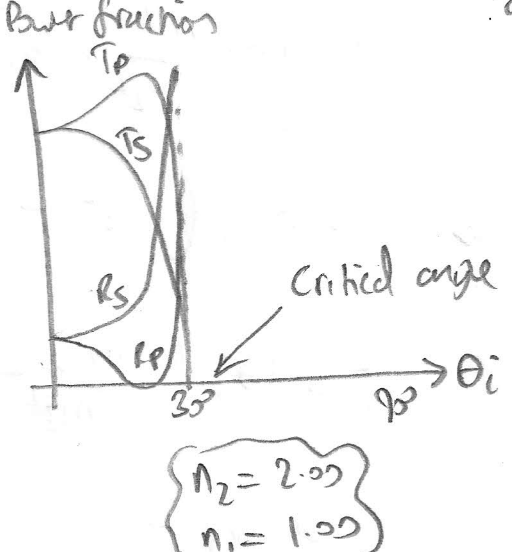
$$\therefore \quad \boxed{\sin^{-1}\left(\sqrt{\frac{9-n^2}{8}}\right) = \theta}$$

(iv) → see spreadsheet.



Q6/ See spreadsheet. (i)





(ii)

Critical angle $\boxed{\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)}$

T_p	Transmitted power fraction	(P-polarization)
R_p	Reflected " "	(" ")
T_s	Transmitted " "	(S-polarization) ← ie E field ⊥ to wavevector plane.
R_s	Reflected " "	(" ")

Brewster angle $\boxed{\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$ is when:

$R_p \to 0$

ie only S-polarized light is reflected. If you can achieve this with windows or sunglasses, this can reduce glare eg in an icy environment.

(iii) $|r_{\parallel}|^2 = \left(\dfrac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} \right)^2$

$|r_{\parallel}|^2 = 0$ at $\theta_i = \theta_B$ Brewster angle

$\therefore \quad n_1 \cos\theta_t = n_2 \cos\theta_i$

$n_1^2 \cos^2\theta_t = n_2^2 \cos^2\theta_i \qquad (*)$

Snell's law $\quad n_1 \sin\theta_i = n_2 \sin\theta_t$

$\therefore \; n_1^2 \sin^2\theta_i = n_2^2 \sin^2\theta_t \qquad \therefore \; \sin^2\theta_t = \left(\dfrac{n_1}{n_2}\right)^2 \sin^2\theta_i$

$(*) \Rightarrow \quad n_1^2 (1 - \sin^2\theta_t) = n_2^2 (1 - \sin^2\theta_i)$

$\therefore \; n_1^2 \left(1 - \dfrac{n_1^2}{n_2^2} \sin^2\theta_i\right) = n_2^2 (1 - \sin^2\theta_i)$

$\Rightarrow \quad \sin^2\theta_i \left(n_2^2 - \dfrac{n_1^4}{n_2^2} \right) = n_2^2 - n_1^2$

$n_1^2 \sin^2\theta_i \left(\dfrac{n_2^2}{n_1^2} - n_1^2/n_2^2 \right) = n_1^2 \left(\dfrac{n_2^2}{n_1^2} - 1 \right)$

$\sin^2\theta_i = \dfrac{n_2^2/n_1^2 - 1}{n_2^2/n_1^2 - n_1^2/n_2^2}$

Now $\quad \sin^2\theta_i + \cos^2\theta_i = 1$

$1 + \dfrac{1}{\tan^2\theta_i} = \dfrac{1}{\sin^2\theta_i} \qquad \left(\tan\theta_i = \sin\theta_i / \cos\theta_i \right)$

$\therefore \quad 1 + \dfrac{1}{\tan^2\theta_i} = \dfrac{n_2^2/n_1^2 - n_1^2/n_2^2}{n_2^2/n_1^2 - 1}$

$\therefore \quad \dfrac{1}{\tan^2\theta_i} = \dfrac{\dfrac{n_2^2}{n_1^2} - \dfrac{n_1^2}{n_2^2} - \dfrac{n_2^2}{n_1^2} + 1}{n_2^2/n_1^2 - 1}$

$$\therefore \quad \tan^2\theta_i = \frac{\frac{n_2^2}{n_1^2} - 1}{1 - \frac{n_1^2}{n_2^2}}$$

$$\tan^2\theta_i = \frac{\frac{n_2^2}{n_1^2}\left(1 - \frac{n_1^2}{n_2^2}\right)}{1 - \frac{n_1^2}{n_2^2}}$$

$$\therefore \quad \boxed{\tan\theta_i = \frac{n_2}{n_1}}$$

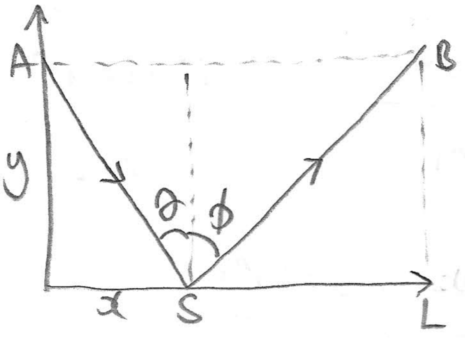
(−ve root has no physical meaning)

So Brewster angle is: $\boxed{\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$

where $|r_{\parallel}|^2 = 0$.

