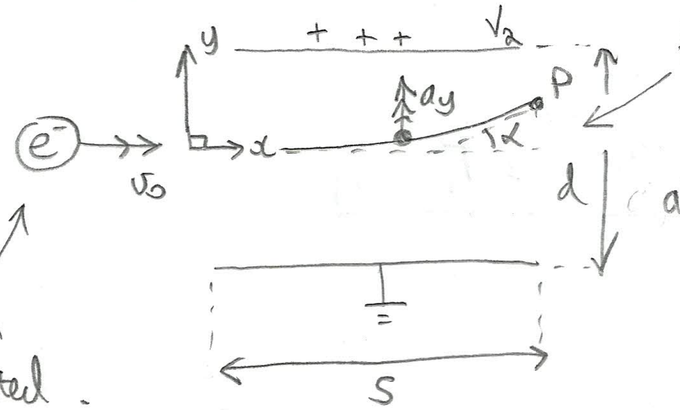
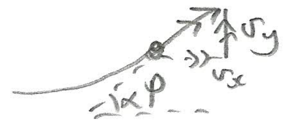
BPhO Round 1 2015 Section 2

(Q1 is Section 1 !)

Q2



$\tan\alpha = \left.\dfrac{v_y}{v_x}\right|_P$



electron accelerated from rest via potential V_1

a) Assume classical physics ($v_0 \ll c$)

$\therefore \underbrace{\frac{1}{2} m_e v_0^2}_{\text{kinetic energy gain of electron}} = \underbrace{eV_1}_{\text{work done by horizontal field}}$

Mass of electron
$m_e = 9.109 \times 10^{-31}$ kg

charge on electron
$e = 1.60 \times 10^{-19}$ C

$\therefore \boxed{v_0 = \sqrt{\dfrac{2eV_1}{m_e}}}$ (i)

b) Constant acceleration motion between horizontal plates

Newton II : $/\!/x$: No acceleration $\therefore \boxed{v_x = v_0}$

$/\!/y$: $m_e a_y = \dfrac{V_2}{d} e$ i.e. $F_y = eE_y$, $E_y = V_2/d$

$\therefore a_y = \dfrac{V_2 e}{m_e d}$

$\therefore$ Now if t is the time between the plates

$v_y = a_y t = \dfrac{V_2 e}{m_e d} t$

$s = v_0 t \quad \therefore t = \dfrac{s}{v_0}$

So $v_y = \dfrac{V_2 e}{m_e d}\dfrac{s}{v_0} = \dfrac{eV_2 s}{m_e v_0 d}$

$\therefore \tan\alpha = \dfrac{v_y}{v_x} = \boxed{\dfrac{eV_2 d}{s m_e v_0^2}}$ Now $m_e v_0^2 = 2eV_1$

$\therefore \quad \tan\alpha = \dfrac{eV_2}{2eV_1}\dfrac{s}{d}$ $\quad \therefore \quad \boxed{\tan\alpha = \frac{1}{2}\frac{V_2 s}{V_1 d}}$

$$\therefore \quad \boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\frac{V_2}{V_1}\frac{s}{d}\right)}$$

(iii) $\quad y_e = \frac{1}{2} a_y t^2$

$$y_e = \frac{1}{2}\frac{V_2 e}{M_e d}\left(\frac{s}{v_0}\right)^2 = \frac{1}{2}\frac{V_2 e s^2}{d M_e v_0^2}$$ ← MS stops here!

Now $\quad M_e v_0^2 = 2eV_1$ $\quad$ So $\quad y_e = \dfrac{V_2 e s^2}{4d eV_1}$

$$\therefore \quad \boxed{y_e = \frac{1}{4}\left(\frac{V_2}{V_1}\right)\left(\frac{s}{d}\right)s}$$

b) $\quad t = \dfrac{s}{v_0} \qquad v_0 = \sqrt{\dfrac{2eV_1}{M_e}}$

So $\quad \boxed{t = \sqrt{\frac{M_e}{2eV_1}} \times s}$

$$\therefore \quad t = \sqrt{\frac{9.109\times10^{-31}}{2\times1.60\times10^{-19}\times 2\times10^{3}}} \times 4\times10^{-2}$$

$$\boxed{t = 1.51\times10^{-9}\,s}$$

c) on exiting the horizontal plates, the electron has velocity

$\begin{pmatrix} v_0 \\ v_y \\ 0 \end{pmatrix}$ $\quad \boxed{v_0 = \sqrt{\frac{2eV_1}{M_e}}}$ $\quad v_y = \dfrac{eV_2}{Mv_0}\dfrac{s}{d}$ ← from above

$$\therefore \quad \boxed{v_y = \frac{eV_2}{M_e}\frac{s}{d}\sqrt{\frac{M_e}{2eV_1}}}$$

This experiences a field $\underline{E} = \hat{z}\,\frac{V_2}{d}$ for a z displacement s

Hence expect v_z upon exit to be same as v_y at point P

$$\therefore \boxed{v_z = \frac{eV_2}{m_e}\,\frac{s}{d}\sqrt{\frac{m_e}{2eV_1}}} = \sqrt{\frac{e}{m_e}}\,\frac{V_2}{\sqrt{V_1}}\,\frac{s}{d}\,\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\,\frac{V_1}{V_1}$$

$$= \boxed{\sqrt{\frac{2eV_1}{m_e}}\,\frac{V_2}{V_1}\,\frac{s}{d}\,\frac{1}{2}}$$

