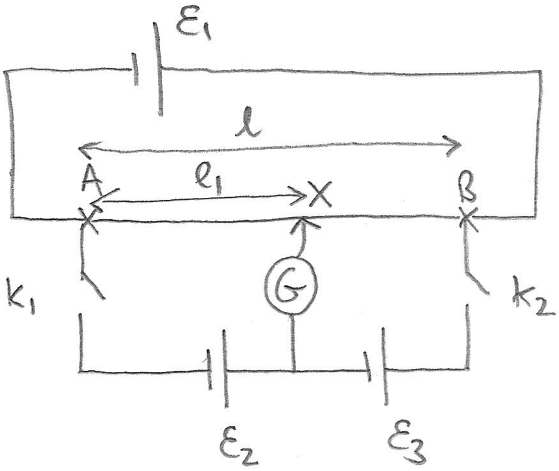
BPhO1 2016 Section 2

[uniform wire between A and B]

Q2/



a) k_1, k_2 open

(i) So PD between X and A (ie 'potential at X') is $\boxed{\mathcal{E} = \frac{l_1}{l}\mathcal{E}_1}$

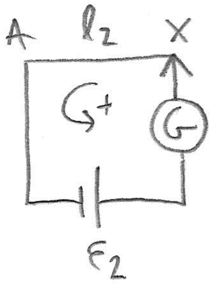
(Since resistance of a wire $\propto$ length)

(ii) Now k_1, k_2 are closed. X is now moved to l_2 from A

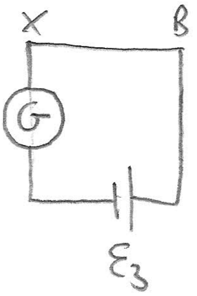
↳ So $\boxed{\mathcal{E} = \frac{l_2}{l}\mathcal{E}_1}$

We are given $\boxed{\mathcal{E}_1 = \mathcal{E}_2 + \mathcal{E}_3}$

Applying Kirchoff II to the loops left and right of the galvanometer:



$\boxed{\mathcal{E}_2 = \frac{l_2}{l}\mathcal{E}_1}$



$\mathcal{E}_3 = \mathcal{E}_1 - \mathcal{E}$

$\boxed{\mathcal{E}_3 = \mathcal{E}_1\left(1 - \frac{l_2}{l}\right)}$

[Not this guarantees $\mathcal{E}_1 = \mathcal{E}_2 + \mathcal{E}_3$ → So what would happen if this was not true? Would internal resistance of the wires mean Kirchoff II satisfied?]

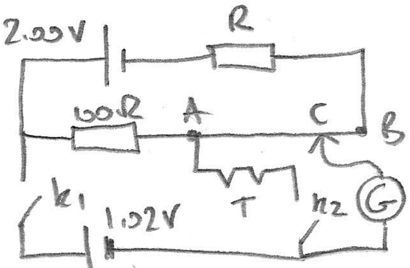
b) AB is a wire of length $\boxed{l = 1.00\,m}$ $\boxed{R_{AB} = 2.00\,\Omega}$

We are given:

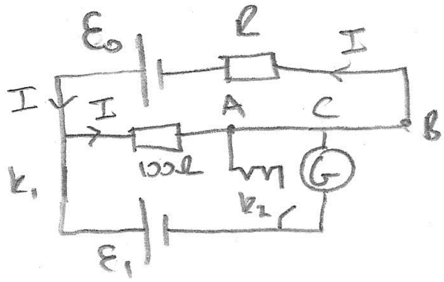
(I) * No current through G k_1 closed k_2 open $AC = 0.90\,m$

(II) * " " " G k_1 open k_2 closed $AC = 0.45\,m$

T is a thermocouple



(I)



$\varepsilon_0 = 2.00V$
$\varepsilon_1 = 1.02V$
(EMFs)

If current through G is zero then all current is in the upper loop

Kirchoff II: (upper loop) $\varepsilon_0 = I(100 + R_{AB} + R)$

$R_{AB} = 2.00\,\Omega$ so $\boxed{\varepsilon_0 = I(102 + R)}$ ①

Kirchoff II (lower loop) $\varepsilon_1 = I(100 + R_{AC})$

$R_{AC} = \frac{0.90}{1.00} \times 2.00\,\Omega = 1.8\,\Omega$

[AC = 0.90m]

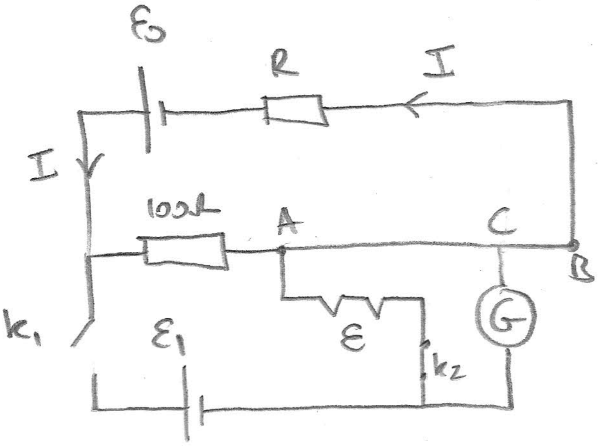
$\therefore \boxed{\varepsilon_1 = I(101.8)}$ ②

①/② : $\frac{\varepsilon_0}{\varepsilon_1} = \frac{102 + R}{101.8}$ $\therefore R = 101.8 \frac{\varepsilon_0}{\varepsilon_1} - 102$

$R = 101.8 \times \frac{2.00}{1.02} - 102$

$\boxed{R = 97.6\,\Omega}$

(II)



[AC = 0.45 m]

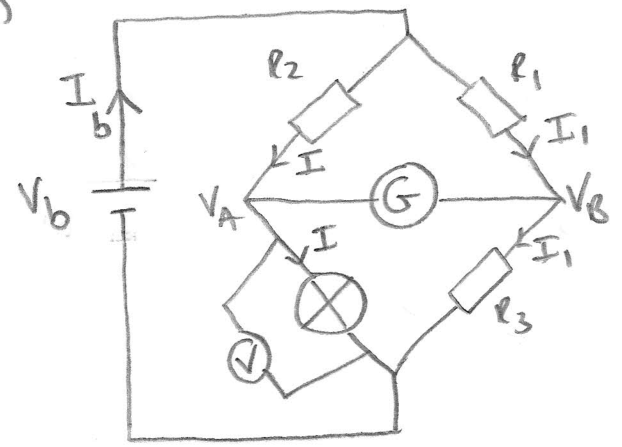
Kirchoff II: (upper loop) $\boxed{\varepsilon_0 = I(102 + R)}$

[ie same as before]

Kirchoff II (thermocouple loop) $\boxed{\varepsilon = I R_{AC}}$ $R_{AC} = \frac{0.45}{1.00} \times 2\,\Omega = \boxed{0.9\,\Omega}$

$\therefore \frac{\varepsilon_0}{\varepsilon} = \frac{102 + R}{0.9}$ $\therefore \varepsilon = \frac{\varepsilon_0 \times 0.9}{102 + 97.6} = \boxed{9.02 \times 10^{-3}\text{ Volts}}$

c)



[Wheatstone Bridge]

$V = 2I + 8I^2$

$R_{\otimes} = V/I = \boxed{2 + 8I}$

$R_2 = R_1 = R_3 = 4\,\Omega$

Assume bridge is balanced
ie no current flows through G

$\therefore\ V_A = V_B$

Now: $V_A = \dfrac{V_b R_{\otimes}}{R_{\otimes} + 4}$

and $V_B = V_b/2$ (Since $R_1 = R_3$)

$\therefore\ \dfrac{R_{\otimes}}{R_{\otimes} + 4} = \dfrac{1}{2}$ $\quad\therefore\quad R_{\otimes} = \frac{1}{2} R_{\otimes} + 2$

$\frac{1}{2} R_{\otimes} = 2$

$\boxed{R_{\otimes} = 4\ \Omega}$

We could have reasoned this straight away!

