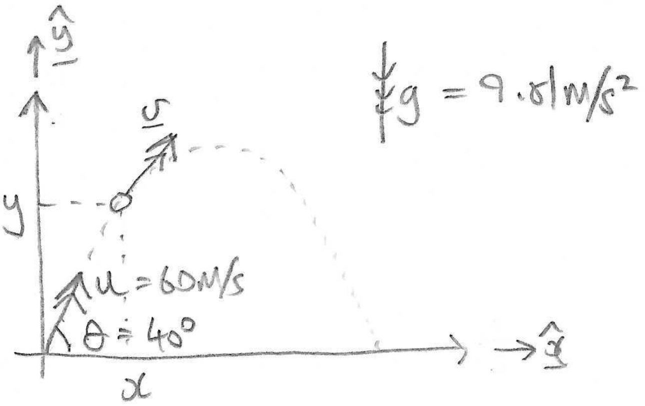
BPhO 1 2019 Section 1 AF SOLUTIONS

a) A golf ball is struck and begins to move at an initial velocity of 60 m s^{-1} at an angle 40° above the horizontal. Determine at time $t = 3$ s after the strike

(i) the velocity of the ball, and

(ii) the position of the ball relative to the origin. } VECTORS

[4]



Ignoring air resistance, and assume ball is struck from (0,0)

$$\underline{v} = \begin{pmatrix} u\cos\theta \\ u\sin\theta - gt \end{pmatrix} \qquad \underline{r} = \begin{pmatrix} ut\cos\theta \\ ut\sin\theta - \frac{1}{2}gt^2 \end{pmatrix}$$

$\boxed{\text{Velocity}}$ $\boxed{\text{position}}$

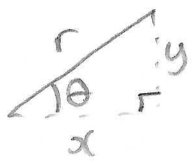
[i.e. constant acceleration motion in both x and y directions in $\hat{x}$ acceleration is ZERO, so constant velocity, in $\hat{y}$, acceleration is g downwards]

So after 3s:

$$\underline{v} = \begin{pmatrix} 60\cos 40° \\ 60\sin 40° - 9.81\times 3 \end{pmatrix} = \begin{pmatrix} 46.0 \\ 9.1 \end{pmatrix} \text{(m/s)}$$

Cartesian x, y Coordinates

$$\underline{r} = \begin{pmatrix} 60\cos 40° \times 3 \\ 60\sin 40° \times 3 - \frac{1}{2}\times 9.81\times 3^2 \end{pmatrix} = \begin{pmatrix} 138 \\ 71.6 \end{pmatrix} \text{(m)}$$



$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\boxed{\text{Velocity:}}$ $v = \sqrt{46^2 + 9.1^2} = \boxed{46.9 \text{ m/s}}$

at an elevation of $\tan^{-1}\left(\frac{9.1}{46}\right) = \boxed{11.2°}$

$\boxed{\text{Position}}$ $r = \sqrt{138^2 + 71.6^2} = \boxed{155}$

at an elevation of $\tan^{-1}\left(\frac{71.6}{138}\right) = \boxed{27.4°}$

} Range and elevation (polar) Coordinates.

Assume in m

b) A drone flies horizontally. The displacement of the drone is given by $\underline{s} = 2t\hat{\imath} + 6t\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors to the East and North respectively. Determine at $t = 2$ s:

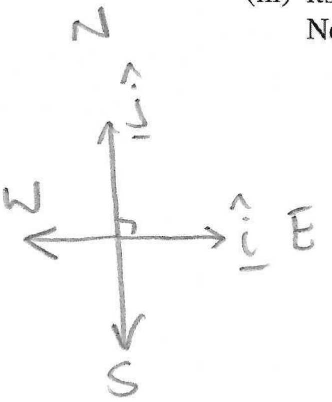
(i) the speed of the drone,

(ii) its bearing in degrees,

(iii) its acceleration.