Hence: $$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$v = \sqrt{\frac{2eV_1}{m_e} + 2\left(\frac{eV_2 s}{m_e d}\right)^2 \frac{m_e}{2eV_1}}$$

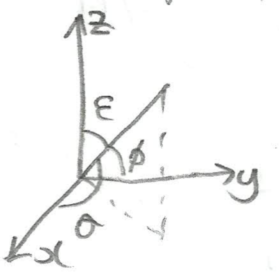
$$v = \sqrt{\frac{2eV_1}{m_e} + \frac{eV_2^2 s^2}{m_e d^2 V_1}}$$

Shortcut!

$$\boxed{v = \sqrt{\frac{2eV_1}{m_e}}\left(1 + \frac{1}{2}\left(\frac{V_2}{V_1}\right)^2\left(\frac{s}{d}\right)^2\right)^{\frac{1}{2}}}$$

let angles between $\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ and axes be $\theta, \phi, \varepsilon$

$$\therefore \quad \underline{v} = v\begin{pmatrix} \cos\theta \\ \cos\phi \\ \cos\varepsilon \end{pmatrix}$$



$$\underline{v} = \sqrt{\frac{2eV_1}{m_e}}\begin{pmatrix} 1 \\ \frac{V_2}{2V_1}\frac{s}{d} \\ \frac{V_2 s}{2V_1 d} \end{pmatrix}$$

So $\boxed{\cos\theta = \frac{v_x}{v} = k}$; $\boxed{\cos\phi = k\,\frac{V_2}{2V_1}\,\frac{s}{d} = \cos\varepsilon}$

where $\boxed{k = \left(1 + \frac{1}{2}\left(\frac{V_2}{V_1}\right)^2\left(\frac{s}{d}\right)^2\right)^{-\frac{1}{2}}}$

Note for the parameters given:

$$k = \left(1 + \frac{1}{2}\left(\frac{50}{2000}\right)^2\left(\frac{4}{1.5}\right)^2\right)^{-\frac{1}{2}}$$

$$= 0.998891\ldots$$

$$\therefore \theta = \cos^{-1} k = \boxed{2.7^\circ}$$

$$\phi = \varepsilon = \cos^{-1}\left(k \times \frac{50}{2\times 2000} \times \frac{4.00}{1.50}\right)$$

$$= \boxed{88.1^\circ}$$

Q3/

Wind force $\boxed{F = Av^2 + B}$ on cyclist travelling at speed v on a straight level road.

To balance F, driving force $D = P/v$ where P is cycle power (well, 'rate of work done'). $\therefore P = Fv$

So predict $\boxed{P = Av^3 + Bv}$

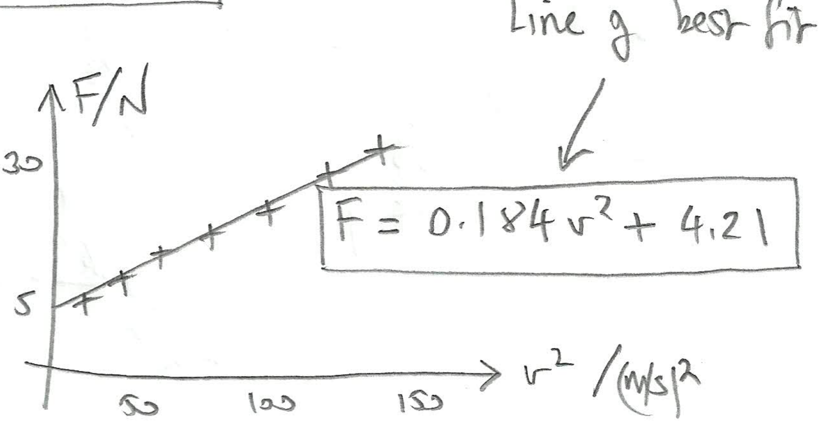
a) See Excel sheet.

So $A = (0.18 \pm 0.01)\ kg\,m^{-1}$

$B = (4.2 \pm 0.2)\ kg\,m\,s^{-2}$ (or N)

[MS gives (4.0 ± 0.6) N]

[$N/m^2s^{-2} = kg\,m\,s^{-2}\,m^{-2}\,s^2 = kg\,m^{-1}$]



b) Av^2 : Air resistance (drag model $\simeq \frac{1}{2}C_D\rho A v^2$)

B : Rolling friction with load

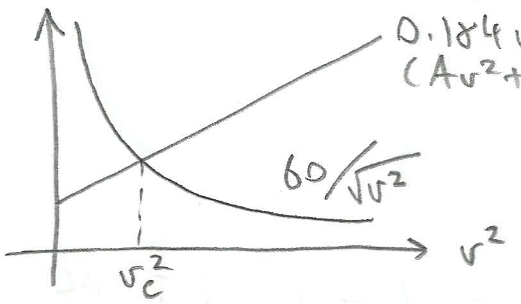
c) $P = 60W$, so $P = Av^3 + Bv$

$\therefore$ $\boxed{Av^3 + Bv - P = 0}$ can be solved for v

[$0.184v^3 + 4.21v - 60 = 0$]