$R_{\otimes} = 2 + 8I$

So $2 + 8I = 4$

$8I = 2$

$\boxed{I = 0.25\,A}$

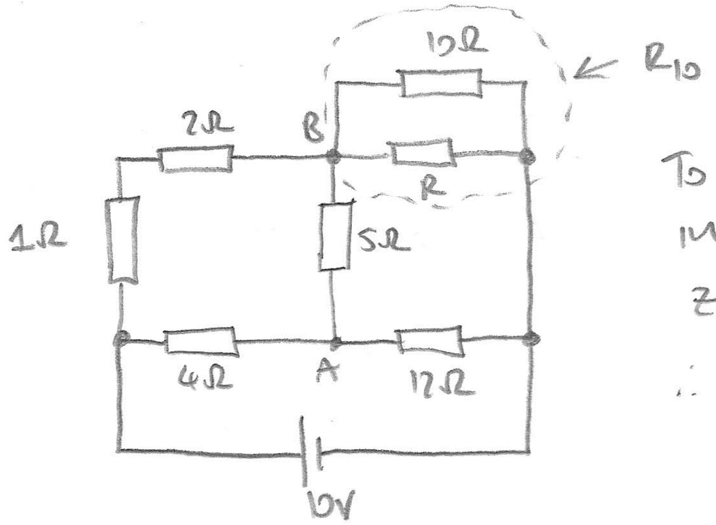
Total resistance of bridge is $\dfrac{1}{\frac{1}{4+4} + \frac{1}{4+4}} = \boxed{4\,\Omega}$

clearly $\boxed{I_b = 2I}$ Since all resistances the same

$\therefore\ V_b = 0.5 \times 4 = \boxed{2\,V}$

This is all that is needed

d)



To minimize heat generated in 5Ω resistor we want zero current passing through it.

$\therefore \; V_{AB} = 0$

$V_{AB} = V_A - V_B$

$$V_B = \frac{R_{10}}{1+2+R_{10}} \times 10\,V$$

$$V_A = \frac{12}{12+4} \times 10V$$

$$\therefore \; \frac{R_{10}}{3+R_{10}} - \frac{12}{16} = 0$$

ie lower and upper loops (potential divider idea)

So $\quad R_{10} = \frac{3}{4}(3+R_{10})$

$$R_{10}\left(1-\frac{3}{4}\right) = \frac{9}{4}$$

$$R_{10}\,\frac{1}{4} = \frac{9}{4}$$

$$\boxed{R_{10} = 9\Omega}$$

$$R_{10} = \frac{1}{\frac{1}{R}+\frac{1}{10}}$$

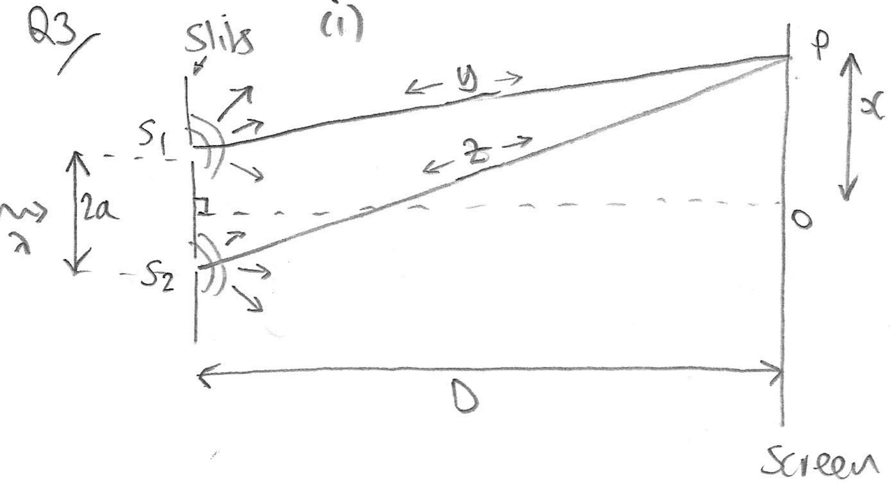
So $\quad \frac{1}{R} + \frac{1}{10} = \frac{1}{9}$

$$\frac{1}{R} = \frac{1}{9} - \frac{1}{10}$$

$$\frac{1}{R} = \frac{10-9}{90} = \frac{1}{90}$$

$$\therefore \; \boxed{R = 90\Omega}$$

Q3/ (i)



ie assume $D \gg a, \lambda$

plane wave from S_1

plane wave from S_2

{ignore spherical wave front effects}

Wave amplitude at P is $\psi = A\cos(ky - \omega t) + A\cos(kz - \omega t)$

$k = \frac{2\pi}{\lambda}$ wavenumber.

Maxima when $\underbrace{k(z-y)}_{\text{phase difference}} = 2\pi n$ where $n \in$ integer

Pythagoras: $z^2 = (x+a)^2 + D^2$

$y^2 = (x-a)^2 + D^2$

$\therefore \quad z^2 = D^2\left(1 + \left(\frac{x+a}{D}\right)^2\right)$

$z = D\left(1 + \left(\frac{x+a}{D}\right)^2\right)^{\frac{1}{2}}$ if $x \ll D$

then $\boxed{z \simeq D\left(1 + \frac{1}{2}\left(\frac{x+a}{D}\right)^2\right)}$

similarly $y = D\left(1 + \left(\frac{x-a}{D}\right)^2\right)^{\frac{1}{2}}$

So $\boxed{y \simeq D\left(1 + \frac{1}{2}\left(\frac{x-a}{D}\right)^2\right)}$

$\therefore \quad z - y \simeq \frac{1}{2D}\left\{(x+a)^2 - (x-a)^2\right\}$

$= \frac{1}{2D}\left(\cancel{x^2} + 2ax + \cancel{a^2} - \cancel{x^2} + 2ax - \cancel{a^2}\right) = \frac{4ax}{2D}$

$$\therefore \boxed{z - y \approx \frac{2ax}{D}}$$

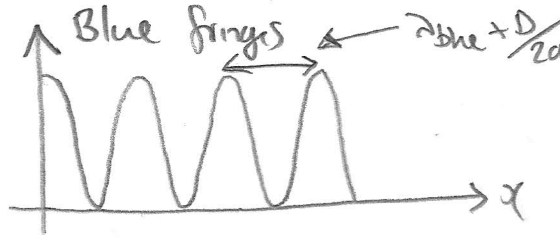
$\therefore$ Maxima when $\frac{2\pi}{\lambda}\left(\frac{2ax}{D}\right) = 2\pi n \quad \therefore \frac{2ax}{D} = n\lambda$

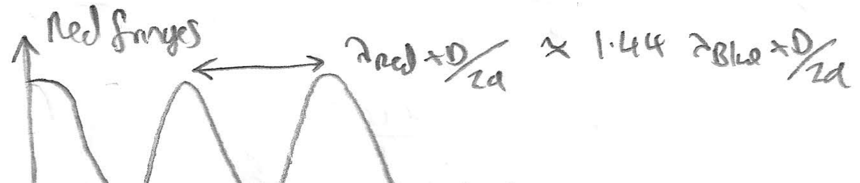
"fringes" are at: $\Rightarrow \quad x_n = \frac{n\lambda D}{2a}$

So fringe spacing $x_{n+1} - x_n = \Delta x = \boxed{\frac{\lambda D}{2a}}$

(ii) White light contains a spectrum of all optical wavelengths.
At $n=0$, all colours superpose, so white spot at 0.
(i.e. $x=0$).