Note: **all** bearings are measured clockwise from North.

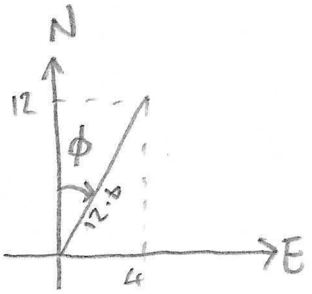
[3]



(i) $\underline{v} = d\underline{s}/dt = \boxed{2\underline{i} + 6\underline{j}}$

So constant speed of $\sqrt{2^2+6^2} = \boxed{6.3}$ (m/s)

(ii) At $t = 2s$ $\quad \underline{s} = 4\underline{i} + 12\underline{j}$



$\therefore$ Bearing $\phi = \tan^{-1}\left(\frac{4}{12}\right) = \boxed{018.4°}$

[Range in m is $\sqrt{4^2+12^2} = 12.6$]

(iii) $\underline{a} = d\underline{v}/dt = \underline{0}$ since

$\underline{v}$ is a constant.

c) Estimate the mass of a piece of paper the size of a pinhead (the blunt end of a sewing pin). Show your calculation.

[2]

Paper is typically defined by:

$$\boxed{\text{Grammage} = \frac{\text{mass in g}}{\text{length/m} \times \text{width/m}}}$$ *

laser printer ("office") paper is $\boxed{80 g/m^2}$

photo paper ≈ $\boxed{300 g/m^2}$

let "paper" be $\boxed{100 g/m^2}$

∴ mass for a pinhead size is

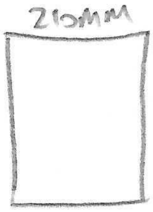
$$\simeq 100 \times 10^{-3} \frac{kg}{m^2} \times (10^{-3} m)^2$$

$$\simeq 10^{2-3-6}\ kg$$

$$\simeq \boxed{10^{-7}\ kg}$$

[Acceptable calculated answers within $10^{-8} \ldots 10^{-6}$ (kg)]

* A4:



$\frac{297}{210} \simeq \sqrt{2}$

mass @ $80 g/m^2$

$= 80 \times 10^{-3} \frac{kg}{m^2} \times 210 \times 297 \times 10^{-6} m^2$

$= \boxed{4.99 \times 10^{-3} kg}$ i.e. ≃ 5g

d) The speed of surface waves of wavelength λ on a liquid of density ρ is given by

$$v = \left[\frac{a\lambda}{2\pi} + \frac{2\pi b}{\rho\lambda}\right]^{\frac{1}{2}}$$

where a and b are constants. Determine the units of a and b.

[2]

$$\Rightarrow \quad v^2 = \frac{a\lambda}{2\pi} + \frac{2\pi b}{\rho\lambda}$$

So $\quad [a\lambda] = m^2 s^{-2}$ $\qquad [\lambda] = m$

$\therefore \quad [a] = \boxed{m s^{-2}}$

[...] notation means 'find the units of'.

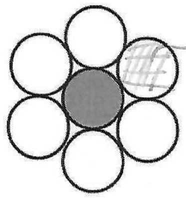
$\left[\frac{b}{\rho\lambda}\right] = m^2 s^{-2}$

$\therefore \quad [b] = m^2 s^{-2} \, kg m^{-3} \, m$

$= \boxed{kg s^{-2}}$

$\rho_A = 3.2\times10^{-8}\,\Omega m$ $\qquad$ $\rho_S = 2.0\times10^{-7}\,\Omega m$

e) **Figure 1** shows the cross section of a high voltage overhead electrical transmission cable. The central strand is of steel and the six outer strands are of aluminium. The resistivity of steel is $2.0\times10^{-7}\,\Omega\,\text{m}$, and that of aluminium $3.2\times10^{-8}\,\Omega\text{m}$. The cross-sectional area of each strand is $5.0\times10^{-4}\,\text{m}^2$. The steel is present to give mechanical strength to the cable and only reduces the resistance of a length ℓ of cable by $1.4\times10^{-4}\,\Omega$ (δR) when it is included. Calculate the length of the cable.



