BPhO 1 2017 Section 1 AF Solutions

a) Physicists sometimes use the approximation that light travels in a vacuum at a speed of 1 foot in 1 ns. What is the percentage error in using this value?

$$(1.000\text{ m} = 1.094\text{ yards and } 1.000\text{ yard} = 3.000\text{ feet})$$

[3]

According to the list of constants provided in the paper:

$$\boxed{c = 3.00 \times 10^8 \text{ m/s}}$$

$$1\text{ ft/ns} = \frac{\frac{1.000}{3 \times 1.094}\text{ m}}{10^{-9}\text{ s}}$$

$$= \boxed{3.047 \times 10^8 \text{ m/s}}$$

So % error is: $\frac{3.047 - 3.00}{3.00}$

$$= \boxed{1.56\%}$$

(3.s.f appropriate precision here)

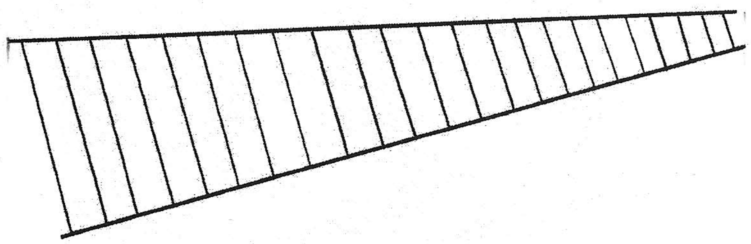
{ if you use $c = 2.998 \times 10^8$ m/s
then % error is $\boxed{1.63\%}$ }

b) A window cleaner's ladder shown in **Figure 1** is narrower at the top than the bottom. It has a weight of 350 N and a length of 5.0 m. When it lies flat on the ground, a force of 80 N is needed to lift the narrow end off the ground.

(i) How far is the centre of mass from the narrow end? y

(ii) What force is required to lift the wide end of the ladder off the ground?

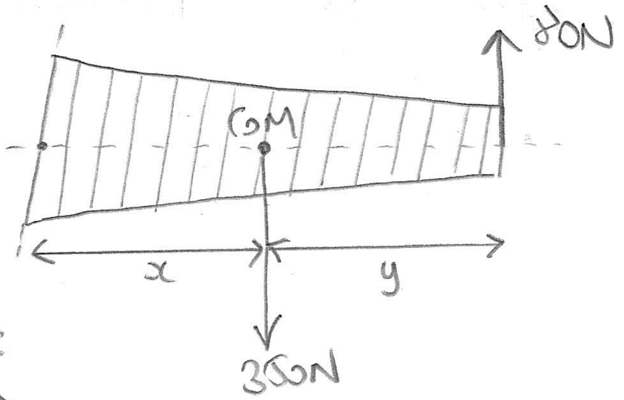
F



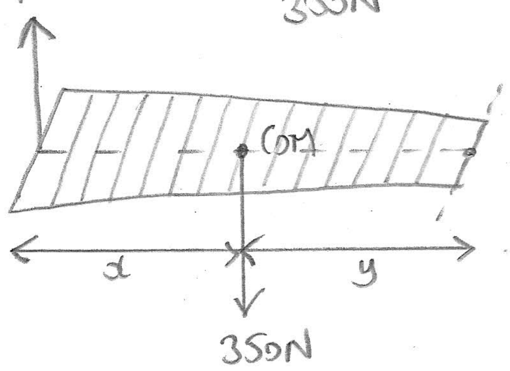
Is this supposed to be a 3D projection?!

Figure 1

[5]



Pivoting about wide end (narrow end rises)



Pivoting about narrow end (wide end rises)

So $80(x+y) = 350x$ ① (Narrow end just lifted)

$F(x+y) = 350y$ ② (wide end just lifted)

Now $\boxed{x+y = 5.0}$ So in ①: $x = \frac{80}{350} \times 5.0$

$\boxed{x = 1.14\text{ m}}$

∴ (i) $y = 5.0 - 1.14 = \boxed{3.86\text{ m}}$

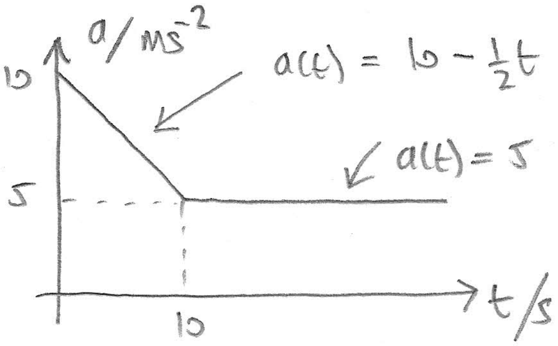
(ii) $F = \frac{350y}{5} = \boxed{270\text{ N}}$

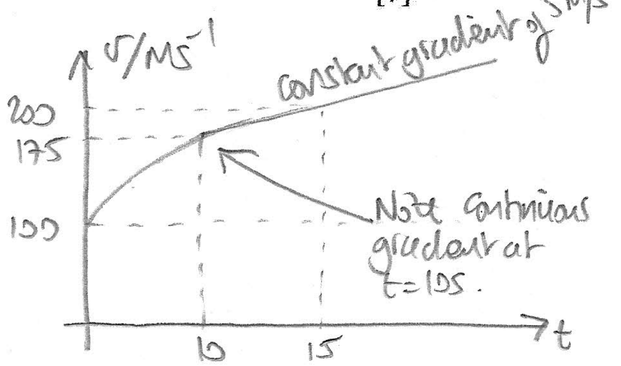
c) A particle moves in a straight line with an intial acceleration of $10\,\mathrm{m\,s^{-2}}$. The acceleration decreases uniformly with time until, after ten seconds, the acceleration is $5\,\mathrm{m\,s^{-2}}$, and from then on the acceleration remains constant. If the intial velocity is $100\,\mathrm{m\,s^{-1}}$,

(i) find when the velocity has doubled;

(ii) sketch a graph of the velocity against time.

[7]





$$v = \int_0^t a(t')\,dt' + 100 = \begin{cases} 10t - \frac{1}{4}t^2 + 100 & 0 \le t \le 10 \\ 175 + 5(t-10) \end{cases} \quad (\text{m/s})$$

$0 \le t \le 10$: $\quad v = \int_0^t (10 - \frac{1}{2}t)\,dt + 100$

$$v = 10t - \frac{1}{4}t^2 + 100$$

$$v(10) = 100 - \frac{1}{4}100 + 100 = 175\ \text{m/s}$$

So v is 200 m/s after 10s.

