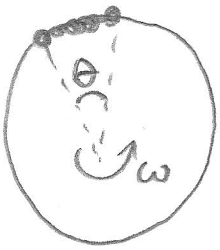
BPhO 2015-2016 Section 1 Paper 1 #

(a) A measurement is carried out to check the speed of a camera shutter of 1/15 s. The camera is focused symmetrically on a rotating turntable which revolves at 33.3 ± 0.1 revolutions per minute and has a spot at its centre and at its circumference. A photograph shows the arc produced by the spot on the circumference subtends an angle of 12.4 ± 0.1° at the centre of rotation. What is the correct exposure time? ω

θ

[3]



θ corresponds to shutter opening time τ

Since speed of rotation ω is constant, spot arc angle $\theta = \omega\tau$

$$\therefore \quad \tau = \frac{\theta}{\omega}$$

$$\tau = \frac{(12.4 \pm 0.1) \text{ degrees}}{(33.3 \pm 0.1) \times \frac{360 \text{ degrees}}{60 \text{ s}}} \quad (s)$$

$$\therefore \quad \frac{12.3 \times 60}{33.4 \times 360} < \tau < \frac{12.5 \times 60}{33.2 \times 360} \quad (s)$$

$$\therefore \quad 6.14 \times 10^{-2} < \tau < 6.28 \times 10^{-2} \quad (s)$$

$$\boxed{\frac{1}{16.3} < \tau < \frac{1}{15.9}}$$

or $$\boxed{\tau = (6.21 \pm 0.07) \times 10^{-2} \text{ s}}$$

(b) The temperature coefficients of resistance, α, of certain alloys are positive and others are negative. They have resistance per unit length of r. This makes it possible to produce a resistor, using the two wires in series, which does not vary with temperature. The values of r, at 0 °C, and α are given in Table 1.b for constantan and manganin. These wire have lengths L_c and L_m respectively at 0 °C. What values of L_c and L_m are required to produce a 5.0 Ω resistor?

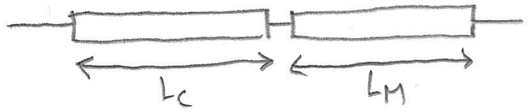
r_0

Wire	r /Ωm^{-1}	α/ °C^{-1}
Constantan	6.3	-3.0 x 10^{-5}
Manganin	5.3	+1.4 x 10^{-5}

Table 1.b

$r(T) = r_0(1+\alpha)$

[5]



So $5.0 = 6.3 L_c + 5.3 L_M$ ① (Resistance equation)

This is true at 0°C

Now let $\boxed{r = r_0(1+\alpha T)}$ where T is temperature in degrees Celsius.

So for total resistance to be 5.0 Ω at $T \neq 0$

$\Rightarrow 5.0 = 6.3(1 - 3.0\times10^{-5}T)L_c + 5.3(1 + 1.4\times10^{-5}T)L_M$ ②

① − ②: $0 = 6.3\times3.0\times10^{-5}T L_c - 5.3\times1.4\times10^{-5}T L_M$

$\therefore L_c = \dfrac{5.3\times1.4}{6.3\times3.0} L_M \Rightarrow \boxed{L_c = 0.393\, L_m}$

Calc memory = A

In ①: $5.0 = 6.3\times A\, L_M + 5.3 L_M$

$\therefore L_M = \dfrac{5.0}{6.3A + 5.3} = \boxed{0.64\text{ m}}$ $\therefore \boxed{L_c = 0.25\text{M}}$

$h = 6.626 \times 10^{-34}$ Js Planck's constant

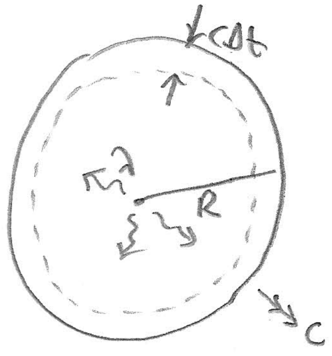
P

(c) A monochromatic sodium lamp, wavelength $\lambda = 6.0 \times 10^{-7}$ m, radiates 100 W of radiation uniformly in all directions. At what distance from the lamp will the photons have an average density of $10^6\ m^{-3}$? **[5]**

Energy/s

- # photons emmited /s is $\dfrac{P}{hc/\lambda}$ ← photon energy
- Photons spread out at $c = 2.998 \times 10^8\ ms^{-1}$ from lamp

