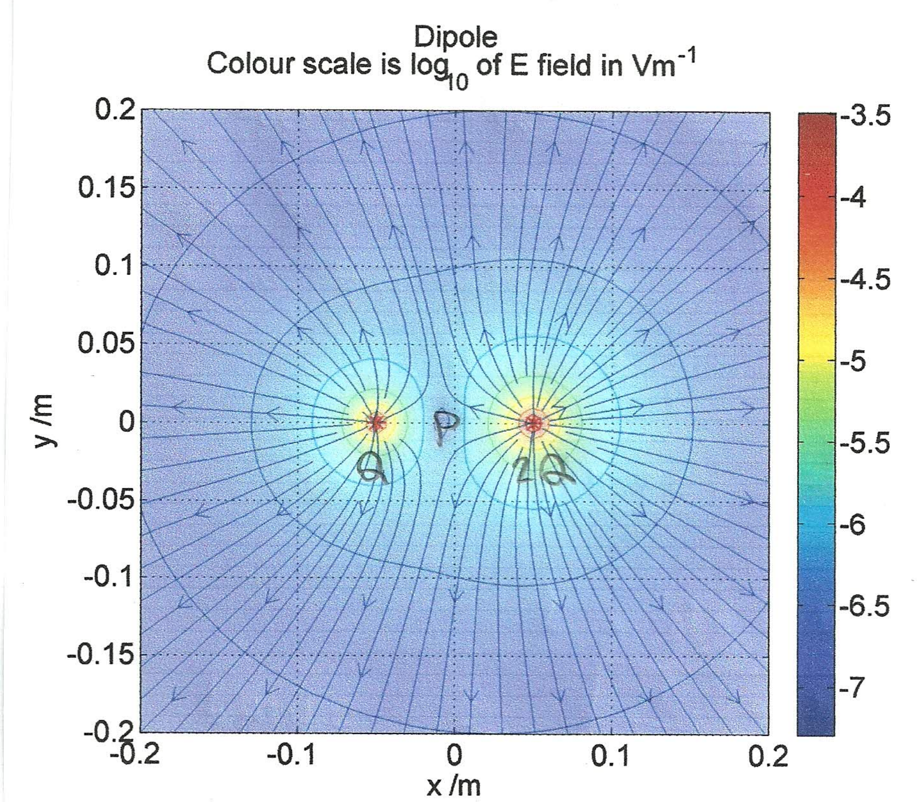
BPhO 2016 Paper 1 Section 1

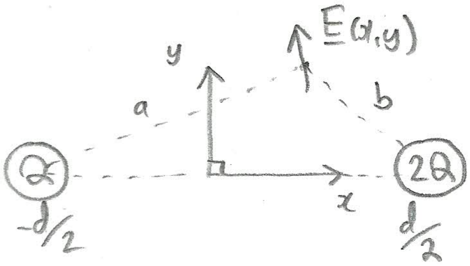
(a) Sketch the electric field lines due to two point charges, of magnitudes $+Q$ and $+2Q$, at A and B, separated by a distance d.

(i) Determine the location of the neutral point, P, where the electric field is zero.

(ii) Why does the magnitude of the electric field vary along a field line?

[5]





Coulomb's law for $E(x,y)$

$$\underline{E} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a^2}\begin{pmatrix} x+\frac{d}{2} \\ y \end{pmatrix}\Big/ a$$

$$+ \frac{2Q}{4\pi\varepsilon_0}\frac{1}{b^2}\begin{pmatrix} x-\frac{d}{2} \\ y \end{pmatrix}\Big/ b$$

$$a^2 = \left(x+\frac{d}{2}\right)^2 + y^2$$

$$b^2 = \left(x-\frac{d}{2}\right)^2 + y^2$$

$\left(x < \frac{d}{2}\right)$ and $\left(x > -\frac{d}{2}\right)$

otherwise opposite signs clearly no zero for $x > \frac{d}{2}$, $x < -\frac{d}{2}$ anyway.

$|\underline{E}| = 0$ at P. clearly $y = 0$

$$\therefore\ 0 = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\left(x+\frac{d}{2}\right)^2} - \frac{2Q}{4\pi\varepsilon_0}\frac{1}{\left(\frac{d}{2}-x\right)^2}$$

in x direction

$$\therefore\ \left(\frac{d}{2}-x\right)^2\Big/2 = \left(x+\frac{d}{2}\right)^2$$

$$\frac{d^2}{4} - dx + x^2 = 2\left\{x^2 + dx + \frac{d^2}{4}\right\}$$

$$\therefore\ x^2 + 3dx + \frac{d^2}{4} = 0$$

$$\left(x + \frac{3d}{2}\right)^2 - \frac{9d^2}{4} + \frac{d^2}{4}$$

$$\left(x + \frac{3d}{2}\right)^2 - \frac{8d^2}{4} = 0$$

$$\left(x + \frac{3d}{2}\right)^2 - 2d^2 = 0$$

$$x + \frac{3d}{2} = \pm d\sqrt{2}$$

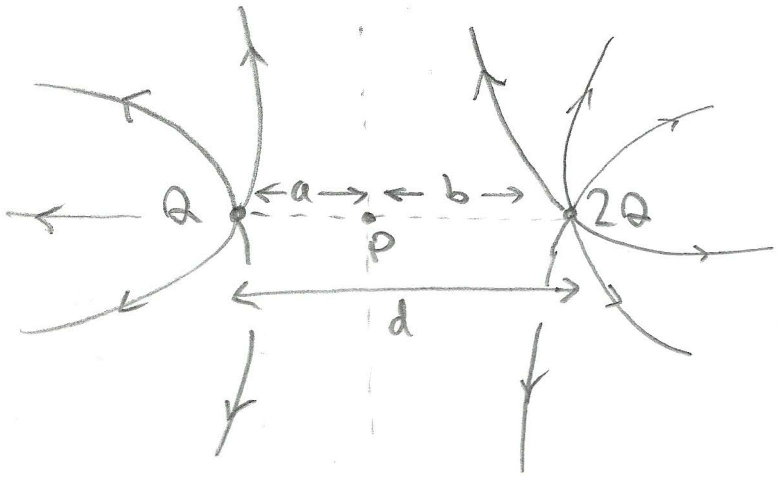
$$x = -\frac{3d}{2} \pm d\sqrt{2}$$

$$\boxed{x = \left(-\frac{3}{2} + \sqrt{2}\right)d}$$

Since $\left(-\frac{3}{2} - \sqrt{2}\right)d$ is $< -\frac{d}{2}$.

