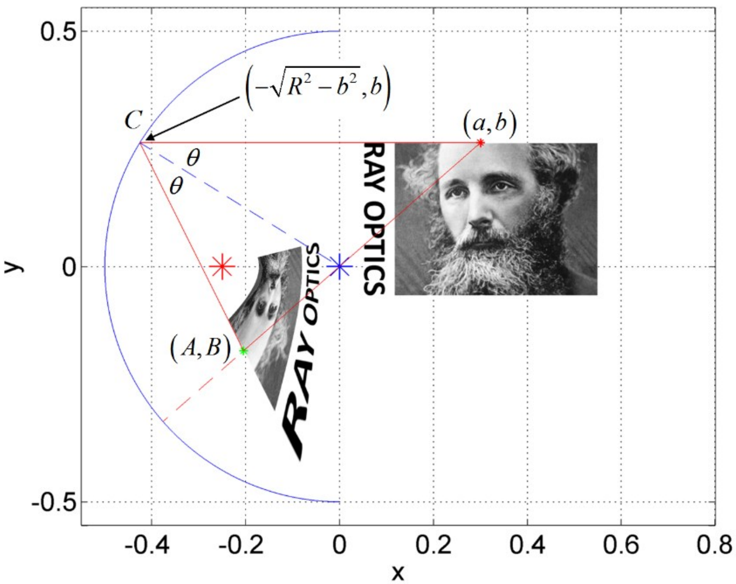
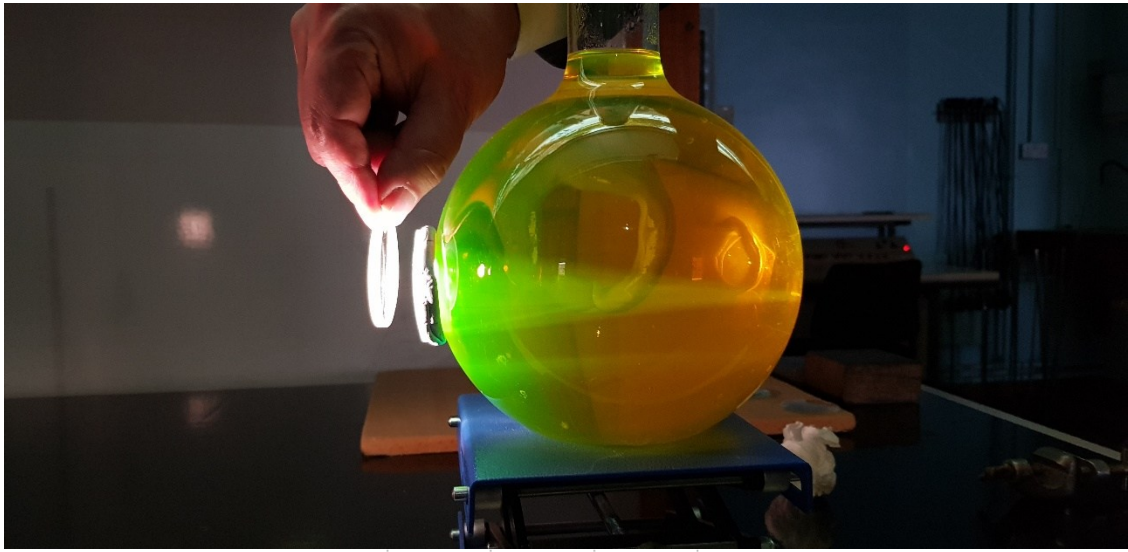
v2 Mar 2025

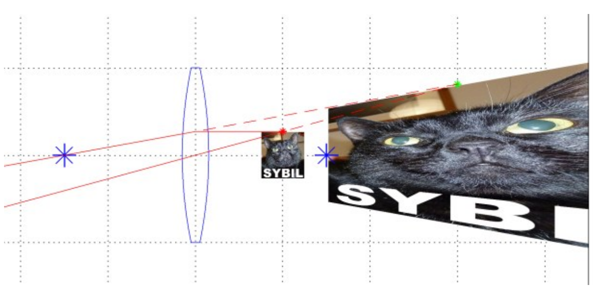
BPhO Computational Challenge

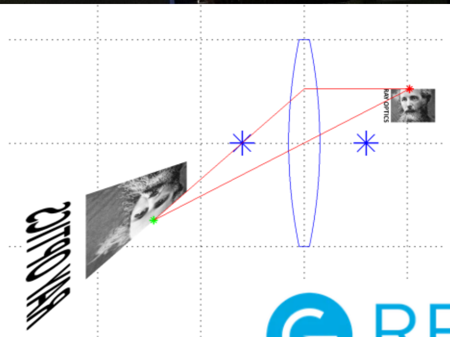
2025 Optics

Reflection in a concave mirror









BPhO British Physics Olympiad

RESEARCH

Instructions: Welcome to the **British Physics Olympiad Computational Challenge 2025.** The goal is to build computer models based upon the instructions in this document. Much can be achieved using a *spreadsheet* such as Microsoft Excel, although you are encouraged to use a *programming language* of your choice* for the more sophisticated models and graphical visualizations.

The challenge runs from **Easter 2025 till August 2025.** To submit an entry you will need to fill in a web form. There may be a small administration charge, payable online as per other BPhO competition entries.

The deliverable of the challenge is to produce a **screencast** of ***maximum length three minutes*** which describes your response to the challenge, i.e. the graphs and the code & spreadsheets and your explanation of these. The videos must be uploaded to **YouTube**, and we recommend you set these as *Unlisted* with *Comments disabled*. **Your entry will comprise a YouTube link.** *Instructions how to do this are on the next slide.* To produce the screencast, we recommend the Google Chrome add-on **Screencastify.**

You can enter the challenge **individually** or in **pairs**. If you opt for the latter, *both* of you must make equal contributions to the screencast.

Gold, Silver or **Bronze** e-certificates will be emailed to each complete entry, and the **top five** Golds will be invited to present their work at a special ceremony. You should receive a result by December 2025. Note no additional feedback will be provided, and the decision of the judges is final.

Bronze: All spreadsheet-based challenge elements completed. Some basic coding attempted.
Silver: All tasks completed in code.
Gold: All tasks completed in code to a high standard, with extension work such as the construction of apps (i.e. programs with graphical user interfaces), and significant development of the models. The highest quality entries will typically contain research papers based upon the models and computational methods. For this challenge we also provide a *Geometric Optics* problem sheet, which could be attempted and written up. Many of the problems link directly to the challenges (*Rainbows*, *Prism dispersion*). **NOTE: YOU CANNOT GET A GOLD IF YOUR VIDEO IS OVER 3 MINUTES.**

*__MATLAB__ or **Python** is recommended, although any system that can easily execute code in loops and plot graphs will do. e.g. **Octave, Java, Javascript, C#, C++...** Use what you can access and feel comfortable with. Programming resources

How to make a screencast using Screencastify and upload this to Youtube

1. Download the Google Chrome web browser
2. Download the Screencastify add-on to Chrome. The free educational version will allow up to 5 minutes of video.
3. When you are ready to make your video (have all the program windows open in advance, and prepare what you are going to say), click on the Screencastify arrow in the corner of your browser. Follow the instructions to record a screen, and a three second countdown will begin.
4. Record your video! Remember, the **maximum length is three minutes.**
5. Export your video to a **.webm** or **.mp4** file. There is also a direct to YouTube upload option.
6. Upload your video to YouTube (you will need to set up an account first and establish a Channel).
7. Navigate to your video and copy to the clipboard the YouTube weblink. Submit this link in your submission form in the BPhO website.
8. It is recommended that (i) you *don't* have a presenter image in your video (you can turn off this in Screencastify) , i.e. **only have a voice-over**. Also *turn off Comments* in YouTube and make the video *Unlisted.* This means nobody can leave comments, and only those with the link will find your video.

Please don't record a video which is well over three minutes, and then submit a speeded-up version. We can tell!
You will certainly not gain a Gold, regardless of quality, and you may be disqualified.

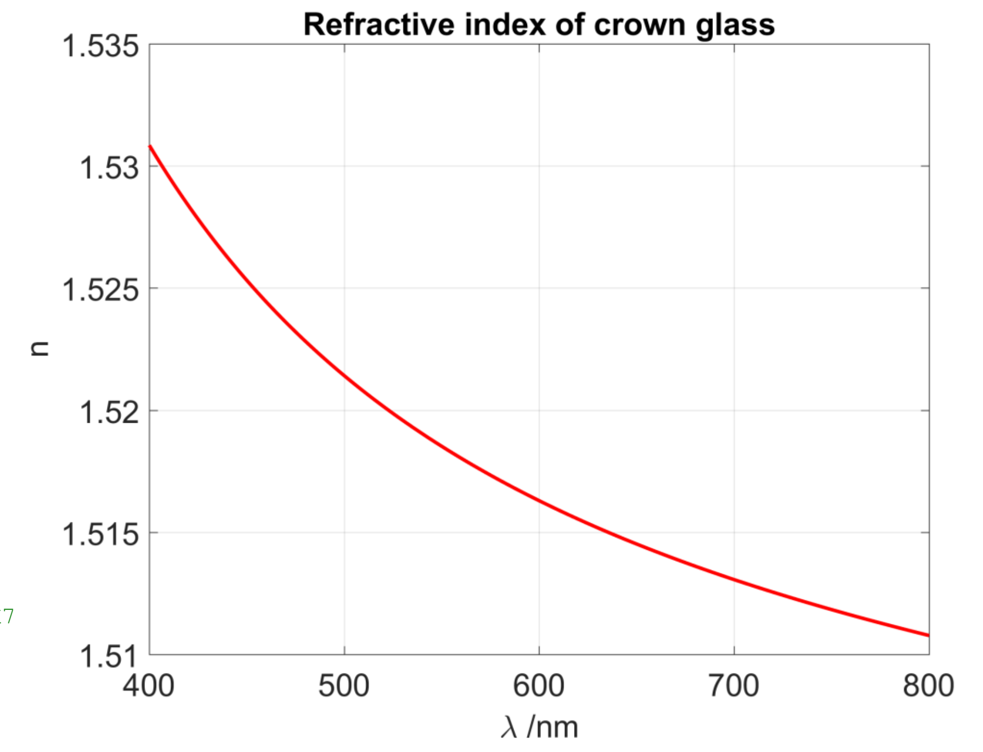
Screencastify

c is the **speed of light in a vacuum.**

$$\boxed{c = 3.00 \times 10^{8}\,\mathrm{ms}^{-1}}$$

n is the **refractive index** of a medium, such that the **wave speed** is

$$\boxed{c_{\mathrm{medium}} = \frac{c}{n}}$$




```
%Dispersion formula for BK7 Crown Glass
% https://refractiveindex.info/?shelf=3d&book=glass&page=BK7
% wavelength lambda is in nm.
% n is the refractive index
function n = crown_glass(lambda)

%Convert to microns
x = lambda/1000;

%Sellmeier coefficients
a = [1.03961212, 0.231792344, 1.01146945];
b = [0.00600069867, 0.0200179144, 103.560653];

%Build up formula for refractive index
y = zeros(size(x));
for k=1:length(a)
    y = y + ( a(k)*x.^2 )./( x.^2 - b(k) );
end
n = sqrt( 1 + y );
```

$$\boxed{n = \sqrt{1 + \sum_{k} \frac{a_k \lambda^2}{\lambda^2 - b_k}}}$$

Challenge #1a: Create a model of the refractive index of crown glass

You could do this in a spreadsheet or in a programming language.

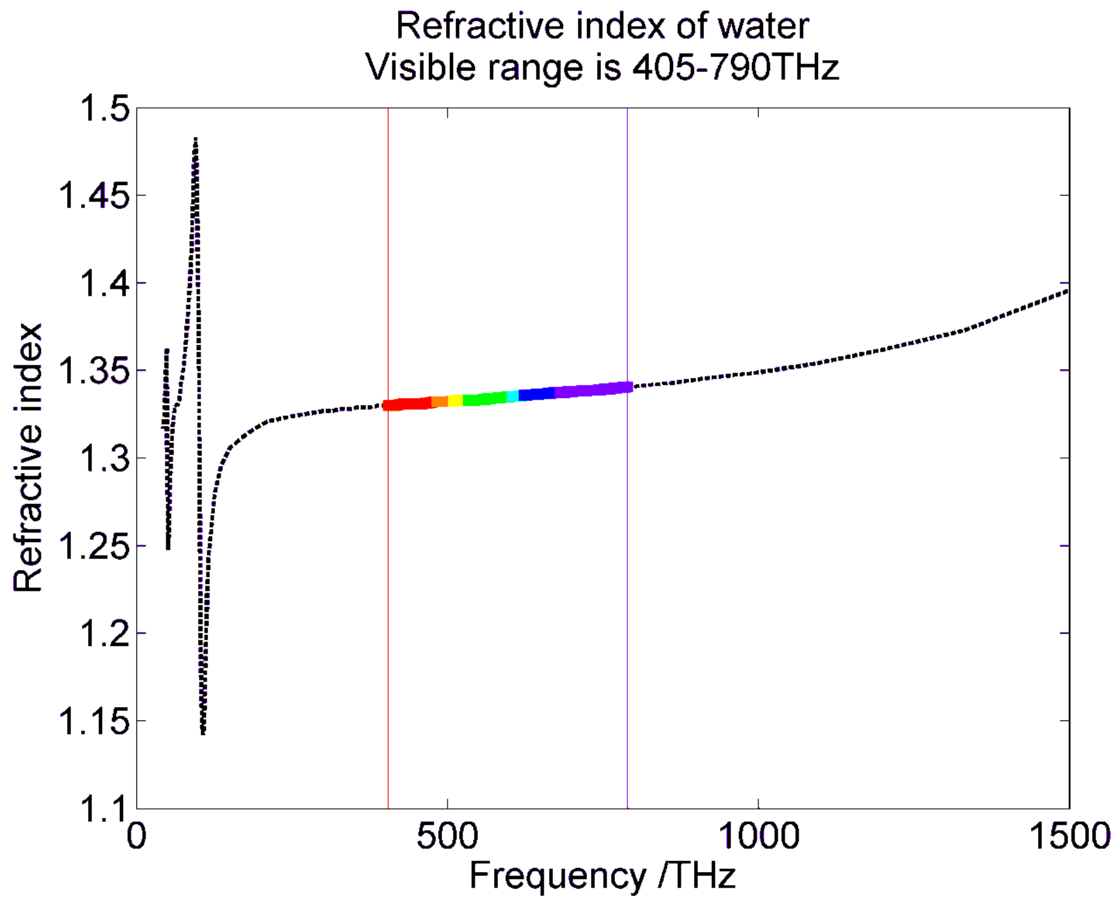
* You'll need this model in the prism challenge *

Refractive index n

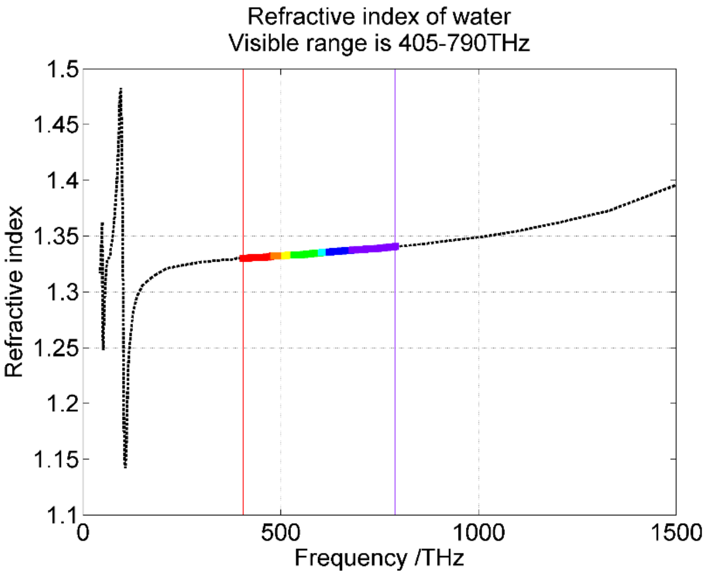
$$\boxed{n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in medium}}}$$

$n \approx 1$ air

$n \approx 1.34$ water

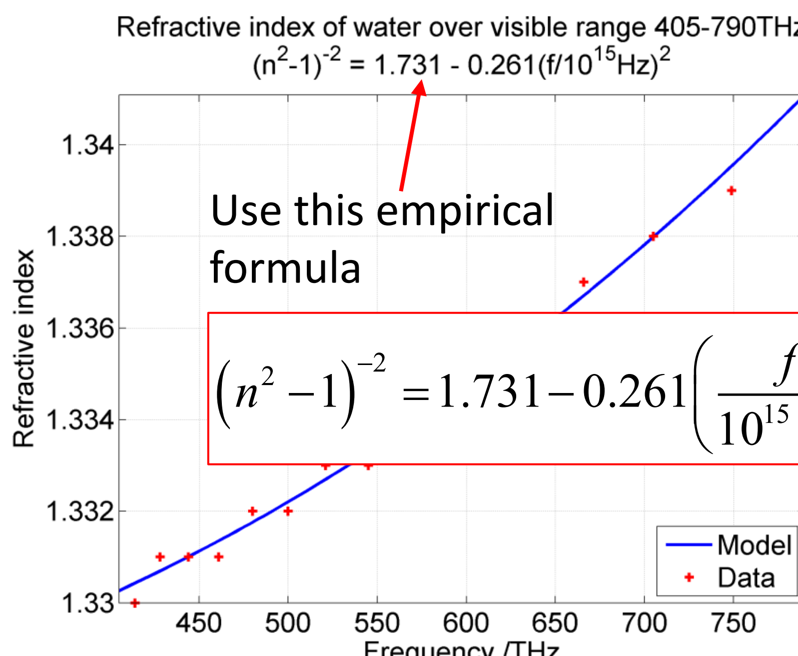


Note n often varies with the frequency of light




```
%colours_from_f
% Function which provides the R,G,B values( within interval [0,1] )
% of visible light depending on the frequency /THz
function [R,G,B,colour_str] = colours_from_f(f)
if f < 405
    R = NaN; G = NaN; B = NaN; colour_str = 'Infra Red';
elseif (f>=405) && ( f < 480 )
    R = 1; G = 0; B = 0; colour_str = 'Red';
elseif (f>=480) && ( f < 510 )
    R = 1; G = 127/255; B = 0; colour_str = 'Orange';
elseif (f>=510) && ( f < 530 )
    R = 1; G = 1; B = 0; colour_str = 'Yellow';
elseif (f>=530) && ( f < 600 )
    R = 0; G = 1; B = 0; colour_str = 'Green';
elseif (f>=600) && ( f < 620 )
    R = 0; G = 1; B = 1; colour_str = 'Cyan';
elseif (f>=620) && ( f < 680 )
    R = 0; G = 0; B = 1; colour_str = 'Blue';
elseif (f>=680) && ( f <= 790 )
    R = 127/255; G = 0; B = 1; colour_str = 'Violet';
else
    R = NaN; G = NaN; B = NaN; colour_str = 'Ultra Violet';
end
```

Challenge #1b: Create a model of the refractive index of water with frequency (and hence wavelength in a vacuum), over the range 405nm to 790nm

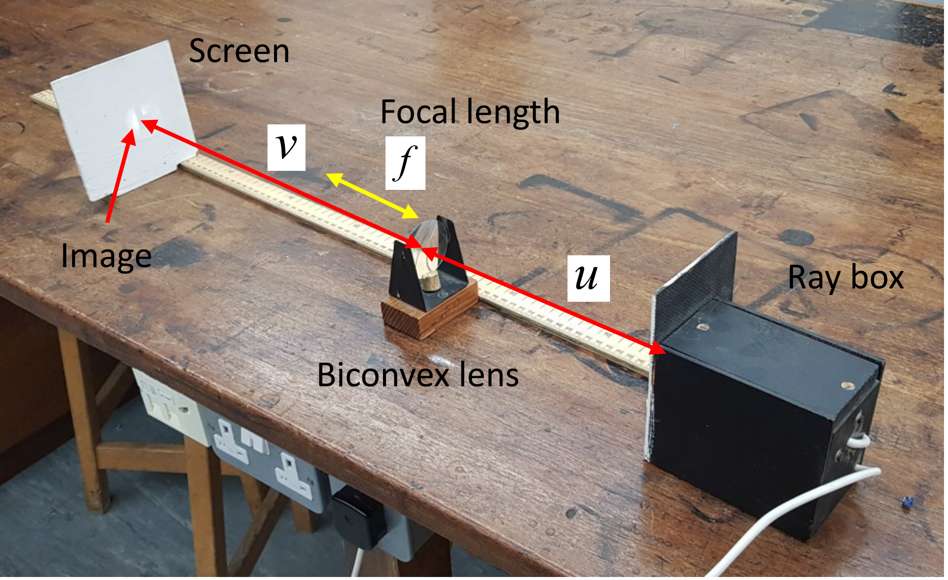


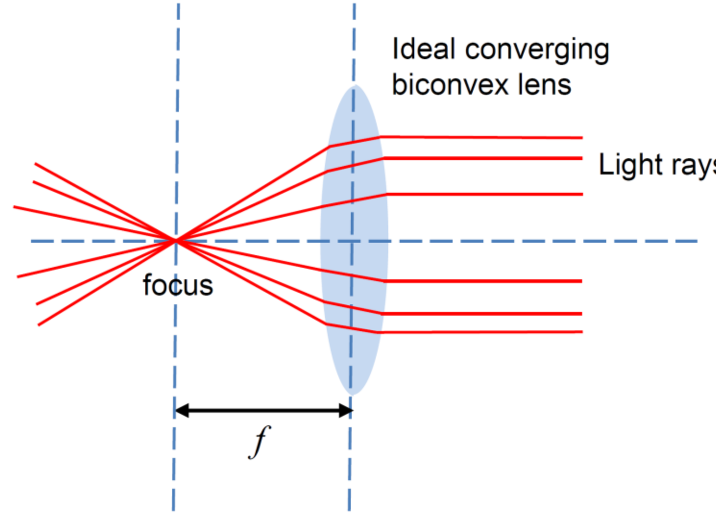
Use this empirical formula

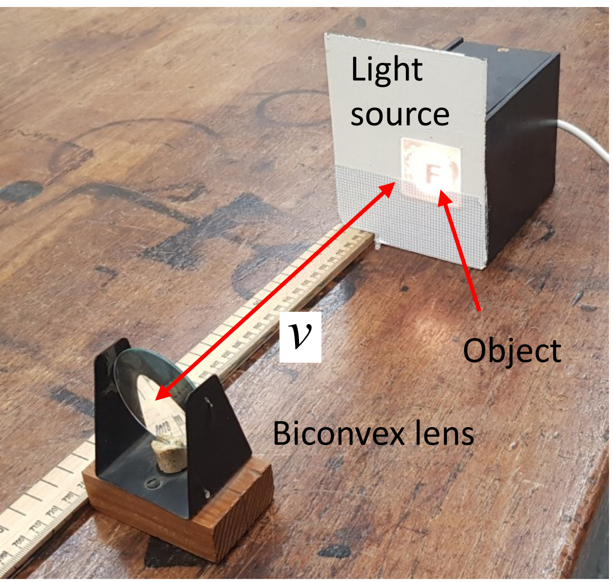
$$\left(n^2-1\right)^{-2} = 1.731 - 0.261\left(\frac{f}{10^{15}\ \text{Hz}}\right)^2$$