Consider a shell of thickness $c\Delta t$ at distance R from lamp; $c\Delta t \ll R$



Volume of shell is $\Delta V \cong 4\pi R^2 c\Delta t$

Now $\dfrac{P\Delta t\lambda}{hc}$ is # photons emmitted in Δt (i.e. # in the shell)

$\therefore$ # photons / unit volume in the shell (ρ) is

$$\rho = \frac{\frac{P\Delta t\lambda}{hc}}{4\pi R^2 c\Delta t} = \boxed{\frac{P\lambda}{4\pi h c^2 R^2}}$$

{Δt cancels!}

So $\boxed{R = \sqrt{\dfrac{P\lambda}{4\pi h c^2 \rho}}}$

So if $\rho = 10^6$ photons/m³ $\Rightarrow$ $R = \sqrt{\dfrac{100 \times 6.0 \times 10^{-7}}{4\pi \times 6.626 \times 10^{-34} \times (2.998 \times 10^8)^2 \times 10^6}}$

$\boxed{R = 283\text{m}}$

charge $e = 1.6 \times 10^{-19}\,C$ V

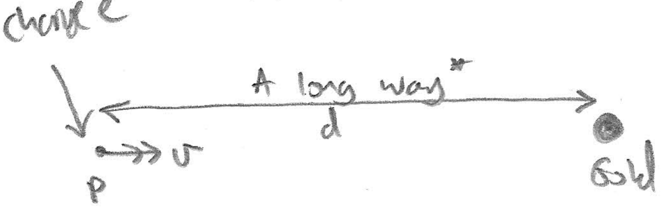
(d) Protons are accelerated from rest through a p.d. of 2.0×10^6 V and fired at a gold ($^{197}_{79}Au$) foil. What is the distance of closest approach of a proton to the gold nucleus? r

[4]

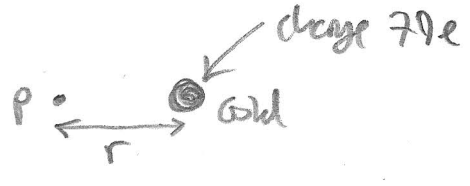
which repels the proton

- Proton energy is eV (at release)
- Work done against electric field of gold nucleus is:

$$\frac{79e^2}{4\pi\varepsilon_0 r}$$ ie at this point the proton has zero KE.



AFTER ACCELERATION



AT CLOSEST APPROACH

$\frac{1}{2}M_p v^2 = eV$

Assume classical physics

[$(\gamma-1)M_p c^2$ in relativistic physics $\gamma = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$]

so $eV = \frac{79e^2}{4\pi\varepsilon_0 r}$

$$\therefore \boxed{r = \frac{79e}{4\pi\varepsilon_0 V}}$$

$$r = \frac{79 \times 1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times 2.0 \times 10^6}$$

$$\boxed{r = 5.7 \times 10^{-14}\,m}$$

* $d \gg r$

{ Atomic nuclei radii are often quoted at about 10^{-15} m, so consistent with this very small value }

(e) Figure 1.e is a section through a smooth parabolic metal bowl, which can be rotated about its vertical axis of symmetry, the y - axis. Its equation, in Cartesian coordinates, is $y = ax^2$. The gradient at the point (x, y) is $2ax$.

There is one angular speed of rotation, ω, of the bowl about the y - axis for which a small metal sphere remains at rest relative to the rotating bowl, wherever it is placed on the inner surface. Determine ω in terms of a and g.

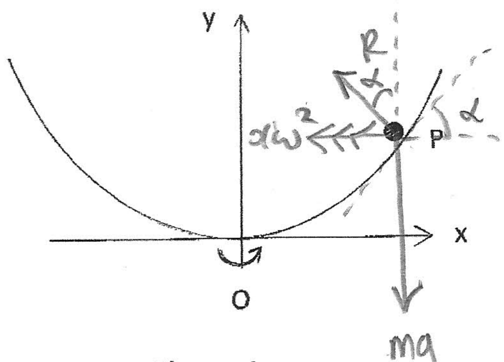


Figure 1.e

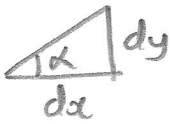
$\downarrow g$

* R is normal contact force with bowl.

* Assume no friction.