(ii) Field lines are // to field vector $\underline{E}$ if $|\underline{E}|$ is not constant then clearly $|\underline{E}|$ will vary along a field line!!

Perhaps easier to visualize if one considers distances a, b from Q and 2Q



$a+b=d$ ①

Now from Coulomb's law

$$\frac{Q}{4\pi\varepsilon_0 a^2} = \frac{2Q}{4\pi\varepsilon_0 b^2} \quad \text{at } P$$

ie each charge repels the other with an equal magnitude.

$\therefore \quad b^2 = 2a^2 \Rightarrow \boxed{b = a\sqrt{2}}$ ②.

$\therefore$ in ① : $a + a\sqrt{2} = d$

$$\boxed{a = \frac{d}{1+\sqrt{2}}} \approx 0.414d$$

$$\boxed{b = \frac{\sqrt{2}}{1+\sqrt{2}}d} \approx 0.586d$$

$\therefore$ using (x,y) coordinates above ; $y_p = 0$

$$x_p = -\frac{d}{2} + \frac{d}{1+\sqrt{2}}$$

$$x_p = -\frac{d}{2} + \frac{d(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$$

$$x_p = -\frac{d}{2} + \frac{d(1-\sqrt{2})}{1-2}$$

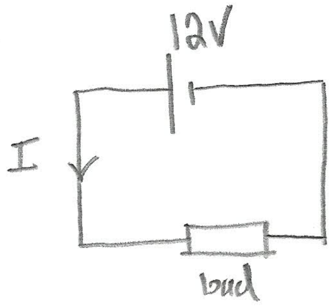
$$x_p = -\frac{d}{2} - d(1-\sqrt{2})$$

$$\boxed{x_p = -\frac{3d}{2} + d\sqrt{2}}$$

which is what we computed previously.

(b) A charge of 0.5 x 10^6 C passes through a 12 V battery when the battery discharges. Assuming that the p.d. across the terminals remains constant, calculate the time for which it can supply 0.45 kW.

[2]



$P = IV = 0.45\text{ kW}$

$I = \dfrac{0.5\times10^6}{t} = \dfrac{Q}{t}$

assuming a steady current discharge (not like a capacitor). Assume V = 12 Volts

so $P = \dfrac{Q}{t}V$ $\therefore$ $\boxed{t = \dfrac{QV}{P}}$

$\therefore$ $t = \dfrac{0.5\times10^6\times12}{0.45\times10^3}$

$t = 1.33\times10^4\text{ s}$

$\boxed{t \simeq 3.70 \text{ hours}}$
(3 hrs 42 minutes)

(c) Draw a general resistive network diagram with:

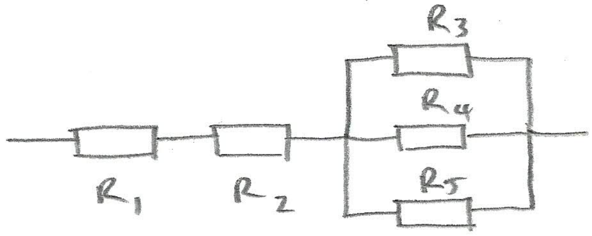
(i) two resistors in series which are, in turn, in series with three resistors in parallel.

(ii) five resistors that are not in series or parallel, or in a combination of series and parallel arrangements.

Calculate the resistance in (i) and (ii) if all the resistors have resistance R.

[5]

(i)

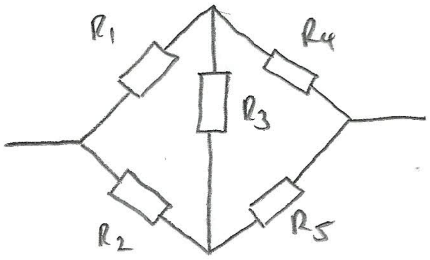


$$R_{TOT} = R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}$$

if $R_1 = R_2 = R_3 = R_4 = R_5 = R$

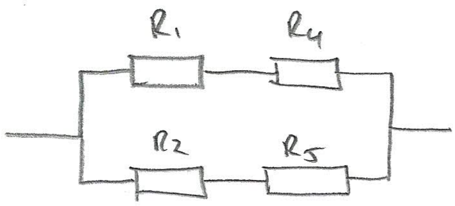
$$R_{TOT} = 2R + \frac{1}{3/R} = 2R + \frac{R}{3} = \boxed{\frac{7R}{3}}$$

(ii)



"Wheatstone bridge" layout

If $R_1 = R_2 = R_3 = R_4 = R_5$ then no current can flow through R_3, hence circuit is equivalent to



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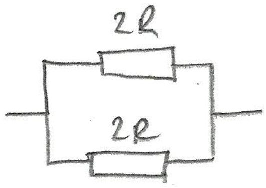


So $R_{TOT} = \frac{1}{\frac{1}{2R} + \frac{1}{2R}} = \boxed{R}$

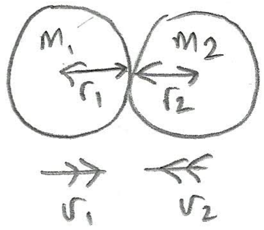
(d) Two spheres, of uniform density, one of mass m_1 and radius r_1 and the other of mass m_2 and radius r_2, attract each other gravitationally. What is their *relative* speed at the instant of collision if they are released from rest when a great distance apart?