You would then see coloured fringes, with blue at more regular intervals than red.





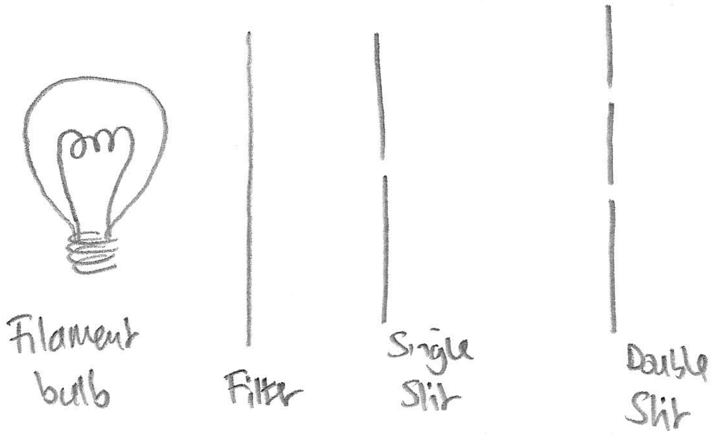
λ_{Blue} : 450 – 490 nm i.e. average of 470 nm

λ_{Red} : 680 nm ↑ 635 – 700 nm

Note λ_{Green} = 560 – 520 nm

$\frac{680}{470} = 1.44$

(iii)



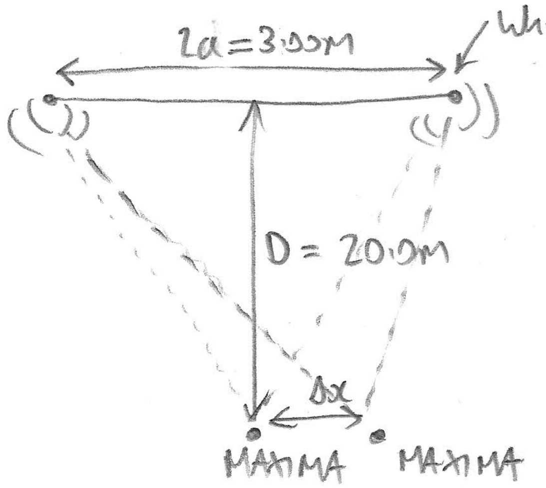
ie it is RANDOM

* Bulb is an $\boxed{\text{incoherent source}}$ which means $\boxed{\text{no phase relationship}}$ between any photons produced. The filter selects a given λ but subsequent photons are not in phase. $\therefore$ $\psi \neq Ae^{i(kx-\omega t)}$ for the waves arriving at the single slit.

there is a random phase term

However, for any given photon, diffraction by the single slit will mean the waves arriving at the double slit will have the same 'source' phase ie although subsequent photons will have random phases, the illumination of the double slit will be coherent.

b)



whistles, emitting frequency of sound $\boxed{f = 2.00\,kHz}$

Same situation as in part (a)

so $\boxed{\Delta x = \frac{\lambda D}{2a}}$

Now $\boxed{c = f\lambda}$ where c is the speed of sound

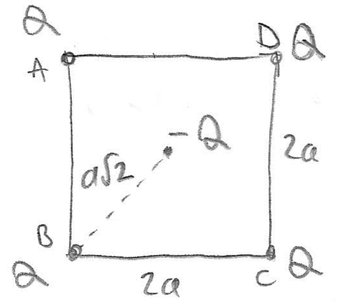
Now $\lambda = \frac{2a\Delta x}{D}$ so $\boxed{c = \frac{2a\Delta x f}{D}}$ $\Delta x = 1.14\,m$

gives this

$\therefore$ $c = \frac{3.00 \times 1.14 \times 2000}{20.0} = \boxed{342\,m/s}$

Q4/

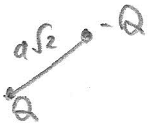
a) (i)



Total potential energy is

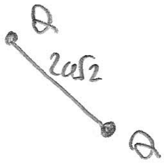
$$V = 4\times\frac{Q^2}{4\pi\varepsilon_0(2a)}$$

$$-\frac{4\times Q^2}{4\pi\varepsilon_0(a\sqrt{2})}$$



ie sum all possible distinct interactions

$$+\frac{2\times Q^2}{4\pi\varepsilon_0(2a\sqrt{2})}$$



$$\therefore V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{\frac{4}{2}-\frac{4}{\sqrt{2}}+\frac{2}{2\sqrt{2}}\right\}$$

$$V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{2-\frac{4}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right\}$$

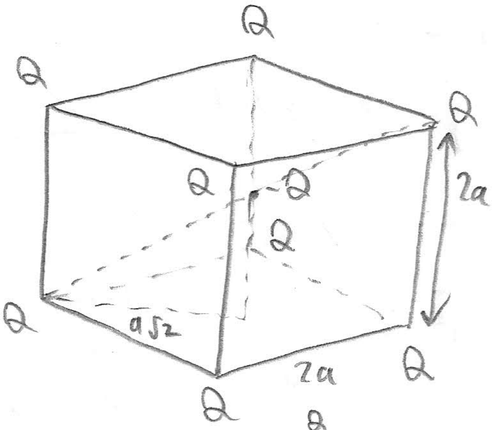
$$V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{2-\frac{3}{\sqrt{2}}\right\}$$

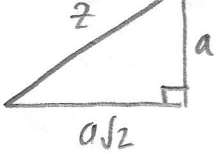
$$V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{\frac{2\sqrt{2}-3}{\sqrt{2}}\right\} \qquad (2\sqrt{2}\simeq 2.83)$$

$$\boxed{V = -\frac{Q^2}{4\pi\varepsilon_0 a}\left(\frac{3-2\sqrt{2}}{\sqrt{2}}\right)}$$ I prefer $\frac{3}{\sqrt{2}}-2\simeq 0.121$

$$\boxed{V \approx -\frac{Q^2}{4\pi\varepsilon_0 a}\times 0.121}$$

(ii)

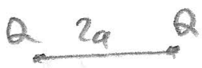




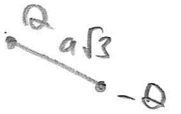
$z^2 = a^2 + 2a^2$

So $\boxed{z = a\sqrt{3}}$

$$\text{So } V = 12\times\frac{Q^2}{4\pi\varepsilon_0(2a)}$$



$$-8\times\frac{Q^2}{4\pi\varepsilon_0(a\sqrt{3})}$$



$$+12\times\frac{Q^2}{4\pi\varepsilon_0(a\sqrt{2}\times 2)}$$

↑ Two diagonals / face

$$+4\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{2a\sqrt{3}}$$

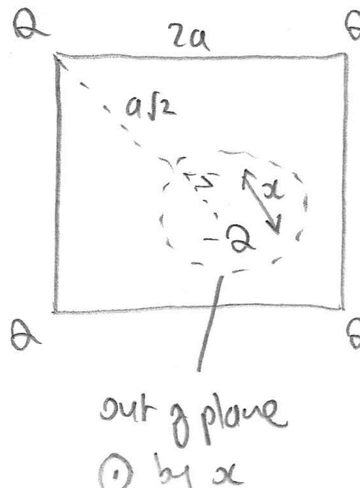
8

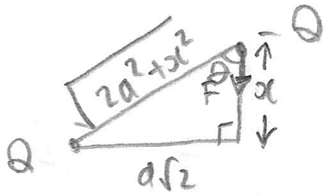
$$\therefore \quad V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{\frac{12}{2} - \frac{8}{\sqrt{3}} + \frac{12}{2\sqrt{2}} + \frac{4}{2\sqrt{3}}\right\}$$

$$\boxed{V = \frac{Q^2}{4\pi\varepsilon_0 a}\left\{6 - \frac{6}{\sqrt{3}} + \frac{6}{\sqrt{2}}\right\}}$$

$$\boxed{V \simeq \frac{Q^2}{4\pi\varepsilon_0 a} \times 6.78}$$

b)