So for $t \geq 10$: $\quad v = 175 + 5(t-10)$ (m/s)

∴ when $v = 200$ m/s

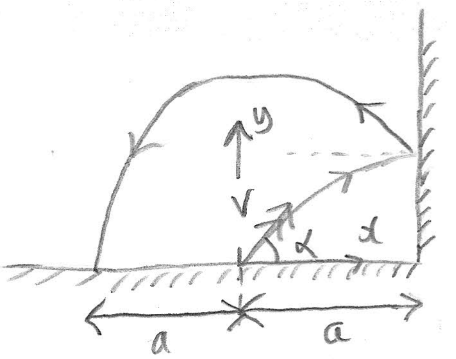
$200 = 175 + 5(t-10) \Rightarrow t = \frac{200-175}{5} + 10$

$$t = 15\text{s}$$

d) A student standing at a distance a from a vertical wall kicks a ball from ground level with velocity V at an angle α to the horizontal in a plane perpendicular to that of the wall. The ball strikes the wall and rebounds. The coefficient of restitution for the collision is $e = 2/3$. The ball first strikes the ground at a distance $2a$ from the wall. e is the ratio of the components of velocity at normal incidence to the wall, before and after collision; $e = \frac{v_{\text{after}}}{v_{\text{before}}} \leq 1$.

Find a in terms of V, α and g, the gravitational field strength.

[6]



$\downarrow g$

Before bounce with wall at $x = a$:

$$x = Vt\cos\alpha$$
$$y = Vt\sin\alpha - \tfrac{1}{2}gt^2$$
$$v_x = V\cos\alpha$$
$$v_y = V\sin\alpha - gt$$

So when $x = a$

$$t_a = a/V\cos\alpha$$

$$y = \frac{Va}{V\cos\alpha}\sin\alpha - \frac{1}{2}g\left(\frac{a}{V\cos\alpha}\right)^2 = a\tan\alpha - \frac{ga^2}{2V^2\cos^2\alpha}$$

Also at $x = a$ (just before bounce)

$$v_x = V\cos\alpha$$
$$v_y = V\sin\alpha - ga/V\cos\alpha$$

Now after bounce; $v_x \to -\tfrac{2}{3}V\cos\alpha$

and assume v_y stays the same.

If τ is now time after bounce: $\left(t = \tau + \frac{a}{V\cos\alpha}\right)$

$$x = a - \tfrac{2}{3}V\tau\cos\alpha$$
$$y = \left(V\sin\alpha - \frac{ga}{V\cos\alpha}\right)\tau - \tfrac{1}{2}g\tau^2 + a\tan\alpha - ga^2/2V^2\cos^2\alpha$$

when $x = -a$, $y = 0$. So $\tfrac{2}{3}V\tau\cos\alpha = 2a$

$$\Rightarrow \tau_a = \frac{3a}{V\cos\alpha} = 3t_a$$

$$\therefore\ 0 = \left(V\sin\alpha - \frac{ga}{V\cos\alpha}\right)\frac{3a}{V\cos\alpha} - \frac{1}{2}g \times \frac{9a^2}{V^2\cos^2\alpha} + a\tan\alpha - \frac{ga^2}{2V^2\cos^2\alpha}$$

$$0 = 4a\tan\alpha - \frac{3ga^2}{V^2\cos^2\alpha} - \frac{5ga^2}{V^2\cos^2\alpha}$$

$$\therefore \quad \frac{8ga^2}{\cos^2\alpha \, V^2} = \frac{4a\sin\alpha}{\cos\alpha}$$

$$\therefore \quad 2ga = V^2 \sin\alpha\cos\alpha$$

$$\therefore \quad 4ga = V^2 \sin 2\alpha \qquad [\sin 2\alpha = 2\sin\alpha\cos\alpha]$$

$$\therefore \quad \boxed{a = \frac{V^2 \sin 2\alpha}{4g}}$$

Perhaps a nice shortcut! y velocity is not affected by collision with wall, so $\boxed{y = Vt\sin\alpha - \frac{1}{2}gt^2}$ for entire trajectory

$\therefore$ Since time of flight $T = t_a + 3t_a$ ← $[x = a - \frac{2}{3}VT\cos\alpha$ and $x = -a$ when $T = 3t_a]$

$$\boxed{T = \frac{4a}{V\cos\alpha}} \qquad \left[t_a = \frac{a}{V\cos\alpha}\right]$$

when $y = 0$: (and $t > 0$)

$$V\sin\alpha = \frac{1}{2}gT$$

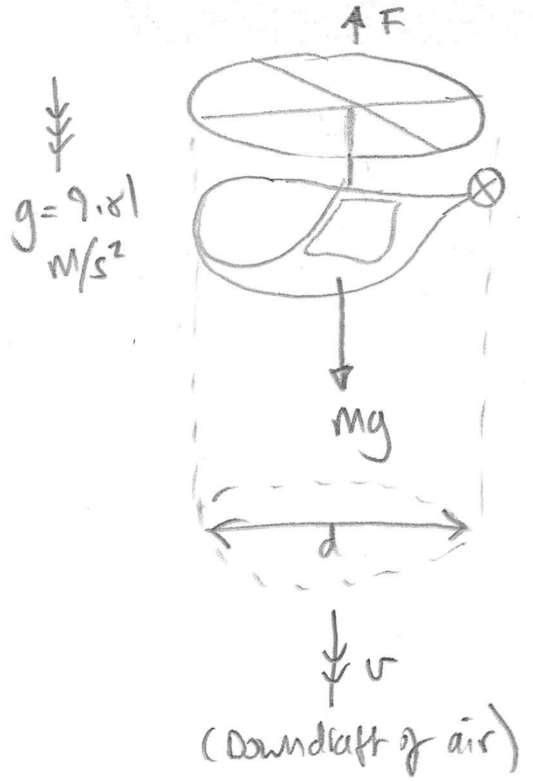
$$\therefore \quad V\sin\alpha = \frac{1}{2}g \times \frac{4a}{V\cos\alpha}$$

$$2V^2\sin\alpha\cos\alpha = 4ga$$

$$\therefore \quad \boxed{a = \frac{V^2\sin 2\alpha}{4g}}$$

e) A helicopter of total mass 1000 kg is able to remain in a stationary position by imparting a uniform downward velocity to a cylinder of air below it of effective diameter 6 m. Assuming the density of air to be $1.2\,\text{kg}\,\text{m}^{-3}$, calculate the downward velocity of the air.

[5]



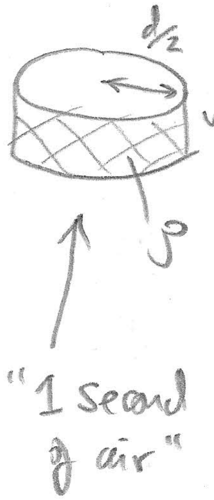
In equilibrium $\boxed{F = mg}$

Rate of change of momentum of air in downdraft = F (Newton III)

= mass of one second of air × v

$= \pi \left(\frac{d}{2}\right)^2 v \rho \times v$

$= \frac{1}{4}\pi d^2 \rho v^2$



So $\boxed{v = \sqrt{\frac{4mg}{\pi d^2 \rho}}}$

$v = \sqrt{\frac{4 \times 1000 \times 9.81}{\pi \times 6^2 \times 1.2}}$

$= \boxed{17\ \text{m/s}}$

f) In this question, distances are measured in nautical miles and speeds in nautical miles per hour. A motor boat sets out at 2 p.m. from a point with position vector $-4\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$ relative to a marker buoy (where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are two fixed perpendicular unit vectors) and travels at a steady speed of magnitude $\sqrt{41}$ in a straight line to intercept a ship **S**. The ship **S** maintains a steady velocity vector $\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and at 3 p.m. is at a position $3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ relative to the buoy. Find

(i) the position vector of the ship at 2 p.m., $\underline{x}_0$

(ii) the velocity vector of the motor boat,

(iii) the time of interception.