[7]

Assume total energy at the point of release is zero (ie KE and GPE = 0)

At point of collision, conservation of energy means

(1) $$\boxed{\underbrace{\tfrac{1}{2}M_1v_1^2 + \tfrac{1}{2}M_2v_2^2}_{KE} - \underbrace{\frac{GM_1M_2}{r_1+r_2}}_{GPE} = 0}$$



Now momentum must also be conserved, since no external forces act. ie since initial momentum = 0

$$\Rightarrow \quad 0 = M_1v_1 - M_2v_2 \quad \text{ie} \quad \boxed{v_2 = \frac{M_1}{M_2}v_1} \quad (2)$$

Hence in (1):
$$\tfrac{1}{2}M_1v_1^2 + \tfrac{1}{2}M_2\left(\frac{M_1}{M_2}\right)^2 v_1^2 = \frac{GM_1M_2}{r_1+r_2}$$

$$\tfrac{1}{2}v_1^2\left(M_1 + \frac{M_2M_1^2}{M_2^2}\right) = \frac{GM_1M_2}{r_1+r_2}$$

$$v_1^2\left(1 + \frac{M_1}{M_2}\right) = \frac{2GM_2}{r_1+r_2}$$

$$v_1^2\left(\frac{M_2+M_1}{M_2}\right) = \frac{2GM_2}{r_1+r_2}$$

$$v_1^2 = \frac{2GM_2^2}{(M_1+M_2)(r_1+r_2)}$$

$$\therefore \quad v_1 = \sqrt{\frac{2GM_2^2}{(M_1+M_2)(r_1+r_2)}}$$

Now $v_2 = \sqrt{\frac{M_1^2}{M_2^2}}\ v_1$

So $\boxed{v_2 = \sqrt{\frac{2GM_1^2}{(M_1+M_2)(r_1+r_2)}}}$

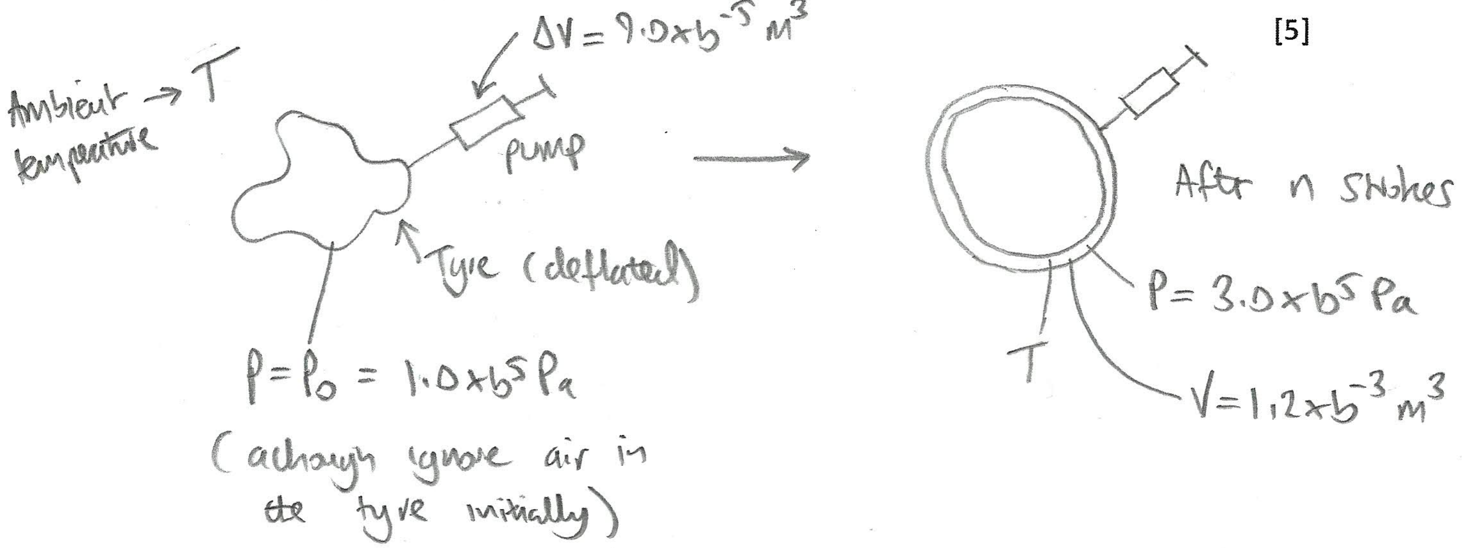
Now relative speed is $v_1 + v_2$, since in opposite directions

ie $v_{rel} = \sqrt{\frac{2G}{(M_1+M_2)(r_1+r_2}}\ (M_1 + M_2)$

So $\boxed{v_{rel} = \sqrt{\frac{2G(M_1+M_2)}{r_1+r_2}}}$

(e) A bicycle tyre has a volume of 1.2×10^{-3} m^3 when fully inflated. A bicycle pump has a working volume of 9.0×10^{-5} m^3. How many strokes, n, of the pump are needed to inflate the completely flat tyre, containing no air, to a pressure of 3.0×10^5 Pa? The atmospheric pressure is 1.0×10^5 Pa. Assume the air is pumped in slowly so that there is no change in temperature.

[5]



Assume air is an ideal gas, for air in the tyre when fully inflated.

So $\boxed{PV = nRT}$

\# moles transferred from pump

Molar gas constant

Now for air in the pump:

$\boxed{P_0 \Delta V = mRT}$ where m is # moles of air in the pump.