$(\times 4)$

By symmetry force is clearly only towards the plane (in equilibrium // plane)

$$F = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{2a^2+x^2}\cos\theta \times 4$$

Now $x = \sqrt{2a^2+x^2}\cos\theta$

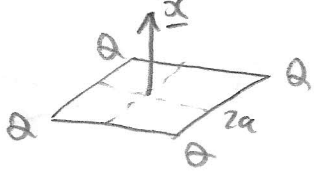
$$\therefore \quad F = \frac{4Q^2}{4\pi\varepsilon_0}\frac{1}{2a^2+x^2}\frac{x}{\sqrt{2a^2+x^2}}$$

{ Here $x>0$ and everything +ve }

$$\therefore \quad \boxed{|F| = \frac{4Q^2}{4\pi\varepsilon_0}\frac{|x|}{(2a^2+x^2)^{3/2}}}$$

Now if $\hat{x}$ is a unit vector out of the plane ie $|\hat{x}| = 1$

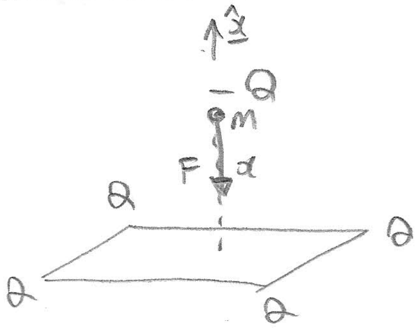
Force on $-Q$ is



$$\boxed{\underline{F} = -\frac{4Q^2}{4\pi\varepsilon_0}\frac{x}{(2a^2+x^2)^{3/2}}\hat{x}}$$

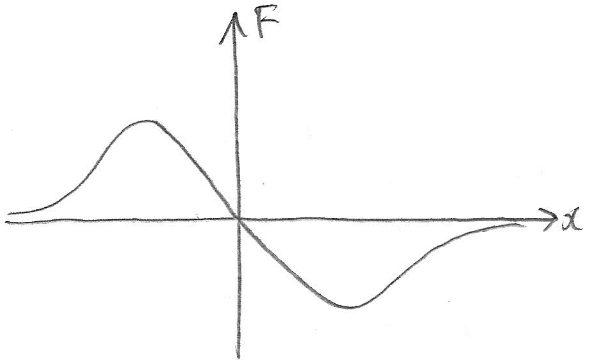
Now x can be -ve or positive ie $-Q$ below the plane too.

let $\underline{F} = F\hat{\underline{x}}$ so $\boxed{F = \frac{-4Q^2}{4\pi\varepsilon_0}\frac{x}{(2a^2+x^2)^{3/2}}}$



Newton II, given charge has mass m

$$\boxed{m\ddot{x} = -\frac{4Q^2}{4\pi\varepsilon_0}\frac{x}{(2a^2+x^2)^{3/2}}}$$



when $x^2 \ll 2a^2$

$$-\frac{x}{(2a^2+x^2)^{3/2}} \propto \frac{-x}{(2a^2)^{3/2}}$$

(i.e. linear)

when $|x| \to \infty$

$$\frac{-x}{(2a^2+x^2)^{3/2}} \to \frac{-x}{x^3} \to -\frac{1}{x^2}$$

i.e. $\to 0$

Now $(2a^2+x^2)^{3/2} = (2a^2)^{3/2}\left(1+\frac{x^2}{2a^2}\right)^{3/2}$

So $F = -\frac{4Q^2}{4\pi\varepsilon_0}(2a^2)^{-3/2}x\left(1 - \frac{3}{2}\frac{x^2}{2a^2} + \ldots\right)$

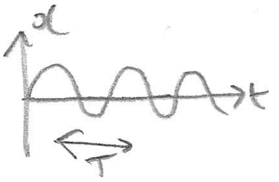
*

To first order in x

$$F \simeq -\frac{Q^2}{\pi\varepsilon_0}(2a^2)^{-3/2}x$$

$$\therefore \ddot{x} \simeq -\frac{Q^2}{\pi m\varepsilon_0}\frac{1}{\sqrt{8a^6}}x$$

$$\boxed{\ddot{x} \simeq -\frac{Q^2}{2\sqrt{2}\pi m\varepsilon_0 a^3}x}$$



T is the period

SHM : $\ddot{x} = -\left(\frac{2\pi}{T}\right)^2 x$

So $\frac{2\pi}{T} = \sqrt{\frac{Q^2}{2\sqrt{2}\pi m\varepsilon_0 a^3}}$ $\therefore$ $T = \sqrt{\frac{4\pi^2\, 2\sqrt{2}\,\pi m\varepsilon_0 a^3}{Q^2}}$

$$\boxed{T = \sqrt{\frac{8\pi^3\sqrt{2}\, m\varepsilon_0 a^3}{Q^2}}}$$

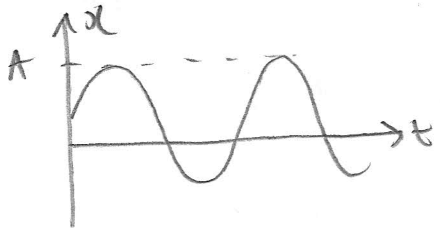
Now from Binomial expansion of F

$$1 \gg \frac{3}{2}\frac{x^2}{2a^2}$$ for linear approximation to be valid

$$\therefore \quad x \ll \sqrt{\frac{4a^2}{3}}$$

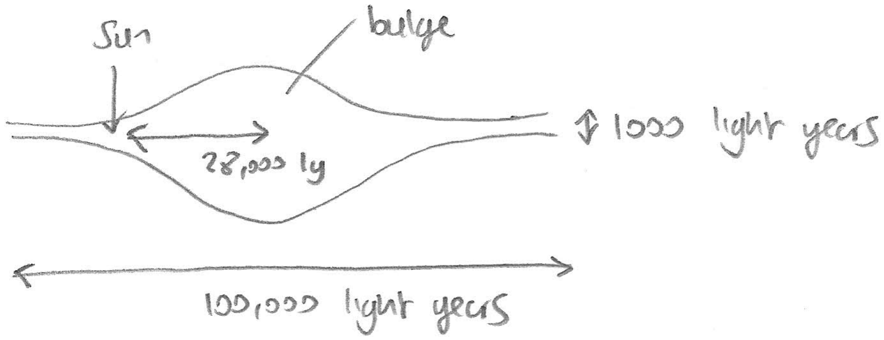
$$x \ll \frac{2}{\sqrt{3}}a$$

So if $x = A\cos(\omega t - \phi)$ is the SHM solution

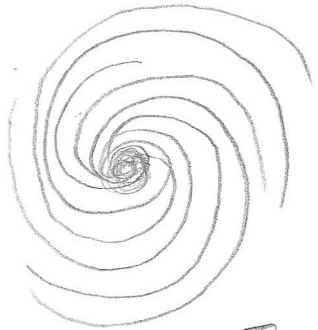


Amplitude $\boxed{A \ll \frac{2a}{\sqrt{3}}}$

Q5/ [Galaxy rotation curve]