[4]

Now gradient at P = $\tan\alpha$



If $y = ax^2$, P has coordinates (x, ax^2) and gradient is $\boxed{2ax = \tan\alpha}$ ①

Now by Newton II:

$//x$: $\underbrace{mx\omega^2}_{}$ $= R\sin\alpha$ ②

mass x acceleration radially inwards

$//y$: $0 = R\cos\alpha - mg$

$\therefore mg = R\cos\alpha$ ③

②/③ : $\dfrac{mx\omega^2}{mg} = \tan\alpha$

$\therefore \dfrac{x\omega^2}{g} = 2ax$

$\therefore \boxed{\omega^2 = 2ag}$

Now $\tan\alpha = 2ax$

So if ω = 60 RPM

$= \dfrac{60 \times 2\pi \text{ rad}}{60 s} = 2\pi \text{ rad}s^{-1}$

$a = \dfrac{\omega^2}{2g} = \dfrac{4\pi^2}{2 \times 9.81}$

$= \boxed{2.012}$

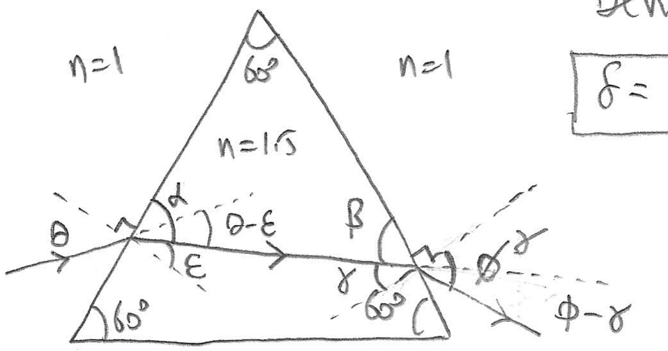
ie $y \approx 2x^2$

(f) A ray of light is incident on a 60°glass prism of refractive index 1.500 at an angle of incidence of 48.59°. Determine;

(i) the angle of emergence, ϕ, from the prism; i.e. the angle between the emergent ray and the normal to the prism face.

(ii) the angle of deviation of the ray, δ.

[5]



Deviation of ray is:

$$\boxed{\delta = \theta - \varepsilon + \phi - \gamma} \quad (1)$$

Snell's law:

$$1\sin\theta = n\sin\varepsilon \quad (2)$$

$$n\sin\gamma = 1\sin\phi \quad (3)$$

So $\varepsilon = \sin^{-1}\left(\frac{\sin 48.59°}{1.5}\right) = \boxed{30.0°}$

So $\alpha = 90° - \varepsilon = 60°$ * $\therefore \beta = 60°$ $\therefore \boxed{\gamma = 30°}$

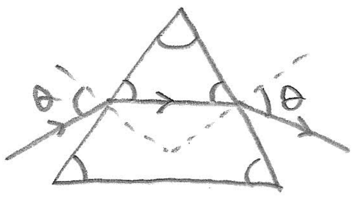
$\therefore \boxed{\phi = 48.59°}$ (i.e. in this case, same as θ)

$\therefore$ Deviation angle $\delta = 48.59° - 30.0° + 48.59° - 30°$

$\delta = 2\theta - 60°$

$\boxed{\delta = 37.18°}$

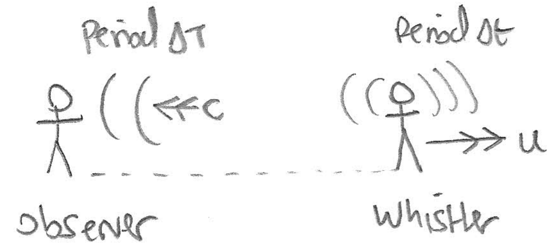
* So actually ray diagram should look like:



(g) A man blowing a whistle of frequency f moves away from a stationary observer at speed u. Derive the formula for the frequency f_1 heard by the observer. The velocity of sound is c.

If a man blowing a whistle of frequency 500 Hz moves away from a stationary observer towards a fixed wall, in a direction perpendicular to the wall at 2.00 m s^{-1}, determine the beat frequency heard by the observer if c = 330 m s^{-1}.

[6]



Time between wave crests heard by observer:

$$\Delta T = \underbrace{\Delta t}_{\text{time at source}} + \frac{u\Delta t}{c}$$

$u\Delta t$ ← Distance travelled by source in Δt

c ← wave speed

ie $\frac{u\Delta t}{c}$ is the extra time delay corresponding to source movement.