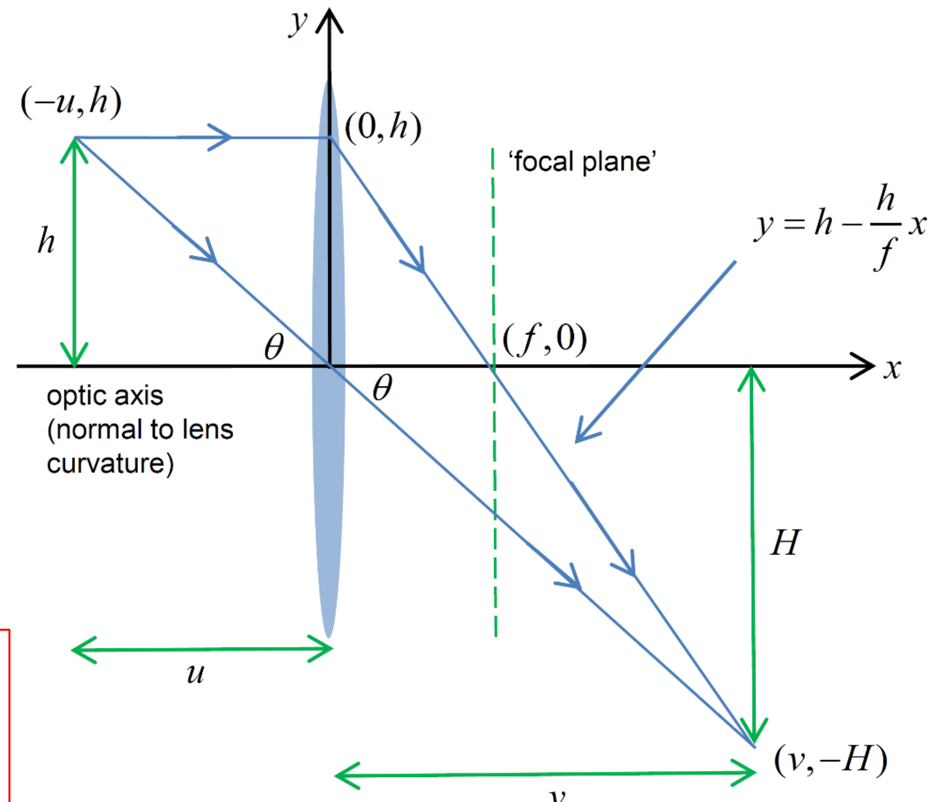
Use the code above to set Red,Green,Blue colours for different frequencies.

Even better, *interpolate* between the colours to make a smooth colour map



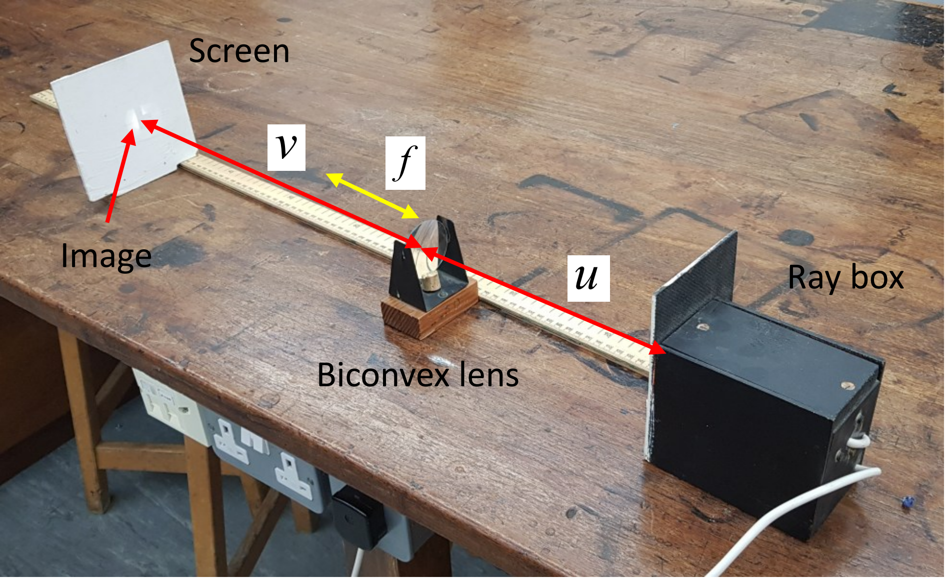






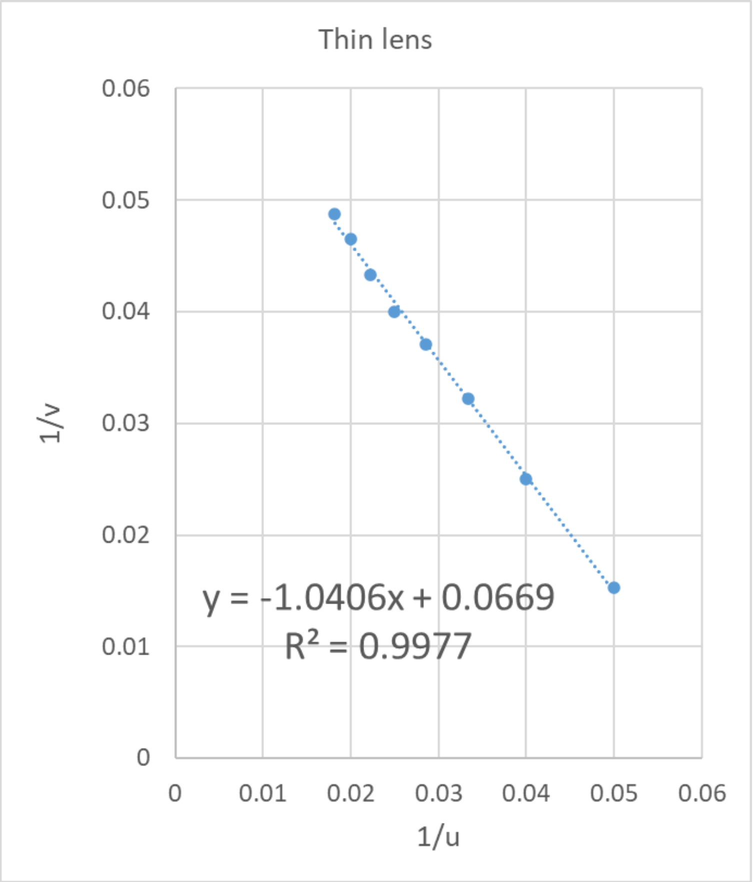
Thin lens equation

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$$



u /cm	v /cm	1/u	1/v
20	65.5	0.05	0.015
25	40	0.04	0.025
30	31	0.033	0.032
35	27	0.029	0.037
40	25	0.025	0.04
45	23.1	0.022	0.043
50	21.5	0.02	0.047
55	20.5	0.018	0.049

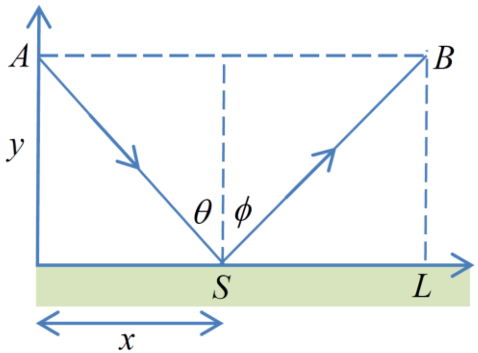
The lens is moved from the ray box by distance u and the screen is moved to v from the lens until the image of the **F** is in sharp focus.



Thin lens equation $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Challenge #2: Create a spreadsheet with the u,v data and plot $1/v$ vs $1/u$. From a line of best fit, assess the veracity of the *thin lens equation*, and determine the focal length f of the lens (in cm).

https://www.youtube.com/watch?v=hRqD6tE5fRY



c is the speed of light in a vacuum.

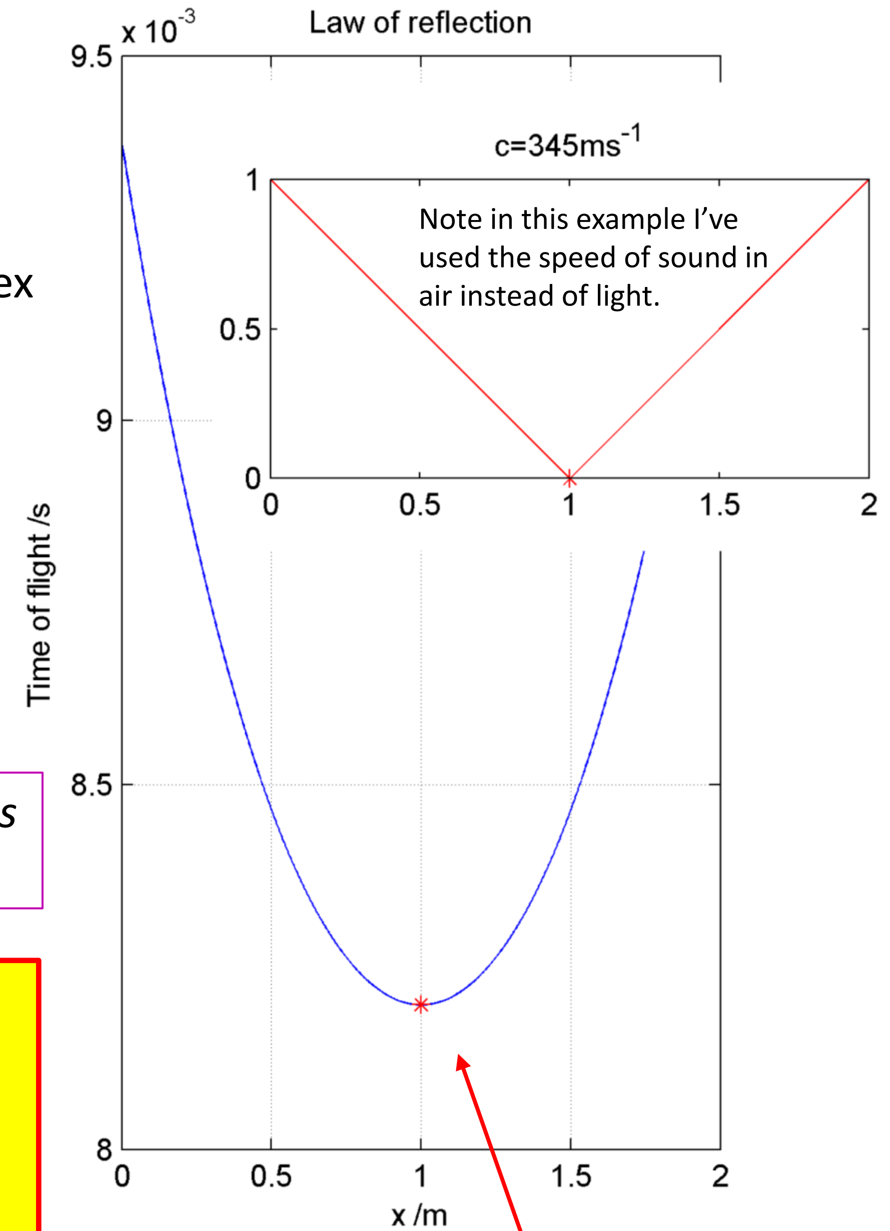
$c = 3.00 \times 10^{8}\,\mathrm{ms}^{-1}$

n is the refractive index of a medium.

Travel time for ray path ASB

$$\boxed{t = \frac{\sqrt{x^2 + y^2}}{c/n} + \frac{\sqrt{(L-x)^2 + y^2}}{c/n}}$$

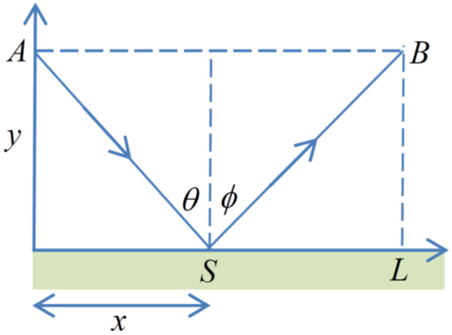
Fermat's principle: Path of light from ASB *minimizes travel time.*



Challenge #3: Plot t vs x and confirm the travel time is minimized when $x = L/2$, which implies the angle of incidence θ equals the angle of reflection ϕ.

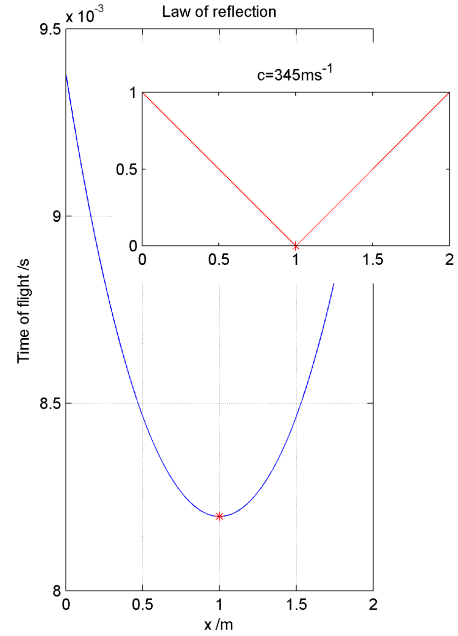
You could do this with a spreadsheet, or use a programming language.

Pierre de Fermat (1607-1665)



$$x = y\tan\theta,$$
$$L - x = y\tan\phi.$$

$$t = \frac{\sqrt{x^2+y^2}}{c/n} + \frac{\sqrt{(L-x)^2+y^2}}{c/n}$$



$$\frac{\partial t}{\partial x} = \frac{n}{c}\left(\frac{\frac{1}{2}2x}{\sqrt{x^2+y^2}} + \frac{\frac{1}{2}2\left(L-x\right)\left(-1\right)}{\sqrt{(L-x)^2+y^2}}\right)$$

$$\Rightarrow \frac{\partial t}{\partial x} = \frac{n}{c}\left(\frac{y\tan\theta}{y\sqrt{\tan^2\theta+1}} - \frac{y\tan\phi}{y\sqrt{\tan^2\phi+1}}\right).$$

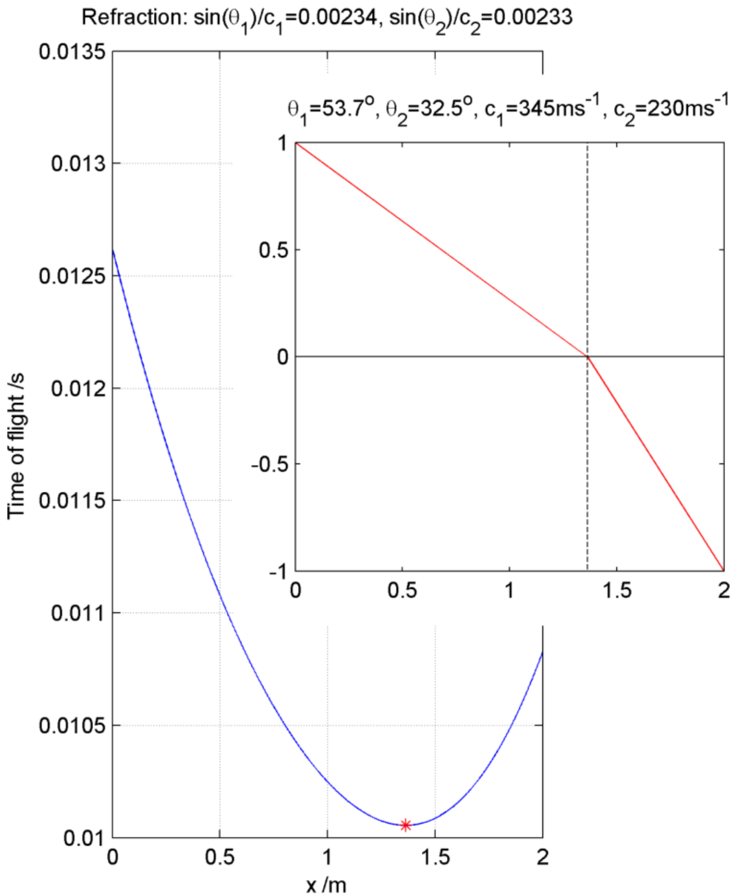
$$\therefore \frac{\partial t}{\partial x} = \frac{n}{c}\left(\sin\theta - \sin\phi\right).$$

Proof of the Law of Reflection by Fermat's principle

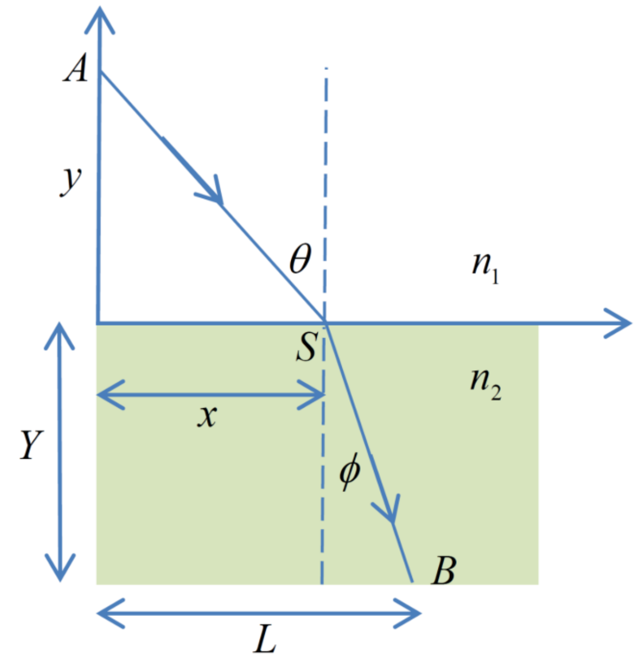
The travel time is minimized when $\frac{\partial t}{\partial x} = 0$, which is when

$$\theta = \phi.$$

i.e. the angle of incidence θ equals the angle of reflection ϕ. This is the *Law of Reflection.*



Note in this example I've used the speed of sound in air instead of light.