Now $P = Fv$ so $F = P/v$ $\therefore$ $\boxed{F = \frac{P}{\sqrt{v^2}}}$

So



Now $F = 0.184v^2 + 4.21$

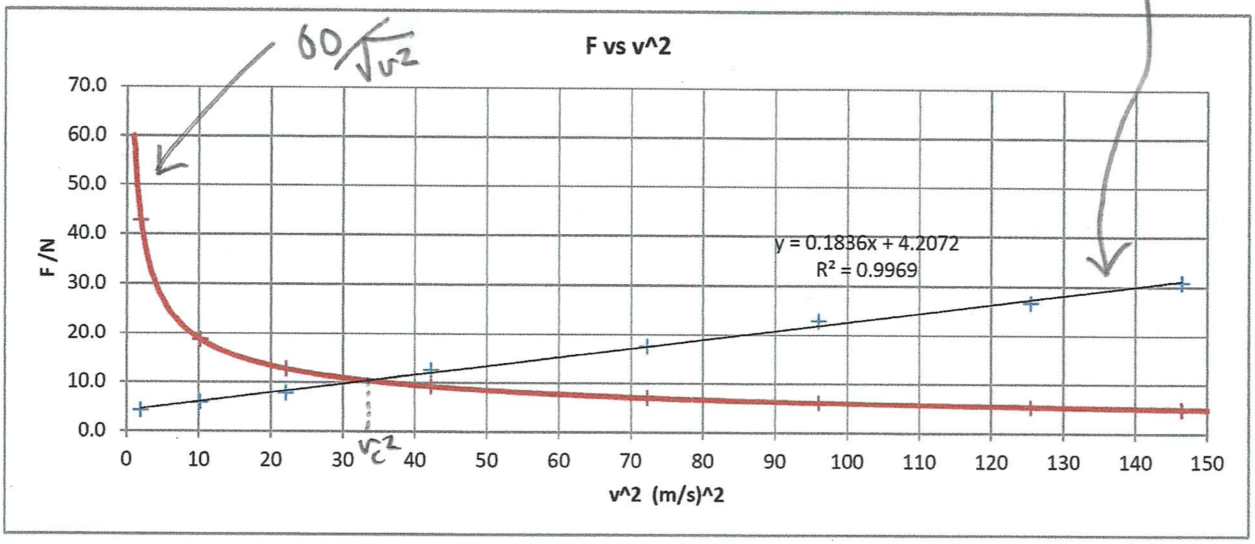
$v_c^2 = 33.3$ $\therefore$ $\boxed{v_c = 5.8\ ms^{-1}}$

(Accuracy suggested by MS is ± 0.4 m/s)

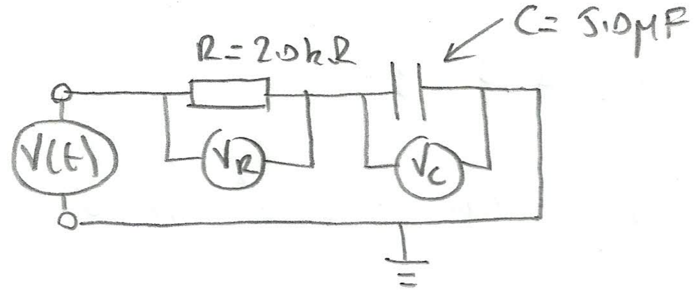
Cyclist problem. BPhO 2015 Section 2 Q3

v /ms^-1	P /W	F = P/v (N)	v^2
1.4	6	4.3	1.96
3.2	19	5.9	10.24
4.7	37	7.9	22.09
6.5	82	12.6	42.25
8.5	149	17.5	72.25
9.8	224	22.9	96.04
11.2	298	26.6	125.44
12.1	373	30.8	146.41

60/sqrt(v^2)
42.86
18.75
12.77
9.23
7.06
6.12
5.36
4.96



Q4/



$$\boxed{V_c = V_0(1 - e^{-t/RC})}$$

for $0 < t < \tau$

$$V(t) = \begin{cases} \underset{V_0}{5.0} \text{ Volts} & 0 < t < \underset{\tau}{0.020s} \\ 0 & t > 0.020s \end{cases}$$