Clearly # strokes is $\dfrac{n}{m} = \dfrac{PV/RT}{P_0\Delta V/RT} = \left(\dfrac{P}{P_0}\right)\left(\dfrac{V}{\Delta V}\right)$

$= 3 \times \dfrac{1.2\times10^{-3}}{9.0\times10^{-5}}$

$= \boxed{40}$

(7)

(f) A van, travelling at constant speed of 80 km hr^{-1} (km/hour), passes a car. The car immediately begins to accelerate at a constant rate of 1.2 m s^{-2} and passes the van 0.50 km further down the road. What is the speed, v, of the car when it passes the van?

w a

[4]

From passing point $\quad x_v = wt \quad$ (van displacement)

$x_c = ut + \frac{1}{2}at^2 \quad$ (car displacement)

Now $\quad x_c = x_v$ when $x_c = 0.5$ km $= d$

So $\quad \boxed{t = d/w} = \dfrac{0.5 \text{ km}}{80 \text{ km/hr}} = 6.25 \times 10^{-3}$ hr $\quad \times 3600$

$= \boxed{22.5 s}$

Now $\quad \dfrac{d - \frac{1}{2}at^2}{t} = u \quad$ (speed of car when van passes)

So $\quad u = \dfrac{d - \frac{1}{2}ad^2/w^2}{d/w} = w - \frac{1}{2}a\,d/w$

$\therefore \quad \boxed{u = w - \dfrac{ad}{2w}} = \dfrac{80 \times 10^3}{3600} - \dfrac{1.2 \times 500}{\frac{2 \times 80 \times 10^3}{3600}}$

$= \boxed{8.72 \text{ m/s}}$

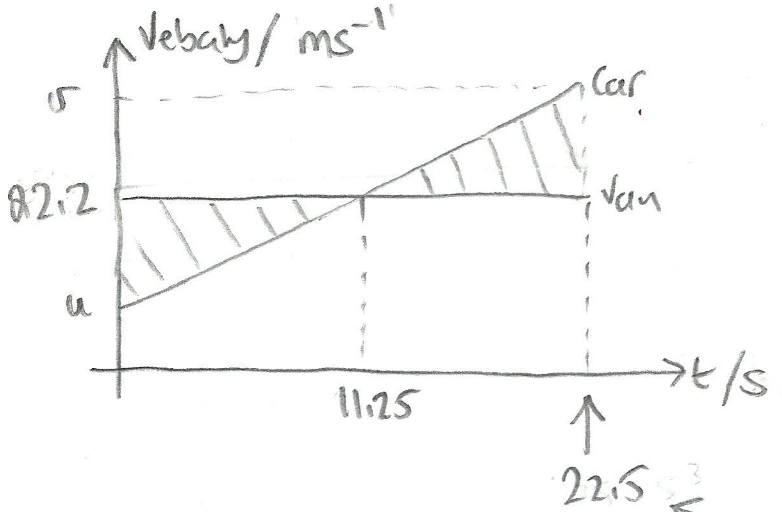
$\therefore$ Car speed when it passes the van is $v = u + at$

$v = w - \dfrac{ad}{2w} + \dfrac{ad}{w}$ $\quad$ {check: $v = 8.72 + 1.2 \times 22.5$ $= 35.7$ m/s}

$\boxed{v = w + \dfrac{ad}{2w}} = 35.7$ m/s $= \boxed{128.6 \text{ km/h}}$

[1 km/hr $= \dfrac{1000 \text{m}}{3600 \text{s}}$ so 1 m/s $= 3.6$ km/h]

Can also solve this graphically $[80\ km/h = 22.2\ m/s]$ $\div 3.6$



areas under van and car graphs = 0.5 km

So shaded areas must equal, and 80 km/h is the mean average of v and u

$t = \frac{500m}{80/3.6\ m/s}$

Now $\frac{v - 22.2}{11.25} = a = 1.2$

So $v = 22.2 + 1.2 \times 11.25 = \boxed{35.7\ m/s}$

or $\boxed{128.5\ km/h}$

ie don't use rounded 22.2 m/s

using $80/3.6$ for van speed we get $\boxed{128.6\ km/h}$

M $\qquad\qquad\qquad\qquad$ T_0

(g) A calorimeter contains 0.800 kg of water at a temperature of 15.0 °C. The heat capacity of the calorimeter is 42.8 J °C^{-1}. 0.400 kg of molten lead is poured into the calorimeter. The final equilibrium temperature is 25.0 °C. What was the initial temperature of the lead?

C $\qquad$ M $\qquad$ T_1 $\qquad$ T

C_{LL} $\qquad\qquad$ C_{SL}

The specific heat of molten lead is 158 J kg^{-1}°C^{-1}, the specific heat of solid lead is 137 J kg^{-1} °C^{-1} and the specific latent heat is 2.323 x 10^4 J kg^{-1}. Lead freezes at 327 °C. The specific heat of water is 4200 J kg^{-1}°C^{-1}.