Using light, where c is the speed of light in a vacuum and n is the refractive index

The travel time for ray path ASB across a boundary between two media of wave speeds c_1 and c_2 is minimized when …

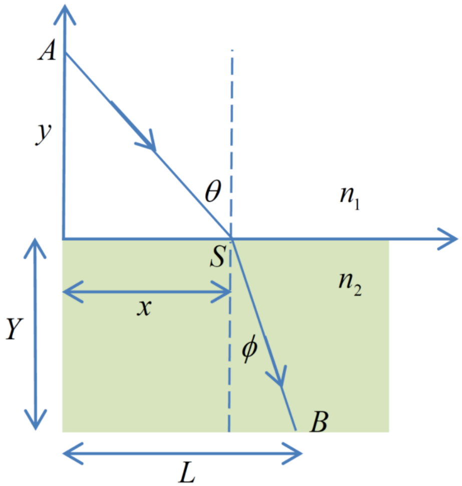
Snell's law

$$\frac{\sin\theta}{c_1} = \frac{\sin\phi}{c_2}$$

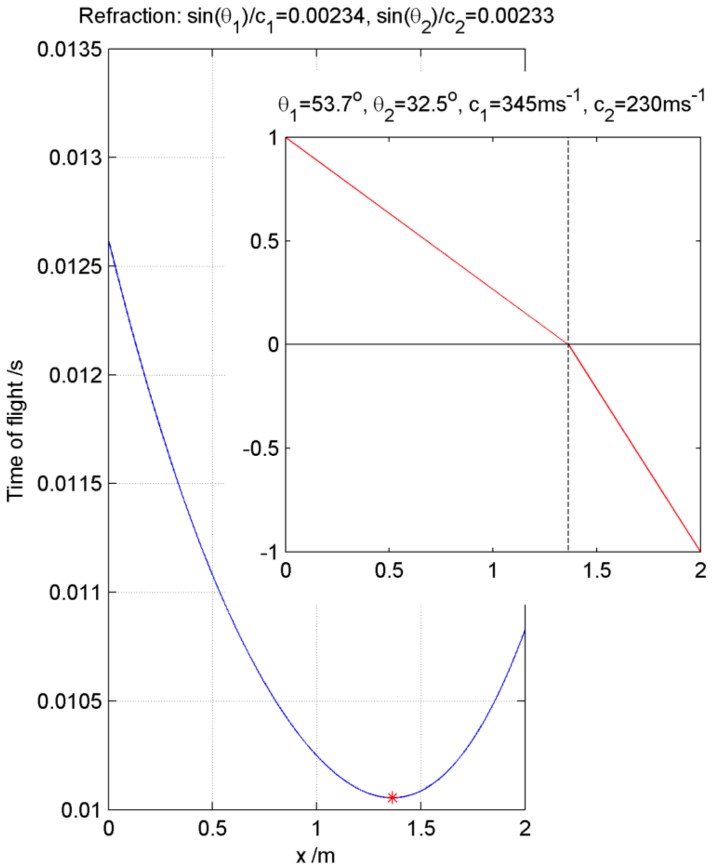
$$n_{1,2} = \frac{c}{c_{1,2}}$$

$$\therefore n_1 \sin\theta = n_2 \sin\phi$$

Challenge #4: Plot travel time t vs x and hence demonstrate *Snell's Law of refraction*. Inputs are wave speeds c_1 and c_2



$$x = y\tan\theta,$$
$$L - x = Y\tan\phi,$$



$$t = \frac{\sqrt{x^2+y^2}}{c/n_1} + \frac{\sqrt{(L-x)^2+Y^2}}{c/n_2}.$$

$$\therefore \frac{\partial t}{\partial x} = \frac{1}{c}\left(\frac{\frac{1}{2}2xn_1}{\sqrt{x^2+y^2}} + \frac{\frac{1}{2}2\,(L-x)\,(-1)n_2}{\sqrt{(L-x)^2+Y^2}}\right).$$

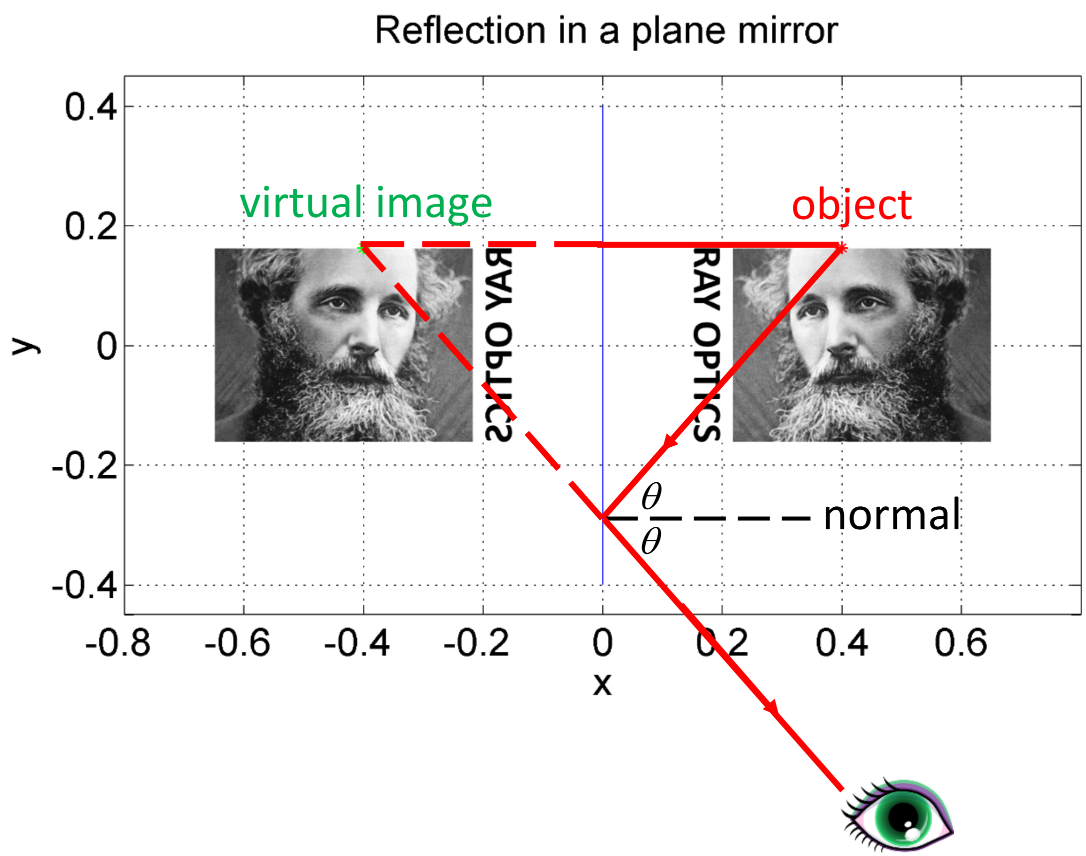
$$\implies \frac{\partial t}{\partial x} = \frac{1}{c}\left(\frac{yn_1\tan\theta}{y\sqrt{\tan^2\theta+1}} - \frac{Yn_2\tan\phi}{Y\sqrt{\tan^2\phi+1}}\right).$$

$$\therefore \frac{\partial t}{\partial x} = \frac{1}{c}\left(n_1\sin\theta - n_2\sin\phi\right).$$

The travel time is minimized when $\frac{\partial t}{\partial x} = 0$, which is when

$$n_1\sin\theta = n_2\sin\phi.$$

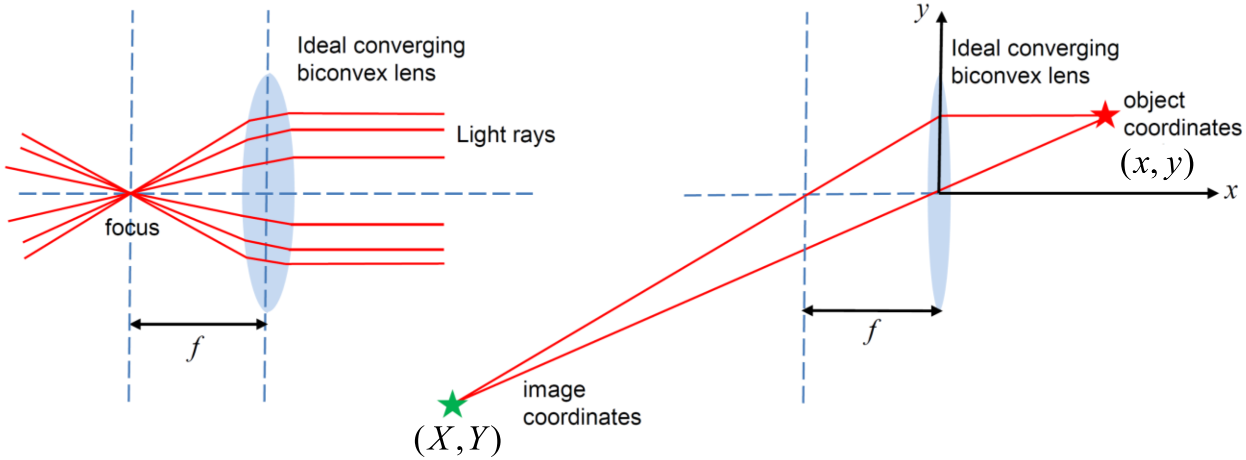
Proof of Snell's Law of Refraction by Fermat's principle



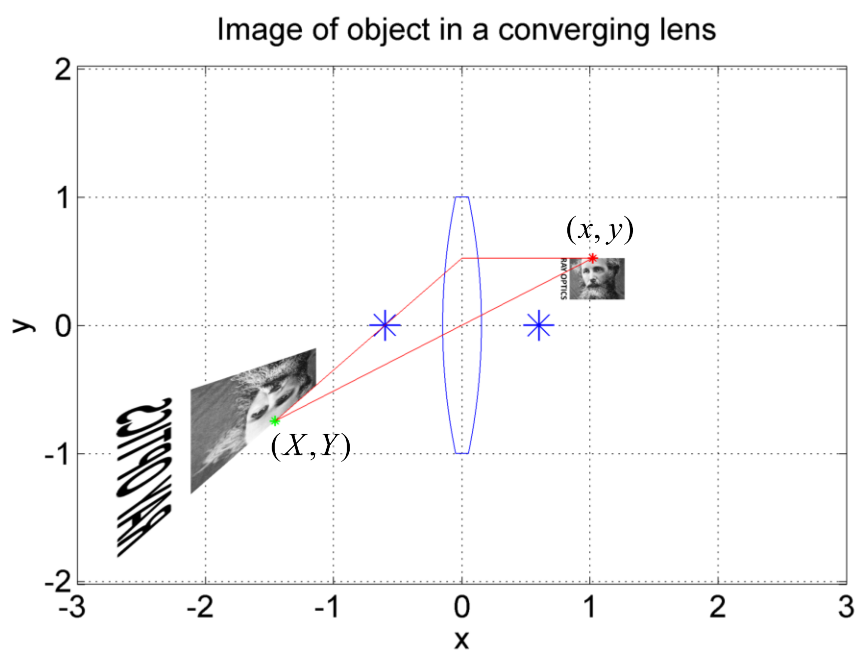
Challenge #5: Write a computer program that <u>imports an image file</u> (the 'object') and then computes the locations of the pixel coordinates that constitute a virtual image in a plane mirror. Use this information to <u>plot the virtual image</u>.

Once you have achieved this, apply the same programming concept in challenges 6-10.

Extra: Make it interactive! Move the object with a mouse/finger/arrow keys and update the virtual image.



Challenge #6: Create an interactive model of the *real*, inverted image of an object placed outside the focal range of an ideal thin lens.



$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

$$u = x$$

$$v = -X$$

Thin lens equation

Coordinate transform between object (x,y) and image (X,Y) associated with a thin lens.

$$\boxed{\begin{aligned} X &= -\frac{f}{x-f}x \\ Y &= \frac{y}{x}X \end{aligned}}$$

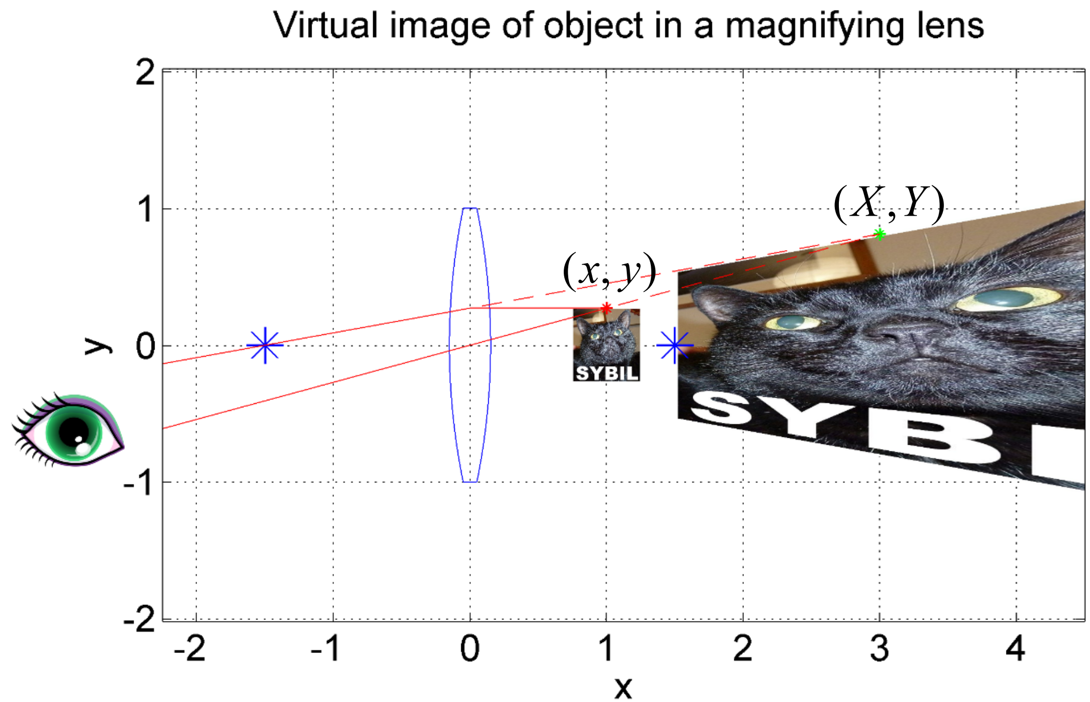
only if $x > f$

$$X = \frac{f}{x - f}x$$

$$Y = \frac{y}{x}X$$

only if

$$0 < x < f$$

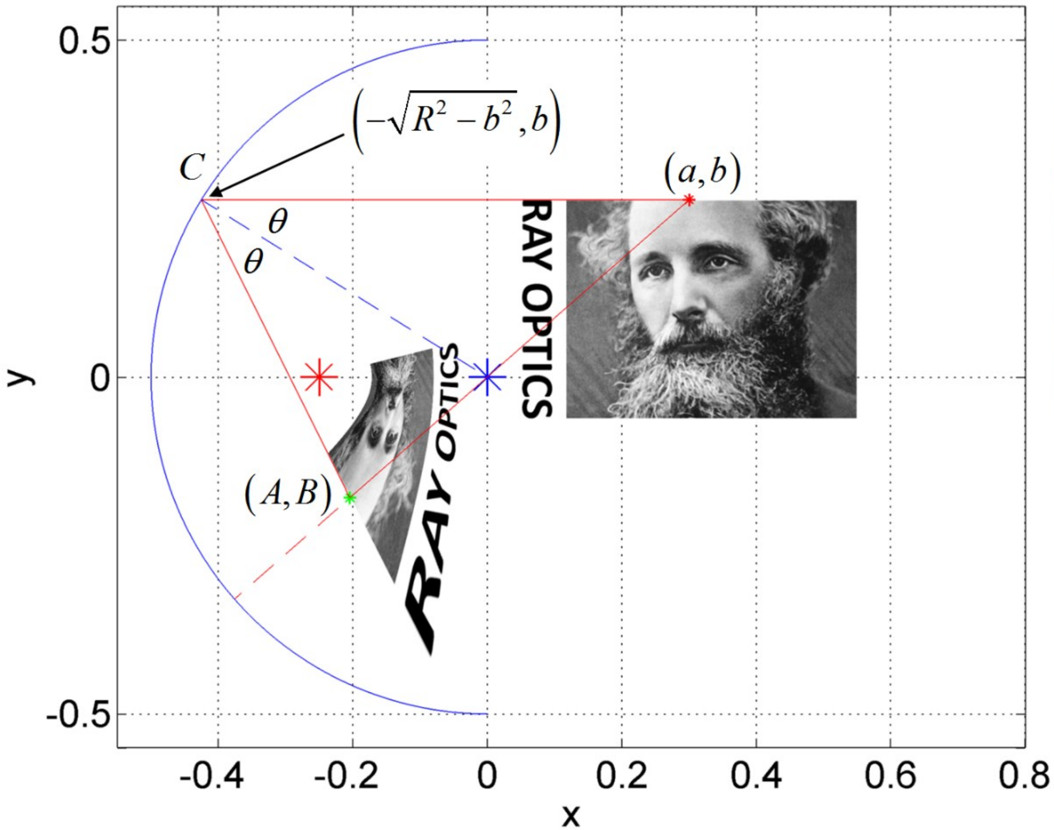


In this case, the **virtual image** of (x,y) is the *apparent source* of diverging rays from the lens.

Challenge #7: Create an interactive model of the *virtual*, enlarged image of an object placed *inside* the focal range of an ideal thin lens.

Perhaps you could combine Challenges 6 and 7 into the *same* computer program, a bit like in the excellent PhET Geometric Optics demo.

Reflection in a concave mirror



Behold the flying dinosaur!

$$X_i = -\frac{m_i\sqrt{R^2-y_i^2}-y_i}{\frac{y_i}{x_i}+m_i},$$

$$Y_i = -\frac{y_i}{x_i}\frac{m_i\sqrt{R^2-y_i^2}-y_i}{\frac{y_i}{x_i}+m_i},$$

$$m_i = \tan 2\theta_i$$

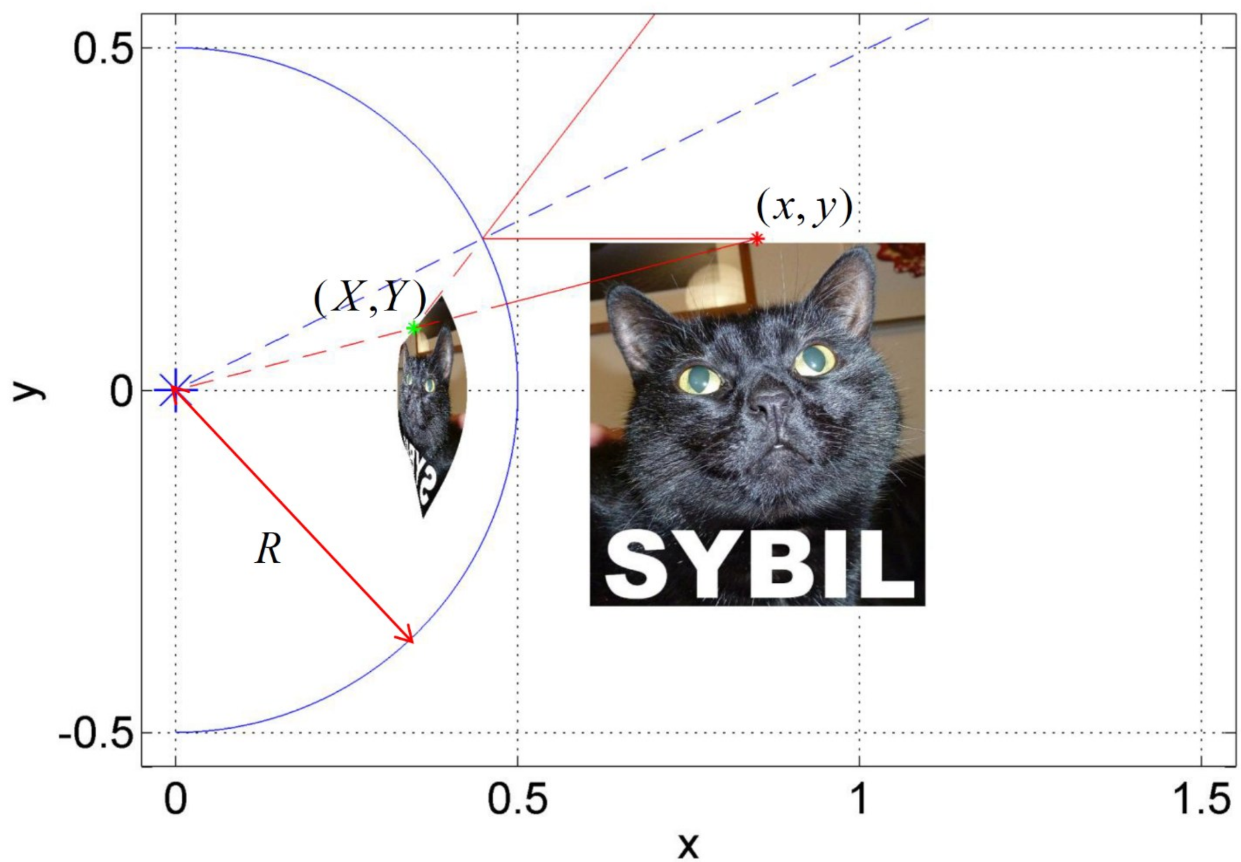
$$\theta_i = \tan^{-1}\left(\frac{y_i}{\sqrt{R^2-x_i^2}}\right).$$

Challenge #8: Create an interactive model of the (surprising!) *real* image of an object in a *concave spherical mirror.*