[7]

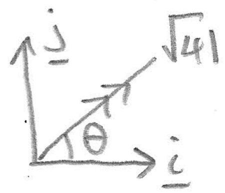
Ship S: $\underline{x}_S = \underline{x}_0 + (\underline{i} + 4\underline{j})t$

t is time in hours after 2pm.

Now $3\underline{i} - \underline{j} = \underline{x}_0 + (\underline{i} + 4\underline{j})(1)$

so $\underline{x}_0 = 3\underline{i} - \underline{j} - \underline{i} - 4\underline{j}$

$\underline{x}_0 = 2\underline{i} - 5\underline{j}$



Motor boat velocity

Motor boat: $\underline{x}_B = -4\underline{i} - 5\underline{j} + \sqrt{41}(\cos\theta\, \underline{i} + \sin\theta\, \underline{j})t$

interception when $\underline{x}_S = \underline{x}_B$

$\therefore \quad 2\underline{i} - 5\underline{j} + (\underline{i} + 4\underline{j})t = -4\underline{i} - 5\underline{j} + \sqrt{41}(\cos\theta\, \underline{i} + \sin\theta\, \underline{j})t$

comparing coefficients of $\underline{i}, \underline{j}$:

$\underline{i}$: $2 + t = -4 + \sqrt{41}\cos\theta\, t$ ①

$\underline{j}$: $-5 + 4t = -5 + \sqrt{41}\sin\theta\, t$ ②

so $24 + 4t = 4\sqrt{41}\cos\theta\, t$ ③

$4t = \sqrt{41}\sin\theta\, t$ ④ $\Rightarrow \sin\theta = \dfrac{4}{\sqrt{41}}$

$\therefore \quad 6 + t = \sqrt{41}\cos\theta\, t$

$\therefore \quad 36 + 12t + t^2 = 41t^2\cos^2\theta$

⑦

$$\therefore \quad 36 + 12t + t^2 = 41t^2(1-\sin^2\theta)$$

$$36 + 12t + t^2 = 41t^2\left(1 - \frac{16}{41}\right)$$

$$36 + 12t + t^2 = 41t^2 - 16t^2$$

$$36 + 12t = 24t^2$$

$$3 + t = 2t^2$$

$$\therefore \quad 2t^2 - t - 3 = 0$$

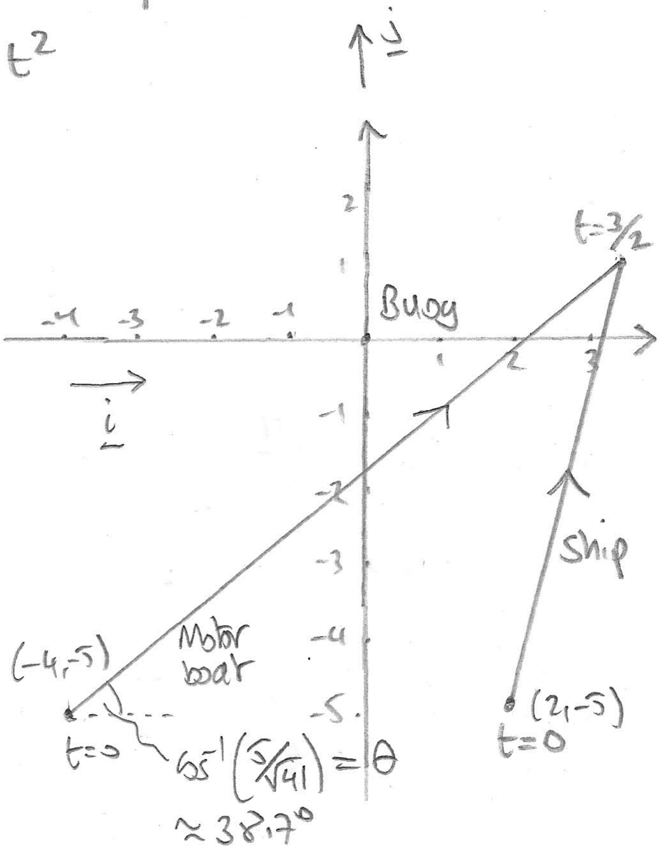
$$t^2 - \tfrac{1}{2}t - \tfrac{3}{2} = 0$$

$$\left(t - \tfrac{1}{4}\right)^2 - \tfrac{1}{16} - \tfrac{3}{2} = 0$$

$$\left(t - \tfrac{1}{4}\right)^2 - \tfrac{25}{16} = 0$$

$$t = \tfrac{1}{4} \pm \tfrac{5}{4}$$

$$\boxed{t = 3/2} \quad \text{(hours)}$$



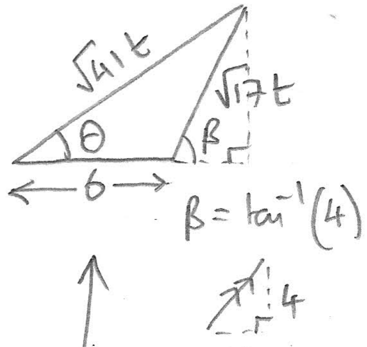
[intersection coordinate is $2\underline{i} - 5\underline{j} + \tfrac{3}{2}(\underline{i} + 4\underline{j})$

$= \boxed{3.5\underline{i} + \underline{j}}$]

Now $\cos^2\theta = 1 - 16/41 = \frac{25}{41}$

$\therefore \quad \boxed{\cos\theta = 5/\sqrt{41}}$

[Note could also solve via:



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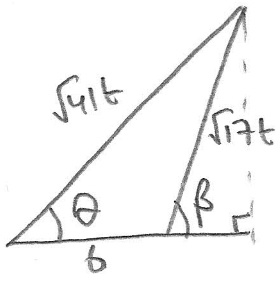
Ship's speed is $\sqrt{1^2+4^2} = \sqrt{17}$

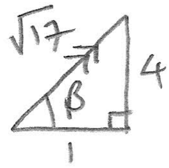
So Motor boat velocity is:

$$\underline{v}_B = \sqrt{41}(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

$$= \boxed{5\underline{i} + 4\underline{j}}$$



$\tan\beta = 4$ $\therefore$



$\boxed{\sin\beta = \frac{4}{\sqrt{17}}}$

cosine rule:

$$17t^2 = 41t^2 + 36 - 2\times 6\times\sqrt{41}t\cos\theta$$

$$\sqrt{41}t\sin\theta = \sqrt{17}t\sin\beta$$

so $41\sin^2\theta = 17\sin^2\beta$

$41\sin^2\theta = 17/17 \times 16$

$$\therefore \boxed{\sin\theta = \frac{4}{\sqrt{41}}}$$

$$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{16}{41}} = \boxed{\frac{5}{\sqrt{41}}}$$

so $0 = 24t^2 + 36 - 12\sqrt{41}t \times 5/\sqrt{41}$

$0 = 24t^2 + 36 - 60t$

$0 = t^2 + \frac{3}{2} - \frac{5t}{2}$

$0 = \left(t - \frac{5}{4}\right)^2 - \frac{25}{16} + 3/2$

$0 = \left(t - 5/4\right)^2 - \frac{1}{16}$

$\therefore\quad t = 5/4 \pm \frac{1}{4}$

so $t = 1, \boxed{3/2}$

why the the tool?