L $\qquad\qquad$ T_F

C_W

[5]

Assume no heat is transferred into or out of the calorimeter once closed

Energy balance:

$$\underbrace{\overbrace{C(T_1-T_0) + MC_W(T_1-T_0)}^{34028\text{J}}}_{\text{Energy gain of calorimeter + water}} = \overbrace{MC_{LL}(T-T_F)}^{\text{Cooling of lead to freezing temp}} + \overbrace{ML}^{9292\text{J}} + \underbrace{MC_{SL}(T_F-T_1)}_{\text{Energy loss from cooling and phase changing lead}}$$

$\swarrow$ 16549.6J

$$\therefore \frac{C(T_1-T_0) + MC_W(T_1-T_0) - ML - MC_{SL}(T_F-T_1) + MC_{LL}T_F}{MC_{LL}} = T$$

$$T = \left[42.8(25-15) + 0.800\times4200(25-15) \underbrace{-\,0.400\times2.323\times10^4}_{} - 0.400\times137(327-25) + 0.400(158)(327)\right] \Big/ 0.400(158)$$

$$= \boxed{457°C}$$

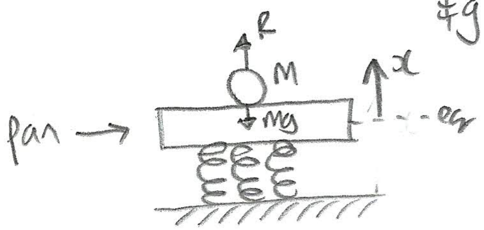
$\Big\{$ check: $MC_{LL}(T-T_F) = 34028 - 9292 - 16549.6$

$$\therefore \quad T = 327 + \frac{34028 - 9292 - 16549.6}{0.4\times158} = 457 \Big\}$$

(h) A small object of mass m rests on a scale-pan which is supported by a spring. The period of vertical oscillations is 0.50 s. When the amplitude of the oscillations exceeds the value, A, the mass leaves the scale-pan. Determine A.

[3]

$g = 9.81\ m/s^2$



Newton II for small mass (assume in contact with pan)

$$\boxed{m\ddot{x} = R - mg}$$

Now assume SHM of system, so $\boxed{\ddot{x} = -\left(\frac{2\pi}{T}\right)^2 x}$

$\therefore\ R = mg - mx\left(\frac{2\pi}{T}\right)^2$ $\qquad (\ddot{x} = -\omega^2 x)$

Mass leaves pan when $R = 0$, when $x = A$

So $A\left(\frac{2\pi}{T}\right)^2 = g$

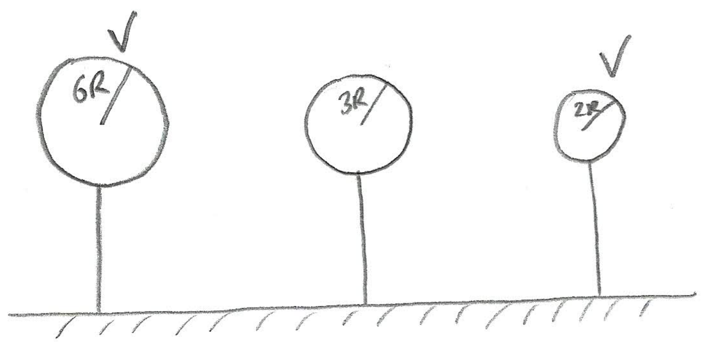
$\therefore\ \boxed{A = \frac{gT^2}{(2\pi)^2}}$ $\qquad \therefore\ A = \frac{9.81 \times 0.50^2}{(2\pi)^2}$ (m)

$\boxed{A = 6.21\ cm}$

(i) Uncharged metallic spheres of radii $6R$, $3R$ and $2R$ are mounted on insulated stands. The spheres of radii $2R$ and $6R$ are charged to a potential V above earth potential. All three spheres are then briefly joined by a copper wire. What, in terms of V, is the subsequent potential of the sphere of radius $3R$?

What fraction of the original total charge is held by the sphere of radius $3R$?

[5]



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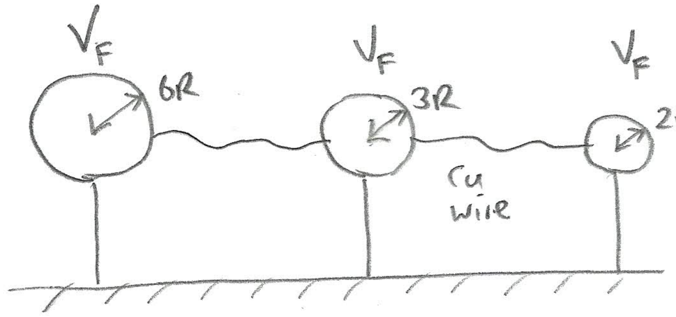


AFTER

charge will flow between spheres until no electric field between the wires $\Rightarrow$ each sphere must be at the same potential V_F

Using $C = 4\pi\varepsilon_0 a$ as the capacitance of a sphere of radius a and $Q = CV$, by conservation of charge:

$$4\pi\varepsilon_0(6R)V + 4\pi\varepsilon_0(2R)V = 4\pi\varepsilon_0 V_F(6R+3R+2R)$$

$$\therefore\ V_F = \frac{6+2}{6+3+2}V \Rightarrow \boxed{V_F = \frac{8}{11}V}$$

$$\therefore\ \frac{\text{Charge in 3R sphere}}{\text{original charge}} = \frac{4\pi\varepsilon_0(3R)\frac{8}{11}V}{4\pi\varepsilon_0(6R)V + 4\pi\varepsilon_0(2R)V}$$

$$= \frac{3\times 8/11}{6+2} = \boxed{\frac{3}{11}}$$

E_k

(j) The maximum kinetic energy of photoelectrons ejected from a tungsten surface by monochromatic light of wavelength 248 nm is 8.60×10^{-20} J.

λ

What is the value of the work function, W, of tungsten?

[3]

$$E_k + W = \underbrace{hc/\lambda}_{\text{photon energy}}$$

Photo electric effect.