$$\boxed{V_c = V_1 e^{-(t-\tau)/RC}} \quad \text{for } t > \tau$$

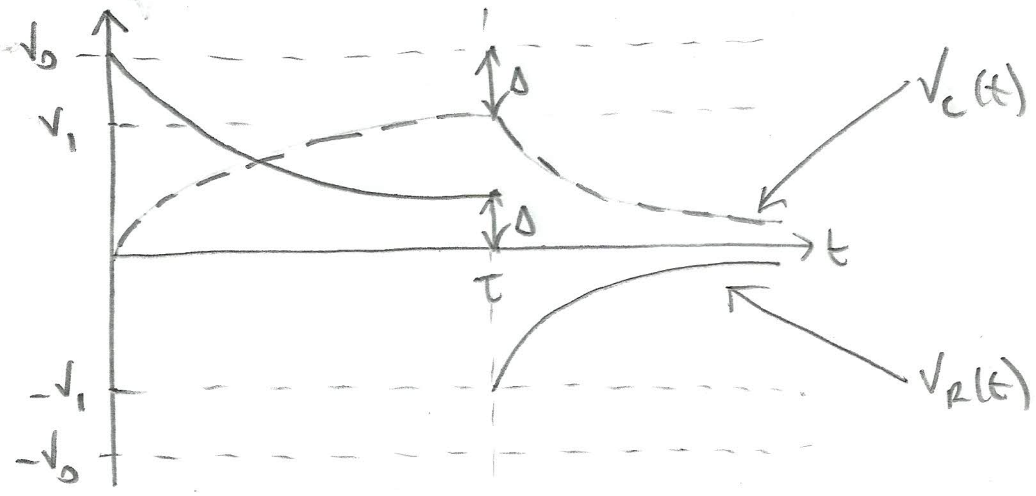
$$V_1 = V_0(1 - e^{-\tau/RC})$$

$$\boxed{V_R = V - V_c}$$

$$V_R = V_0 - V_0 + V_0 e^{-t/RC} \quad \text{for } 0 < t < \tau$$

$$\boxed{V_R = V_0 e^{-t/RC}}$$

And $\boxed{V_R = -V_1 e^{-(t-\tau)/RC}}$ for $t > \tau$ $\qquad [RC = 0.01s]$

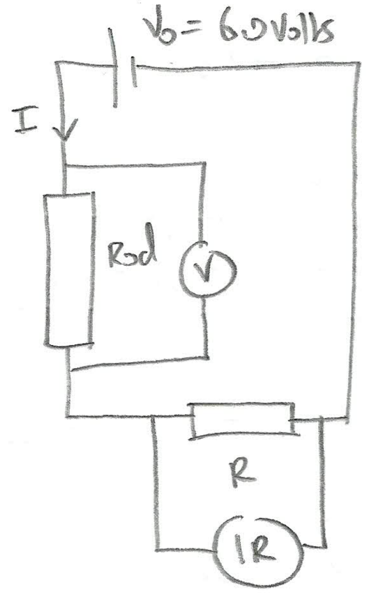


$[\Delta = V_0 e^{-\tau/RC}]$

$$\begin{bmatrix} V_0 = 5.0 \text{ Volts} \\ \tau = 0.020s \\ V_1 = 0.68 \text{ Volts} \end{bmatrix}$$

$$\left[\frac{dV_c}{dt} = \frac{1}{RC} e^{-t/RC} \quad \text{so} \quad \left.\frac{dV_c}{dt}\right|_{t=0} = \frac{1}{RC} \right]$$

b)



Rod: $I = 0.20\,V^3 \quad \therefore \quad V = \left(\frac{I}{0.20}\right)^{\frac{1}{3}}$

So: $V_0 = V + IR$

$\therefore \quad V_0 = \left(\frac{I}{0.20}\right)^{\frac{1}{3}} + IR$

$\therefore \quad \boxed{R = \dfrac{V_0 - \left(I/0.20\right)^{\frac{1}{3}}}{I}}$

(i) $I = 0.40\,A \quad \therefore \quad R = \dfrac{6.0 - \left(\frac{0.4}{0.2}\right)^{1/3}}{0.4} = \boxed{11.9\,\Omega}$

(ii) $VI = 2I^2R$ (power dissipated in rod = twice that of resistor)

$\therefore \left(\frac{I}{0.20}\right)^{\frac{1}{3}} I = 2I^2R$ ① using $V = \left(I/0.20\right)^{\frac{1}{3}}$

Now $V = V_0 - IR$ so $(V_0 - IR)I = 2I^2R$

$\therefore \quad V_0 I = 3I^2R$ ②

$\therefore \quad \dfrac{\left(\frac{I}{0.20}\right)^{\frac{1}{3}}}{V_0} = \frac{2}{3} \quad \therefore \quad I = \left(\frac{2}{3}V_0\right)^3 \times 0.20$

$\therefore \quad I = \left(\frac{2}{3} \times 6.0\right)^3 \times 0.20$

$\boxed{I = 12.8\,A}$ (!)

$\therefore R = \dfrac{6.0 - \left(\frac{12.8}{0.2}\right)^{1/3}}{12.8} = \boxed{0.16\,\Omega}$

So $IR = 2.0$ volts

So $\boxed{V_R = \frac{1}{2}V}$

and since I the same ⇒ power dissipated is twice as much in rod

Check: $V = 6.0 - 12.8 \times 0.16 = \boxed{4.0\text{ Volts}}$

Quick answer: If $IV_{rod} = 2IV_R$

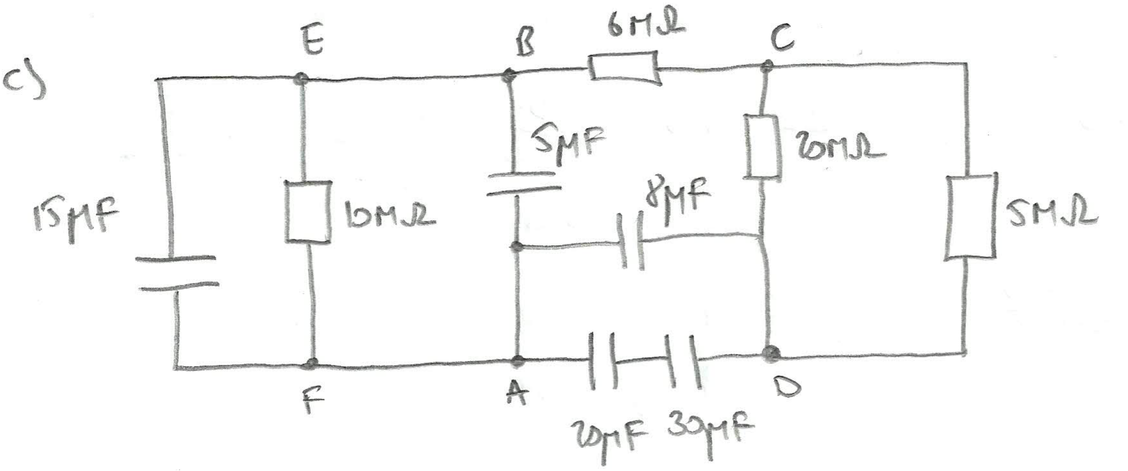
$\Rightarrow \quad V_{rod} = 2V_R$

Now $V_R + V_{rod} = 6$ so $\boxed{V_R = 2,\ V_{rod} = 4}$ (volts)