[Note $\sqrt{41}\times 1\cos\theta$
$= \sqrt{41}\times 5/\sqrt{41} = 5$
which is < 6, so can't be valid]

g) In a factory heating system, water enters the radiators at 60 °C and leaves at 38 °C. The system is replaced by one in which steam at 100 °C is condensed in the radiators, the condensed steam leaving at 82 °C. What mass of steam will supply the same heat energy as 1.00 kg of hot water described in the first instance? (The latent heat of vaporisation of water is 2.260×10^6 J kg^{-1} at 100 °C. The specific heat capacity of water is 4200 J kg^{-1} °C^{-1}.)

L

C

[4]

Original design

Energy released is: $\Delta E = c m_1 \Delta T_1$

$\Delta E = 4200 \times 1.00 \times (60 - 38)$ (J)

$= 92400 \text{ J}$

New design

$\Delta E = m_2 L + c m_2 \Delta T_2 = m_2 (L + c \Delta T_2)$

So if ΔE is the same; and $\Delta T_2 = (100 - 82)$ K

$\Rightarrow \quad m_2 = \dfrac{92400}{2.260 \times 10^6 + 4200 \times (100 - 82)}$

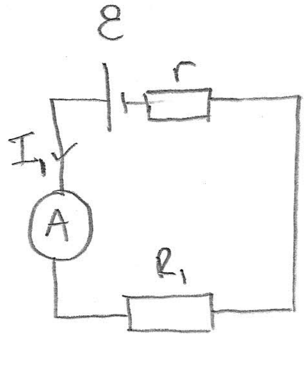
$m_2 = 0.0396$ kg or 39.6 g

[m_2 is the mass of steam, m_1 is the mass of water]

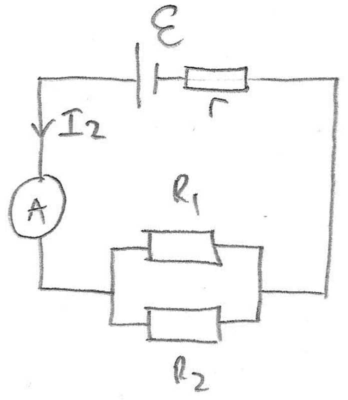
h) A cell, a resistor and an ammeter of negligible resistance are connected in series and a current of 0.80 A is observed to flow when the resistor has a value of 2.00 Ω. When a resistor of 5.00 Ω is connected in parallel with the 2.00 Ω resistor, the ammeter reading is 1.00 A.
Calculate the emf and the internal resistance of the cell.

r

[5]



(a)



(b)

$R_1 = 2.00\ \Omega$ $\quad I_1 = 0.80\text{ A}$

$R_2 = 5.00\ \Omega$ $\quad I_2 = 1.00\text{ A}$

For (a) $\quad \varepsilon = I_1(r + R_1)$ $\quad$ (Kirchoff II and ohm's law)

(b) $\quad \varepsilon = I_2\left(r + \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right)$

Hence: $\quad I_1 r + I_1 R_1 = I_2 r + \dfrac{I_2}{\frac{1}{R_1} + \frac{1}{R_2}}$

$$r(I_2 - I_1) = I_1 R_1 - I_2 \Big/ \frac{1}{R_1} + \frac{1}{R_2}$$

$$r = \frac{I_1 R_1 - I_2 \Big/ \frac{1}{R_1} + \frac{1}{R_2}}{I_2 - I_1}$$

$$= \frac{(0.80)(2.00) - (1.00) \Big/ \frac{1}{2} + \frac{1}{5}}{1.00 - 0.80}$$

$$= \boxed{0.86\ \Omega}$$

($6/7\ \Omega$ as a fraction)

So $\varepsilon = I_1(r + R_1)$

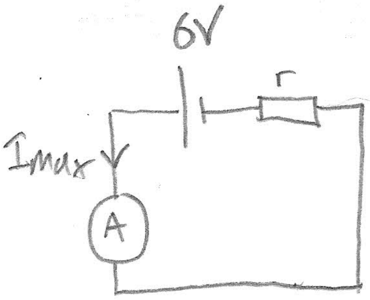
$= 0.8\left(\frac{6}{7} + 2.00\right)$

$= \boxed{2.29\text{ V}}$

($16/7$ volts)

i) A battery with an emf of 6 V can produce a maximum current of 3 A. A resistor is connected to the terminals whose value is such that the power dissipated in it is a maximum. Calculate the maximum energy which can be dissipated in the external resistor in one minute.

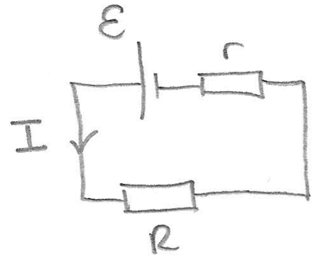
[4]



"Short"

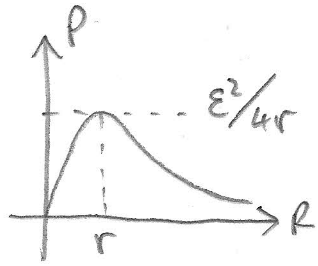
$I_{max} = 3A$

$I_{max} = \frac{6V}{r}$ $\therefore$ $\boxed{r = 2\Omega}$



Power dissipated in R is $\boxed{I^2 R = P}$

$I = \frac{\varepsilon}{r+R}$ so $\boxed{P = \frac{\varepsilon^2 R}{(r+R)^2}}$



$$\frac{dP}{dR} = \frac{(r+R)^2\varepsilon^2 - 2\varepsilon^2 R(r+R)}{(r+R)^4}$$

$\frac{dP}{dR} = 0$ when $r + R - 2R = 0 \Rightarrow \boxed{r = R}$

"Maximum power theorem" so $\boxed{P_{max} = \frac{\varepsilon^2}{4r}}$

So max energy in Δt seconds dissipated in our system is $\Delta E = \frac{\varepsilon^2 \Delta t}{4r} = \frac{6^2 \times 60}{4 \times 2}$

$= \boxed{270\ J}$

($P_{max} = 4.5\ W$)

Δt

j) Calculate the number of photons emitted in a one nanosecond (10^{-9} s) pulse of light from a 0.5 mW laser of wavelength 639 nm.