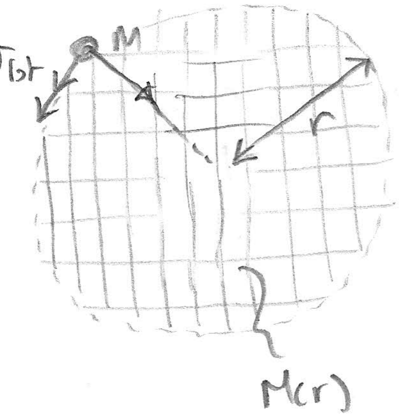


Milky way Schematic (Side on view)



Top view

a) Consider a star of mass m orbiting a spherically symmetric mass M enclosed within orbital radius r



Newton II (assume uniform circular motion)

$$\frac{m v_{rot}^2}{r} = \frac{GMm}{r^2}$$

So $$\boxed{v_{rot} = \sqrt{\frac{GM(r)}{r}}}$$

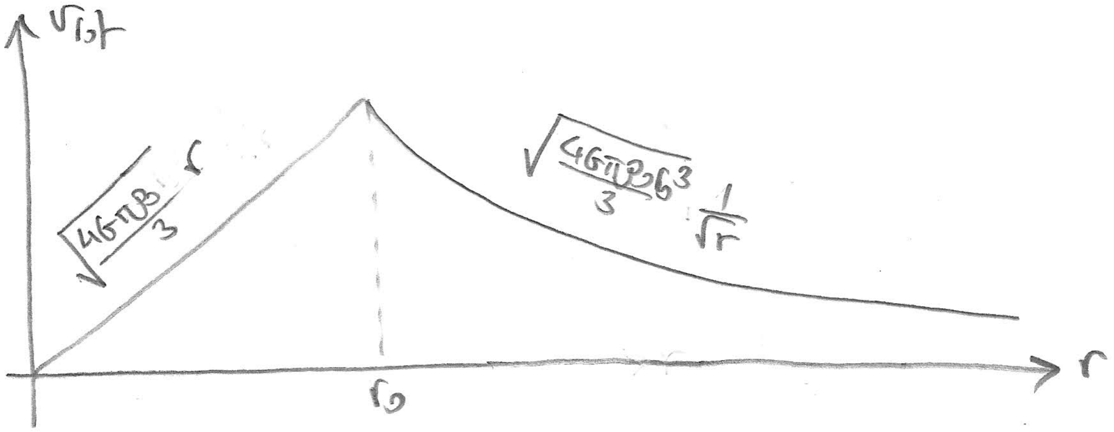
b) Assume $M(r)$ constitutes a spherical bulge of radius r_0 and uniform density ρ_0

So $M(r) = \frac{4}{3}\pi r_0^3 \rho_0 \qquad r \geq r_0$

$M(r) = \frac{4}{3}\pi r^3 \rho_0 \qquad r < r_0$

$$\therefore \quad v_{rot} = \begin{cases} \sqrt{\frac{4G\pi\rho_0}{3} \, r_0^3/r} & r \geq r_0 \\ \sqrt{\frac{4G\pi\rho_0}{3} r^2} & r < r_0 \end{cases}$$

ie $\quad v_{rot} = \begin{cases} \sqrt{\dfrac{4G\pi\rho_0 r_0^3}{3}} \dfrac{1}{\sqrt{r}} & r \geq r_0 \\ \sqrt{\dfrac{4G\pi\rho_0}{3}}\, r & r \leq r_0 \end{cases}$

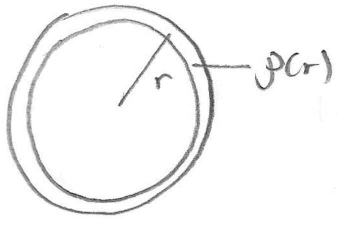


c) In reality, astronomers observe $v_{rot} \rightarrow$ constant for $r > r_0$

Postulate $\boxed{\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}}$

$v_{rot} = \sqrt{\dfrac{GM(r)}{r}} \qquad M(r) = \displaystyle\int_0^r \rho(r) \times 4\pi r^2 \, dr$

$\therefore M(r) = \displaystyle\int_0^r \frac{4\pi\rho_0}{r_0^{-\alpha}} r^{2-\alpha} \, dr$



$M(r) = 4\pi\rho_0 r_0^{\alpha} \left[\dfrac{r^{3-\alpha}}{3-\alpha}\right]_0^r$

ie consider spherical shells of density $\rho(r)$

$$\boxed{M(r) = \frac{4\pi\rho_0 r_0^{\alpha}}{3-\alpha} r^{3-\alpha}}$$

$$\therefore \boxed{v_{rot} = \sqrt{\frac{4\pi G\rho_0 r_0^{\alpha} r^{2-\alpha}}{3-\alpha}}}$$

So if $\alpha = 2$

$\Rightarrow v_{rot} = \sqrt{\dfrac{4\pi G\rho_0 r_0^{\alpha}}{3-2}}$

ie v_{rot} = constant

$$\boxed{v_{rot} = \sqrt{4\pi G \rho_0 b^2}}$$

(Note this implies v_{rot} = constant $r < b$ as well as $r > b$).

d) At $r = 2.8 \times 10^5$ ly, Sun has an orbital velocity of $v_{rot} = 220$ km/s. $\sqrt{\frac{GM}{r}} = v_{exp} = 70$ km/s

based upon visible mass. Let M be visible mass and M_D be Dark Matter.

$$\therefore \quad v_{rot} = \sqrt{\frac{G(M+M_D)}{r}}$$

$$v_{exp} = \sqrt{\frac{GM}{r}}$$

$$\therefore \quad v_{rot}^2 / v_{exp}^2 = \frac{M+M_D}{M} \qquad \therefore \quad M_D = \left(\frac{v_{rot}^2}{v_{exp}^2} - 1\right) M$$

$$= 1 + \frac{M_D}{M}$$

Now $M = v_{exp}^2 \, r/G$ $\quad \therefore \quad \boxed{M_D = \left(v_{rot}^2 - v_{exp}^2\right) \frac{r}{G}}$

Total mass $M + M_D = v_{rot}^2 \, r/G$

So % dark matter $\eta = \dfrac{100 M_D}{M+M_D}$

$$\boxed{\eta = 100\left(1 - \frac{v_{exp}^2}{v_{rot}^2}\right)}$$

So $\eta = \left(1 - 70^2/220^2\right) \times 100$

$$\boxed{\eta = 89.9\%}$$

$$\frac{M_D}{M_\odot} = \left(v_{rot}^2 - v_{exp}^2\right) \times \frac{r}{G M_\odot}$$

$$= \left[\left(220\times10^3\right)^2 - \left(70\times10^3\right)^2\right] \times \frac{2.8\times10^5 \times \overbrace{2.998\times10^8 \times 365\times24\times3600}^{1\text{ light year}/m}}{6.67\times10^{-11}\times 1.99\times10^{30}}$$

$$= \boxed{8.7\times10^{11}} \quad \text{ie dark matter mass}$$

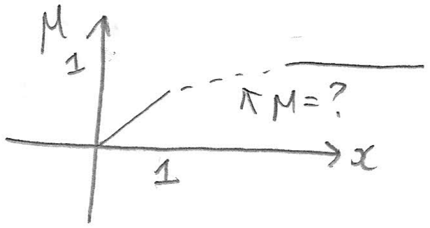
[so total galactic mass is $\simeq \dfrac{8.7\times10^{11}}{0.899} = \boxed{9.7\times10^{11} M_\odot}$]