Note in the diagram $(a,b)\rightarrow(A,B)$ is an example of general pixel coordinate transformation

$$(x_i,y_i)\rightarrow(X_i,Y_i)$$

Reflection in a convex mirror



$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{x}\right)$$

$$k = \frac{x}{\cos(2\alpha)}$$

$$Y = \frac{k\sin\alpha}{\frac{k}{R} - \cos\alpha + \frac{x}{y}\sin\alpha}$$

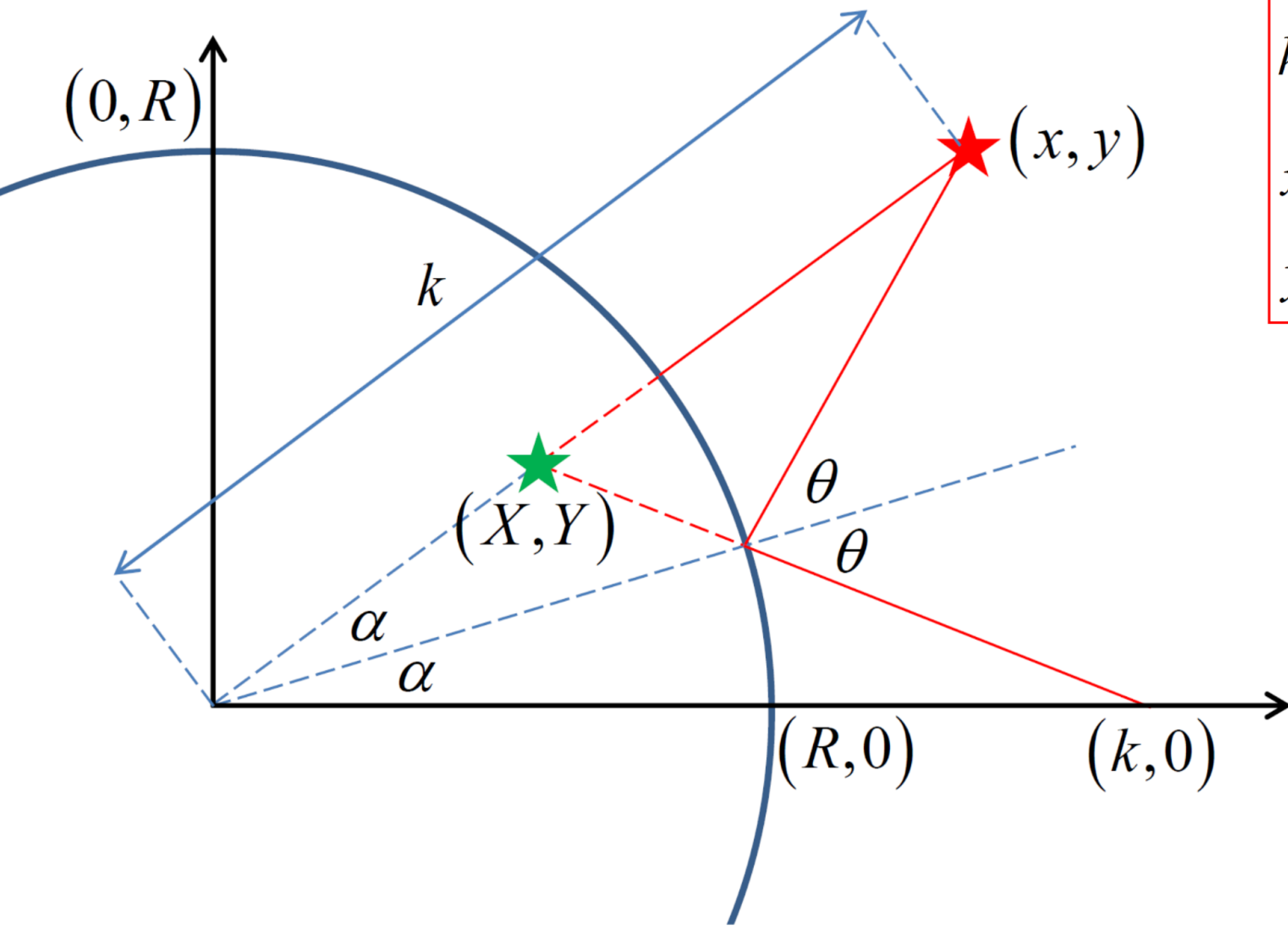
$$X = x\frac{Y}{y}$$

Virtual image from object coordinates

We see an upright, distorted *virtual image* in a cylindrical mirror.

i.e. the *apparent source* of (diverging) light rays from the mirror

Challenge #9: Create an interactive model of the virtual image of an object in a *convex spherical mirror.*



$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{Y}{X}\right)$$

$$k = \frac{R\left(Y\cos\alpha - X\sin\alpha\right)}{Y - R\sin\alpha}$$

$$x = k\cos\left(2\alpha\right)$$

$$y = k\sin\left(2\alpha\right)$$

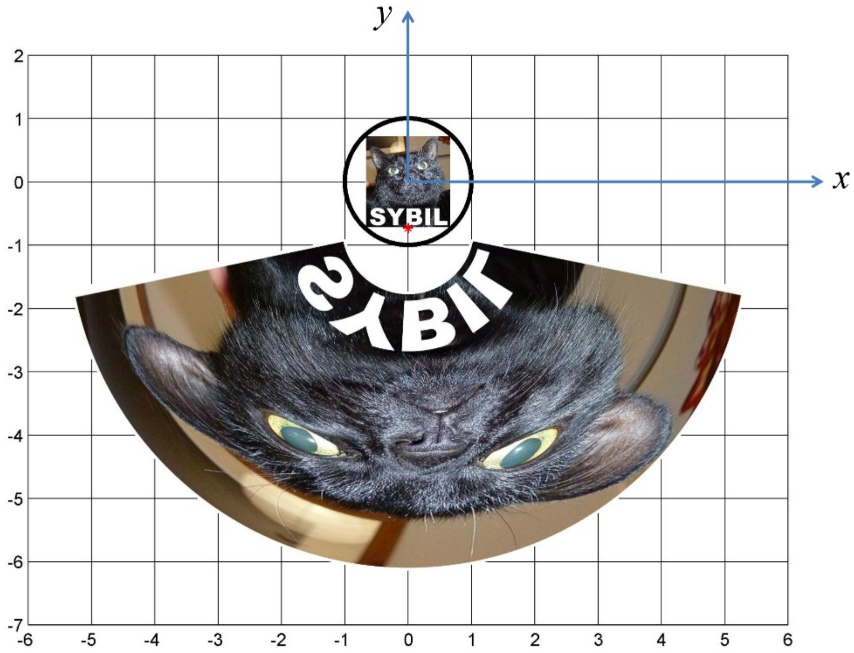
$$\alpha = \tfrac{1}{2}\tan^{-1}\left(\tfrac{y}{x}\right)$$

$$k = \frac{x}{\cos\left(2\alpha\right)}$$

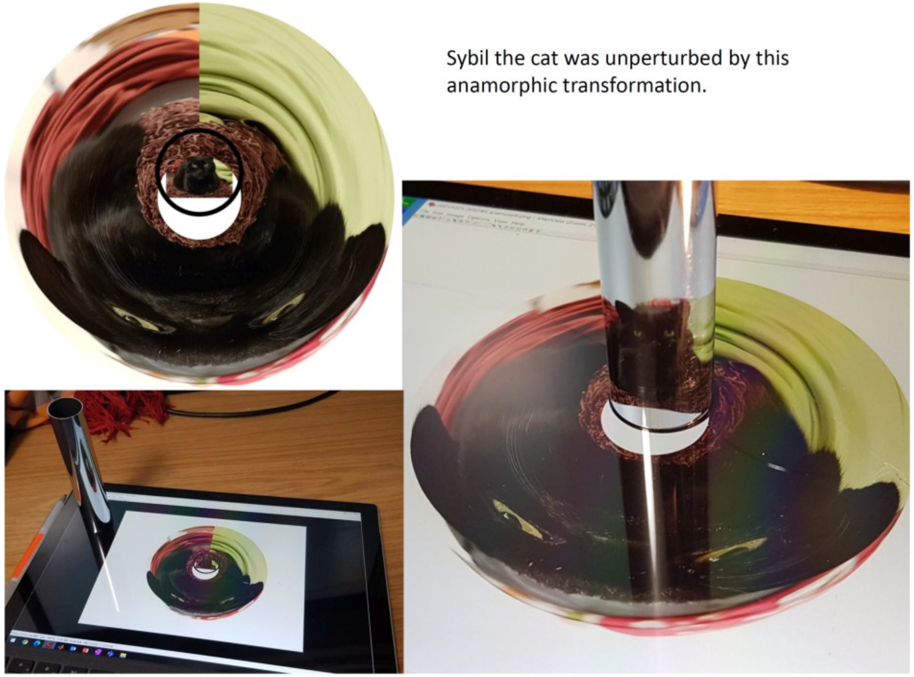
$$Y = \frac{k\sin\alpha}{\tfrac{k}{R} - \cos\alpha + \tfrac{x}{y}\sin\alpha}$$

$$X = x\frac{Y}{y}$$

Use this figure to derive the transformation of object (x,y) coordinates to virtual image (X,Y) coordinates, and indeed the inverse transformation.

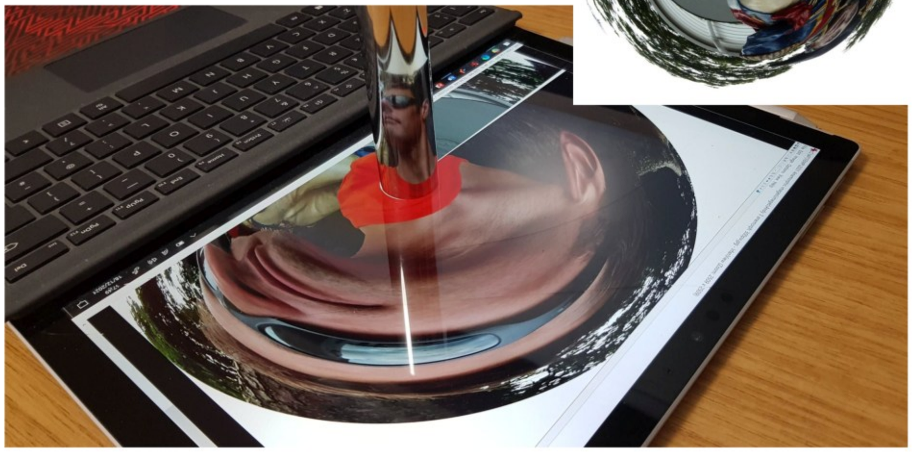


arc_deg = 160   Rf = 3



Sybil the cat was unperturbed by this anamorphic transformation.

'Fear and loathing' in Portmeirion ...



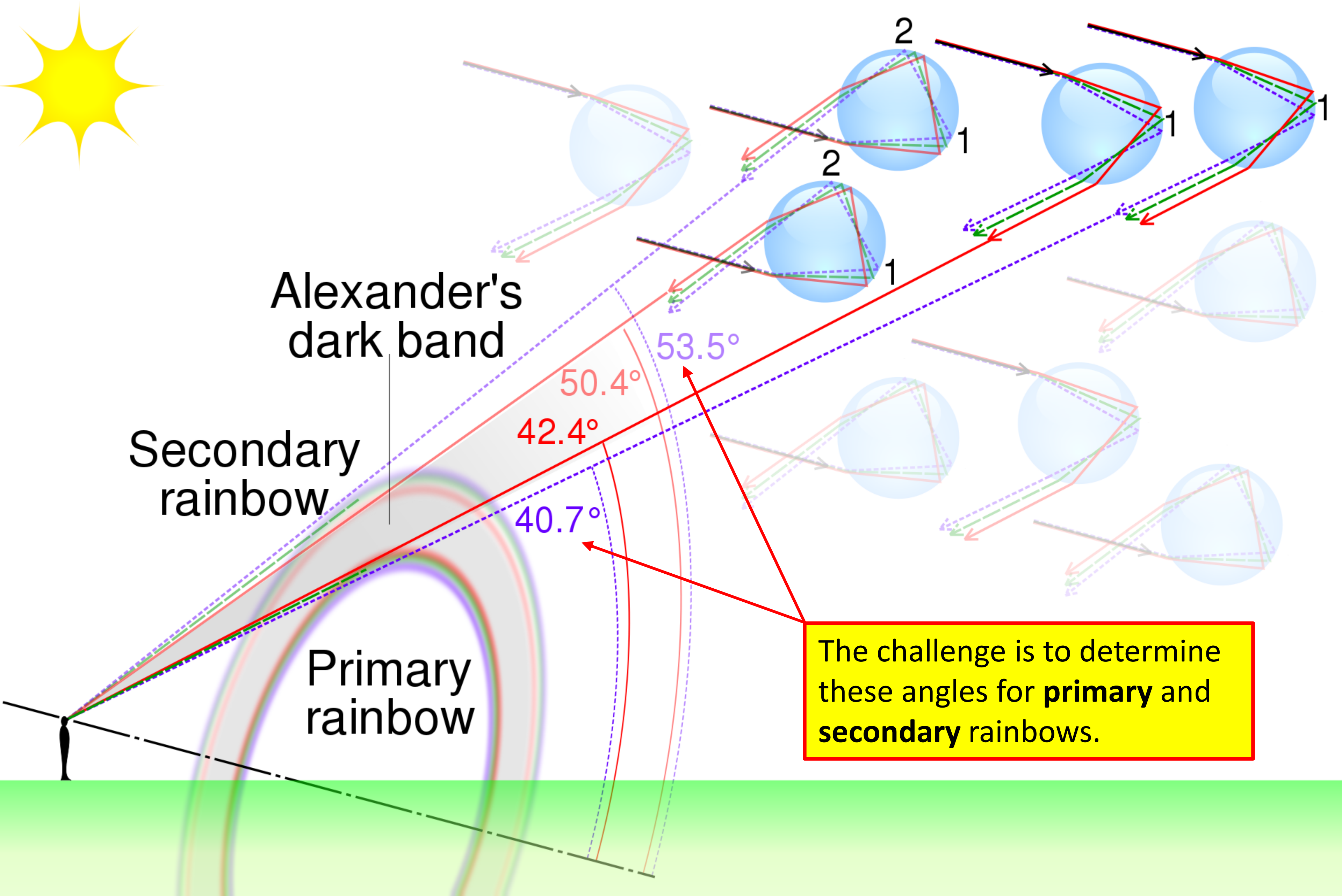
Challenge #10: Create a mapping of pixel coordinates (that are fitted into a unit circle) to an arc of a circle with radius Rf, centred at the base of the object (the red star).

If you place a polished cylinder over the unit circle, you will create an *anamorphic image.* It will appear to look somewhat three dimensional.

https://en.wikipedia.org/wiki/Rainbow

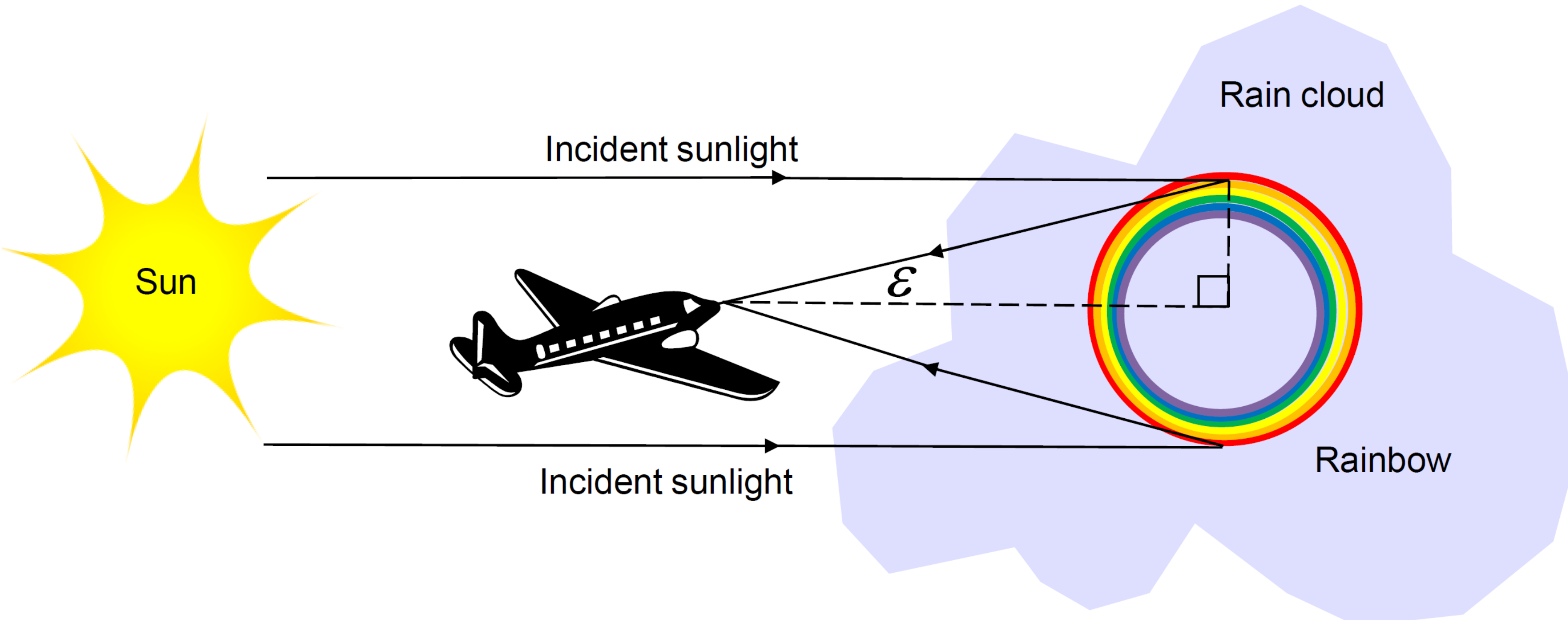
RAINBOW PHYSICS

https://en.wikipedia.org/wiki/Rainbow

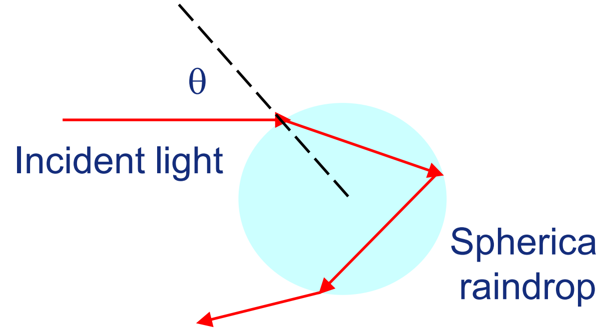


The challenge is to determine these angles for **primary** and **secondary** rainbows.

See a *circular* rainbow when flying since rain cloud is illuminated above and below aircraft

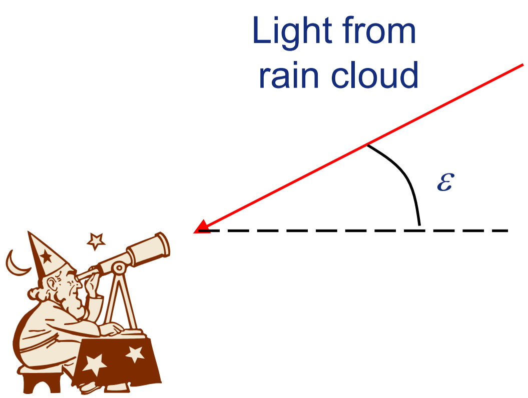


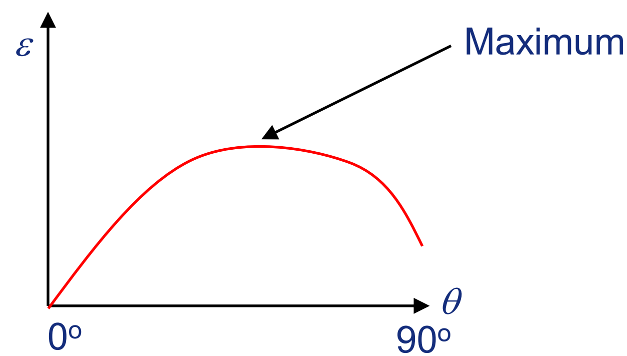
Descartes theory of RAINBOW PHYSICS



René Descartes (1596-1650)

- Light is **internally reflected** off the interior of **spherical** raindrops.
- The wavelength of light is much smaller than the dimensions of the raindrop, so **interference effects** can be ignored.
- The mathematical relation between the angle that light is bent by the raindrops and the angle of incidence to the raindrop, has an '*extremum*'.* This results in the **focussing** of light of particular wavelengths into particular angles.

