

- (a) The circuit in Figure 1.(a) contains a cell of emf E , a known variable resistance R_0 , an unknown resistance R and an ammeter. When X and Y are short circuited $E = I_0 R_0$.
When R is inserted the current is αI_0 , where α is a constant.

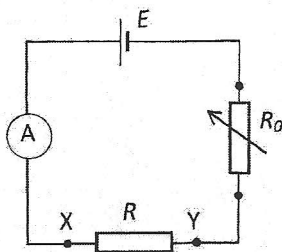


Figure 1.(a)

- (i) Express R in terms of R_0 and α , giving the range of validity of R and α .
(ii) In order to extend the range of α , modify the circuit by putting R in parallel with R_0 . Determine the ranges of R and α for the modified circuit.

[4]

(i) Kirchoff II (or simply Ohm's law for entire circuit)

$$E = I(R + R_0)$$

Now: $E = I_0 R_0$
 $I = \alpha I_0$

} This is given in the text.

So $I_0 R_0 = \alpha I_0 (R + R_0)$

$\therefore R + R_0 = \frac{R_0}{\alpha}$

$$\therefore R = R_0 \left(\frac{1}{\alpha} - 1 \right)$$

[MS gives this as $R = R_0 \left(\frac{1 - \alpha}{\alpha} \right)$ which is the same expression]

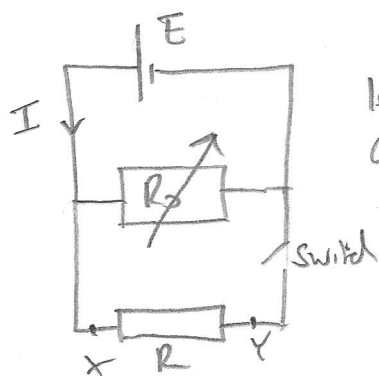
Now $R \geq 0$ so $\alpha \leq 1$ (if $\alpha > 1$ then $R < 0$)

$I \geq 0$ so $\alpha \geq 0$

$$\therefore 0 \leq \alpha \leq 1$$

So using $R = R_0 \left(\frac{1}{\alpha} - 1 \right) \Rightarrow 0 \leq R \leq \infty$
 $\alpha = 1$ at $R = 0$, $\alpha = 0$ at $R = \infty$

(ii)



let $I = \alpha I_0$ as before, and with $x \rightarrow Y$ cut (via a switch) $E = I_0 R_0$.

Total resistance is $R_{tot} = \left(\frac{1}{R_0} + \frac{1}{R} \right)^{-1}$

and $E = I R_{tot}$

$$\underbrace{I_0 R_0}_E = \underbrace{\alpha I_0}_I \times \left(\frac{1}{R_0} + \frac{1}{R} \right)^{-1}$$

$$\frac{1}{R_0} + \frac{1}{R} = \frac{\alpha}{R_0}$$

$$\therefore \frac{1}{R} = \frac{1}{R_0} (\alpha - 1)$$

$$\therefore R = \frac{R_0}{\alpha - 1}$$

Now

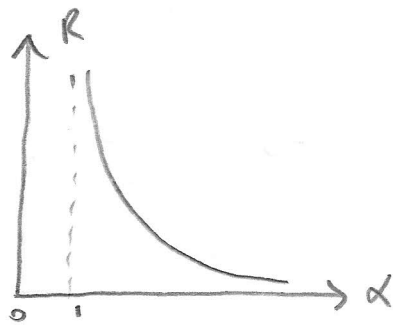
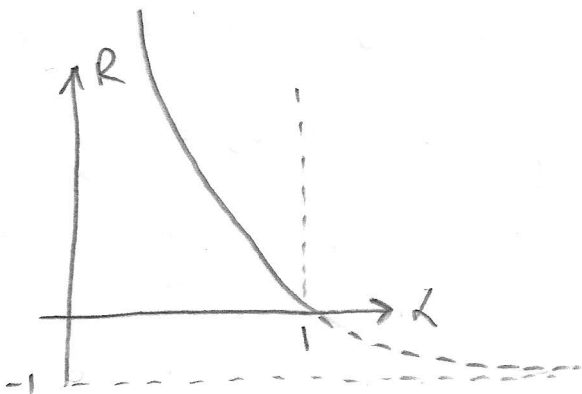
$$0 \leq R \leq \infty$$

\uparrow $\alpha = 1$ \uparrow $\alpha = \infty$

and

$$1 \leq \alpha \leq \infty$$

(i)



$$R = R_0 \left(\frac{1}{\alpha} - 1 \right)$$

$$R = \frac{R_0}{\alpha - 1}$$