$A = 5.0\times10^{-4}\,m^2$

Figure 1

[5]

Al strand resistance : $\boxed{R_A = \rho_A l/A}$

Steel " " : $\boxed{R_S = \rho_S l/A}$

Resistance of cable without steel one is $\boxed{R_A/6}$ (since six in parallel).

" " " with steel one is :

$$\boxed{R_A/6 - \delta R = \frac{1}{\left(\frac{R_A}{6}\right)^{-1} + \frac{1}{R_S}}}$$

$$\left(\frac{R_A}{6} - \delta R\right)\left(\frac{6}{R_A} + \frac{1}{R_S}\right) = 1$$

$$\cancel{1} - \frac{6\delta R}{R_A} + \frac{R_A}{6R_S} - \frac{\delta R}{R_S} = \cancel{1}$$

$$\frac{R_A}{6R_S} = \delta R\left(\frac{1}{R_S} + \frac{6}{R_A}\right)$$

Now solve for l $\rightarrow$ $$\frac{\rho_A}{6\rho_S} = \frac{\delta R\, A}{l}\left(\frac{1}{\rho_S} + \frac{6}{\rho_A}\right)$$

$$l = \frac{6\rho_S A}{\rho_A}\,\delta R\left(\frac{1}{\rho_S} + \frac{6}{\rho_A}\right)$$

$$\boxed{l = \frac{6A\delta R}{\rho_A}\left(1 + \frac{6\rho_S}{\rho_A}\right)}$$

$\therefore\ l = 6\times5.0\times10^{-4}$
$\times\ 1.4\times10^{-4}$
$\times\left(1 + 6\times\frac{20}{3.2}\right)$
$\times\ \frac{1}{3.2\times10^{-8}}$ (m)
$= \boxed{505\text{ m}}$

so 500m to 2 s.f.

ρ_P

f) Platinum (symbol Pt) and potassium (symbol K) have densities of $21.5\,\mathrm{g\,cm^{-3}}$ and $0.89\,\mathrm{g\,cm^{-3}}$ respectively. How many cubic centimetres (cm^3) of platinum could be attached to $10.0\,\mathrm{cm}^3$ of potassium before the combination sinks in mercury of density $13.6\,\mathrm{g\,cm^{-3}}$? Ignore any chemical reactions. ρ_K

ρ_M

[4]

Pt + K object sinks in M when overall density $> \rho_M$.

$V_k = 10.0\,\mathrm{cm}^3$
$V_p =$ volume of Pt

So $$\frac{\rho_P V_P + \rho_k V_k}{V_P + V_k} > \rho_M$$

$$\therefore \quad \rho_P V_P + \rho_k V_k > \rho_M (V_P + V_k)$$

$$V_P(\rho_P - \rho_M) > V_k(\rho_M - \rho_k)$$

$$\boxed{V_P > V_k \frac{(\rho_M - \rho_k)}{\rho_P - \rho_M}}$$

Nice ratio result!

Stick with algebra till the final calculation!

$$\therefore \quad V_P > 10.0\,\mathrm{cm}^3 \times \frac{13.6 - 0.89}{21.5 - 13.6}$$