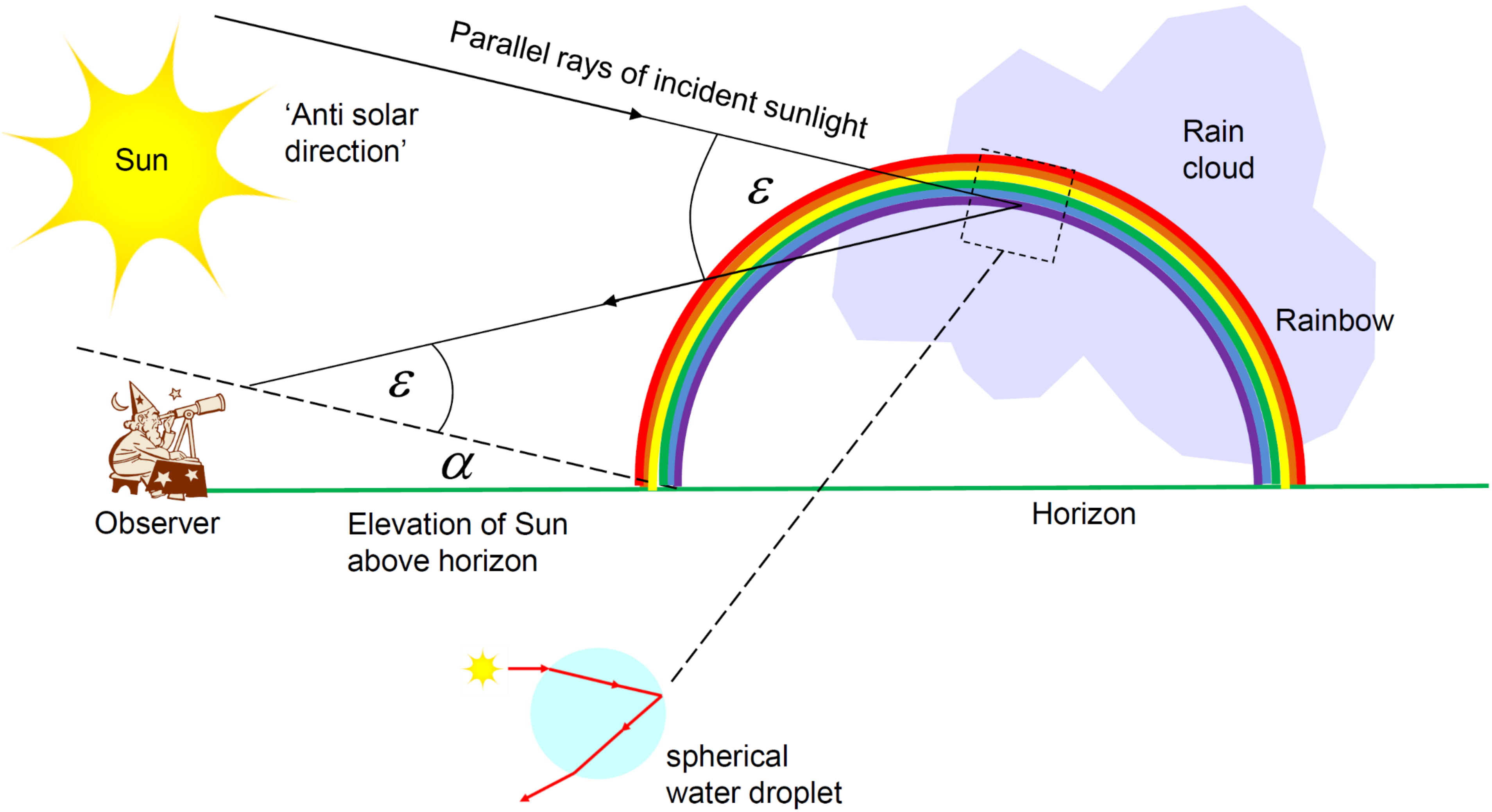




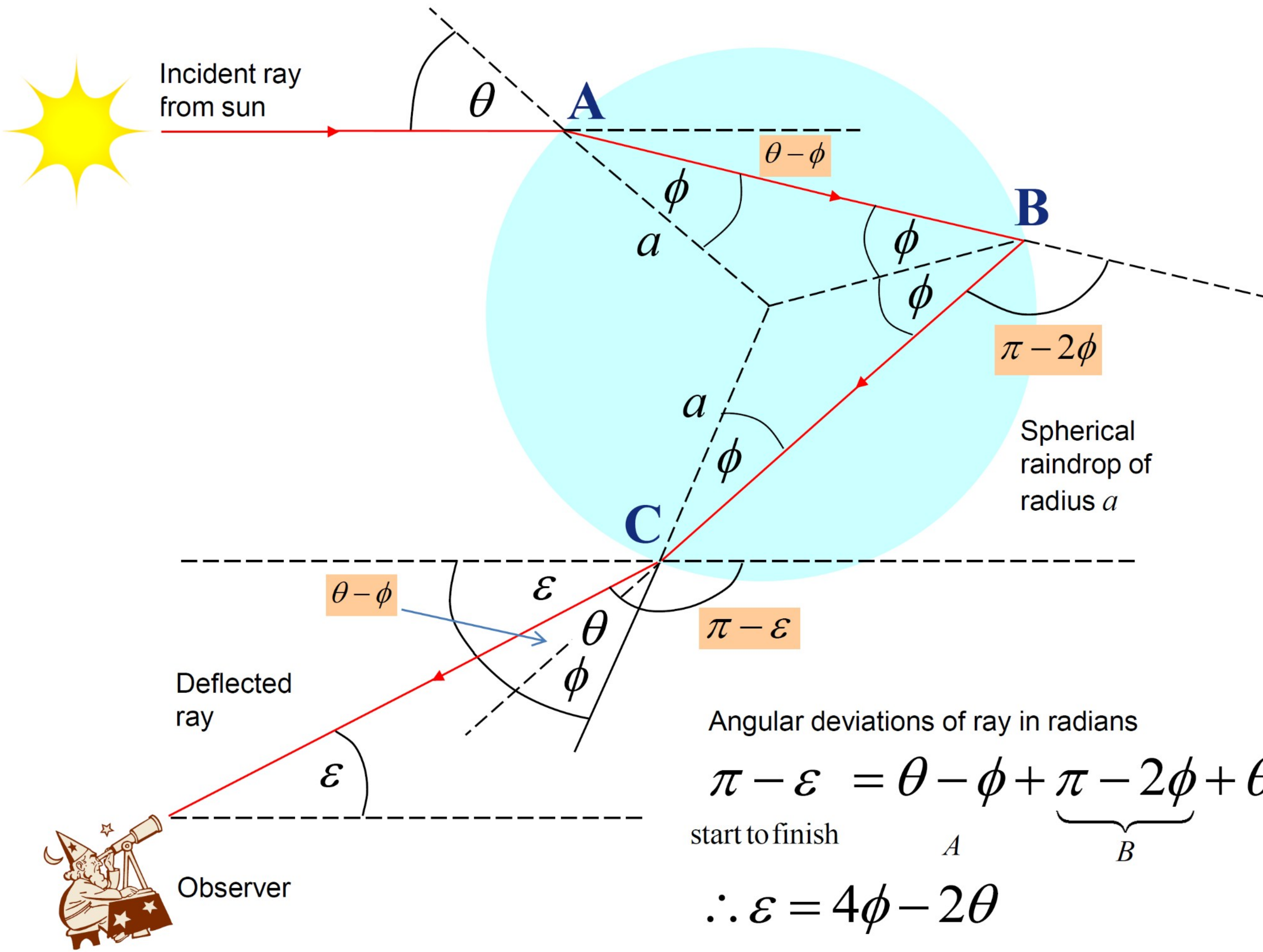
*i.e. a max or min.

So we need to find out $\varepsilon(\theta)$

Schematic of a rainbow



Primary rainbow – a single internal reflection at **B**

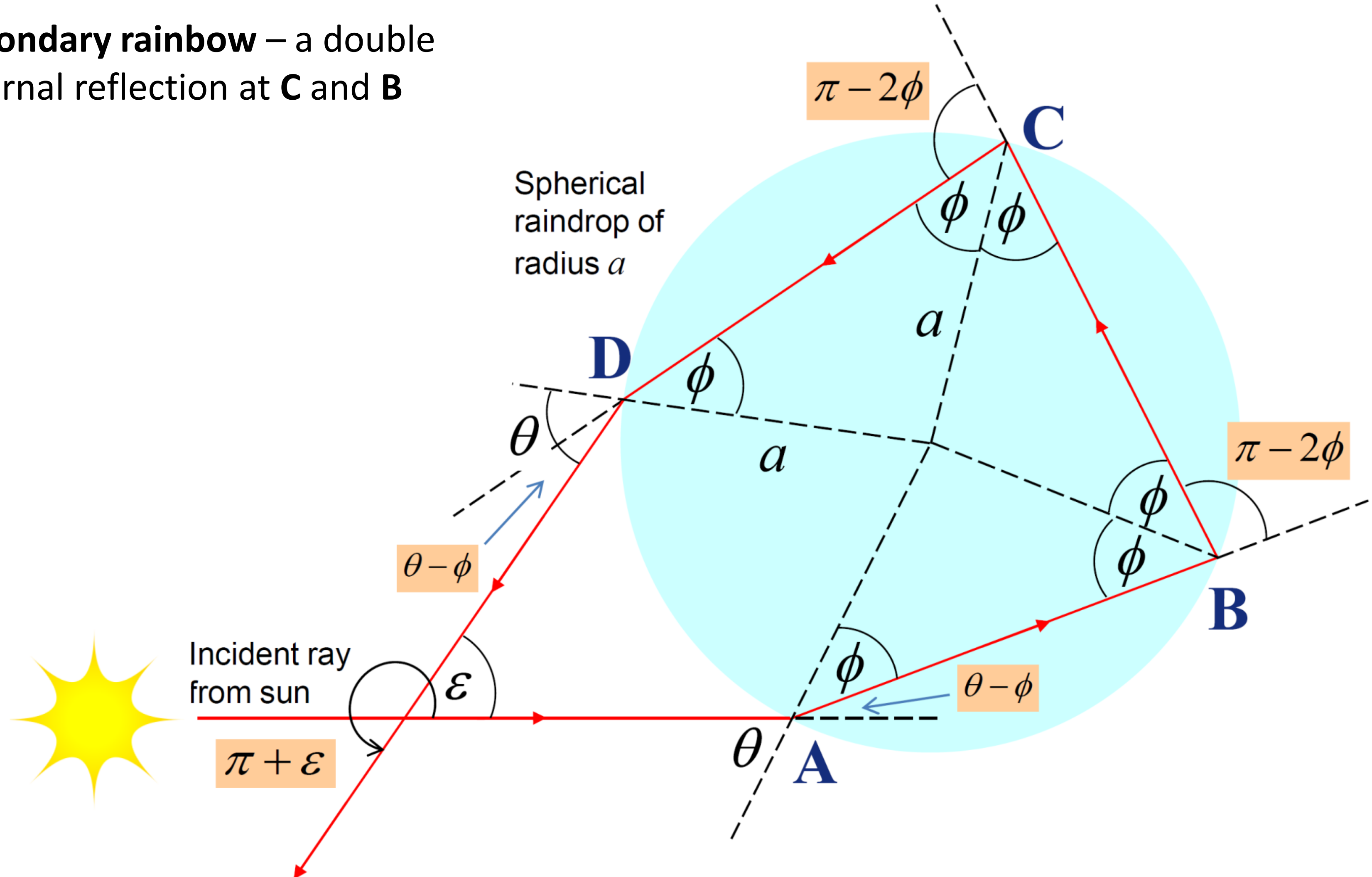


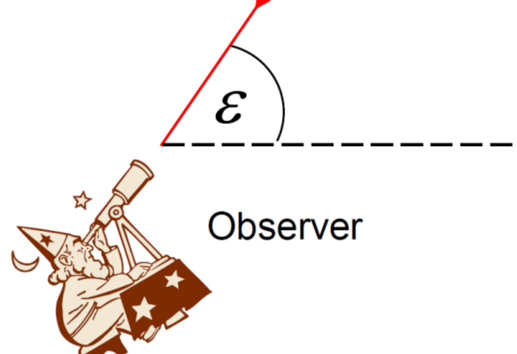
Angular deviations of ray in radians

$$\underset{\text{start to finish}}{\pi-\varepsilon} = \underbrace{\theta-\phi}_{A} + \underbrace{\pi-2\phi}_{B} + \underbrace{\theta-\phi}_{C}$$

$$\therefore \varepsilon = 4\phi - 2\theta$$

Secondary rainbow – a double internal reflection at **C** and **B**

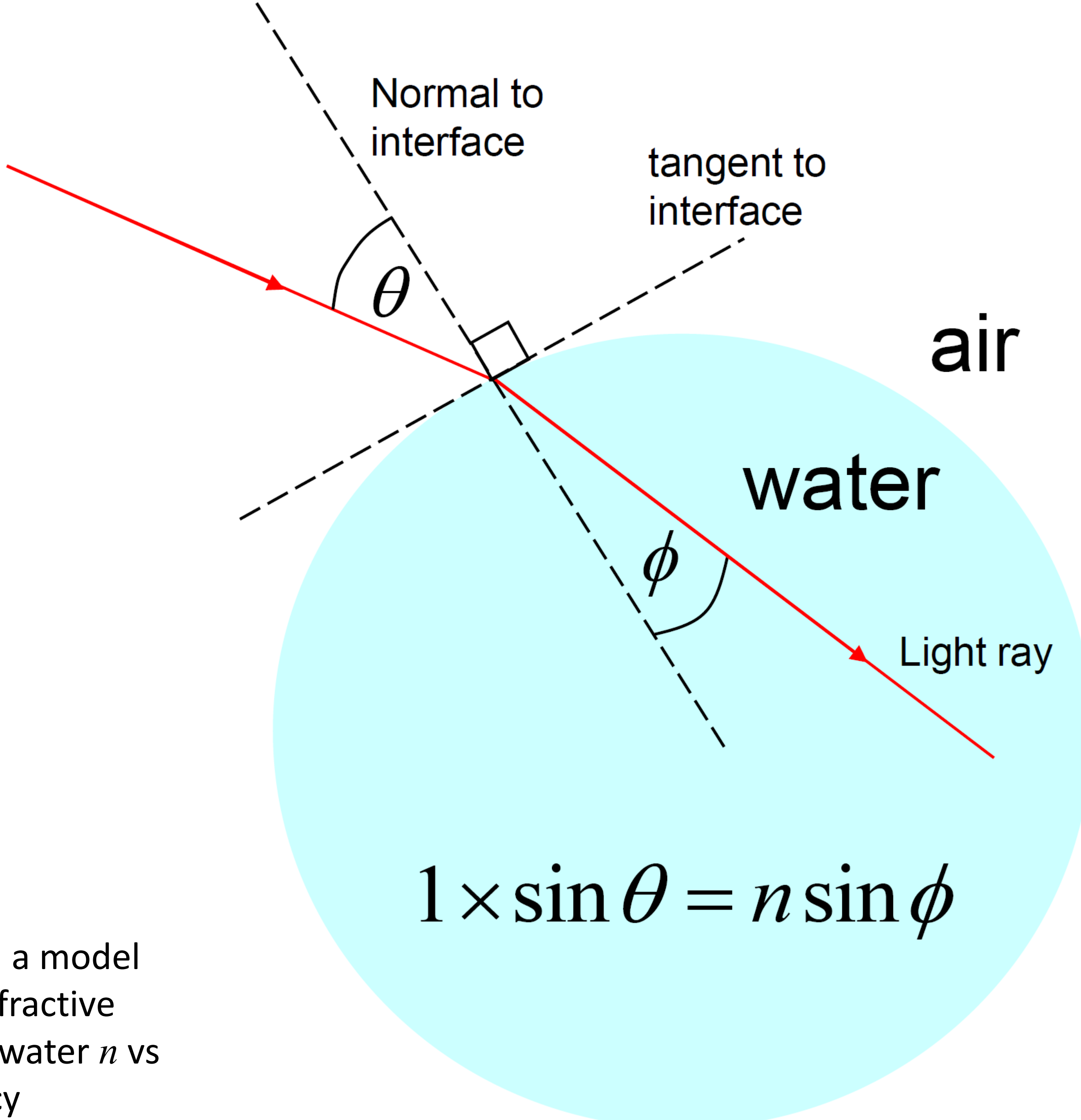




Angular deviations of ray in radians

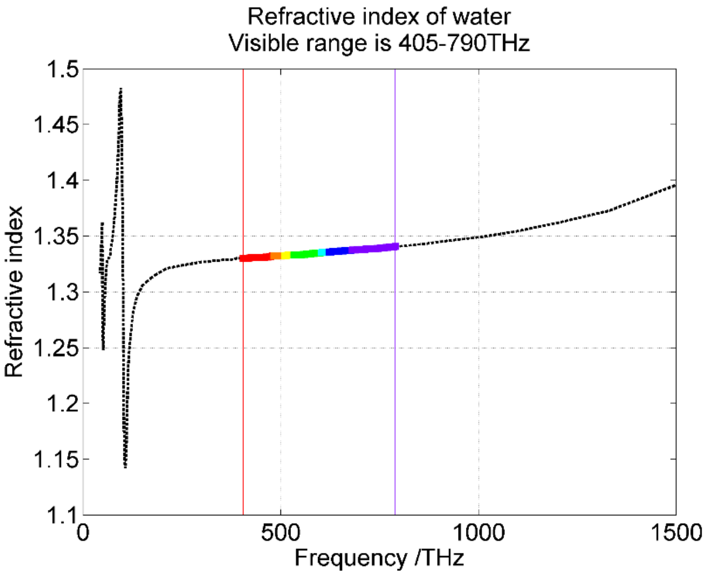
$$\underset{\text{start to finish}}{\pi + \varepsilon} = \underset{A}{\theta - \phi} + \underbrace{\pi - 2\phi}_{B} + \underbrace{\pi - 2\phi}_{C} + \underset{D}{\theta - \phi}$$

$$\therefore \varepsilon = \pi - 6\phi + 2\theta$$



$$1 \times \sin\theta = n \sin\phi$$

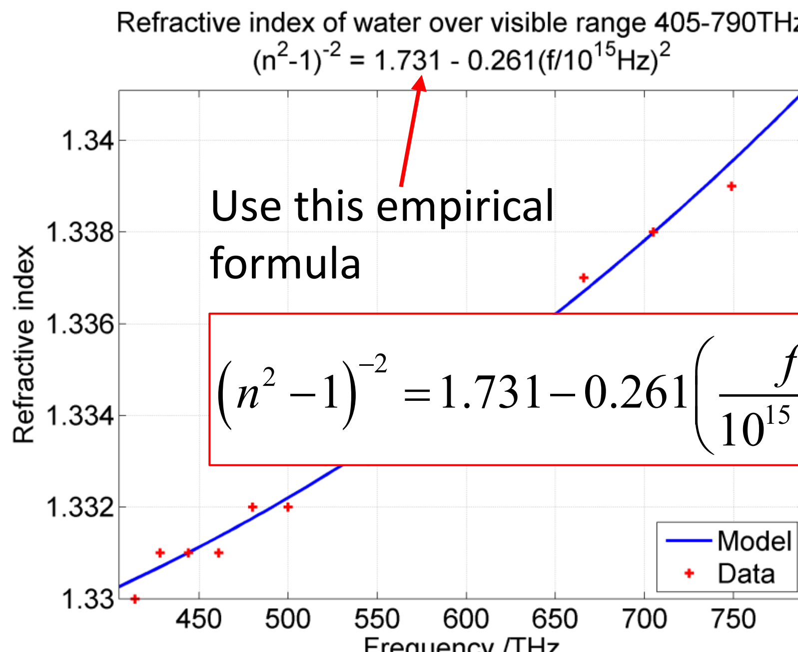
We need a model
of the refractive
Index of water n vs
frequency




```
%colours_from_f
% Function which provides the R,G,B values( within interval [0,1] )
% of visible light depending on the frequency /THz
function [R,G,B,colour_str] = colours_from_f(f)
if f < 405
    R = NaN; G = NaN; B = NaN; colour_str = 'Infra Red';
elseif (f>=405) && ( f < 480 )
    R = 1; G = 0; B = 0; colour_str = 'Red';
elseif (f>=480) && ( f < 510 )
    R = 1; G = 127/255; B = 0; colour_str = 'Orange';
elseif (f>=510) && ( f < 530 )
    R = 1; G = 1; B = 0; colour_str = 'Yellow';
elseif (f>=530) && ( f < 600 )
    R = 0; G = 1; B = 0; colour_str = 'Green';
elseif (f>=600) && ( f < 620 )
    R = 0; G = 1; B = 1; colour_str = 'Cyan';
elseif (f>=620) && ( f < 680 )
    R = 0; G = 0; B = 1; colour_str = 'Blue';
elseif (f>=680) && ( f <= 790 )
    R = 127/255; G = 0; B = 1; colour_str = 'Violet';
else
    R = NaN; G = NaN; B = NaN; colour_str = 'Ultra Violet';
end
```

RECAP!