So $$\boxed{W = \frac{hc}{\lambda} - E_k}$$

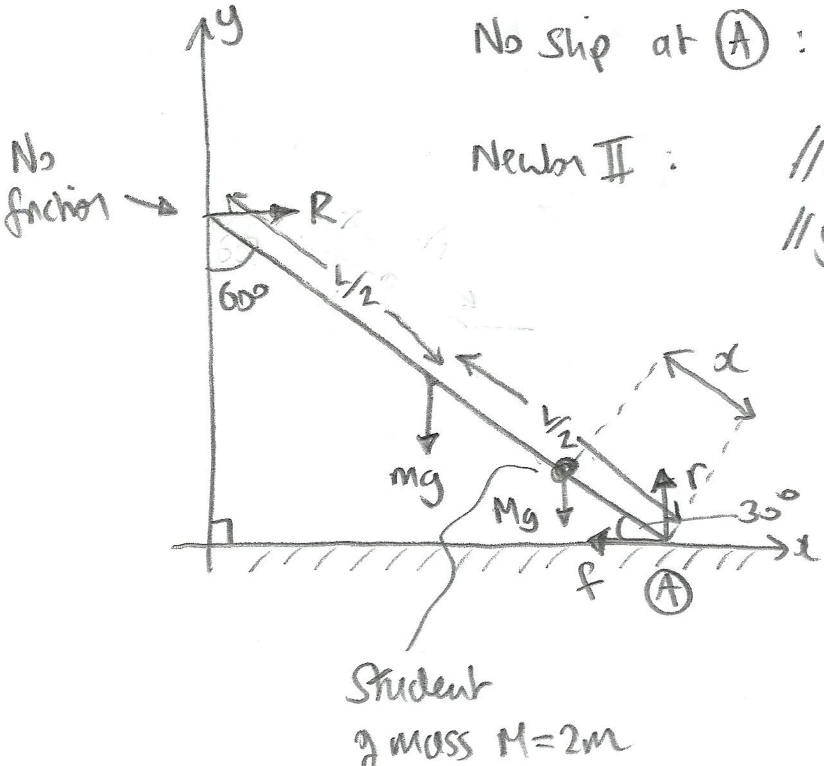
$$W = \frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{248 \times 10^{-9}} - 8.60 \times 10^{-20}$$

$$\boxed{W = 7.15 \times 10^{-19}\ \text{J}}$$

h Planck's constant

c speed of light.

(k) A ladder of length L and mass m, with a uniform density, rests against a frictionless vertical wall at an angle of 60° to the wall. The lower end rests on a flat surface with a coefficient of static friction of $\mu_s = 0.40$. A student with a mass $M = 2m$ attempts to climb the ladder. What fraction of the distance up the ladder will the student have reached when the ladder begins to slip? $\boxed{\text{Assume equilibrium}}$ **[5]**



No slip at A : $\boxed{f \leq \mu_s r}$ ①

Newton II : $/\!/x$: $0 = R - f$

$/\!/y$: $0 = r - mg - Mg$

So $\boxed{R = f}$ ②

and $\boxed{r = (m+M)g}$ ③

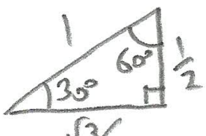
$\circlearrowleft$+ moments about A

$0 = R \times L\cos 60°$

$- mg\frac{L}{2}\cos 30° - Mgx\cos 30°$

$\therefore \quad R\frac{L}{2} = mg\frac{L}{2}\frac{\sqrt{3}}{2} + Mgx\frac{\sqrt{3}}{2}$

So $\boxed{R = mg\frac{\sqrt{3}}{2} + Mg\frac{x}{L}\sqrt{3}}$ ④



Now in ①

$f \leq \mu_s r$

$R \leq \mu_s (m+M)g$

$R = f$ $\qquad$ $r = (m+M)g$

$\therefore \quad mg\frac{\sqrt{3}}{2} + Mg\frac{x}{L}\sqrt{3} \leq \mu_s (m+M)g$

$$\frac{x}{L} \leq \frac{\mu_s (m+M)g - mg\sqrt{3}/2}{Mg\sqrt{3}}$$

$$\boxed{\frac{x}{L} \leq \mu_s\left(\frac{m}{M}+1\right)\frac{1}{\sqrt{3}} - \frac{1}{2}\frac{m}{M}}$$

$\therefore \quad \frac{x}{L} \leq \mu_s \left(\frac{m}{M} + 1 \right) \frac{1}{\sqrt{3}} - \frac{1}{2} \frac{m}{M}$

So if $M = 2m$, $\mu_s = 0.4$

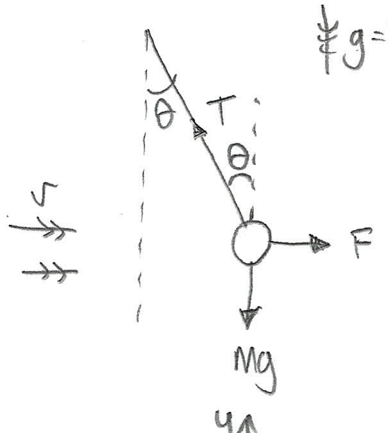
$\therefore \quad \frac{x}{L} \leq 0.4 \left(\frac{1}{2} + 1 \right) \frac{1}{\sqrt{3}} - \frac{1}{2} \left(\frac{1}{2} \right)$

$\frac{x}{L} \leq 0.096$ or $\boxed{9.6\%}$

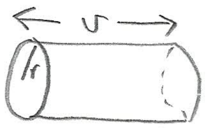
(I) A smooth ball of radius 10.0 cm, mass 0.600 kg, hangs by a weightless string from a support. What is the speed of a horizontal wind necessary to keep the string inclined at 39° to the vertical? Make the assumption that the wind speed drops to zero on collision with the ball. The density of the air is 1.293 kg m^{-3}.