$$\boxed{V_P > 16\,\mathrm{cm}^3}$$

T_0

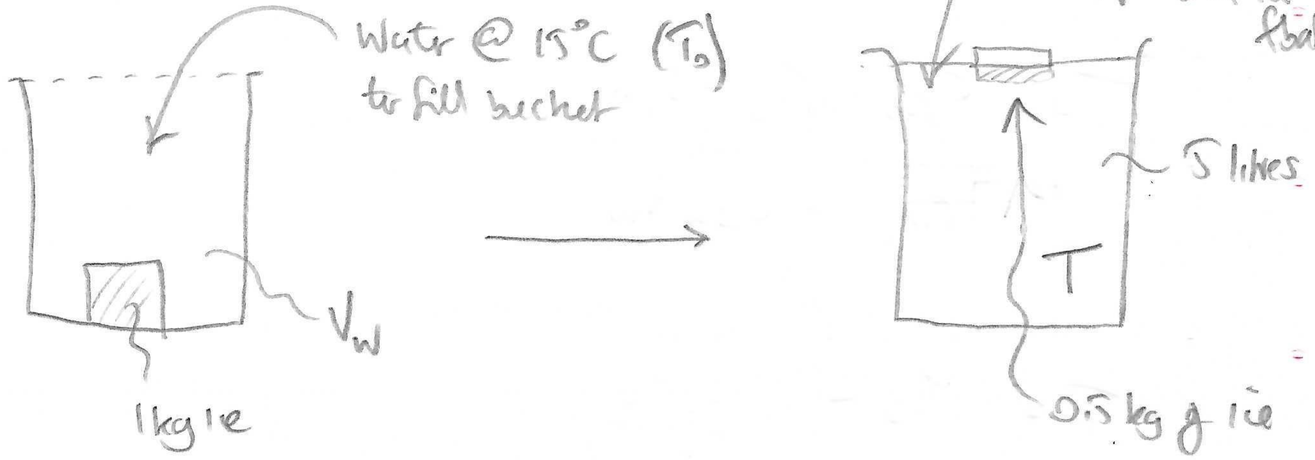
Assume quickly before ice has melted.

g) One kilogram of ice at 0 °C is placed in a thermally insulated bucket of volume 5 litres. Water at 15 °C is added until the bucket is completely filled. Calculate the temperature of the water when half of the ice has melted.

1 litre = 1000 cm^3
Latent heat of fusion of ice, $L_{\text{ice}} = 3.34 \times 10^5$ J kg^{-1}
Specific thermal capacity of water, $c_{\text{ice}} = 4180$ J kg^{-1} K^{-1}
Densities: $\rho_{\text{ice}} = 920$ kg m^{-3} and $\rho_{\text{water}} = 1000$ kg m^{-3}

[5]

Since liquid water is more dense than ice water level will drop slightly, and half melted ice will float..



Energy balance (assume no losses)

Energy lost from water added = latent heat to melt 0.5kg of ice + energy to heat melted ice to T °C.

But first... How much water is added?

$$V_w = 5000\text{ cm}^3 - \underbrace{\frac{1000\text{g}}{0.920\text{g/cm}^3}}_{\text{Volume of 1 kg of ice}}$$

$$= \boxed{3913\text{ cm}^3} = \boxed{3.913 \times 10^{-3}\text{ m}^3}$$

$m_i = 0.5$ kg

$$\therefore \; c\rho_w V_w (T_0 - T) = L_i m_i + c m_i T$$

{Assume ice is at 0°C}

$$\therefore \; c\rho_w V_w T_0 - L_i m_i = T(c m_i + c\rho_w V_w)$$

$$\therefore \; \boxed{T = \frac{c\rho_w V_w T_0 - L_i m_i}{c m_i + c\rho_w V_w}} = \frac{4180 \times 1000 \times 3.913 \times 10^{-3} \times 15 - 3.34 \times 10^5 \times 0.5}{4180 \times 0.5 + 4180 \times 1000 \times 3.913 \times 10^{-3}}$$