P

[3]

Energy of a photon of wavelength λ is

$$E_p = hf = \frac{hc}{\lambda}$$

Energy of laser pulse is $E = P\Delta t$

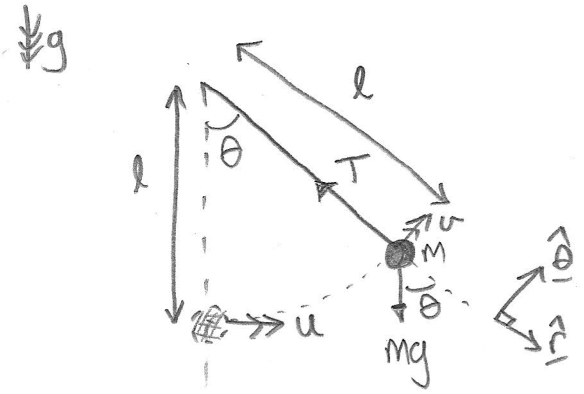
Hence # photons is $E/E_p = \dfrac{P\Delta t\lambda}{hc}$

$$= \left(\frac{6.63\times10^{-34}\times3.00\times10^{8}}{639\times10^{-9}\times0.5\times10^{-3}\times10^{-9}}\right)^{-1}$$

$$= 1.61\times10^{6} \text{ photons}$$

k) A lead ball is attached to the end of a light metal rod of length l, the other end being attached to a horizontal axle of negligble friction. The rod is given an intial impulse and swings round in a vertical circle. When it is at the top of the circle, the tension in the rod is zero. What is the tension in the rod at the lowest point of its swing?

[6]



Newton II :

$$\hat{r}: \quad -mv^2/l = -T + Mg\cos\theta \qquad (1)$$

$$\hat{\theta}: \quad ml\ddot{\theta}^2 = -mg\sin\theta \qquad (2)$$

Conservation of energy:

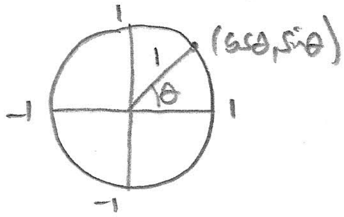
$$\tfrac{1}{2}mu^2 = \tfrac{1}{2}mv^2 + mgl(1-\cos\theta) \qquad (3)$$

so $u^2 = v^2 + 2gl(1-\cos\theta)$ ∴ $v^2 = u^2 - 2gl(1-\cos\theta)$

∴ $T = mg\cos\theta + \frac{m}{l}\left(u^2 - 2gl(1-\cos\theta)\right)$ [from (1)]

$T = mg\cos\theta + \frac{mu^2}{l} - 2mg + 2mg\cos\theta$

$$T = 3mg\cos\theta + \frac{mu^2}{l} - 2mg$$



Now $T=0$ when $\theta = \pi$, i.e. $\cos\theta = -1$

so $0 = -3mg + mu^2/l - 2mg$

∴ $mu^2/l = 5mg$ ⇒ $u^2 = 5gl$ ⇒ $u = \sqrt{5gl}$

∴ $T = 3mg\cos\theta + 3mg$

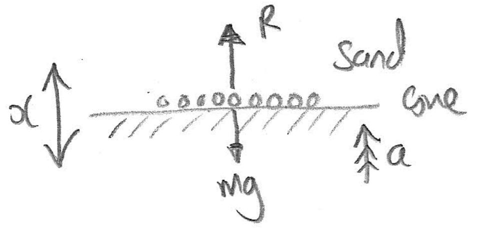
$$T = 3mg(1+\cos\theta)$$

∴ when $\theta = 0$, $\cos\theta = 1$

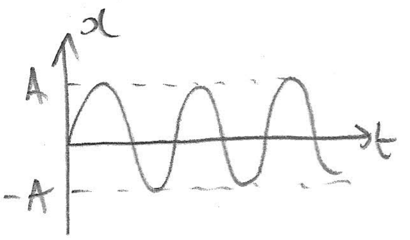
⇒ $T_{\theta=0} = 6mg$

1) Some sand is sprinkled onto a loudspeaker cone which is pointing vertically upwards. The louspeaker is driven in simple harmonic motion when attached to a signal generator and the frequency is gradually raised. At a particular frequency, when the amplitiude of oscillation is 0.20 mm, the sand begins to lose contact with the cone. At what frequency does this occur?
A

[3]



Cone is accelerating s.t $\boxed{a = -\omega^2 x}$ (SHM)



let $x = A\sin\omega t$ (cone displacement)

$\therefore \boxed{a = -\omega^2 A\sin\omega t}$ $(= \ddot{x})$

Assuming sand particle of mass M is in contact with the cone

Newton II: $Ma = R - Mg$

$\therefore R = Mg + Ma$

$\boxed{R = Mg - M\omega^2 A\sin\omega t}$

Contact force $R \geq 0$ if sand is in contact with cone.

so $mg - m\omega^2 A\sin\omega t \geq 0$

Now minimum value of $mg - m\omega^2 A\sin\omega t$ is when $\sin\omega t = 1$ i.e. $mg - \omega^2 A m$

so always in contact throughout motion if

$g - \omega^2 A \geq 0$

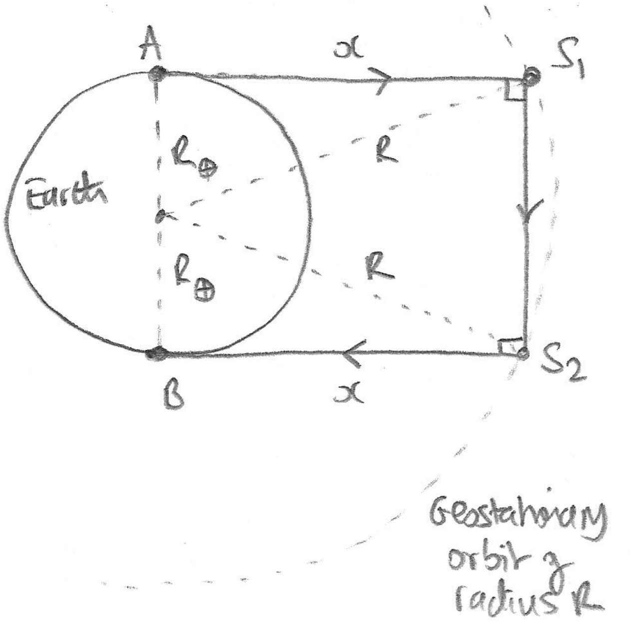
$\Rightarrow \boxed{\omega \leq \sqrt{g/A}}$

$\therefore f_{max} = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.2\times10^{-3}}} = \boxed{35.2\text{ Hz}}$

A and B

m) Two radio stations on the equator, diametrically opposite each other, communicate by sending and receiving radio signals that are tangential to the Earth's surface via two geostationary satellites S_1, S_2 in circular orbits at 3.59×10^4 km above the Earth's surface. Calculate the time delay between sending and receiving a signal.