$\therefore \quad I = 0.20\,V_{rod}^3 \quad \therefore \quad I = 0.20 \times 4^3 = \boxed{12.8\,A}$

$V_R = IR \quad \therefore \quad R = \frac{2}{12.8} = \boxed{0.16\ \Omega}$

c)



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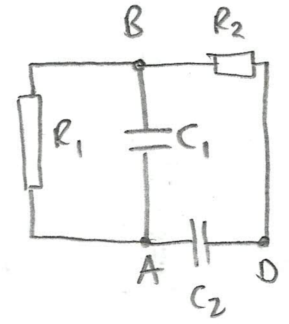
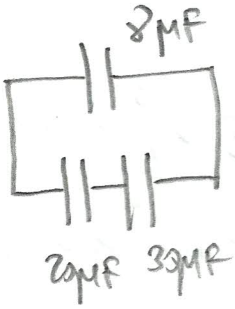


Now

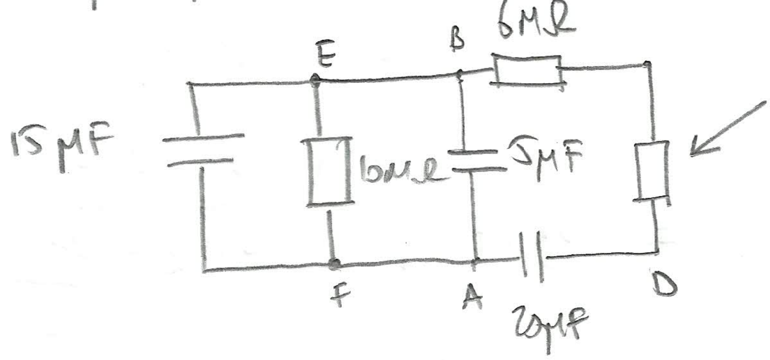
$\frac{1}{\frac{1}{20} + \frac{1}{30}} = \boxed{12\mu F}$

$\therefore$



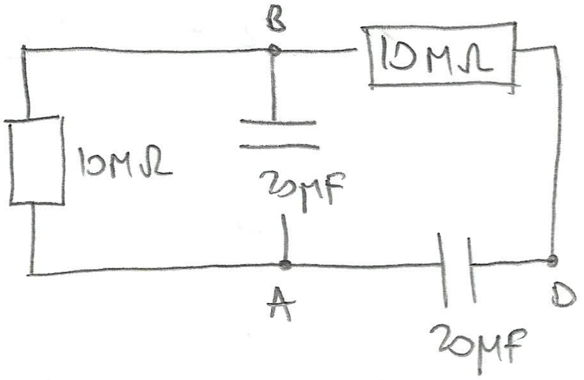
= $\boxed{20\mu F}$

So



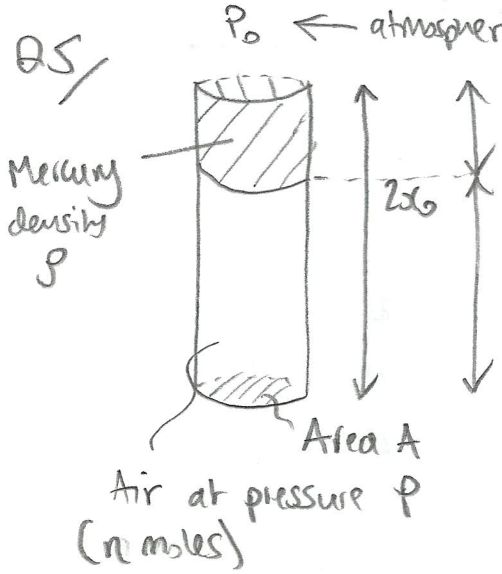
$\frac{1}{\frac{1}{20} + \frac{1}{5}} = 4M\Omega$

Now the $15\mu F$ and $5\mu F$ capacitors are in parallel is equivalent to a $20\mu F$ capacitor. Note $6M\Omega$ and $4M\Omega$ equivalent resistors are in series, so:



So $\boxed{\begin{matrix} R_1 = R_2 = 10\,M\Omega \\ C_1 = C_2 = 20\,\mu F \end{matrix}}$

Q5/



$2x_0 - x_0 - x = x_0 - x$ (height of mercury)

$V_0 = Ax_0$ (original volume)

a) $V = A(x_0 + x)$

$V_0 = Ax_0$

$x_R = x/x_0$

$\therefore \frac{V}{V_0} = V_R = 1 + \frac{x}{x_0} = \boxed{1 + x_R}$

Now $\boxed{P = P_0 + \rho g(x_0 - x)}$ {Atmospheric pressure + head of mercury}

$\therefore P_R = \frac{P}{P_0} = 1 + \frac{\rho g(x_0 - x)}{P_0}$

Now $\boxed{P_0 = \rho g x_0}$ "x_0 mercury is the same as P_0"