$\Rightarrow$ $\boxed{T = 4.25\,^{\circ}C}$

[Slightly nicer way to write T:

$$T = \frac{c\rho_w V_w T_0 - L_i m_i}{c m_i + c\rho_w V_w}$$

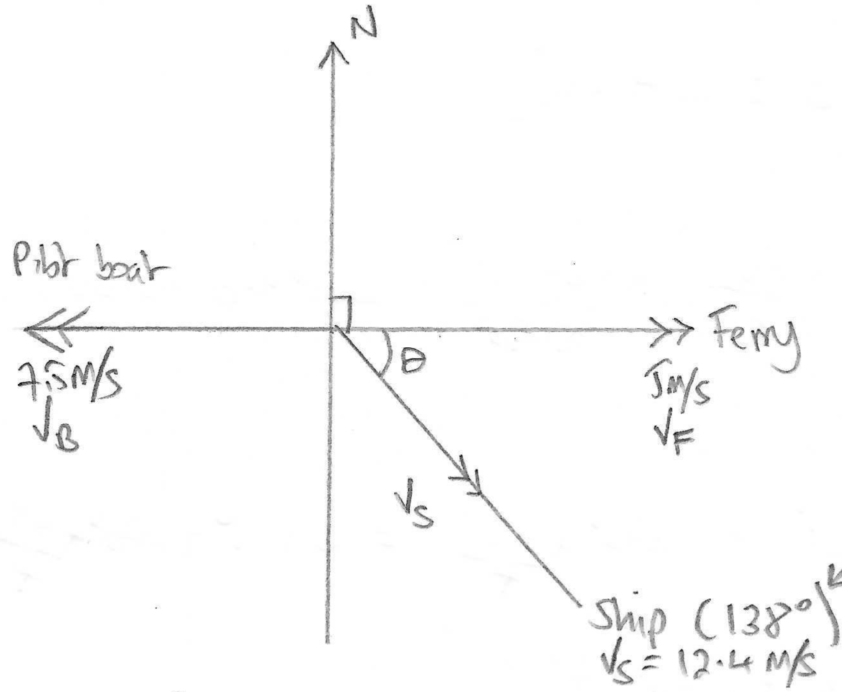
$$\boxed{T = \frac{T_0 - \dfrac{L_i m_i}{c\rho_w V_w}}{\dfrac{m_i}{\rho_w V_w} + 1}}$$

]

* NASTY! *

h) This question concerns three vessels at sea: a ferry (**F**), a container ship (**C**), and a pilot boat (**P**). The ferry is sailing on a bearing of 090° at $5\,\mathrm{m\,s^{-1}}$. Relative to the ferry, the container ship is sailing on a bearing of 160°. The pilot boat is sailing on a bearing of 270° at $7.5\,\mathrm{m\,s^{-1}}$, and the pilot boat observes the container ship moving on a bearing of 120°.
Determine the speed and direction of the container ship relative to the water.
Note: **all** bearings are measured clockwise from North.

[7]



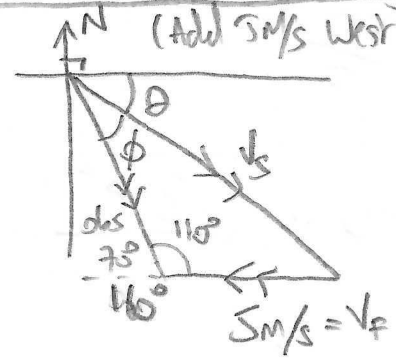
This is what we want is all three velocity relative to the water

We need to find ship speed v_S and bearing $90° + \theta$.

160°

observation from Ferry

(Add 5M/s West to Ship velocity)

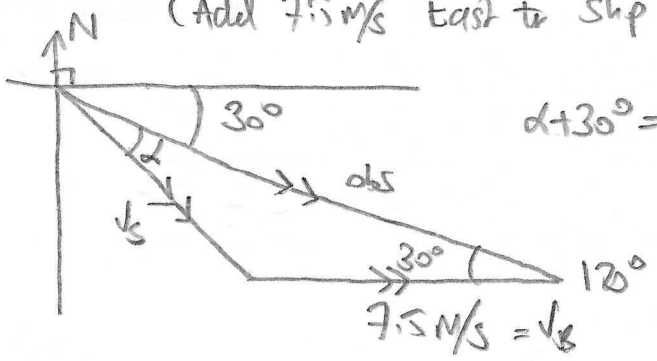


$$\theta + \phi = 160° - 90° = \boxed{70°}$$

$$\boxed{\frac{\sin(70° - \theta)}{v_F} = \frac{\sin 110°}{v_S}} \quad (1)$$

observation from Pilot boat

(Add 7.5 m/s East to Ship velocity)