[4]

$g = 9.81 \, m/s^2$



One second of air colliding with the ball has volume $\pi r^2 v$



∴ Rate of change of momentum of the ball (= the F) is $\pi r^2 v \rho \times v$

ie $\boxed{F = \pi r^2 \rho v^2}$

(In reality it will be some aerodynamic factor (< 1) times this)

To maintain equilibrium in x, y directions:

$T \sin\theta = F$ (x)

$T \cos\theta = Mg$ (y)

$$\therefore \tan\theta = \frac{\pi r^2 \rho v^2}{Mg}$$

$$\therefore \boxed{v = \sqrt{\frac{Mg \tan\theta}{\pi r^2 \rho}}}$$

$$\therefore v = \sqrt{\frac{0.600 \times 9.81 \times \tan 39^\circ}{\pi \times (0.1)^2 \times 1.293}} = \boxed{10.8 \, m/s}$$

A

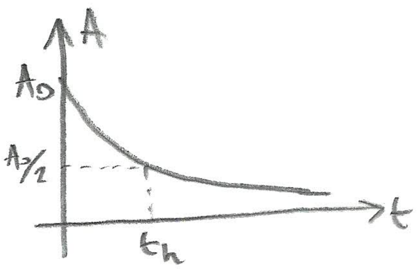
(m) The activity of polonium, Po, fell to one eighth of its initial value in 420 days. Calculate the half-life, t_h, of polonium.

Give the numerical values of a, b, c, d, e, and f in the nuclear equation

$$^{a}_{b}\mathrm{Po} \rightarrow {}^{c}_{d}\alpha + {}^{206}_{82}\mathrm{Pb} + {}^{e}_{f}\gamma$$

[4]

$$\boxed{A = A_0 \times 2^{-t/t_h}} \qquad \therefore\ 2^{t/t_h} = A_0/A$$



$$\therefore\ \frac{t}{t_h} \ln 2 = \ln A_0/A$$

$$\therefore\ \boxed{t_h = \frac{t \ln 2}{\ln A_0/A}}$$

So $$t_h = \frac{420 \ln 2}{\ln 8} \text{ (days)}$$ (ie, $A = A_0/8$ so $A_0/A = 8$)

$$\boxed{t_h = 140 \text{ days}}$$

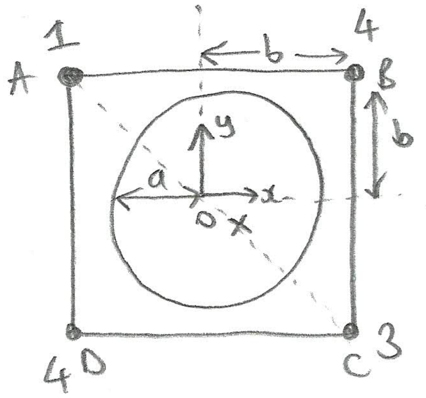
$\boxed{\begin{array}{l} c = 4 \\ d = 2 \end{array}}$ ie $^{4}_{2}\alpha$ (He nucleus)

$\boxed{\begin{array}{l} e = 0 \\ f = 0 \end{array}}$ Since γ ray has no charge (ie zero proton #) and no mass

$\boxed{\begin{array}{l} a = 210 \\ b = 84 \end{array}}$ so $\boxed{^{210}_{84}\mathrm{Po} \rightarrow {}^{4}_{2}\alpha + {}^{206}_{82}\mathrm{Pb} + {}^{0}_{0}\gamma}$

(n) Four masses of 1 kg, 4 kg, 3 kg, and 4 kg are arranged cyclically at the corners of a square of side $2b$ and centre O. A thin circular metal ring has radius a, mass 8 kg, and with the same centre O lies in the same plane as the square. Determine the position of the centre of mass of the system from O.

[3]



$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{\left[1\begin{pmatrix}-b\\ b\end{pmatrix} + 4\begin{pmatrix}b\\ b\end{pmatrix} + 3\begin{pmatrix}b\\ -b\end{pmatrix} + 4\begin{pmatrix}-b\\ -b\end{pmatrix}\right]}{1+4+3+4+8}$$

Note it doesn't matter if $a>b$ or $b>a$ since C.O.M of ring is at $\begin{pmatrix}0\\ 0\end{pmatrix}$

$$\therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{b}{20}\begin{pmatrix}-1+4+3-4\\ 1+4-3-4\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{b}{20}\begin{pmatrix}2\\ -2\end{pmatrix} = \boxed{\begin{pmatrix}b/10\\ -b/10\end{pmatrix}}$$

ie along line AC at a distance $\frac{\sqrt{2}}{10}b$ from O.

$f' = 1/T'$

(o) A trumpeter travelling in an open car sounds a note at 440 Hz. A stationary pedestrian directly in the path of the car hears a note at frequency 466 Hz. What is the speed of the car? The velocity of sound is 331 ms^{-1}. $f = 1/T$

c

[3]

Period of waves received by stationary observer is:

$$T = T' - \frac{vT'}{c}$$

T' ↑ period of waves emitted at source

$\frac{vT'}{c}$ ↖ reduction in time since the source is vT' closer over one period T'. If the sound waves travel at speed c then $\frac{vT'}{c}$ is the corresponding time difference.

$$\therefore \frac{1}{f} = \frac{1}{f'}\left(1 - \frac{v}{c}\right)$$

$$\therefore \frac{f'}{f} = 1 - \frac{v}{c}$$ ← classical Doppler shift formula. *

$$\therefore \frac{v}{c} = 1 - \frac{f'}{f}$$

$$\therefore \boxed{v = c\left(1 - \frac{f'}{f}\right)}$$

$$\therefore v = 331 \times \left(1 - \frac{440}{466}\right)$$

$$\boxed{v = 18.5 \text{ m/s}}$$

[* let $\Delta f = f - f'$ $\therefore \frac{\Delta f}{f'} = \frac{f}{f'} - 1$

$\therefore$ Since $\frac{f}{f'} = \left(1 - \frac{v}{c}\right)^{-1} = \left(\frac{c-v}{c}\right)^{-1} = \frac{c}{c-v}$ $\therefore \frac{\Delta f}{f'} = \frac{c}{c-v} - 1$