(Note assume classical physics. In Special Relativity, Δt is the time as measured in the observer frame. The wave period in the whistler frame is $\Delta t' = \Delta t/\gamma$ where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$)

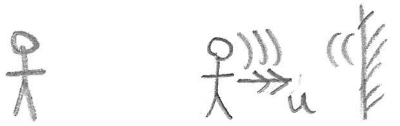
$\Delta t'$ ← ie this is the $\frac{1}{500}$ s

So $\Delta T = \Delta t\left(1 + \frac{u}{c}\right)$

$\therefore\ f_1 = \frac{1}{\Delta T} = f\left(1 + \frac{u}{c}\right)^{-1}$

$\therefore\ \boxed{f_1 = \frac{fc}{c+u}}$

Now if whistler moves towards a wall, which reflects the sound



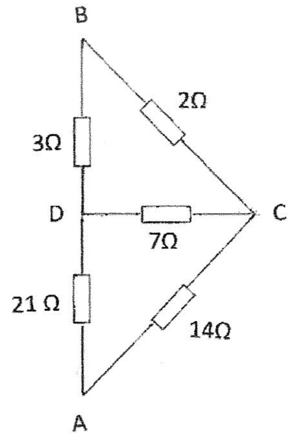
The reflected sound will have frequency $f_2 = \frac{fc}{c-u}$ at observer. $\therefore$ Beat frequency between whistle and reflection is $\Delta f = f_2 - f_1 = fc\left(\frac{1}{c-u} - \frac{1}{c+u}\right) = fc\left(\frac{c+u-c+u}{c^2-u^2}\right)$

$\Rightarrow \boxed{\Delta f = \frac{2fuc}{c^2-u^2}} = \frac{2 \times 500 \times 2.00 \times 330}{330^2 - 2.00^2} = \boxed{6.06\ \text{Hz}}$

1 2

(h) Determine, in Figure 1.h, the total resistances, R_{TBC}, across BC, R_{TBD} across BD and R_{TBA} across AB.

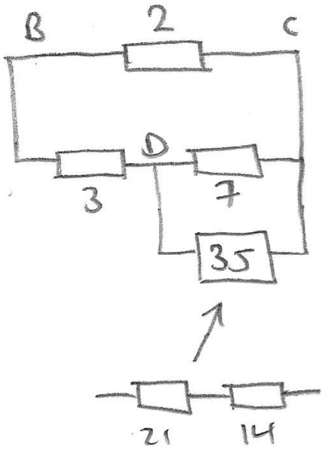
3



Equivalent diagram: 1

Figure 1.h

[6]

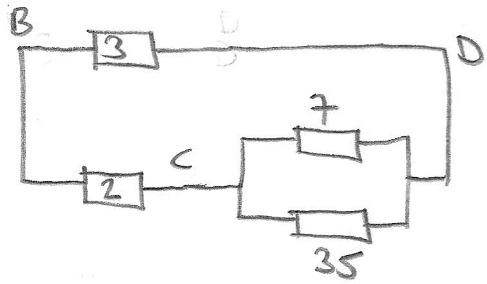


So $$R_{TBC} = \cfrac{1}{\frac{1}{2} + \cfrac{1}{3 + \cfrac{1}{\frac{1}{7}+\frac{1}{35}}}}$$

$$= \cfrac{1}{\frac{1}{2} + \cfrac{1}{3 + \frac{35}{6}}} = 1\tfrac{41}{65}$$

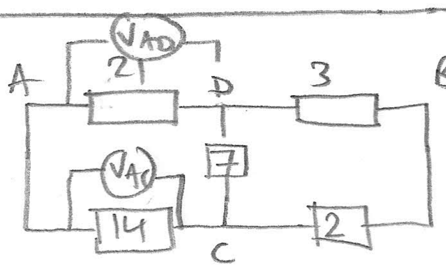
$$\simeq \boxed{1.63\,\Omega}$$

2



$$R_{TBD} = \cfrac{1}{\frac{1}{3} + \cfrac{1}{2 + \cfrac{1}{\frac{1}{7}+\frac{1}{35}}}} = 2\tfrac{11}{65}\,\Omega$$

$$\simeq \boxed{2.17\,\Omega}$$



3

If we temporarily sever DC:

$V_{AD} = \frac{21}{24} V_{AB} = \frac{7}{8} V_{AB}$

$V_{AC} = \frac{14}{16} V_{AB} = \frac{7}{8} V_{AB}$

$\therefore \boxed{V_{DC} = 0}$ so $\boxed{7\,\Omega}$ between DC is irrelevant! $\therefore R_{TBA} = \cfrac{1}{\frac{1}{24} + \frac{1}{16}}$

$= 9\frac{3}{5}\,\Omega = \boxed{9.60\,\Omega}$

m

(i) What mass of radium, mass number 226, half-life of 1620 years, and an α emitter, is required to produce an average of 10 α particles per second?