So $P_R = 1 + 1 - \rho g x / P_0$

$P_R = 2 - \frac{\rho g x}{\rho g x_0}$

$$\boxed{P_R = 2 - x_R}$$

Now $V_R = 1 + x_R \therefore x_R = V_R - 1$

$\therefore P_R = 2 - V_R + 1 \quad \therefore \boxed{P_R = 3 - V_R}$ (i)

(ii) Ideal gas equation: $PV = nRT$

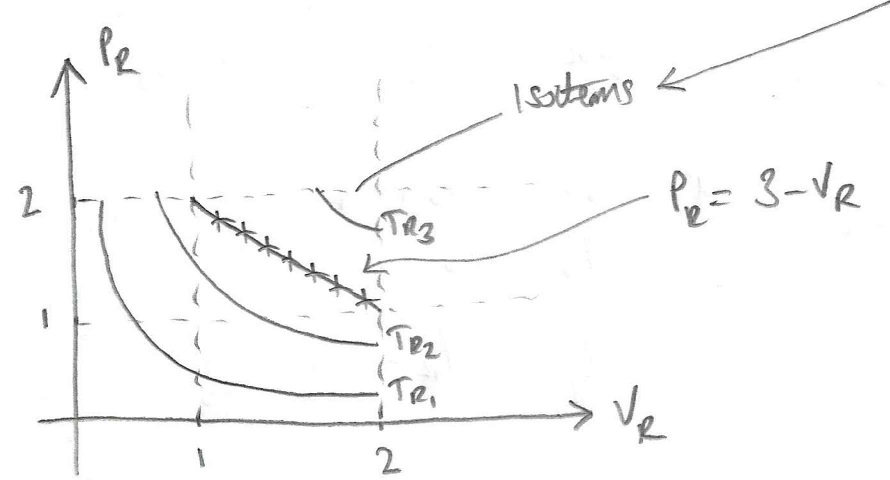
let $T_0 = \dfrac{P_0 V_0}{nR}$

so $\dfrac{PV}{P_0 V_0} = \dfrac{nRT}{nRT_0} \Rightarrow \boxed{T_R = P_R V_R}$

$T_R = T/T_0$

b) From above: $\boxed{P_R = 3 - V_R}$

and for isotherms (T_R = constant) $\boxed{P_R = T_R / V_R}$

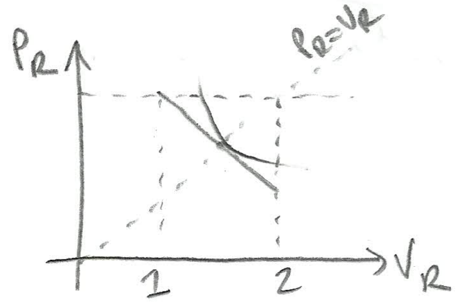


Note symmetric about line $P_R = V_R$

$\boxed{\text{Range of } V_R:}$ Initially $V_R = 1$ ($V = V_0$) $\therefore P_R = 2$

As air expands it pushes mercury out. Eventually $V = 2V_0$ (no mercury). At this point $P = P_0$ $\therefore P_R = 1$

ie $V_R = 2$ and $P_R = 3 - 2 = 1$ ✓



Highest possible temperature when $P_R = V_R$

so $P_R = 3 - P_R$ $\therefore P_R = 3/2$, $V_R = 3/2$

$\therefore T_R = P_R V_R = \boxed{9/4}$

(6)

c) let ΔT_R be a small change in reduced temperature.

We are given: $\boxed{\frac{5}{2}\Delta T_R = S_R\Delta T_R - P_R\Delta V_R}$

{ 1^{st} Law of thermodynamics $\Delta U = \Delta Q - P\Delta V$ ← work done }

internal energy change ↗ ; ↑ Heat change

S_R is the specific heat / mole of air in the tube

Now $\boxed{P_R = 3 - V_R}$ ∴ $T_R = P_R V_R = \boxed{3V_R - V_R^2}$

∴ $\frac{dT_R}{dV_R} = 3 - 2V_R$ so for small changes

$$\boxed{\Delta T_R \simeq (3 - 2V_R)\Delta V_R}$$

∴ $S_R = \dfrac{\frac{5}{2}\Delta T_R + P_R\Delta V_R}{\Delta T_R}$

∴ $S_R = \dfrac{\frac{5}{2}(3-2V_R)\Delta V_R + (3-V_R)\Delta V_R}{(3-2V_R)\Delta V_R}$