[6]



Signal path length A → B via S_1, S_2 is:

$$\delta = 2x + 2R_\oplus$$

time delay is $\Delta t = \delta / c$

where c = speed of light (3.00×10^8 m/s)

$R_\oplus = 6.37 \times 10^6$ m
Earth radius

Pythagoras: $R^2 = R_\oplus^2 + x^2$

$\therefore x = \sqrt{R^2 - R_\oplus^2}$

$$\therefore \Delta t = \frac{2}{c}\left(\sqrt{R^2 - R_\oplus^2} + R_\oplus\right)$$

$$\therefore \Delta t = \frac{2}{3 \times 10^8}\left(\sqrt{(4.23 \times 10^7)^2 - (6.37 \times 10^6)^2} + 6.37 \times 10^6\right)$$

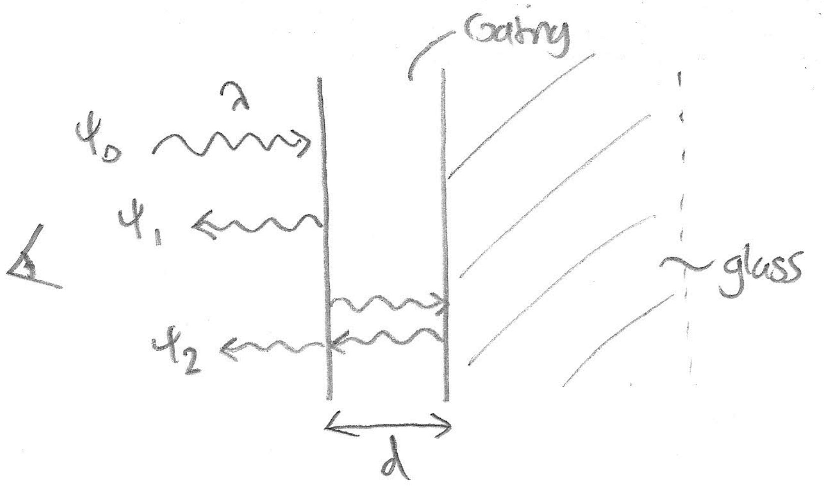
$= 0.321$ s

($R = 3.59 \times 10^7 + 6.37 \times 10^6$ m)
$= 4.23 \times 10^7$ m
since 3.59×10^7 m above Earth's surface.

16

n) A thin film of transparent material of refractive index 1.52 and thickness 0.42 μm forms a thin coating on glass of refractive index 1.60. It is viewed by reflection with white light at normal incidence. What visible wavelength in vacuo is most strongly reflected?

[5]



Phase of ψ_1 is π

Phase of ψ_2 is $\pi + 2\pi \times \frac{2d}{\lambda'}$

λ' is wavelength in Coating. [f = light frequency]

Since f is a constant and $f = \frac{c'}{\lambda'} = \frac{c}{n\lambda'}$

$f = \frac{c}{\lambda}$ $\therefore$ $\frac{1}{\lambda} = \frac{1}{n\lambda'}$ $\Rightarrow$ $\lambda = n\lambda'$ or $\boxed{\lambda' = \frac{\lambda}{n}}$

so phase difference between ψ_1 and ψ_2 is

$$\boxed{\Delta\phi = 4\pi n d / \lambda}$$

constructive interference when $\Delta\phi = 2\pi m$, $m \in \mathbb{Z}^+$. (ie $m = 1, 2, 3 \ldots$)

so $2\pi m = 4\pi n d / \lambda$

$$\Rightarrow \boxed{\lambda_m = 2nd / m}$$

$2nd = 2 \times 1.52 \times 0.42 \times 10^{-6} = 1.28 \times 10^{-6}$ m

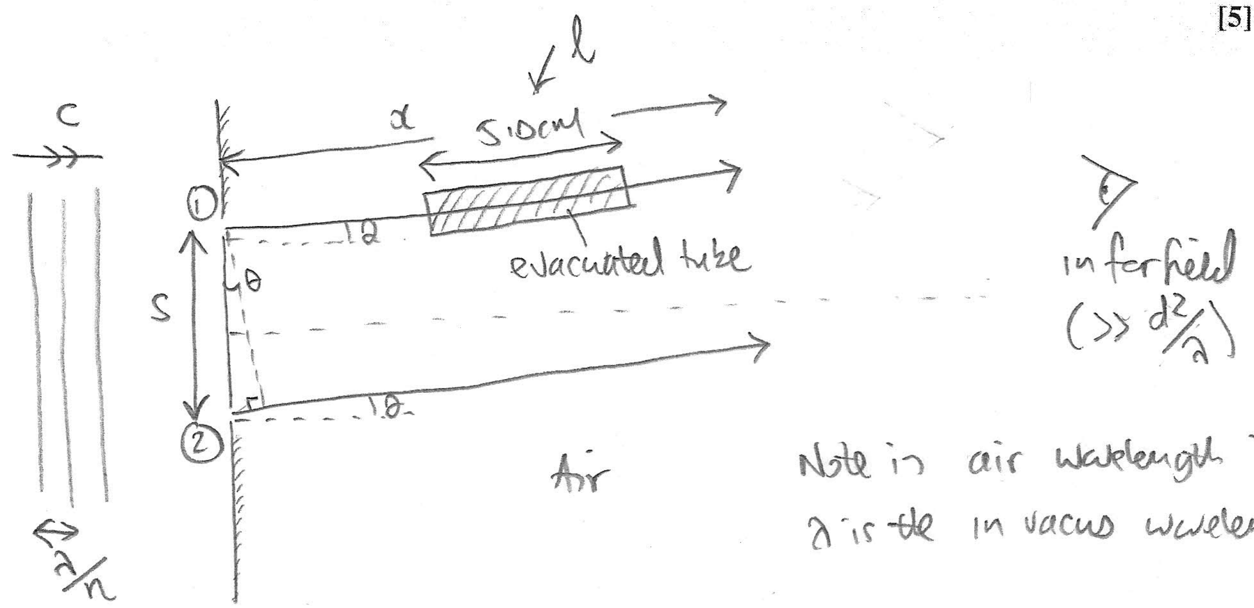
$= \boxed{1280 \text{ nm}}$ So $\lambda_m = \frac{1280 \text{ nm}}{m}$

Visible range is $380 \text{ nm} < \lambda < 740 \text{ nm}$, so if $m = 2$

$\boxed{\lambda_2 = 638 \text{ nm}}$ ie red light

o) Monochromatic light of wavelength 600 nm is incident on two vertical slits hence producing two coherent sources. Before the light leaving these slits overlaps and interferes, each beam passes through a tube 5.0 cm long. One of the tubes is now gradually evacuated and it is noted that the fringe pattern shifts 25 fringes. Calculate the refractive index of air.

[5]



Note in air wavelength is λ/n, λ is the in vacuo wavelength.