Also $\dfrac{M}{M_\odot} = v_{exp}^2 \times \dfrac{r}{G M_\odot} = \boxed{9.8\times10^{10}}$ ie visible mass

e) Modified Newtonian Dynamics (MOND)

$$\Rightarrow \underset{\text{Force of gravity}}{F} = \underbrace{m\mu\left(\frac{a}{a_0}\right)a}_{\text{modified mass}\times\text{acceleration}}$$

$$\mu(x) = \begin{cases} x & x \ll 1 \\ 1 & x \gg 1 \end{cases}$$



when $a/a_0 \ll 1 \Rightarrow \mu\left(\frac{a}{a_0}\right) \rightarrow \frac{a}{a_0}$

$$\therefore \boxed{F \simeq m\,a^2/a_0}$$

For circular motion $a = v_{rot}^2/r$

So MOND:
$$\frac{G M(r) \cancel{m}}{\cancel{r^2}} = \frac{\cancel{m}\, v_{rot}^4}{a_0 \cancel{r^2}}$$

$$\therefore \boxed{\left(G M a_0\right)^{1/4} = v_{rot}}$$

Now if $\alpha = 2$ as before with $\rho(r) = \rho_0\left(\frac{r}{r_0}\right)^{-\alpha}$

$M(r) = 4\pi\rho_0 r_0^2 r$

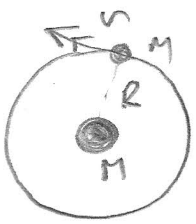
so $v_{rot} \neq$ constant.

$\therefore$ Assume $M(r) \rightarrow$ constant at large radii *

$\therefore v_{rot} \rightarrow$ constant according to MOND.

* ie $\alpha > 3$ if exponential model is correct.

Q6/



a) Kepler III : $\dfrac{R^\alpha}{T^\gamma} = \text{constant}$

(we know that $\gamma = 2$, $\alpha = 3$ ... but let's obtain these from the provided data)

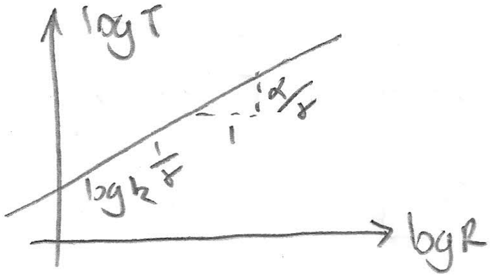
$R_J = 7.76 \times 10^{11}\,m$ $\quad T_J = 11.8$ years $\quad$ (Jupiter)

$R_E = 1.50 \times 10^{11}\,m$ $\quad T_E = 1.0$ years $\quad$ (Earth)

So Kepler III $\Rightarrow$ $R^\alpha = kT^\gamma$ $\quad$ (k = constant)

$\Rightarrow \quad R^{\frac{\alpha}{\gamma}} = k^{\frac{1}{\gamma}}\,T$

$\Rightarrow \quad \dfrac{\alpha}{\gamma}\log R = \log k^{\frac{1}{\gamma}} + \log T$



ie expect a graph of $\log T$ vs $\log R$ to be linear, with gradient $\frac{\alpha}{\gamma}$

Using the data:

$$\frac{\alpha}{\gamma} = \frac{\log T_J - \log T_E}{\log R_J - \log R_E}$$

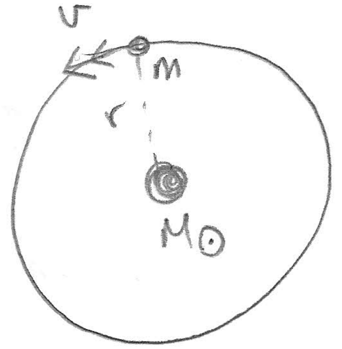
$$\frac{\alpha}{\gamma} = \frac{\log 11.8 - \log 1.0}{\log(7.76\times10^{11}) - \log(1.50\times10^{11})}$$

$$= \frac{\log 11.8}{\log(7.76/1.50)}$$

$$= \boxed{1.50}$$

(As expected since $\frac{\alpha}{\gamma} = \frac{3}{2}$).

(ii)



Assuming a circular orbit

$$\frac{mv^2}{r} = \frac{GmM_\odot}{r^2}$$

$$\therefore \quad M_\odot = \frac{rv^2}{G}$$

Now $v = \dfrac{2\pi r}{T}$ so $\boxed{M_\odot = \dfrac{4\pi^2 r^3}{T^2 G}}$

Using Jupiter data:

$$M_\odot = \frac{4\pi^2 \times (7.76\times 10^{11})^3}{(11.8\times 365\times 24\times 3600)^2 \times 6.67\times 10^{-11}}$$

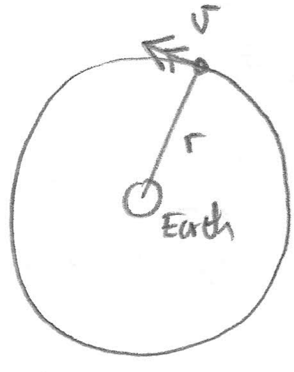
$$= \boxed{2.00\times 10^{30}\ \text{kg}}$$

Using Earth data:

$$M_\odot = \frac{4\pi^2 \times (1.50\times 10^{11})^3}{(1.00\times 365\times 24\times 3600)^2 \times 6.67\times 10^{-11}}$$

$$= \boxed{2.00\times 10^{30}\ \text{kg}}$$

b) For a geostationary ("synchronous") orbit of the Earth



$T = 24\times 3600\,\text{s}$ is 1 day

$$\frac{v^2}{r} = \frac{GM_E}{r^2}$$

$$v^2 = \frac{GM_E}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM_E}{r}$$

$$r^3 = \frac{GM_E}{4\pi^2}T^2$$

(e.g. Kepler III)

$$\boxed{r = \left(\frac{GM_E}{4\pi^2}\right)^{1/3} T^{2/3}}$$

$$r = \left(\frac{6.67\times 10^{-11}\times 5.98\times 10^{24}}{4\pi^2}\right)^{1/3} \times (24\times 3600)^{2/3}$$

$$= \boxed{4.23\times 10^{7}\ \text{m}}$$

$$v = \frac{2\pi r}{T} = \boxed{3073\ \text{m/s}}$$

c) (i) To escape from the gravitational influence of a mass M, the total energy when r (the distance from the mass) $\to \infty$ is > 0

$$\therefore \underbrace{\frac{1}{2} m v^2 - \frac{GmM}{r}}_{\text{Energy total}} > 0$$

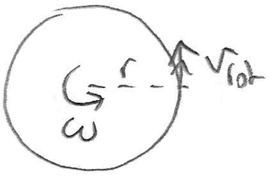
So if r is the radius of a spherical mass (e.g a planet) and v is the launch velocity

$$\boxed{v_{escape} = \sqrt{\frac{2GM}{r}}}$$

(ii) For Earth: $v_{escape} = \sqrt{\dfrac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.38 \times 10^{6}}}$