A

[4]

Number of radioactive particles left after t years is $N = N_0 e^{-\lambda t}$ $\therefore$ Decay rate is $\boxed{\frac{dN}{dt} = -\lambda N}$

So activity $= \lambda N$

Now $N = N_0 \times 2^{-t/t_{1/2}}$ (by definition of half life)

so $2^{-t/t_{1/2}} = e^{-\lambda t}$

$\therefore -\frac{t}{t_{1/2}} \ln 2 = -\lambda t$

So $\boxed{\lambda = \frac{\ln 2}{t_{1/2}}}$

$\therefore 10 = A = \lambda N = \frac{\ln 2}{t_{1/2}} N$ $\therefore$ $\boxed{N = \frac{A t_{1/2}}{\ln 2}}$

Now if radium has mass M, $N = \frac{M}{226u}$

$\therefore$ $\boxed{M = \frac{226 u A t_{1/2}}{\ln 2}} = \frac{226 \times 1.660 \times 10^{-27} \times 10 \times 1620 \times *}{\ln 2}$

$* = 365 \times 24 \times 3600 s$

$= \boxed{2.77 \times 10^{-13} \text{ kg}}$

(j) An a.c. voltmeter displays the rms value of the voltage, V, for a.c. signals and also for periodic signals that are not sinusoidal. What reading will it display if connected to a periodic voltage, period $4T$, that changes instantaneously from +10 V to -2 V to +4V repeatedly, the voltages lasting, respectively, for T, T and $2T$?

Determine the following;

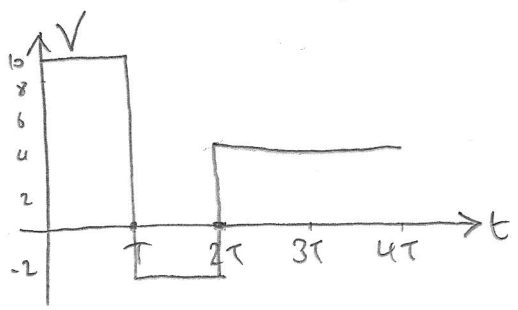
(i) the mean voltage i.e. V_m.

(ii) the rms voltage i.e. V_{rms}.

(iii) the rms value of deviation from the mean, $(V - V_m)$ which is V_{rmsm}.

[4]

plot one period g the signal:



(i) Mean Voltage $V_M = \dfrac{\int_0^{4T} V\,dt}{4T}$

$\therefore\ V_M = \dfrac{10T - 2T + 8T}{4T} = \dfrac{16}{4} = \boxed{4 \text{ Volts}}$

(ii) $V_{rms} = \sqrt{\dfrac{\int_0^{4T} V^2\,dt}{4T}} = \sqrt{\dfrac{10^2 T + 2^2 T + 32T}{4T}}$

$= \sqrt{34} \text{ Volts} \approx \boxed{5.83 \text{ Volts}}$

(iii) $V_{rmsm} = \sqrt{\dfrac{\int_0^{4T} (V - V_m)^2\,dt}{4T}} = \sqrt{\dfrac{(10-4)^2}{4} + \dfrac{(-2-4)^2}{4} + \dfrac{(4-4)^2 \times 2}{4}}$

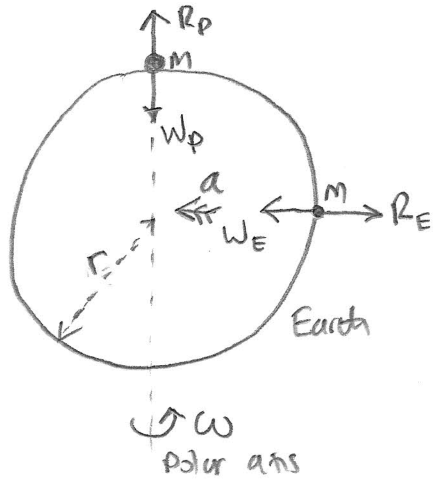
$= \dfrac{1}{2}\sqrt{36 + 36} = \dfrac{1}{2}\sqrt{72}$