∴ $S_R = \dfrac{15 - 10V_R + (3-V_R)\times 2}{6 - 4V_R}$

∴ $\boxed{S_R = \dfrac{21 - 12V_R}{6 - 4V_R}}$ ✱

$V_R = 1$, $S_R = \frac{21-12}{6-4} = \boxed{4.5}$

$V_R = 2$, $S_R = \frac{21-24}{6-8} = \frac{3}{2} = \boxed{1.5}$

✱ $S_R = A + \dfrac{B}{3-2V_R}$

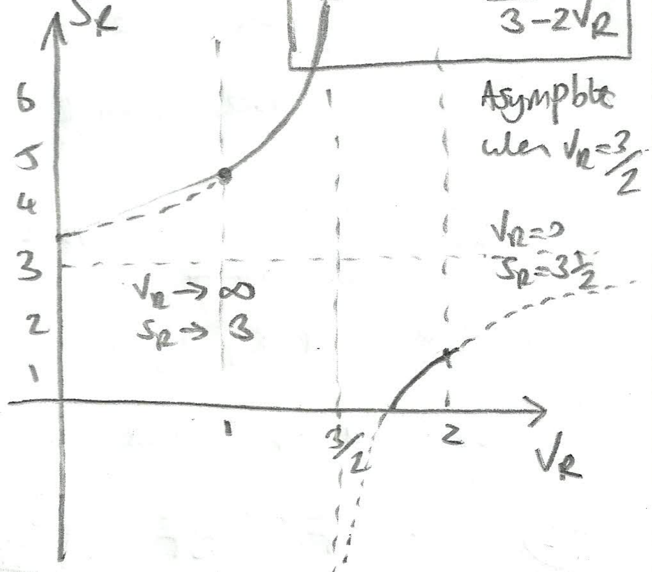
$S_R = \dfrac{(3-2V_R)A + B}{3-2V_R}$ $\frac{\times 2}{2}$

so $6A + 2B = 21$ ①

$4A = 12$ ∴ $\boxed{A = 3}$

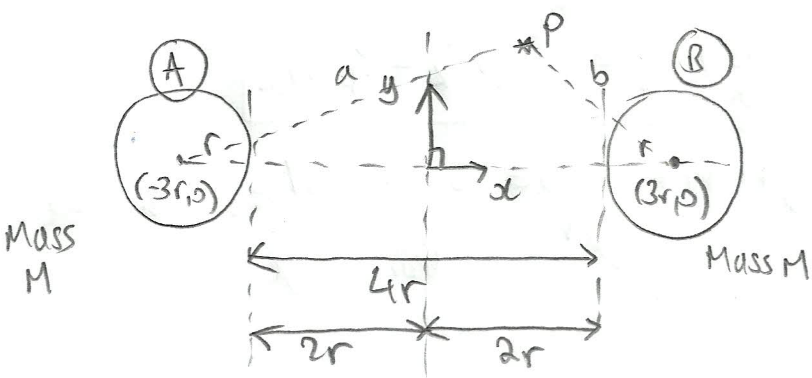
$B = \frac{21-18}{2} = \boxed{\frac{3}{2}}$

∴ $\boxed{S_R = 3 + \dfrac{3/2}{3-2V_R}}$



[— lines mean $1 \le V_R \le 2$]

Q6/ * Ignore motion of stars *



Let star coordinates be $(-3r,0)$ and $(3r,0)$

Potential $V(x,y) = -\dfrac{GM}{a} - \dfrac{GM}{b}$ $\quad a = \sqrt{(x+3r)^2+y^2}$

$$\boxed{V(x,y) = -GM\left[\frac{1}{\sqrt{(x+3r)^2+y^2}} + \frac{1}{\sqrt{(3r-x)^2+y^2}}\right]}$$

$b = \sqrt{(3r-x)^2+y^2}$

a) (i) As you go around the surface of B you get further or closer to A $\therefore$ a circle round B cannot be an equipotential surface $V =$ constant.

(ii) Now $V < 0$ so largest V is when it $\to 0$ which is when $x, y \to \infty$

Now most negative V ("smallest") is clearly when $x = \pm 2r$ ie on surface of B or A
Note assume 'inside stars' is out of bounds!

$\boxed{\text{Algebraic way:}}$ Want to minimize $(x+3r)^2+y^2$ and $(3r-x)^2+y^2$. Clearly set $y=0$

So largest value of $\dfrac{1}{\sqrt{(x+3r)^2}} + \dfrac{1}{\sqrt{(3r-x)^2}} = z$

is $\dfrac{1}{r} + \dfrac{1}{5r} = \boxed{\dfrac{6}{5r}}$ (ie $x = \pm 2r$)

$\therefore$ Most negative V is $\boxed{\dfrac{-6GM}{5r}}$ $= -\dfrac{GM}{3r} \times \dfrac{6\times3}{5}$

$= \boxed{-\dfrac{GM}{3r} \times 3\tfrac{3}{5}}$

(iv) when $x=0, y=0$; $V(0,0) = -GM\left(\dfrac{1}{3r} + \dfrac{1}{3r}\right)$

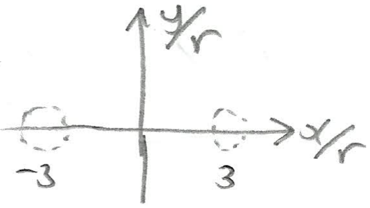
$$\boxed{V(0,0) = \frac{-2GM}{3r}}$$

b) See below. [Next page!] (Not sure about $V_1 = -10\,GM/3r \ldots$)

Now: $$V = -\frac{GM}{3r}\left[\frac{3r}{\sqrt{(x+3r)^2+y^2}} + \frac{3r}{\sqrt{(3r-x)^2+y^2}}\right]$$

$$V = -\frac{GM}{3r}\left[\frac{3}{\sqrt{(\frac{x}{r}+3)^2+(\frac{y}{r})^2}} + \frac{3}{\sqrt{(3-\frac{x}{r})^2+y^2}}\right]$$

So plot generic surface in terms of



with heights in units of $-\dfrac{GM}{3r}$

And we want contours at $10\,(?)$, $2/3$ and 1

[I think max is $3\frac{3}{5}$] ↑ Should be $3\frac{3}{5}$

c) [Guided by MS...] * If a body of mass M is launched from B to A, it must overcome the potential at $(0,0)$ ie $\dfrac{-2GM}{3r}$. (Note this doesn't mean it will have to pass ~~through~~ $(0,0)$).

i.e $V(0,0)$ is the potential when a mass is minimally equidistant from Stars A and B.