$= \boxed{11.2 \text{ km/s}}$

(iii) If you launch from the equator in the direction of the Earth's rotation you are moving at the maximum speed already relative to the centre of the Earth



$v_{rot} = r \times \dfrac{2\pi}{T} = \dfrac{6.38 \times 10^6 \times 2\pi}{24 \times 3600}$

$= \boxed{464 \text{ m/s}}$

(iv) $\therefore$ Minimum launch speed to escape the Earth is

$v_L = v_{escape} - v_{rot}$

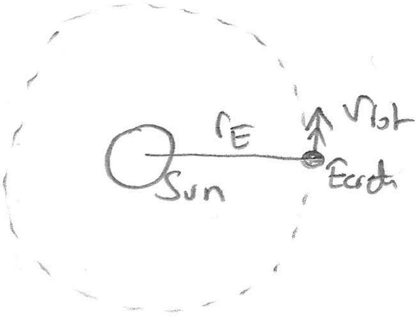
$= 11.2 - 0.464$ km/s

$= \boxed{10.7 \text{ km/s}}$

d) If one launches from Earth and desires to escape the gravitational influence of the Sun, we can use a scaled up argument from c) (iii) to work out the minimum launch speed.

$$v_L = v_{escape} - v_{rot}$$

$$v_L = \sqrt{\frac{2GM_\odot}{r_E}} - r_E \times \frac{2\pi}{T}$$



v_{rot} in this case is the orbital speed (rather than the rotation speed) ← we will ignore the small effect of the Earth's rotation

{and we start from r_E rather than the solar "surface"!}

$$v_L = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 2.00\times 10^{30}}{1.50\times 10^{11}}} - \frac{1.50\times 10^{11}\times 2\pi}{365\times 24\times 3600}$$

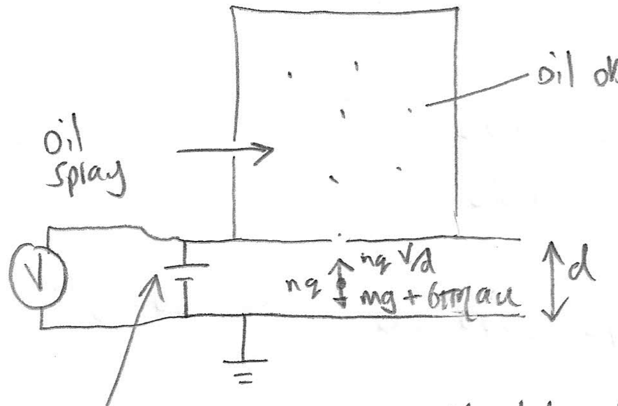
$$= 42.2 - 29.9 \text{ km/s}$$

$$= \boxed{12.3 \text{ km/s}}$$

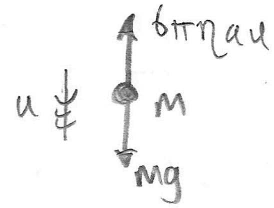
ie quite a difference if one makes use of the orbital velocity.

Q7

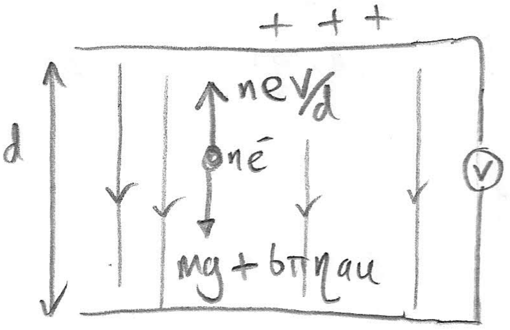
Millikan's oil drop experiment schematic



* let drops fall between plates separated by distance d with the electric field, terminal speed u will quickly be obtained ie viscous drag = weight so no net acceleration,



* Now assume oil drop contains n electrons. ∴ they will be attracted to the top plate (+ve charged)
Set the voltage between the plates such that the electrons rise at constant speed. For 'reasonable' voltages this will occurs rapidly since the viscous drag will now add to the weight, opposing motion upwards



Newton II: $\boxed{ne\frac{V}{d} = mg + 6\pi\eta a u}$ *

* Assume all drops are the same mass m
* " $a = 2.76 \times 10^{-6}$ m
* Assume $u = \frac{\delta}{T}$ where δ is the distance the drops are moving at constant speed.

Electric field $E = \frac{V}{d}$

Electric force neE

T is the time taken for the drops to ascend δ. Note $d > \delta$

(So for $d - \delta$ m, the dynamic equilibrium is being established). metres!

$$\therefore \quad \boxed{n = \frac{mgd}{eV} + \frac{6\pi\eta a d}{eV}\frac{\delta}{T}}$$

For measurement 1, $T = 42s$

" 2, $T = 78s$

$d = 1.50cm$, $\delta = 1.00cm$, $V = 5000$ Volts, $e = 1.60 \times 10^{-19}C$

$\eta = 1.82 \times 10^{-5} \, kg\,m^{-1}s^{-1}$, $a = 2.76 \times 10^{-6} m$

$$\text{so} \quad \boxed{n_1 - n_2 = \frac{6\pi\eta a d \delta}{eV}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

$$n_1 - n_2 = \frac{6\pi \times 1.82\times10^{-5} \times 2.76\times10^{-6} \times 1.50\times10^{-2} \times 1.00\times10^{-2}}{1.60\times10^{-19}\times 5000}\left(\frac{1}{42} - \frac{1}{78}\right)$$

$$= \boxed{1.95}$$

So about $\boxed{\text{2 electrons of charge}}$ is the difference.

b) Consider the coalescence of two drops of the same density ρ and with radii r_1, r_2, charges Q_1, Q_2 and terminal velocities u_1, u_2 [in question $Q_1 = Q_2 = Q$]

From $*$: $Q\frac{V}{d} = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r u$

So ① $Q_1 \frac{V}{d} = \frac{4}{3}\pi r_1^3 \rho g + 6\pi\eta r_1 u_1$ Drop 1

② $Q_2 \frac{V}{d} = \frac{4}{3}\pi r_2^3 \rho g + 6\pi\eta r_2 u_2$ Drop 2

③ $\frac{V}{d}(Q_1 + Q_2) = \frac{4}{3}\pi R^3 \rho g + 6\pi\eta R v$ Coalesced drop (conserve charge) ↳ ie $Q_1 + Q_2$

Conserve mass: $\frac{4}{3}\pi R^3 \rho = \left(\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3\right)\rho$

$$\therefore \quad \boxed{R^3 = r_1^3 + r_2^3}$$

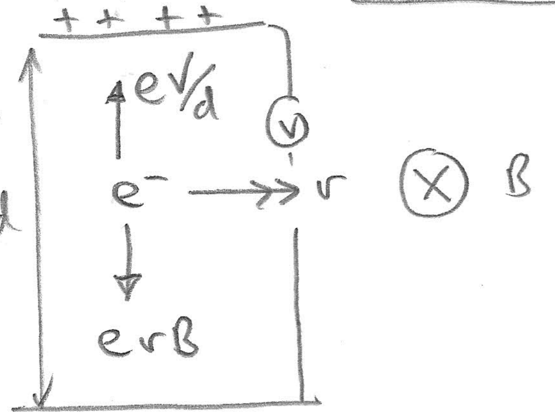
So since ① + ② = ③

$$\frac{4}{3}\pi \cancel{R^3 \rho g} + 6\pi\eta R v = \frac{4}{3}\pi \cancel{\rho g (r_1^3 + r_2^3)} + 6\pi\eta (r_1 u_1 + r_2 u_2)$$

$$\therefore \quad R v = r_1 u_1 + r_2 u_2$$

$$\therefore \quad \boxed{v = \frac{r_1 u_1 + r_2 u_2}{\sqrt[3]{r_1^3 + r_2^3}}}$$

c)