Q7/ See attached note "Dispersion of light via a prism".

Q8/ **Proof of the law of reflection**



Time to get to B from A via S is $\Delta t_{ASB} = \frac{AS}{c} + \frac{SB}{c}$

where c is the wave speed.

Pythagoras: $AS = \sqrt{y^2 + x^2}$

$SB = \sqrt{y^2 + (L-x)^2}$

$$\therefore \quad \Delta t_{ASB} = \frac{1}{c}\left[\sqrt{y^2 + x^2} + \sqrt{y^2 + (L-x)^2}\right]$$

Minimize travel time when $\partial \Delta t_{ASB}/\partial x = 0$, as x is the only variable parameter. (It defines the bounce point between $[0, L]$).

$$\frac{\partial \Delta t_{ASB}}{\partial x} = \frac{1}{c}\left[\frac{\frac{1}{2}(2x)}{\sqrt{y^2+x^2}} + \frac{\frac{1}{2}(2(L-x))(-1)}{\sqrt{y^2+(L-x)^2}}\right]$$

$$= \frac{1}{c}\left[\frac{x}{\sqrt{y^2+x^2}} - \frac{L-x}{\sqrt{y^2+(L-x)^2}}\right]$$

You can clearly see that $\partial \Delta t_{ASB}/\partial x = 0$ when $\boxed{x = L/2}$

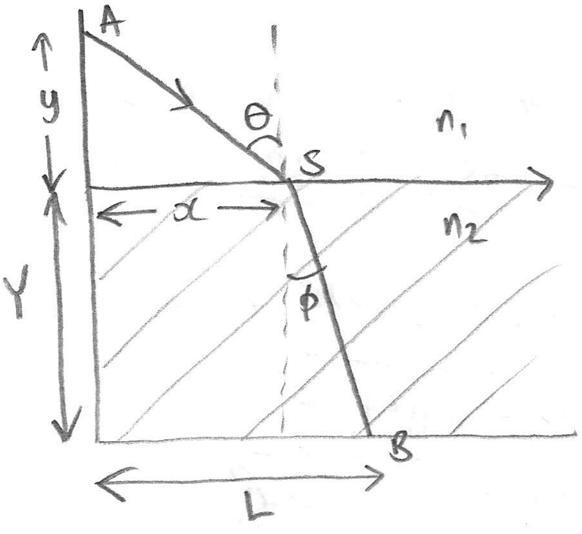
[Also: $\sqrt{y^2+x^2}\sin\theta = x$ and $\sqrt{y^2+(L-x)^2}\sin\phi = L-x$

so $\frac{\partial \Delta t_{ASB}}{\partial x} = \frac{1}{c}(\sin\theta - \sin\phi)$. This is 0 if $\theta = \phi$]

Now $x = y\tan\theta$ and $L-x = y\tan\phi$

so if $x = L/2 \Rightarrow \boxed{\theta = \phi}$ (Also clear from symmetry if S is the half way point of $[0, L]$).

Proof of the law of refraction



Travel time $A \to S \to B$ is:

$$\Delta t_{ASB} = \frac{\sqrt{y^2+x^2}}{c/n_1} + \frac{\sqrt{Y^2+(L-x)^2}}{c/n_2}$$

$$\therefore \frac{\partial \Delta t_{ASB}}{\partial x} = \frac{1}{c}\left\{\frac{\frac{1}{2}(2x)n_1}{\sqrt{y^2+x^2}} + \ldots\right.$$

$$\left. + \ldots \frac{\frac{1}{2}2(L-x)(-1)n_2}{\sqrt{Y^2+(L-x)^2}}\right\}$$

Now $x = \sqrt{y^2+x^2}\sin\theta$

$L-x = \sqrt{Y^2+(L-x)^2}\sin\phi$ $\Rightarrow \frac{\partial \Delta t_{ASB}}{\partial x} = \frac{1}{c}(n_1\sin\theta - n_2\sin\phi)$

So $\partial \Delta t_{ASB}/\partial x = 0$ when $\boxed{n_1\sin\theta = n_2\sin\phi}$ Snell's law of refraction.