$\therefore \frac{\Delta f}{f'} = \frac{c - c + v}{c - v}$ $\therefore \boxed{\Delta f = \frac{v}{c-v} f'}$

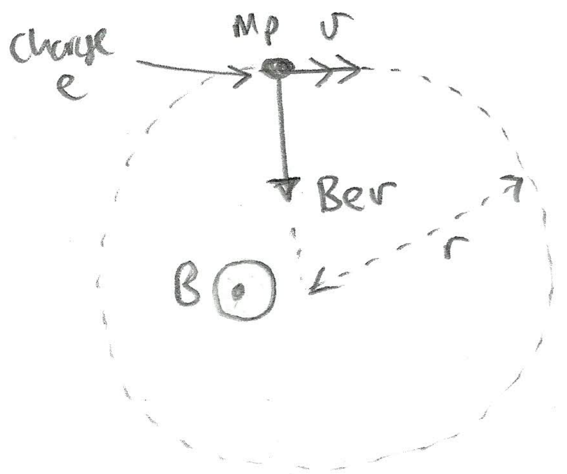
Note if $v \ll c$
$\Delta f \approx \frac{v}{c} f'$]

(p) A beam of protons is accelerated from rest through a potential difference of 2000 V ✓ and enters a uniform magnetic field which is perpendicular to the direction of the proton beam. If the flux density is 0.2000 T, calculate the radius of the path of the beam.

How is the result modified for deuterons?

↑ mass of $2M_p$

[4]



$$\boxed{\text{Lorentz force} \quad \underline{F} = q(\underline{E} + \underline{v} \times \underline{B})}$$

let $q = e$ $\quad e = 1.6 \times 10^{-19}\,C$

$\underline{E} = 0$ ie no electric field

let $\underline{B}$ be uniform out of the page. $\therefore$ $\underline{v} \times \underline{B}$ is radially inwards.

Circular motion, $\therefore$ by Newton II:

$$\boxed{\frac{M_p v^2}{r} = Bev} \qquad \therefore \quad \frac{M_p v}{Be} = r$$

Now assuming $v \ll c$, (Classical physics)

$$\frac{1}{2} M_p v^2 = eV \qquad \therefore \quad v = \sqrt{\frac{2eV}{M_p}}$$

So $$\boxed{r = \frac{M_p}{Be}\sqrt{\frac{2eV}{M_p}}}$$

ie write in terms of one power of M_p

Better, given second part:

$$r = \frac{1}{B}\sqrt{\frac{M_p^2}{e^2} \cdot \frac{2eV}{M_p}}$$

$$\therefore \quad \boxed{r = \frac{1}{B}\sqrt{\frac{2M_pV}{e}}}$$

ie $r = \frac{1}{0.200}\sqrt{\frac{2 \times 1.67 \times 10^{-27} \times 2000}{1.6 \times 10^{-19}}}$

$= 0.0323$ m

$= \boxed{3.23 \text{ cm}}$

So if $M_p \to 2M_p$, $r \to \sqrt{2}\, r$

ie $\boxed{4.57 \text{ cm}}$

(q) A particle, mass m, slides down the smooth track, **Figure 1(q)**, from a height H under gravity. It is to complete a circular trajectory of radius ~~R~~ r when reaching its lowest point. Determine the smallest value of H.

[3]

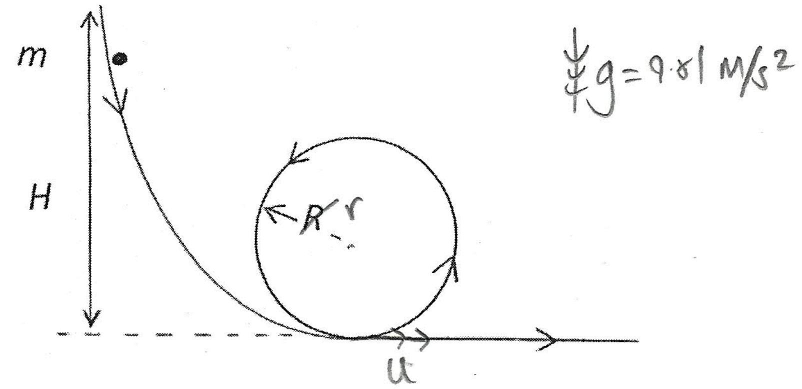
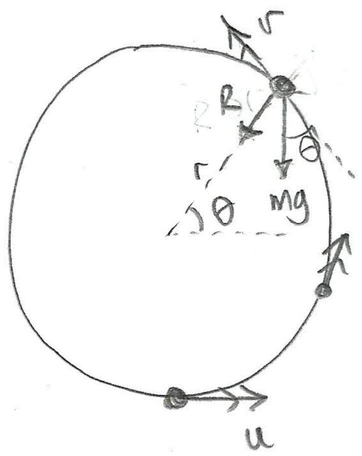


Figure 1(q).

Conservation of energy: $\boxed{mgH = \frac{1}{2}mu^2}$



Let R be the reaction force on the particle at angle θ from horizontal

Newton II radially:

$$\frac{mv^2}{r} = R + mg\sin\theta$$

$$\therefore \boxed{R = \frac{mv^2}{r} - mg\sin\theta}$$

Now to complete a circular trajectory $R > 0$

The minimum value of R is when $\theta = 90^\circ$ i.e. $\sin\theta = 1$

So $\frac{mv^2}{r} - mg > 0 \quad \therefore \boxed{v^2 > rg}$ $\{v$ is at $\theta = 90^\circ\}$

Now by conservation of energy $\frac{1}{2}mv^2 + mg(2r) = \frac{1}{2}mu^2$

Since v above corresponds to

$= mgH$

$$\therefore v^2 = 2gH - 4gr$$

Hence $2gH - 4gr > rg$

$$\therefore \boxed{H > \frac{5r}{2}}$$