So $\frac{1}{2}mv^2 + mV_L > -\frac{2}{3}\frac{GMm}{r}$

Now for v to be minimized we want V_L to be the least negative possible! We must be on the surface of B, so this means launch at $(4r, 0)$

ie $V_L = \frac{-GM}{3r}\left[\frac{3}{\sqrt{(4+3)^2}} + \frac{3}{\sqrt{(3-4)^2}}\right]$

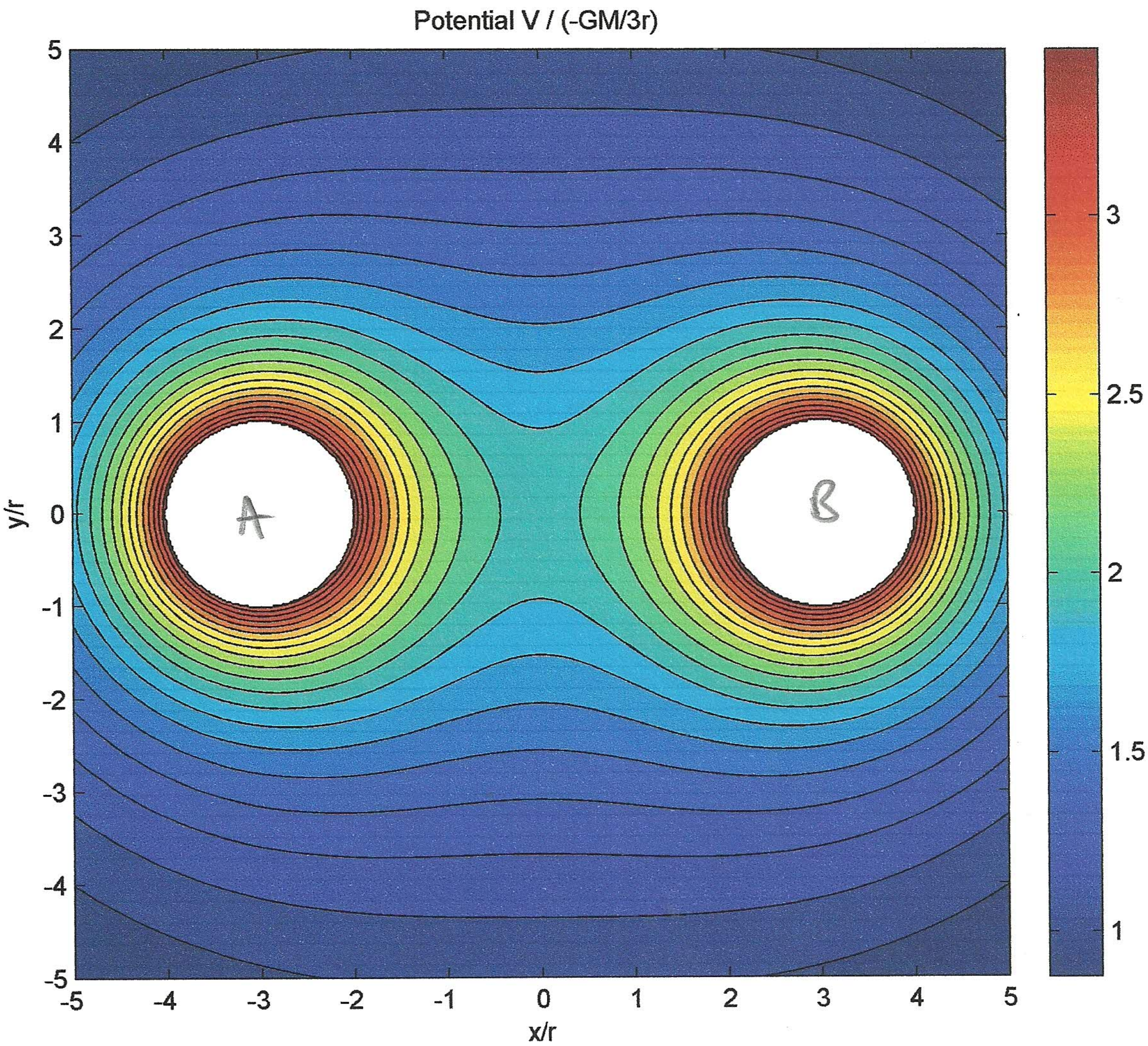
$= \frac{-GM}{3r}\left[\frac{3}{7} + 3\right]$

$= \frac{-GM}{r} \times \frac{8}{7}$ $\therefore$ $\boxed{V_L = \frac{-8GM}{7r}}$

So $v^2 > \left(-\frac{2}{3} + \frac{8}{7}\right) \times 2\frac{GM}{r}$

$\boxed{v^2 > \frac{20GM}{21r}}$

ie minimum launch speed to get from $A \to B$ is $v_{min} = \sqrt{\frac{20GM}{21r}}$.



$$V = -\frac{GM}{3r}\left[\frac{3}{\sqrt{\left(\frac{x}{r}+3\right)^2+y^2}} + \frac{3}{\sqrt{\left(3-\frac{x}{r}\right)^2+y^2}}\right]$$