Set up a balance where electric force on an electron beam is eV/d and this is balanced by a magnetic force evB

So $\boxed{\frac{eV}{d} = evB}$ ①

Now let electron beam be accelerated by potential $\mathcal{E}$

$$\therefore \quad \mathcal{E} e = \frac{1}{2} m_e v^2 \qquad \therefore \quad \boxed{v^2 = \frac{2\mathcal{E} e}{m_e}} \quad ②$$

$$①: \quad v = \frac{V}{Bd} \qquad \therefore \quad v^2 = \frac{V^2}{B^2 d^2}$$

$$\therefore \text{ equating with ②:} \qquad \frac{2\mathcal{E} e}{m_e} = \frac{V^2}{B^2 d^2}$$

$$\therefore \quad \boxed{\frac{e}{m_e} = \frac{V^2}{2\mathcal{E} B^2 d^2}}$$

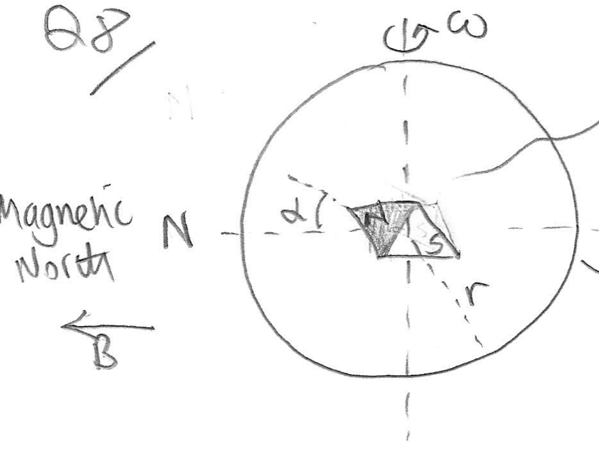
We are given: $V = 3000$ Volts, $d = 2.0$ cm, $B = 2.5 \times 10^{-3}$ T

$\mathcal{E} = 10{,}000$ Volts.

$$\therefore \quad \frac{e}{m_e} = \frac{3000^2}{2 \times 10{,}000 \times (2.5 \times 10^{-3})^2 \times (2.0 \times 10^{-2})^2}$$

$$= \boxed{1.80 \times 10^{11} \text{ C/kg}}$$

Q8/



$r = 0.125\,m$

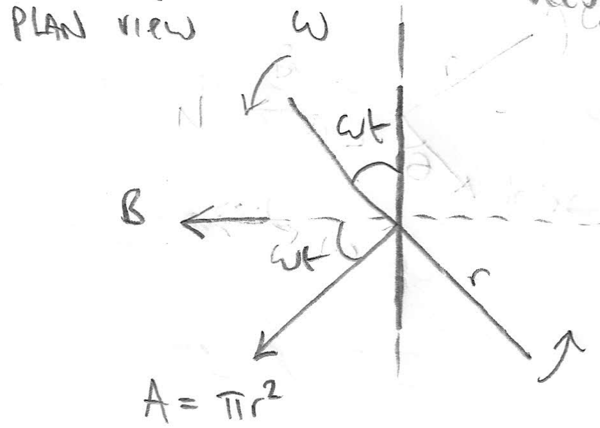
when $\omega = 10 \times 2\pi$ rad s^{-1}

$\alpha \simeq 2.00^\circ$

let horizontal component of Earth's magnetic field be B (Note in 2018 field points towards Magnetic North ie it is actually a South pole!)

a) Magnetic flux through the ring is

$\Phi = \int \underline{B} \cdot d\underline{A}$ ie projection of magnetic field on vector area of loop



$$\boxed{\Phi = \pi r^2 B \cos\omega t}$$

b) Ohm's law $V = IR$

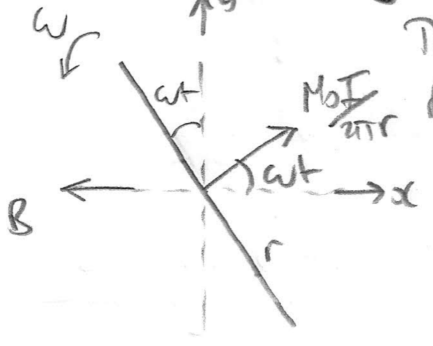
Now by Faraday's Law of induction EMF V induced is $= -\frac{d\Phi}{dt}$

so $V = +\pi r^2 \omega B \sin\omega t$

$\therefore \; I = V/R \; \therefore \; \boxed{I = \frac{\pi r^2 \omega B}{R} \sin\omega t}$

c) current in ring results in a magnetic field $\boxed{B_r = \frac{\mu_0 I}{2\pi r}}$

This must oppose the Earth's magnetic field (Lenz's Law)



$\therefore$ Net field is

$$B_x = \frac{\mu_0 I}{2\pi r}\cos\omega t - B$$

$$B_y = \frac{\mu_0 I}{2\pi r}\sin\omega t$$

∴ using $I = \frac{\pi r^2 \omega B}{R} \sin\omega t$

$$B_x = \frac{\mu_0}{2\cancel{\pi r}} \frac{\cancel{\pi} r^2 \omega B}{R} \sin\omega t \cos\omega t - B$$

$$B_y = \frac{\mu_0}{2\pi r} \frac{\pi r^2 \omega B}{R} \sin^2\omega t$$

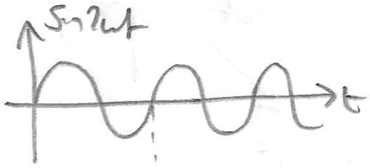
∴ $$\boxed{B_x = \frac{\mu_0 r \omega B}{4R} \sin 2\omega t - B}$$

$$\boxed{B_y = \frac{\mu_0}{4R} r \omega B \left(1 - \cos 2\omega t\right)}$$

$[2\sin\omega t \cos\omega t = \sin 2\omega t]$

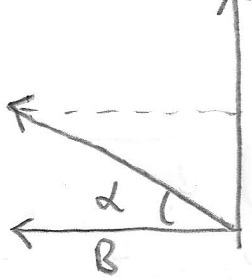
$[\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)]$

d) $$\overline{\sin 2\omega t} = \overline{\cos 2\omega t} = 0$$



$2\omega t = 2\pi$
$t = \pi/\omega$

So $$\boxed{\overline{B_y} = \frac{\mu_0 r \omega B}{4R} \qquad \overline{B_x} = -B}$$



$$\overline{B_y} = \frac{\mu_0 r \omega B}{4R}$$

e) From diagram on the left

$$\tan\alpha = \frac{\overline{B_y}}{B}$$

$$\tan\alpha = \mu_0 r \omega / 4R$$

So $$\boxed{R = \frac{\mu_0 r \omega}{4 \tan\alpha}}$$

∴ $$R = \frac{4\pi \times 10^{-7} \times 0.125 \times 10 \times 2\pi}{4 \times \tan 2^\circ}$$

$$= \boxed{7.1 \times 10^{-5}\ \Omega}$$

$2.2 \times 10^{-4}\ \Omega$
(Note MS misses out a π if it is π too large)