$= \boxed{4.24 \text{ Volts}}$

(k) Assuming the Earth is a homogeneous sphere, calculate the fractional difference Δ between the acceleration due to free fall at the Earth's equator and at the poles, indicating which is the greater. You may not use a value of g given in the constants table as that is an average value, neither correct at the poles nor at the equator.

Mass of the $M_E = 5.98 \times 10^{24}$ kg Radius of the Earth $\cancel{R_E}$ r $= 6.38 \times 10^6$ m

[8]



Newton II at pole:

$$0 = R_p - W_p \quad (1)$$

Newton II at equator:

$$M r\omega^2 = W_E - R_E \quad (2)$$

R is normal contact force on mass at rest relative to Earth surface.

$\omega = \dfrac{2\pi}{T}$

where $T = 24 \times 3600$ s

Now define effective strength of gravity (that we associate with weight / unit mass)

$$M g_p = R_p \quad (3) \qquad M g_E = R_E \quad (4)$$

And by Newton's law of gravitation $W_p = W_E = \dfrac{G M_E M}{r^2}$ (5)

{slightly different if Earth is not a homogeneous sphere!}

$\therefore$ $\boxed{g_p = \dfrac{G M_E}{r^2}}$ $\boxed{g_E = \dfrac{G M_E}{r^2} - r\omega^2}$ So clearly $\boxed{g_p > g_E}$

$$\therefore \Delta = \frac{g_p - g_E}{g_p} = 1 - \left(1 - \frac{r\omega^2}{GM_E/r^2}\right)$$

$$\boxed{\Delta = \frac{r^3\omega^2}{G M_E}}$$

ie $1 - \dfrac{g_E}{g_p} = \Delta$

$$\therefore \Delta = \frac{(6.38 \times 10^6)^3 \times \left(\dfrac{2\pi}{24 \times 3600}\right)^2}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}} = 3.44 \times 10^{-3}$$

$$= \boxed{0.34\%}$$

(I) A horizontal square wire loop of side 4.00 cm has a resistance, R, of $2.00 \times 10^{-3}\ \Omega$. The loop is situated in a vertical downward magnetic field of 0.700 T. When the field is switched off, it decreases to zero, at a uniform rate, in 0.800 s.

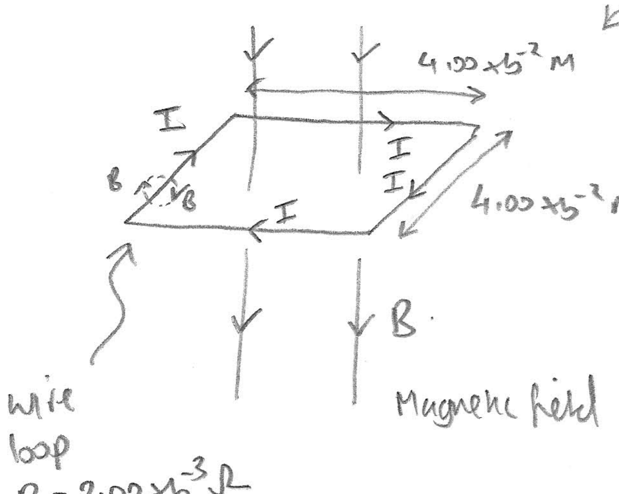
Determine:

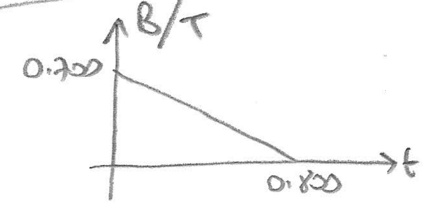
(i) the induced current, I, and its direction in the loop.

(ii) the energy dissipated in the loop.