Challenge #1b: Create a model of the refractive index of water with frequency (and hence wavelength in a vacuum), over the range 405nm to 790nm

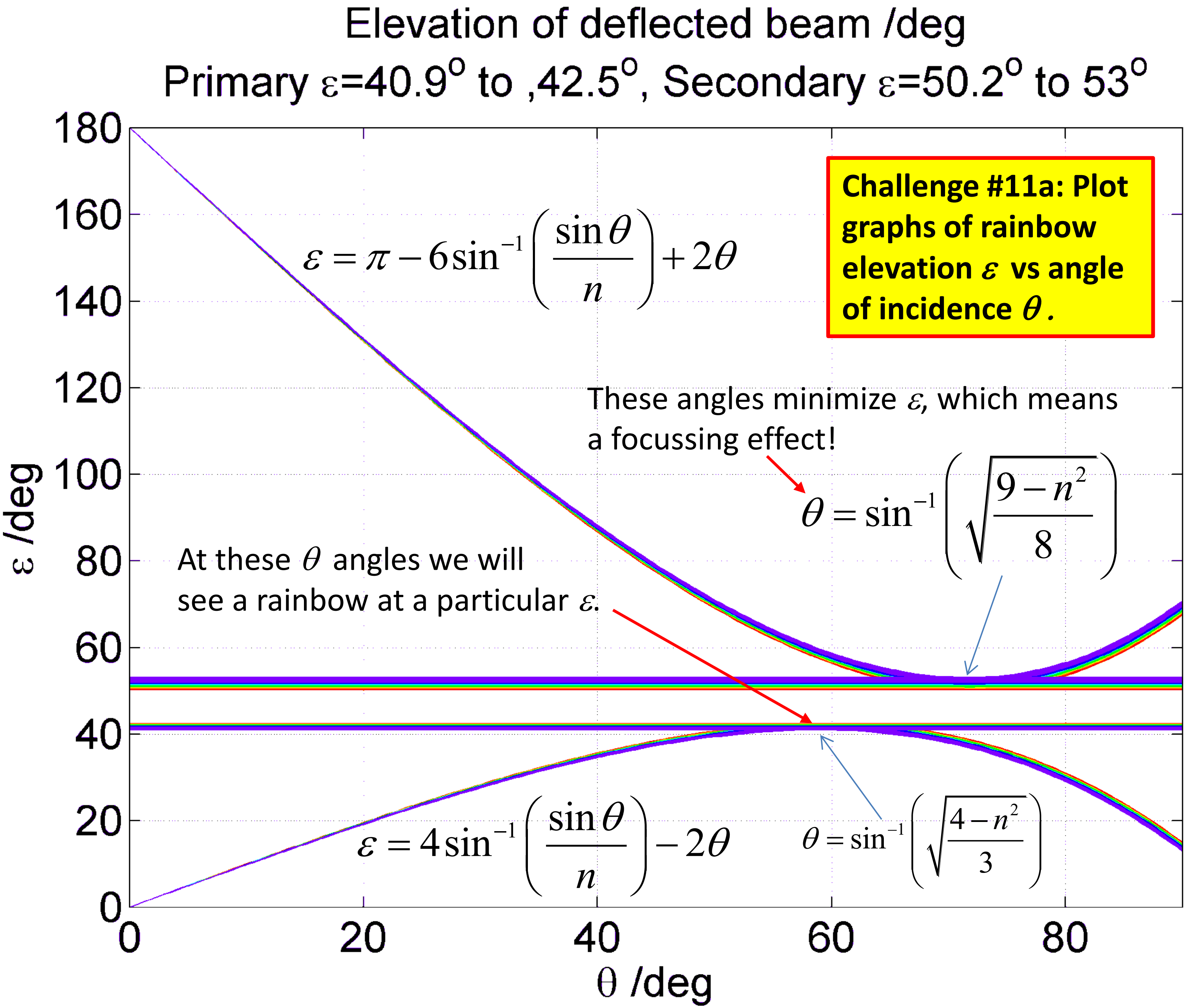


Use this empirical formula

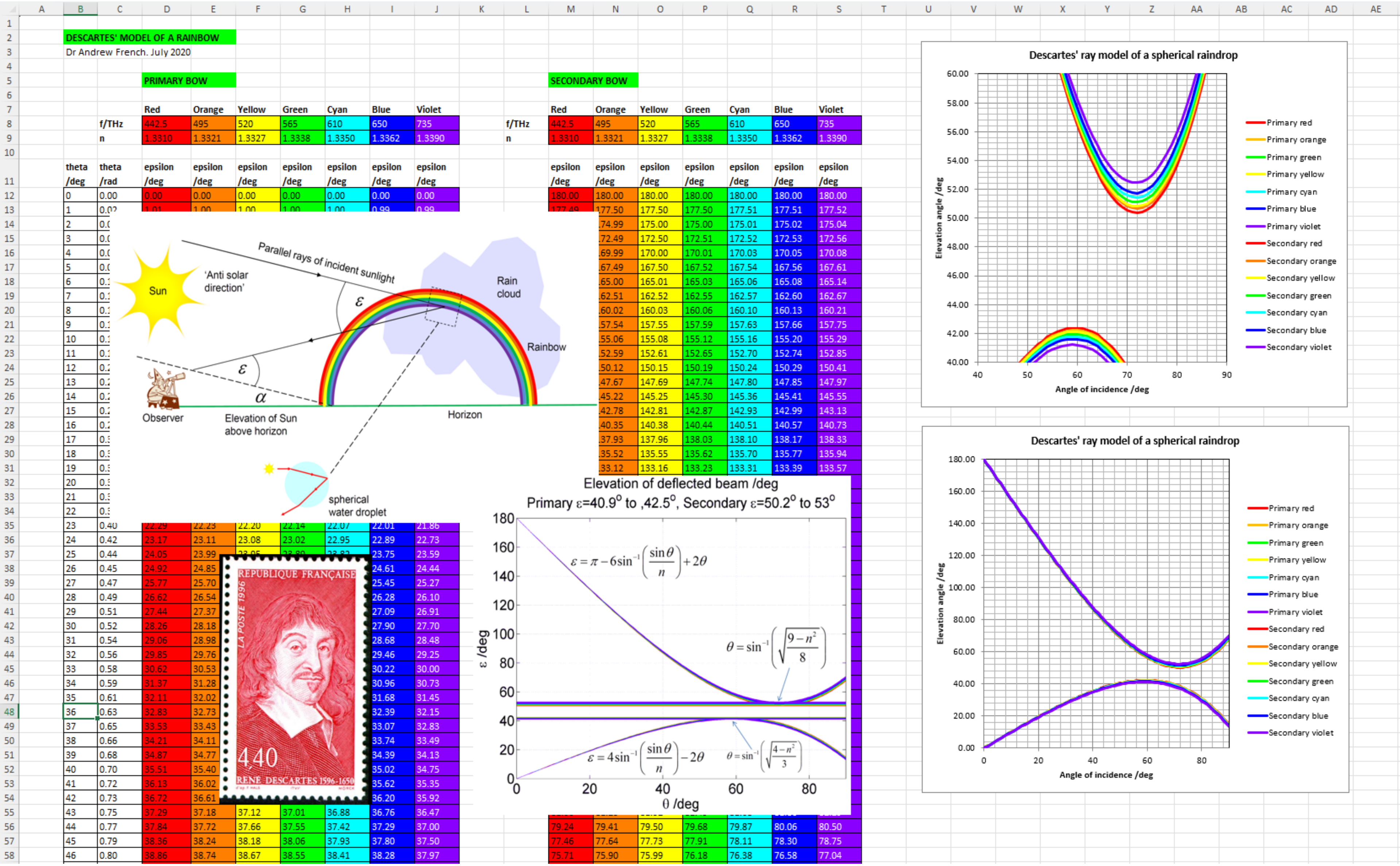
$$\left(n^2-1\right)^{-2} = 1.731 - 0.261\left(\frac{f}{10^{15}\ \text{Hz}}\right)^2$$

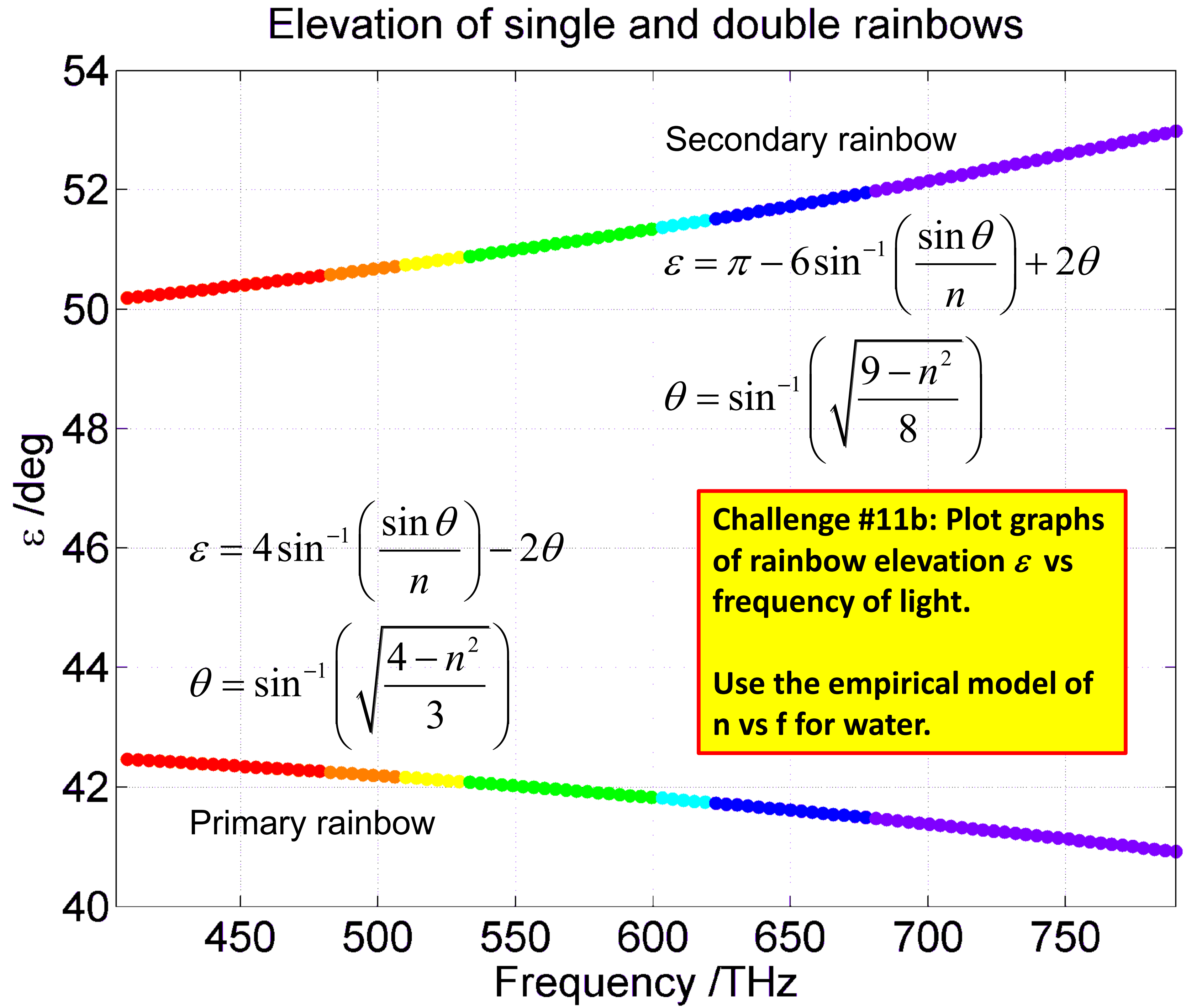
Use the code above
to set Red,Green,Blue
colours for different frequencies.

Even better, *interpolate* between
the colours to make a smooth colour map


Elevation of deflected beam /deg
Primary ε=40.9° to ,42.5°, Secondary ε=50.2° to 53°
Challenge #11a: Plot graphs of rainbow elevation ε vs angle of incidence θ.
$\varepsilon = \pi - 6\sin^{-1}\left(\frac{\sin\theta}{n}\right) + 2\theta$
These angles minimize ε, which means a focussing effect!
$\theta = \sin^{-1}\left(\sqrt{\frac{9-n^2}{8}}\right)$
At these θ angles we will see a rainbow at a particular ε.
$\varepsilon = 4\sin^{-1}\left(\frac{\sin\theta}{n}\right) - 2\theta$
$\theta = \sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right)$
ε /deg
θ /deg
180
160
140
120
100
80
60
40
20
0
0
20
40
60
80

You can do it in Excel, although we recommended a programming language!





Challenge #11b: Plot graphs of rainbow elevation ε vs frequency of light.

Use the empirical model of n vs f for water.

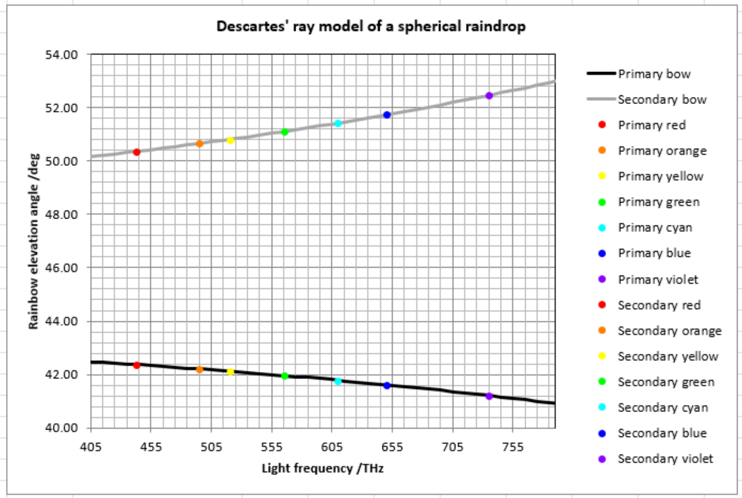
You can do it in Excel, although we recommended a programming language!

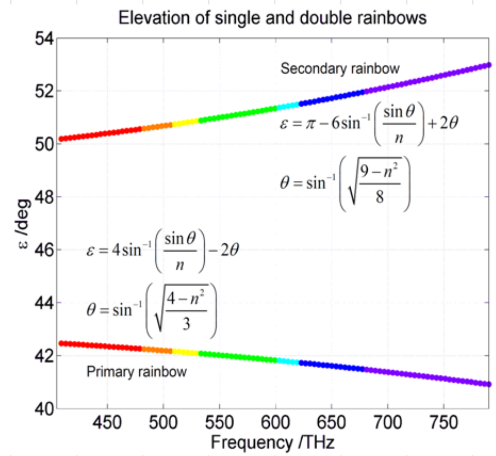
DESCARTES' MODEL OF A RAINBOW

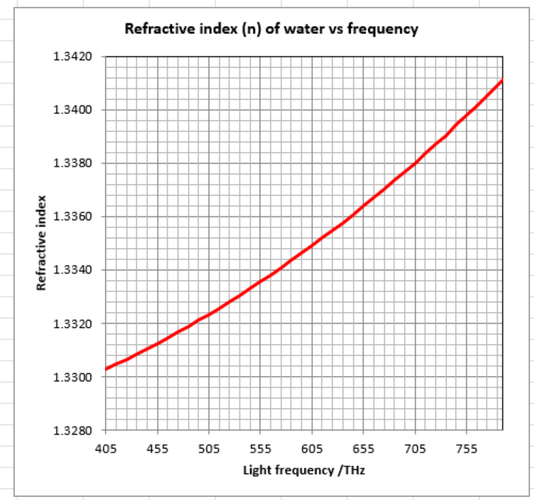
Dr Andrew French. July 2020

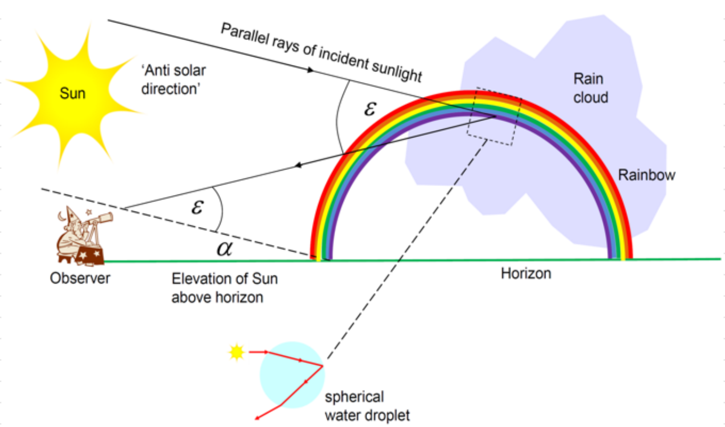
Light frequency /THz	Refractive index of water	Primary bow theta /deg	Primary bow elev /deg	Secondary bow theta /deg	Secondary bow elev /deg
405	1.3303	59.57	42.48	71.93	50.18
415	1.3305	59.56	42.45	71.92	50.22
425	1.3307	59.55	42.42	71.92	50.27
435	1.3308	59.54	42.39	71.91	50.32
445	1.3310	59.52	42.36	71.91	50.38
455	1.3312	59.51	42.33	71.90	50.43
465	1.3315	59.50	42.30	71.89	50.48
475	1.3317	59.49	42.27	71.89	50.54
485	[illegible]	[illegible]	[illegible]	[illegible]	50.60
495	[illegible]	[illegible]	[illegible]	[illegible]	50.66
505	[illegible]	[illegible]	[illegible]	[illegible]	50.72
515	[illegible]	[illegible]	[illegible]	[illegible]	50.78
525	[illegible]	[illegible]	[illegible]	[illegible]	50.84
535	[illegible]	[illegible]	[illegible]	[illegible]	50.90
545	[illegible]	[illegible]	[illegible]	[illegible]	50.97
555	[illegible]	[illegible]	[illegible]	[illegible]	51.03
565	[illegible]	[illegible]	[illegible]	[illegible]	51.10
575	[illegible]	[illegible]	[illegible]	[illegible]	51.17
585	[illegible]	[illegible]	[illegible]	[illegible]	51.24
595	[illegible]	[illegible]	[illegible]	[illegible]	51.31
605	[illegible]	[illegible]	[illegible]	[illegible]	51.39
615	[illegible]	[illegible]	[illegible]	[illegible]	51.46
625	[illegible]	[illegible]	[illegible]	[illegible]	51.54
635	[illegible]	[illegible]	[illegible]	[illegible]	51.61
645	[illegible]	[illegible]	[illegible]	[illegible]	51.69
655	1.3364	59.21	41.59	71.73	51.77
665	1.3367	59.20	41.54	71.72	51.85
675	1.3370	59.18	41.50	71.71	51.94
685	1.3373	59.16	41.45	71.70	52.02
695	1.3377	59.14	41.40	71.69	52.11
705	1.3380	59.12	41.36	71.68	52.19
715	1.3383	59.10	41.31	71.67	52.28
725	1.3387	59.08	41.26	71.66	52.37
735	1.3390	59.06	41.21	71.65	52.46
745	1.3394	59.04	41.16	71.64	52.56
755	1.3398	59.02	41.10	71.62	52.65
765	1.3401	59.00	41.05	71.61	52.75
775	1.3405	58.97	41.00	71.60	52.84
785	1.3409	58.95	40.94	71.59	52.94
795	1.3413	58.93	40.89	71.58	53.04

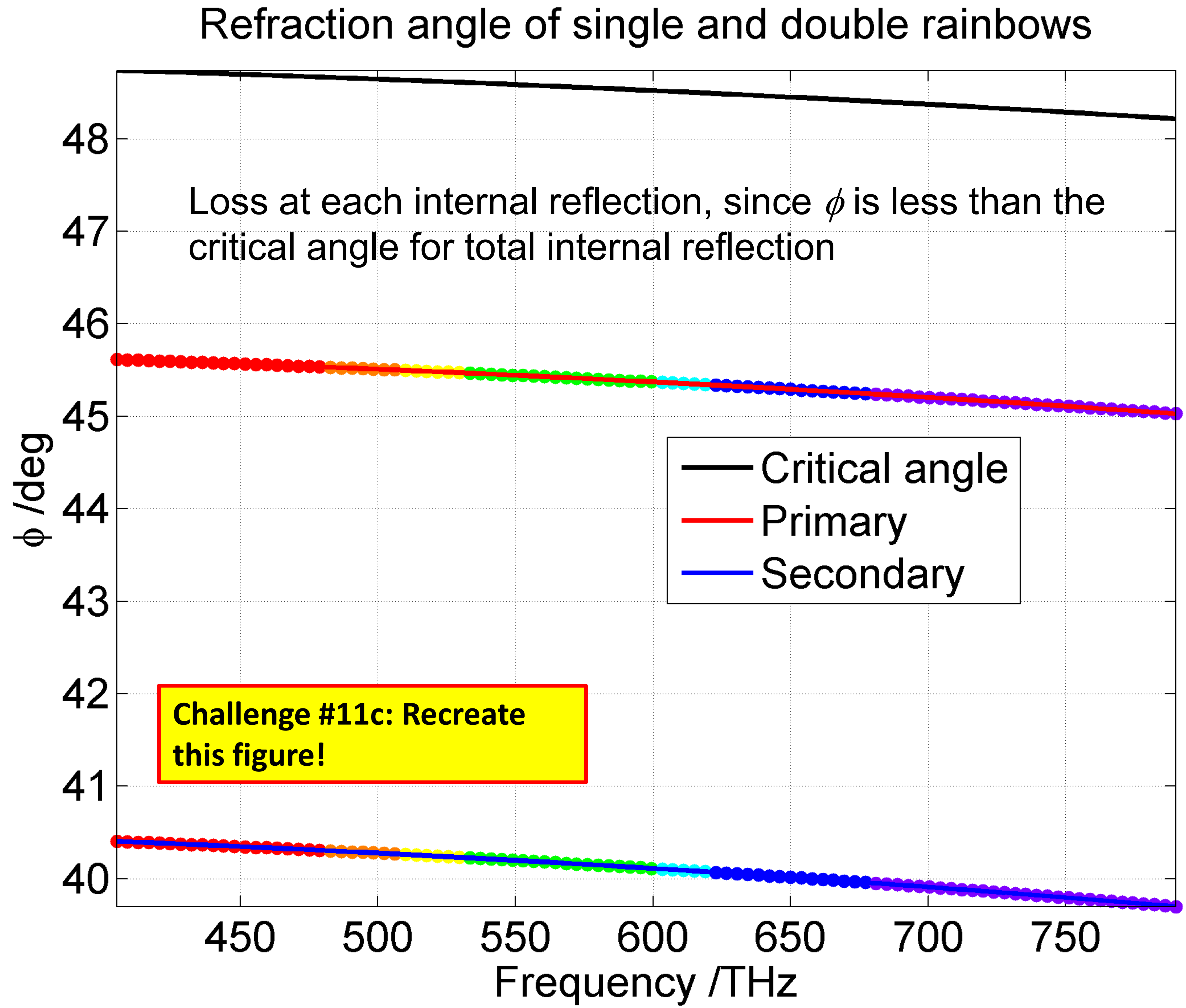
	Red	Orange	Yellow	Green	Cyan	Blue	Violet
f/THz	442.5	495	520	565	610	650	735
n	1.3310	1.3321	1.3327	1.3338	1.3350	1.3362	1.3390
Primary bow /rad	1.0389	1.0378	1.0372	1.0361	1.0348	1.0336	1.0308
Primary elevation /deg	42.3709	42.2082	42.1238	41.9606	41.7825	41.6114	41.2066
Secondary theta /rad	1.2550	1.2544	1.2541	1.2534	1.2527	1.2521	1.2505
Secondary elevation /deg	50.3631	50.6563	50.8083	51.1026	51.4238	51.7326	52.4637