θ ≈ receives light from ① which travels $x - l$ through air and l through tube.

ie phase change of $\Delta\phi_1 = \frac{2\pi}{\lambda/n}(x-l) + \frac{2\pi}{\lambda} l$

θ ≈ receives light from ② which travels $x - s\sin\theta$ through air

ie phase change of $\Delta\phi_2 = \frac{2\pi}{\lambda/n}(x - s\sin\theta)$

so overall phase change following interference is

$$\Delta\phi = \Delta\phi_1 - \Delta\phi_2 = \frac{2\pi}{\lambda}\left(l - nl + ns\sin\theta\right)$$

If tube length $= 0 \Rightarrow \Delta\phi_0 = \frac{2\pi}{\lambda} ns\sin\theta$

So phase change due to tube is $\Delta\phi - \Delta\phi_0 = \boxed{\frac{2\pi}{\lambda} l(1-n)}$

{ $n > 1$ so −ve phase shift

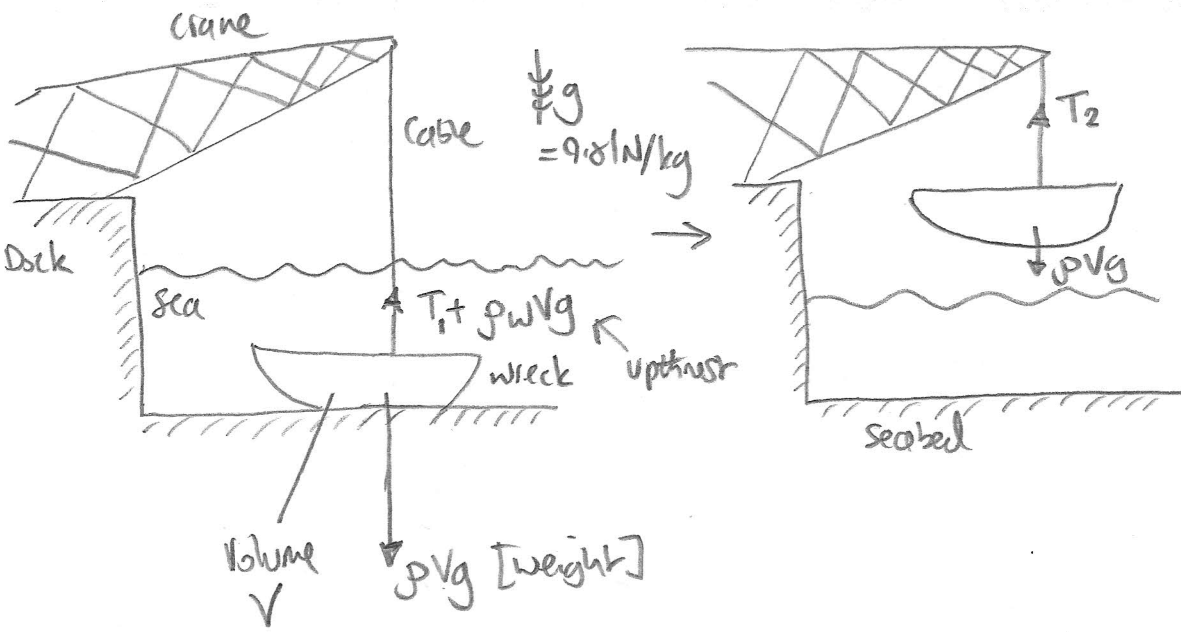
If this is $25 = N$ fringes this means $\Delta\phi - \Delta\phi_0 = -N \times 2\pi$.

$\therefore$ $\boxed{-N = \frac{l}{\lambda}(1-n)} \Rightarrow \boxed{1 + \frac{N\lambda}{l} = n}$ $\therefore n = 1 + \frac{25 \times 600 \times 10^{-9}}{5.0 \times 10^{-2}}$

$\boxed{n = 1.0003}$

p) A submerged wreck is lifted from a dock basin by means of a crane to which is attached a steel cable 10 m long of cross-sectional area 5.0 cm^2 and Young modulus 5.0 × 10^{10} Pa. The material being lifted has a mass of 1.0 × 10^4 kg and mean density 8000 kg m^{-3}. Find the change in extension of the cable as the load is lifted clear of the water. Assume that at all times the tension in the cable is the same throughout its length. (Density of water is 1000 kg m^{-3}.)

[5]



Assume wreck is lifted without acceleration:

$\therefore\ T_1 + \rho_w V g - \rho V g = 0$ ①

$T_2 - \rho V g = 0$ ② } Newton II

$\therefore\ T_1 + \rho_w V g = T_2$

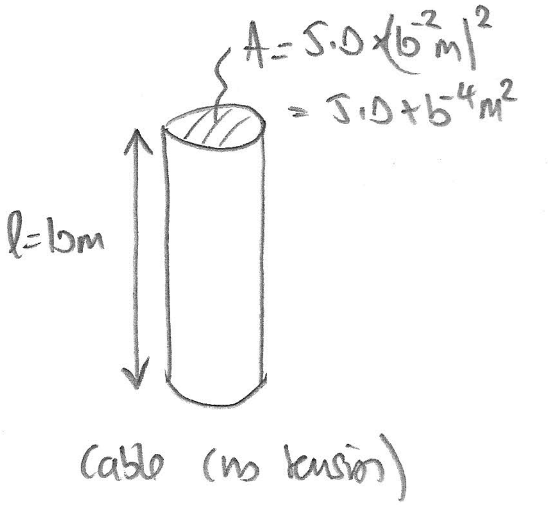
So extra tension in cable once lifted clear of sea is

$\Delta T = T_2 - T_1 = \boxed{\rho_w V g}$

Now $m = \rho V$ so $V = m/\rho$

$\therefore\ \boxed{\Delta T = \frac{\rho_w}{\rho} m g}$

"Stress / Strain = Young's modulus"



If cable extension under load is x

$$\frac{T_1/A}{x_1/l} = Y = 5.0 \times 10^{10} \text{ Pa}$$

$$\frac{T_2/A}{x_2/l} = Y \quad \text{also.}$$

We want $\boxed{x_2 - x_1}$

So: $x_2 = T_2 \, l / AY$

$x_1 = T_1 \, l / AY$

{ Assume A does not change following extension i.e. Poisson ratio = 1 }

$\therefore \quad \Delta x = x_2 - x_1$

$= (T_2 - T_1) \, l / AY$

$$\therefore \quad \boxed{\Delta x = \frac{\rho_w}{\rho} mg \, l / AY}$$

$$\therefore \quad \Delta x = \frac{1000}{8000} \times \frac{1 \times 10^4 \times 9.81 \times 10}{5.0 \times 10^{-4} \times 5.0 \times 10^{10}}$$

$= 4.9 \times 10^{-3}$ m $\approx$ $\boxed{4.9 \text{ mm}}$

q) A uniform beam AOB, O being the midpoint of AB, mass M, rests on three vertical springs with stiffness constants k_1, k_2, k_3 at A, O and B respectively. The bases of the springs are fixed to a horizontal platform. Determine the compression of the springs and their compressional forces in the following two instances:

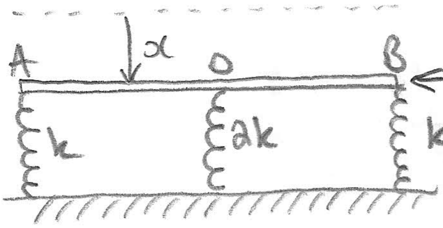
(i) $k_1 = k_3 = k$ and $k_2 = 2k$

(ii) $k_1 = k, k_2 = 2k$ and $k_3 = 3k$

$g = 9.81\ m/s^2$

[8]

(i)



← Beam of mass M

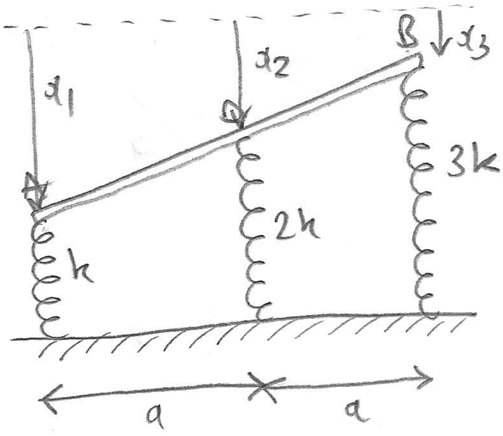
clearly symmetric!

let each spring be compressed by x.

To balance weight $Mg \Rightarrow kx + 2kx + kx = Mg$

$\therefore \boxed{x = Mg/4k}$

(ii) In this case spring 2 will be compressed less than #1, and 3 less than #2.



Now to balance weight

$\boxed{Mg = kx_1 + 2kx_2 + 3kx_3}$ (1)

But since bar is straight and AO = OB

$\boxed{x_1 - x_2 = x_2 - x_3}$ (2)

Now to be in equilibrium, no net moment about any point. let's choose O.

$\therefore \circlearrowright + \quad 3kx_3a - kx_1a = 0 \Rightarrow \boxed{3x_3 = x_1}$ (3)

$\therefore$ in (2): $3x_3 = 2x_2 - x_3 \Rightarrow \boxed{2x_3 = x_2}$ (4)

$\therefore$ in (1): $Mg = k(3x_3) + 2k(2x_3) + 3x_3k \Rightarrow \boxed{x_3 = Mg/10k}$

(21)

$$\therefore \boxed{x_2 = \frac{Mg}{5k}, \quad x_1 = \frac{3Mg}{10k}}$$

{ Note in official solutions they state ② as:

$$x_1 - x_3 = 2(x_2 - x_3) \quad ✓$$

$$\therefore x_1 - x_3 + 2x_3 = 2x_2$$

$$\boxed{x_1 + x_3 = 2x_2}$$

But they use $2x_2 = x_1 + (2x_3)$
↑ error

.... this results in $x_1 = \frac{3Mg}{11k}$, $x_2 = \frac{5Mg}{22k}$, $x_3 = \frac{Mg}{11k}$ }

so compressional forces are (and extensions)

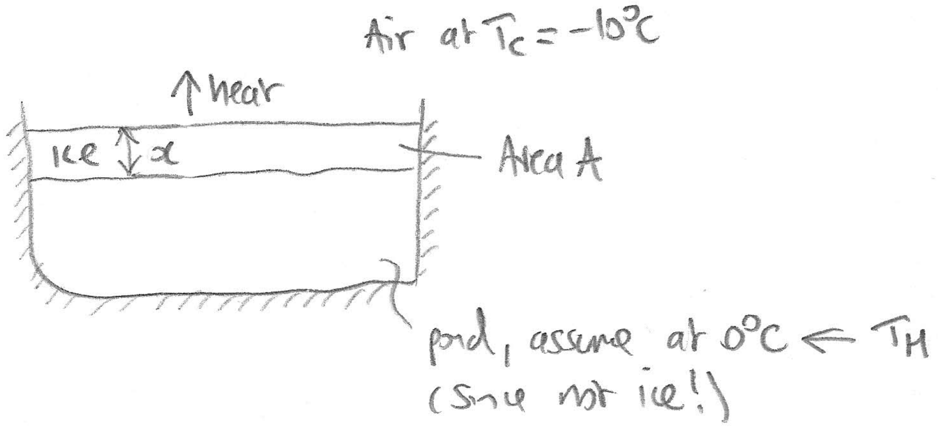
Spring	extension	compressional force
1	$3Mg/10k$	$3Mg/10$
2	$Mg/5k$	$\frac{2}{5}Mg$
3	$Mg/10k$	$\frac{3}{10}Mg$

r) A pond is covered by a layer of ice 5 cm thick. How long will it be before the ice is 10 cm thick if the air temperature stays constant at $-10\,°\mathrm{C}$?

Assume the density of ice $= 900\,\mathrm{kg\,m^{-3}}$; the latent heat of fusion of ice $= 330\,\mathrm{kJ\,kg^{-1}}$; the thermal conductivity of ice $= 2.1\,\mathrm{W\,m^{-1}\,K^{-1}}$.

The power flowing perpendicular to the faces through a uniform material is given by power flow $P = \lambda A \frac{(T_H - T_C)}{x}$, in which λ is the thermal conductivity of the material, T_H is the hotter temperature at one face of the material, T_C is the colder temperature on the other face, A is the area of a face, and x is the thickness of the material.

[7]



{ Perhaps more elegant to define ΔT as $T_{freeze} - T_{air}$ rather than $|T_c|$ }

Heat loss from ice is $\lambda A |T_c| / x$ $\mathrm{J\,s^{-1}}$

Assume ice grows rather than simply gets colder

∴ in δt seconds there is a loss of $\delta m L$ Joules due to latent heat of fusion of ice.

$\delta m = A \delta x \rho$, where δx is the extra ice thickness in δt

So balancing heat: $+\frac{\lambda A |T_c|}{x}\delta t = A \delta x \rho L$

$\Rightarrow \frac{\delta x}{\delta t} = \frac{\lambda A |T_c|}{A \rho L}\frac{1}{x}$ $\Rightarrow \frac{dx}{dt} = \frac{\lambda |T_c|}{\rho L}\frac{1}{x}$

So $\int_{x_0}^{x} x'\,dx' = \frac{\lambda |T_c|}{\rho L}\int_0^t dt'$

$\Rightarrow \frac{1}{2}(x^2 - x_0^2) = \frac{\lambda |T_c|}{\rho L} t$

$\Rightarrow \boxed{t = \frac{\rho L}{2\lambda |T_c|}(x^2 - x_0^2)}$

So if $x = 0.1\,\mathrm{m}$, $x_0 = 0.05\,\mathrm{m}$

$\Rightarrow t = \frac{900 \times 330 \times 10^3}{2 \times 2.1 \times 10}(0.1^2 - 0.05^2)$

$= 5.3 \times 10^4\,\mathrm{s} \simeq \boxed{14.7\text{ hours}}$