So area flux linked is $A = (4.00 \times 10^{-2})^2\ m^2 = \boxed{1.6 \times 10^{-3}\ m^2}$

[7]





Faraday's law of EM induction

$$\boxed{\left|\underbrace{\frac{d}{dt}(BA)}_{\text{Rate of change of flux}}\right| = \underbrace{IR}_{\text{EMF induced}}}$$ **

Note by Lenz's law, current induced must produce an opposing B field* ⇒ I is clockwise

(i)

* change - and note B is reducing

Now $\frac{d}{dt}(BA) = 1.6 \times 10^{-3} \times \frac{0.700}{0.800}$, so $I = \frac{1.6 \times 10^{-3} \times 0.700}{0.800 \times 2.00 \times 10^{-3}}$

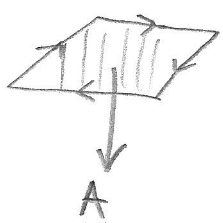
$= \boxed{0.700 \text{ Amps}}$

(ii) Hence energy dissipated is $I^2R \times 0.800$
(Since I = constant, we don't need to write $\int_0^{0.800} I^2 R\,dt$)

$= 0.700^2 \times 2.00 \times 10^{-3} \times 0.800$

$= \boxed{7.84 \times 10^{-4}\ J}$

** $-\frac{d}{dt}(BA) = IR$ is better. And BA is really $\underline{B}.\underline{A}$ and if $\boxed{\text{vector area}}$ points down, loop is clockwise.

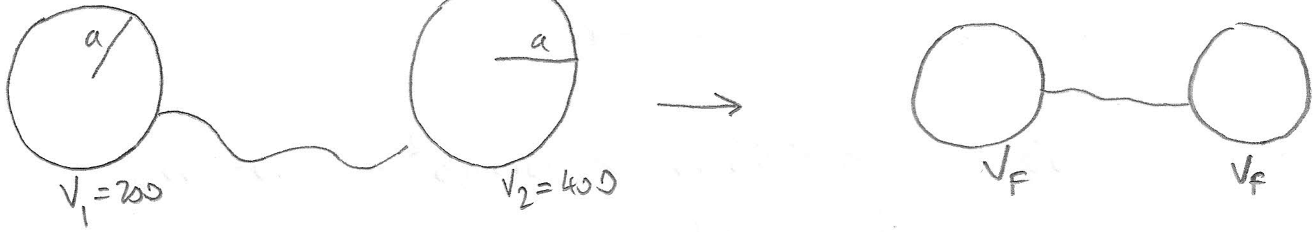


$Q = CV \qquad E = \frac{1}{2}CV^2$

a

(m) Two well separated identical conducting spheres of radius 10.0 cm are charged to +200 V and +400 V. If they are joined by a long wire, how much heat is generated?

[10]



Initial energy stored on spheres is $\boxed{\frac{1}{2}C(V_1^2 + V_2^2)}$

$C = 4\pi\varepsilon_0 a$ $\qquad \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$

$\uparrow$ $\qquad$ Permittivity of free space

Capacitance of a uniform conductive sphere

Now when wire joins, charge flows until voltage is the same.

$\therefore$ Final energy is $\boxed{\frac{1}{2}C(V_F^2 + V_F^2) = CV_F^2}$

$\therefore$ Energy dissipated ΔE $= \boxed{\frac{1}{2}C(V_1^2 + V_2^2) - CV_F^2}$

Now by conservation of charge:

Intuitive

$\boxed{CV_1 + CV_2 = 2CV_F}$ $\quad \therefore \quad$ $\boxed{V_F = \frac{1}{2}(V_1 + V_2)}$

$$\text{So} \quad \Delta E = C\left[\frac{V_1^2 + V_2^2}{2} - \frac{1}{4}(V_1 + V_2)^2\right]$$

$$= \frac{1}{4}C\left[2V_1^2 + 2V_2^2 - V_1^2 - V_2^2 - 2V_1V_2\right]$$

$$= \frac{1}{4}C\left[V_1^2 + V_2^2 - 2V_1V_2\right] \Rightarrow \boxed{\Delta E = \frac{C(V_2 - V_1)^2}{4}}$$

$$\therefore \quad \Delta E = \frac{4\pi\varepsilon_0 a}{4}(V_2 - V_1)^2$$

$$\therefore \quad \boxed{\Delta E = \pi\varepsilon_0 a (V_2 - V_1)^2}$$

$$\therefore \quad \Delta E = \pi \times 8.85 \times 10^{-12} \times 10.0 \times 10^{-2} \times (400 - 200)^2$$

$$= \boxed{1.11 \times 10^{-7} \text{ J}}$$