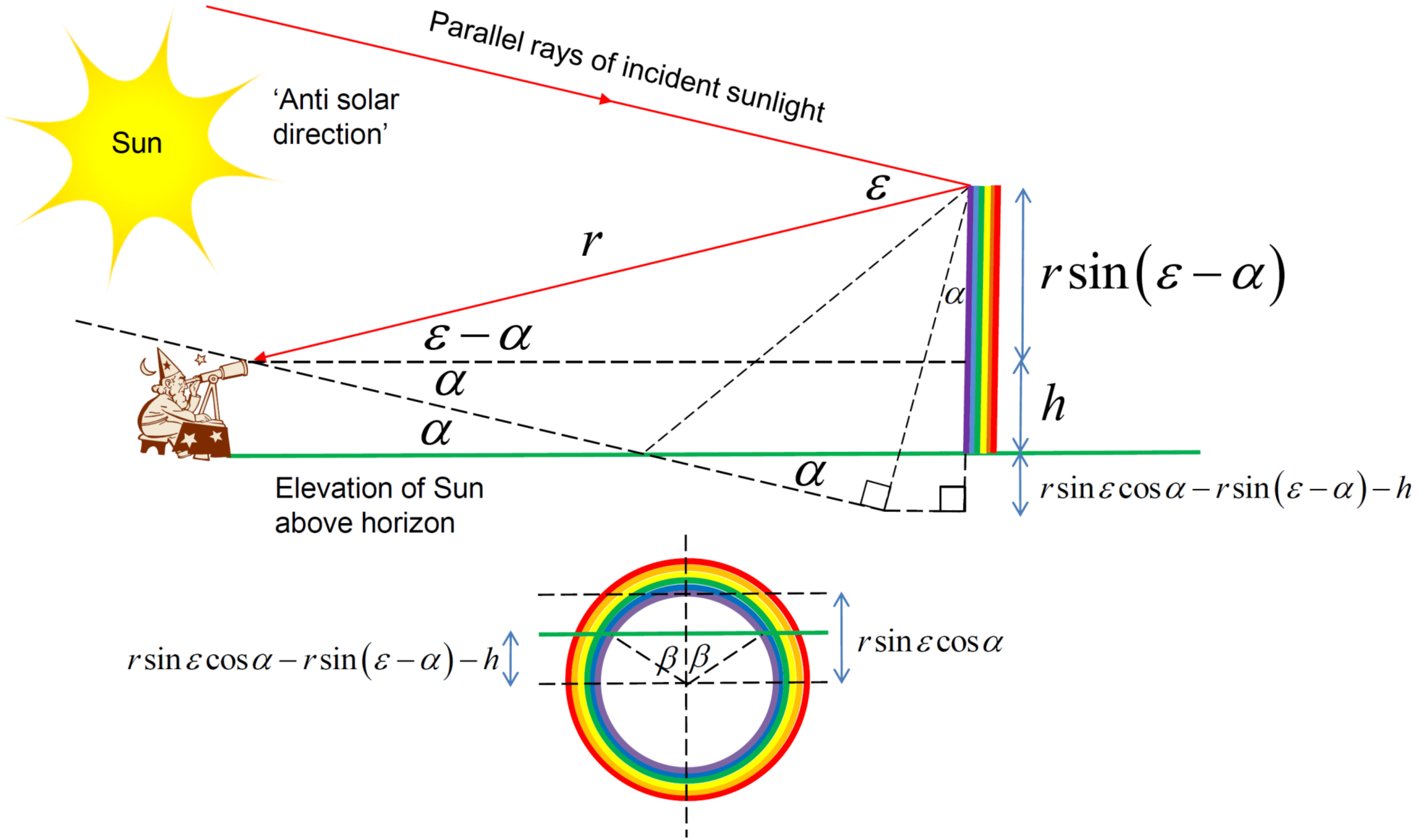




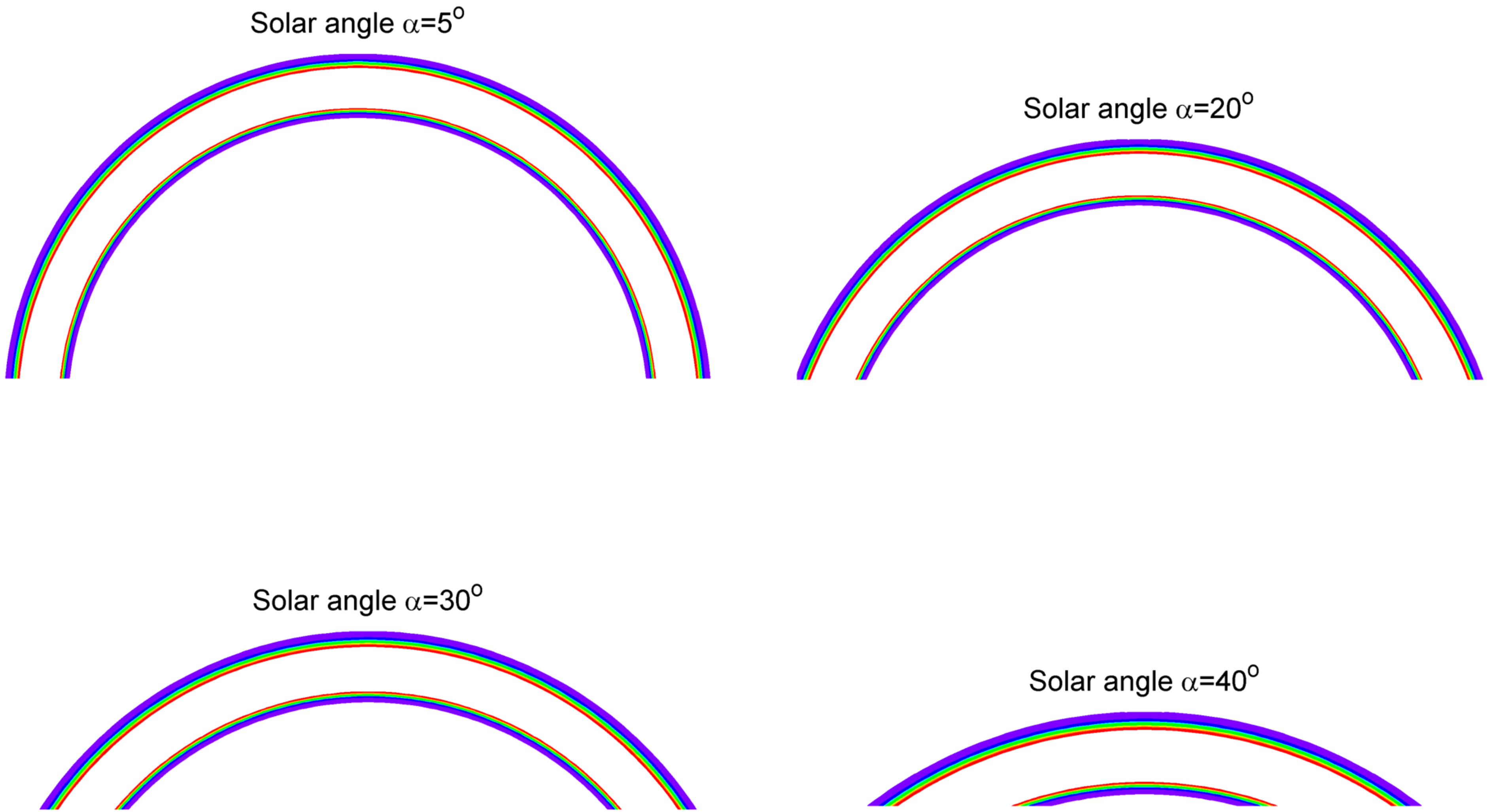




Refraction angle of single and double rainbows
Loss at each internal reflection, since ϕ is less than the critical angle for total internal reflection
Critical angle
Primary
Secondary
Challenge #11c: Recreate this figure!
ϕ /deg
Frequency /THz
48
47
46
45
44
43
42
41
40
450
500
550
600
650
700
750

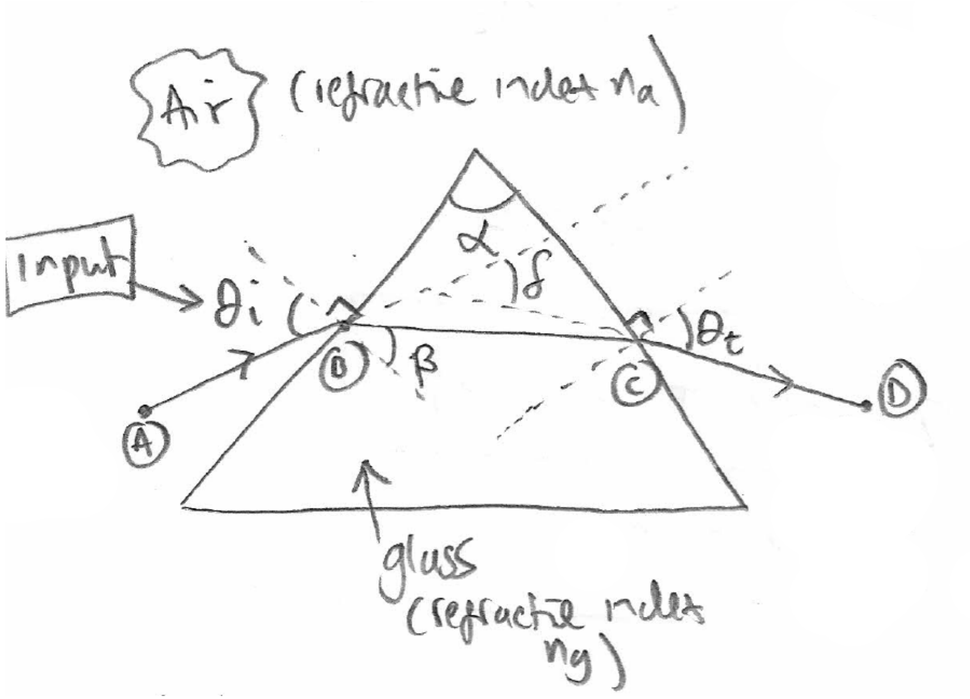


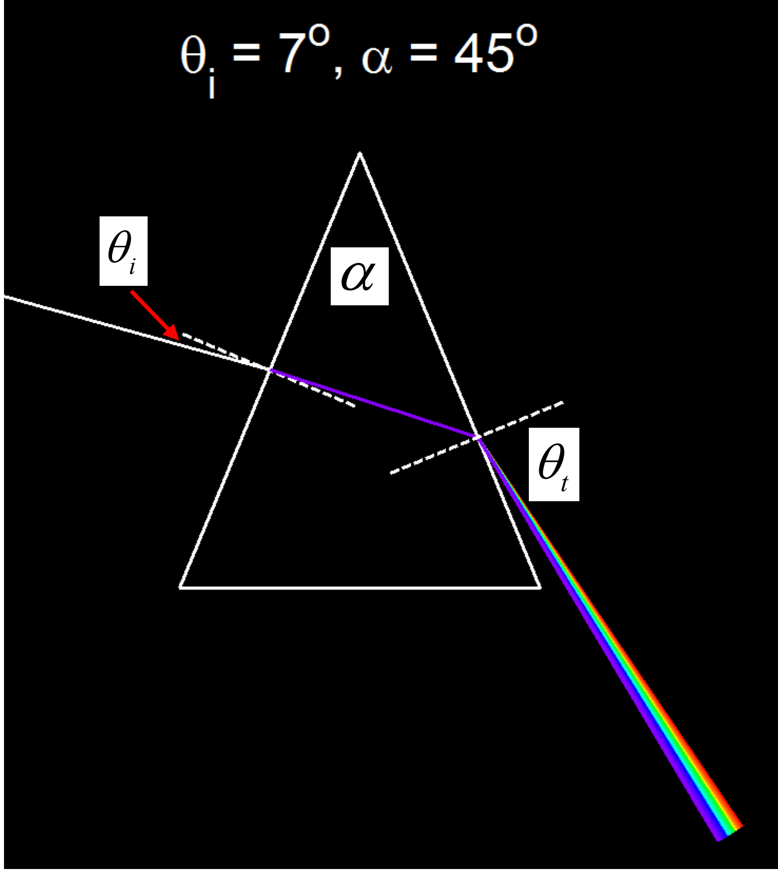
Parallel rays of incident sunlight
Sun
'Anti solar direction'
ε
r
α
$r\sin(\varepsilon-\alpha)$
$\varepsilon-\alpha$
α
α
h
α
Elevation of Sun above horizon
$r\sin\varepsilon\cos\alpha - r\sin(\varepsilon-\alpha) - h$
$r\sin\varepsilon\cos\alpha - r\sin(\varepsilon-\alpha) - h$
β β
$r\sin\varepsilon\cos\alpha$



Challenge #11d: Create a model of primary and secondary rainbows that you would see at sea level (with no topographic obstructions) for different angles of (anti) solar elevation.

Challenge #12a: Create a dynamic model of the path of a beam of white light through a triangular prism





$$\sin\theta_t = \sqrt{n^2 - \sin^2\theta_i}\,\sin\alpha - \sin\theta_i\cos\alpha$$

$$\delta = \theta_i + \theta_t - \alpha$$

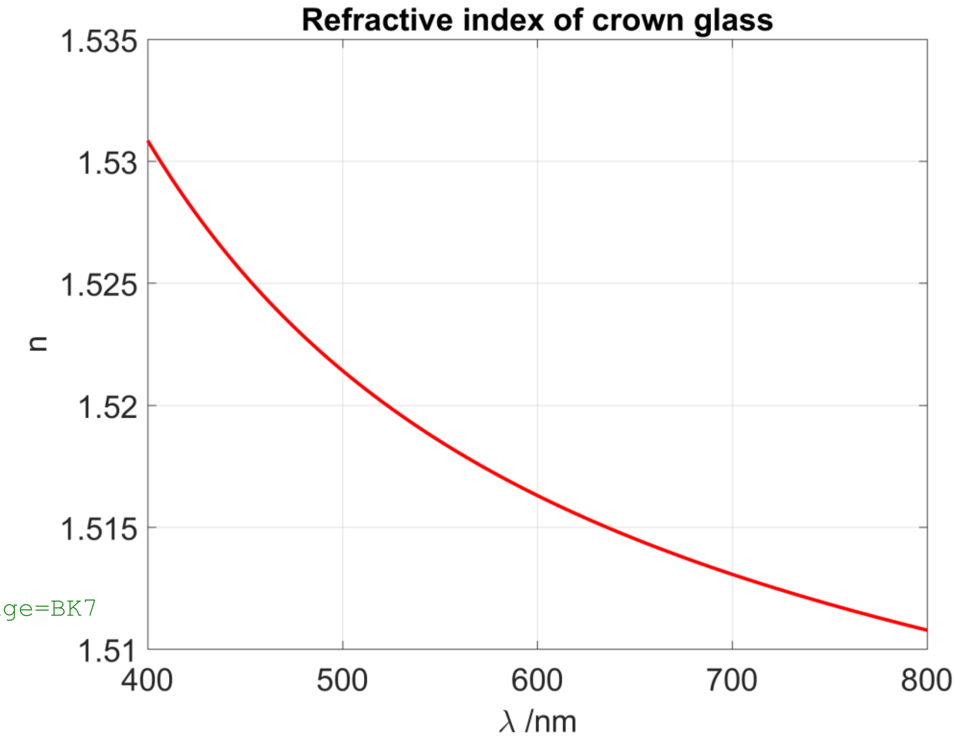
Deflection of (white) light ray

c is the **speed of light in a vacuum.**

$$\boxed{c = 3.00 \times 10^{8}\,\mathrm{ms}^{-1}}$$

n is the **refractive index** of a medium, such that the **wave speed** is

$$\boxed{c_{\mathrm{medium}} = \frac{c}{n}}$$




```
%Dispersion formula for BK7 Crown Glass
% https://refractiveindex.info/?shelf=3d&book=glass&page=BK7
% wavelength lambda is in nm.
% n is the refractive index
function n = crown_glass(lambda)

%Convert to microns
x = lambda/1000;

%Sellmeier coefficients
a = [1.03961212, 0.231792344, 1.01146945];
b = [0.00600069867, 0.0200179144, 103.560653];

%Build up formula for refractive index
y = zeros(size(x));
for n=1:length(a)
    y = y + ( a(n)*x.^2 )./( x.^2 - b(n) );
end
n = sqrt( 1 + y );
```

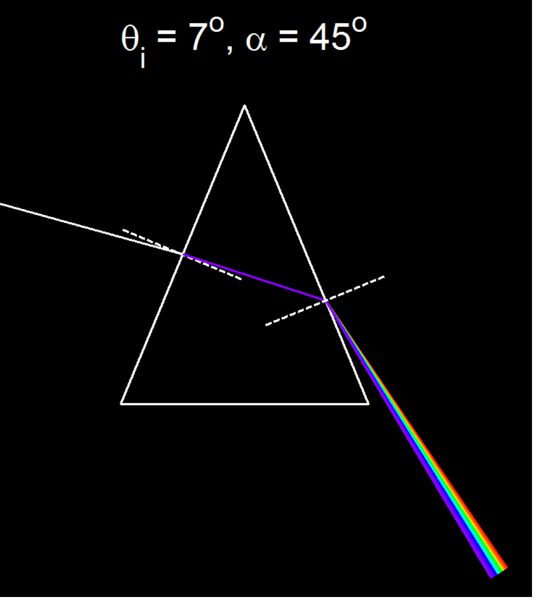
$$\boxed{n = \sqrt{1 + \sum_{k} \frac{a_k \lambda^2}{\lambda^2 - b_k}}}$$

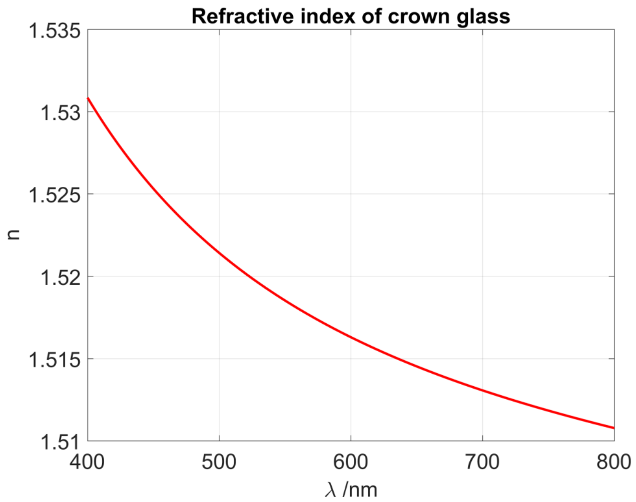
RECAP!

Challenge #1a: Create a model of the refractive index of crown glass

You could do this in a spreadsheet or in a programming language.

You'll need this model in the prism challenge






```
%colours_from_f
% Function which provides the R,G,B values( within interval [0,1] )
% of visible light depending on the frequency /THz
function [R,G,B,colour_str] = colours_from_f(f)
if f < 405
    R = NaN; G = NaN; B = NaN; colour_str = 'Infra Red';
elseif (f>=405) && ( f < 480 )
    R = 1; G = 0; B = 0; colour_str = 'Red';
elseif (f>=480) && ( f < 510 )
    R = 1; G = 127/255; B = 0; colour_str = 'Orange';
elseif (f>=510) && ( f < 530 )
    R = 1; G = 1; B = 0; colour_str = 'Yellow';
elseif (f>=530) && ( f < 600 )
    R = 0; G = 1; B = 0; colour_str = 'Green';
elseif (f>=600) && ( f < 620 )
    R = 0; G = 1; B = 1; colour_str = 'Cyan';
elseif (f>=620) && ( f < 680 )
    R = 0; G = 0; B = 1; colour_str = 'Blue';
elseif (f>=680) && ( f <= 790 )
    R = 127/255; G = 0; B = 1; colour_str = 'Violet';
else
    R = NaN; G = NaN; B = NaN; colour_str = 'Ultra Violet';
end
```

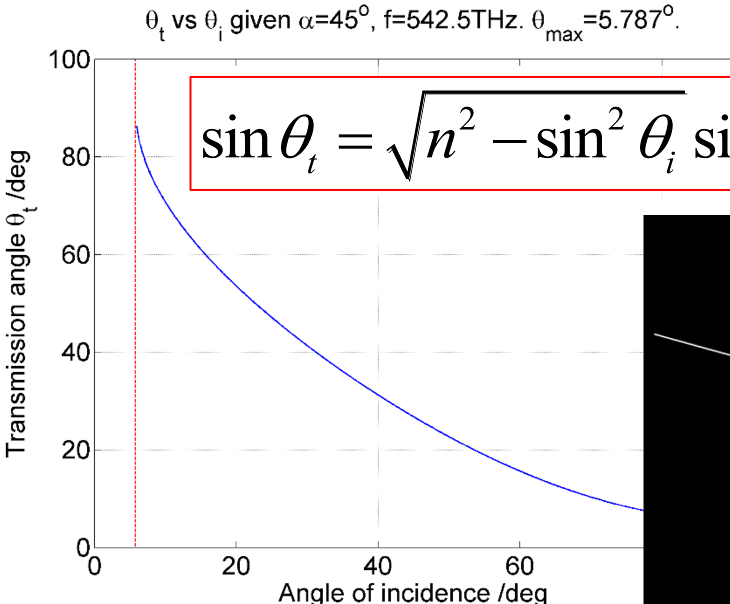
RECAP!

Challenge #1a: Create a model of the refractive index of glass with frequency (and hence wavelength in a vacuum), over the range 405nm to 790nm

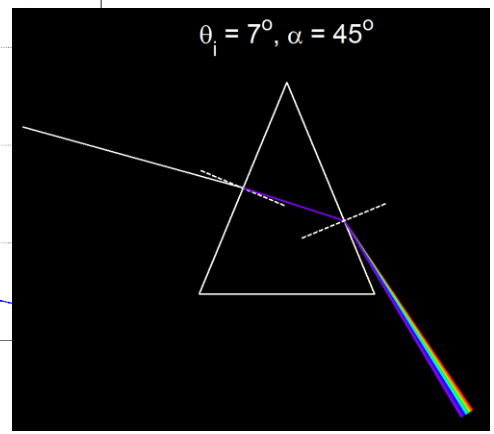
Use the code above to set Red,Green,Blue colours for different frequencies.

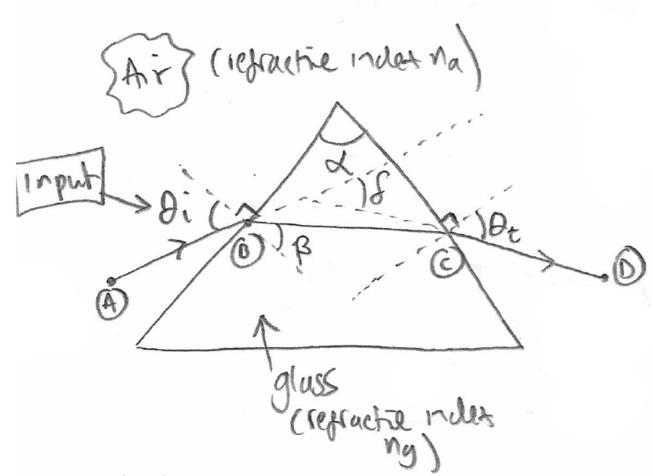
Even better, *interpolate* between the colours to make a smooth colour map

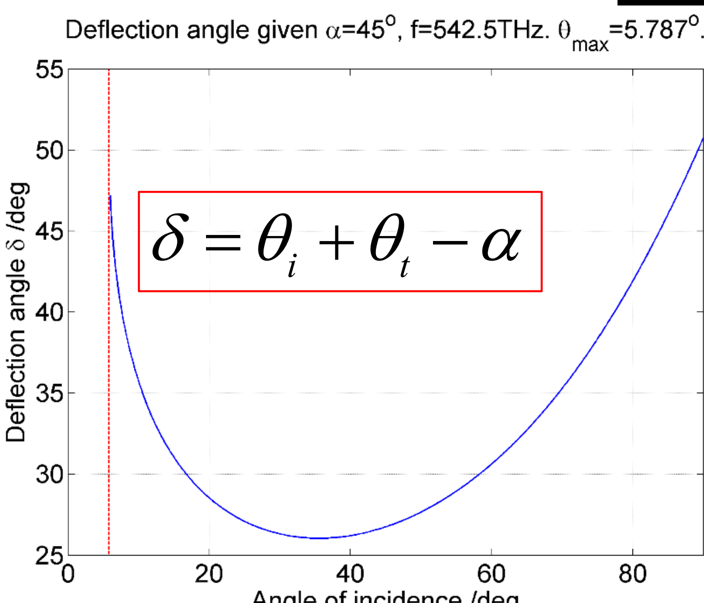
Challenge #12b: See if you can recreate these graphs using your model of glass refractive index and dispersion in a triangular prism



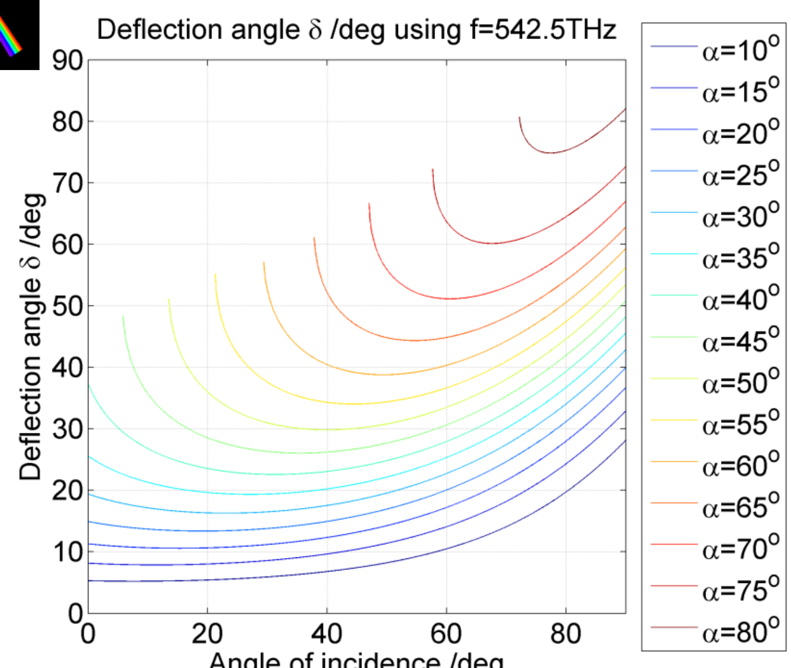
$$\sin\theta_t = \sqrt{n^2 - \sin^2\theta_i}\,\sin\alpha - \sin\theta_i\cos\alpha$$







$$\delta = \theta_i + \theta_t - \alpha$$



Extension opportunities:

• Answer the questions in the **Ray Optics problem sheet** (provided with this document) and write these up in an illustrated paper. Many link to the challenge tasks, so you can re-use your models. A good opportunity to learn LaTeX – which is the typesetting language used to write most technical papers and books in the physical sciences. Including ***Science by Simulation*** *

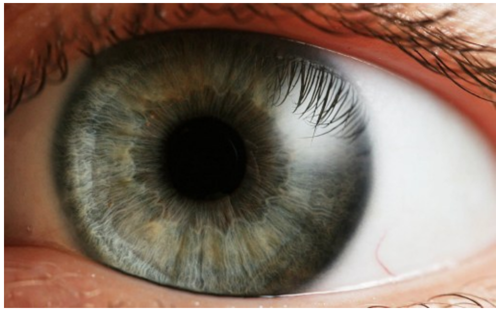
• Write a **graphical user interface** (GUI) for some of the ray optics models (e.g. the thin biconvex lens, the concave spherical mirror, prism dispersion, rainbows) and encode these as an 'app'. Coding up an iOS/Android smartphone app will particularly impress the judges.

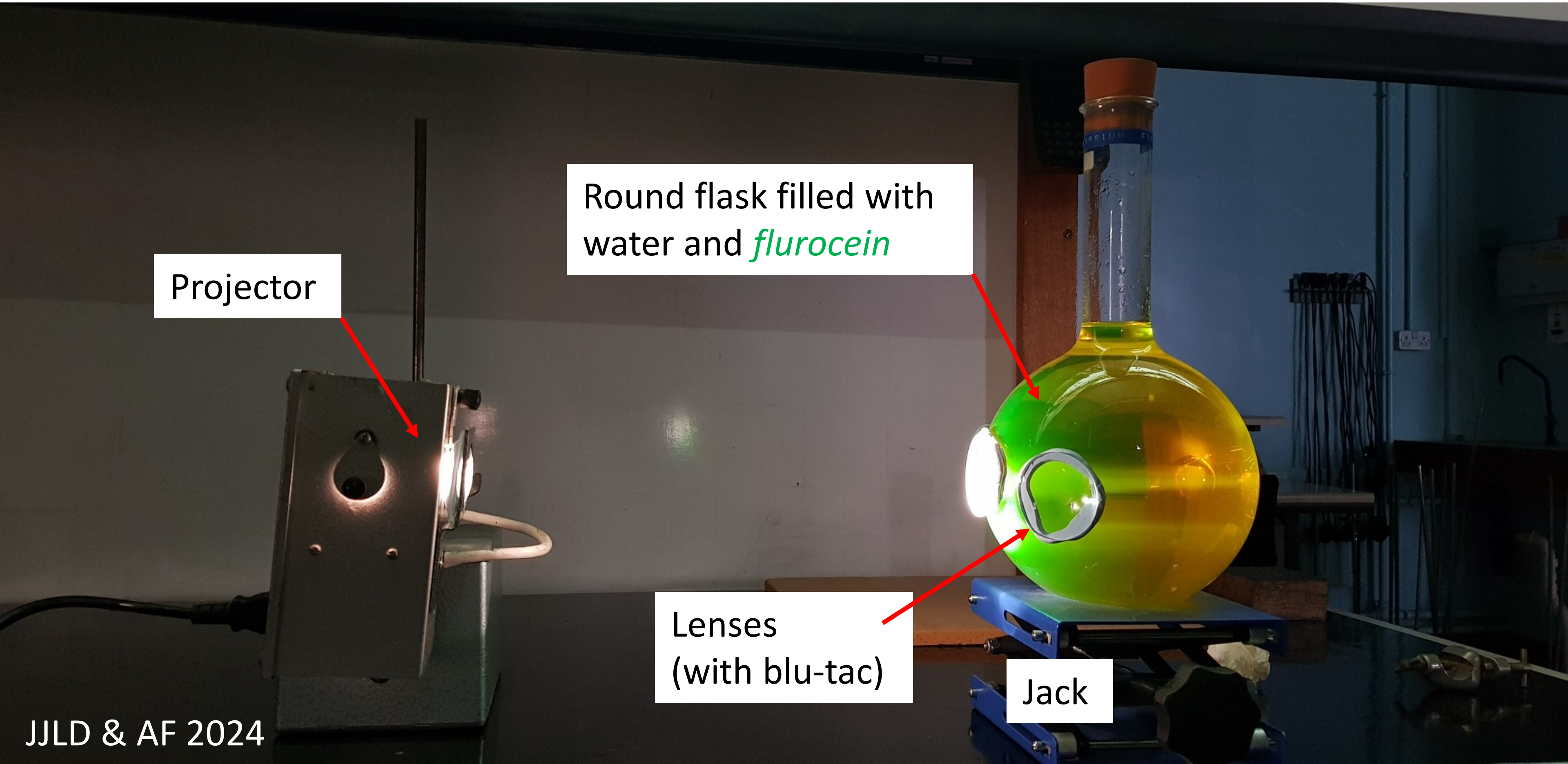
• Create a dynamic model in an app to demonstrate **short and long sight** (and how to correct for it) – see the next two slides for inspiration!

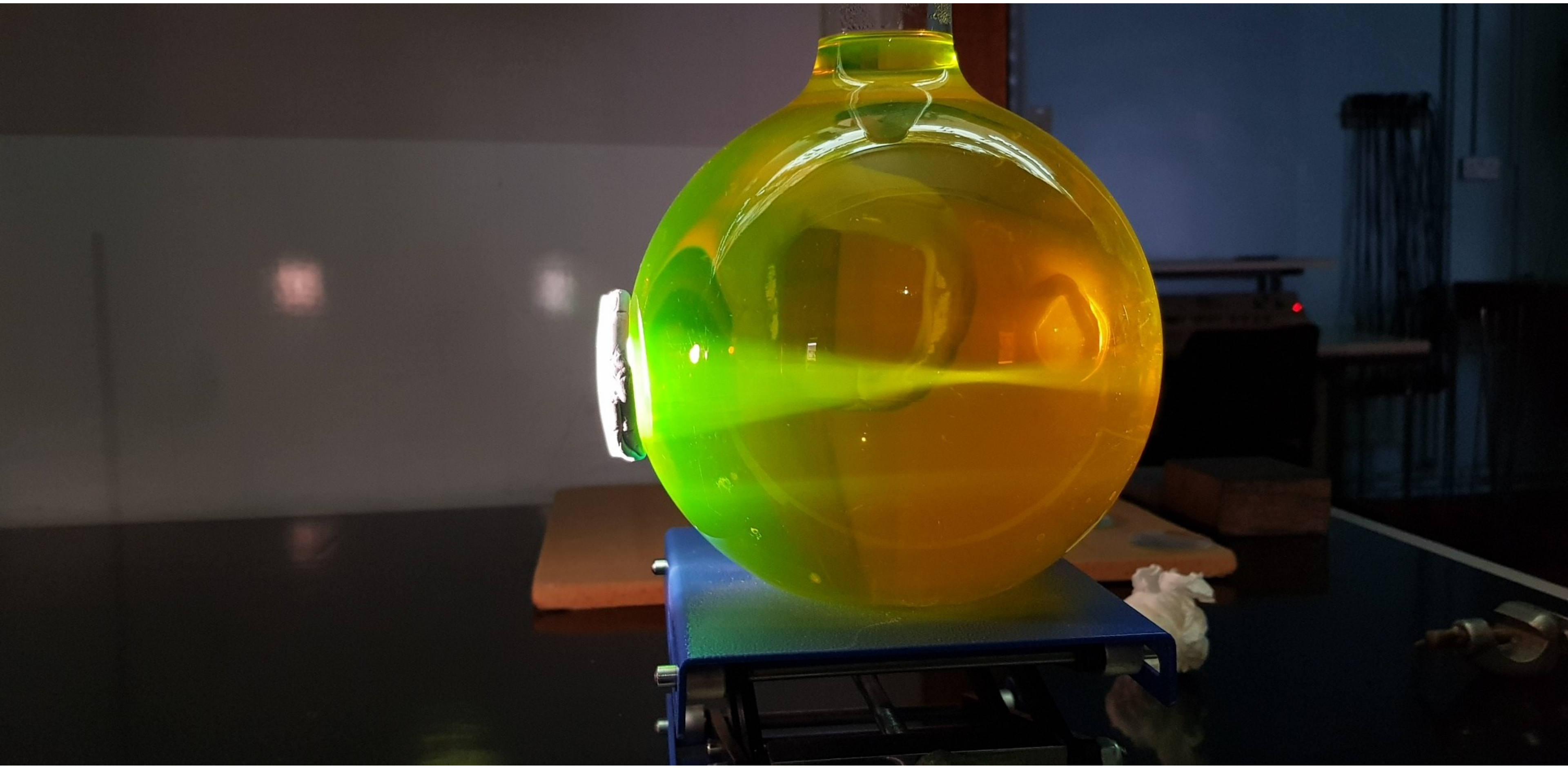
Don't forget to include any extension projects in your video, as this is the only way you will gain credit for your work in the BPhO Computational Challenge.
I'm afraid we cannot accept any other files. **Submit only the YouTube link to your three-minute screencast.**

* *ScibySim* was created in Scientific Word. There are lots of other LaTeX-based tools available. Find one that works for you!

Long and short sight (and corrective lenses) demonstration

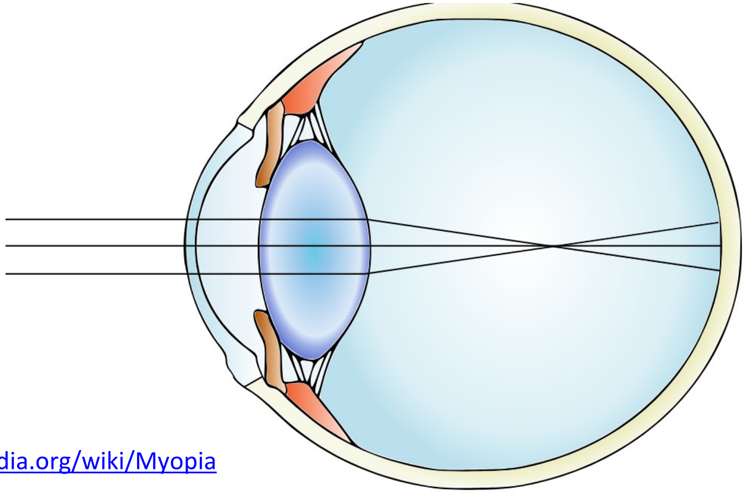




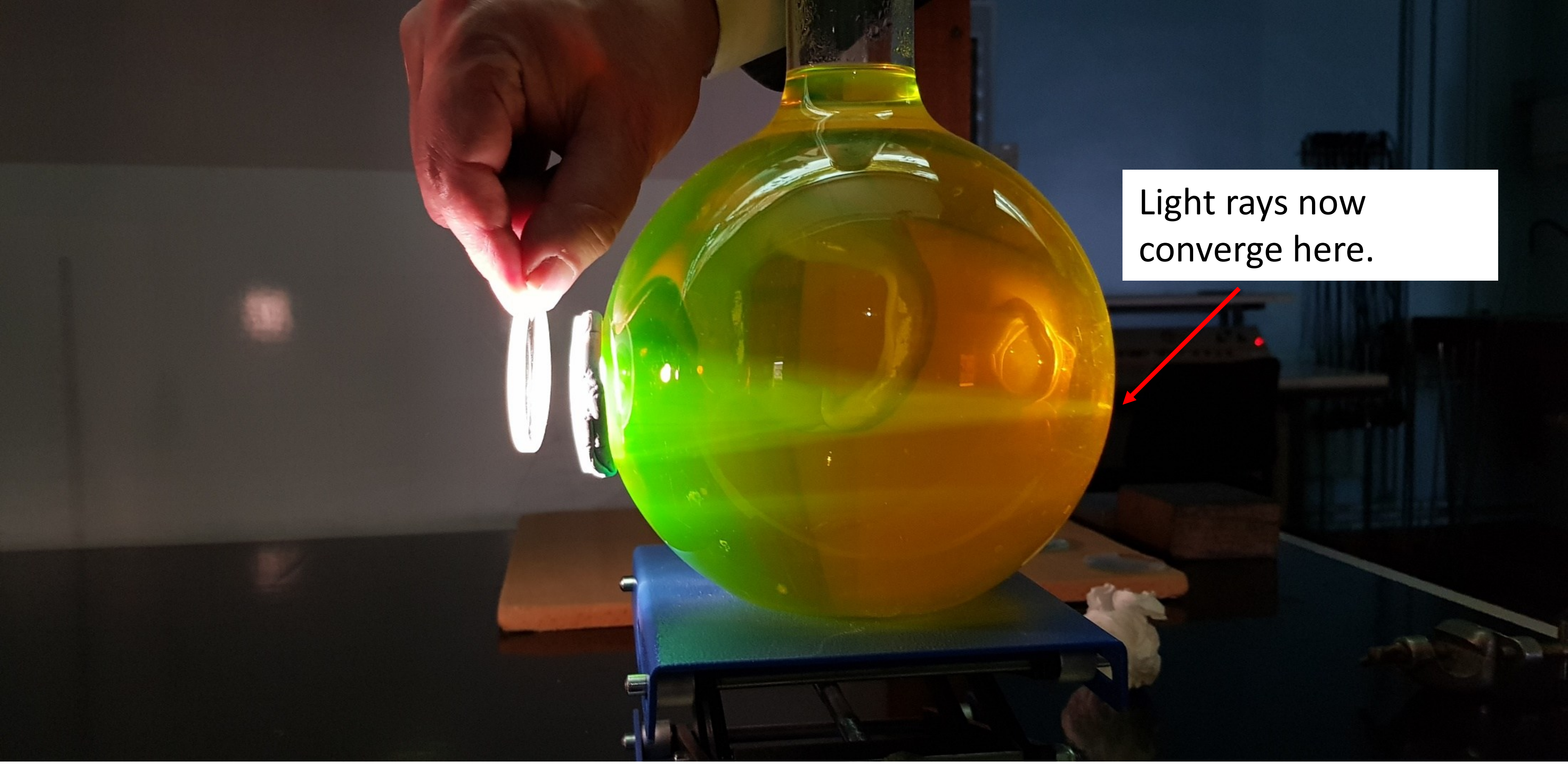


Rotate sphere such that the new lens has a smaller focal length. This simulates **short sight.**

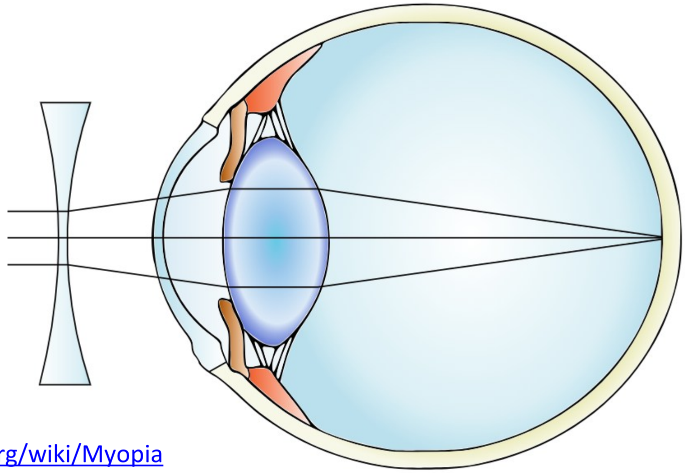
(i.e. too large eyeball, or too powerful lens).



https://en.wikipedia.org/wiki/Myopia



Correct **short sight** with a *concave* lens (i.e. a pair of glasses or contact lens for a real eye).



https://en.wikipedia.org/wiki/Myopia