$\alpha + 30° = \theta$

$$\boxed{\frac{\sin(\theta - 30°)}{v_B} = \frac{\sin 30°}{v_S}} \quad (2)$$

$$\frac{(1)}{(2)}: \quad \frac{\sin(70° - \theta)}{\sin(\theta - 30°)} \frac{v_B}{v_F} = \frac{\sin 110°}{\sin 30°}$$

$$\therefore \sin(70^\circ-\theta)\,\frac{v_B}{v_F} = \frac{\sin 110^\circ}{\sin 30^\circ}\sin(\theta-30^\circ)$$

$$\left[\sin 70^\circ\cos\theta - \cos 70^\circ\sin\theta\right]\frac{v_B}{v_F} = \frac{\sin 110^\circ}{\sin 30^\circ}\left(\sin\theta\cos 30^\circ - \cos\theta\sin 30^\circ\right)$$

$$\left[\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B\right]$$

$$\therefore \left(\sin 70^\circ - \cos 70^\circ\tan\theta\right)\frac{v_B}{v_F} = \frac{\sin 110^\circ}{\sin 30^\circ}\left(\cos 30^\circ\tan\theta - \sin 30^\circ\right)$$

$$\therefore \tan\theta\left(\frac{\cos 30^\circ\sin 110^\circ}{\sin 30} + \cos 70^\circ\times\frac{7.5}{5}\right) = \sin 110^\circ + \sin 70^\circ\times\frac{7.5}{5}$$

$$\therefore \theta = \tan^{-1}\left(\frac{\sin 110^\circ + \sin 70^\circ\times 7.5/5}{\frac{\cos 30^\circ\sin 110^\circ}{\sin 30^\circ} + \cos 70^\circ\times\frac{7.5}{5}}\right)$$

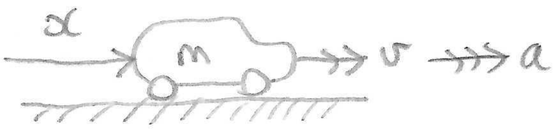
$$= \boxed{47.7^\circ}$$ so bearing is $\boxed{138^\circ}$

From ①: $$v_S = \frac{v_F\sin 110^\circ}{\sin(70^\circ-\theta)} = \frac{5\times\sin 110^\circ}{\sin(70^\circ-47.7^\circ)}$$

$$= \boxed{12.4\,m/s}$$

i) A car accelerates from a standing start. If the mass of the car is m, and the car is driven at constant driving power P, find an expression for the velocity of the car v as a function of distance travelled from a standing start, x. Ignore resistive effects and inefficiencies in power transmission.

[4]



Driving force $\boxed{F = P/v}$ Since $P = F\,dx/dt$

(work done $F\,dx$ in time dt)

NII: $\boxed{ma = \frac{P}{v}}$ Since no resistive forces.

Now $a = \frac{v\,dv}{dx}$ $\left[a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{dx}{dx} \right.$

chain rule method $\left. = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{v\,dv}{dx} \right]$

$\therefore\ mv^2\,dv = P\,dx$

$\therefore\ m\int_0^v v'^2\,dv' = Px$

$\frac{1}{3}mv^3 = Px$ $\therefore\ \boxed{v = \left(\frac{3Px}{m}\right)^{\frac{1}{3}}}$

[Note $Fv = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$ ie input power = rate of change of KE

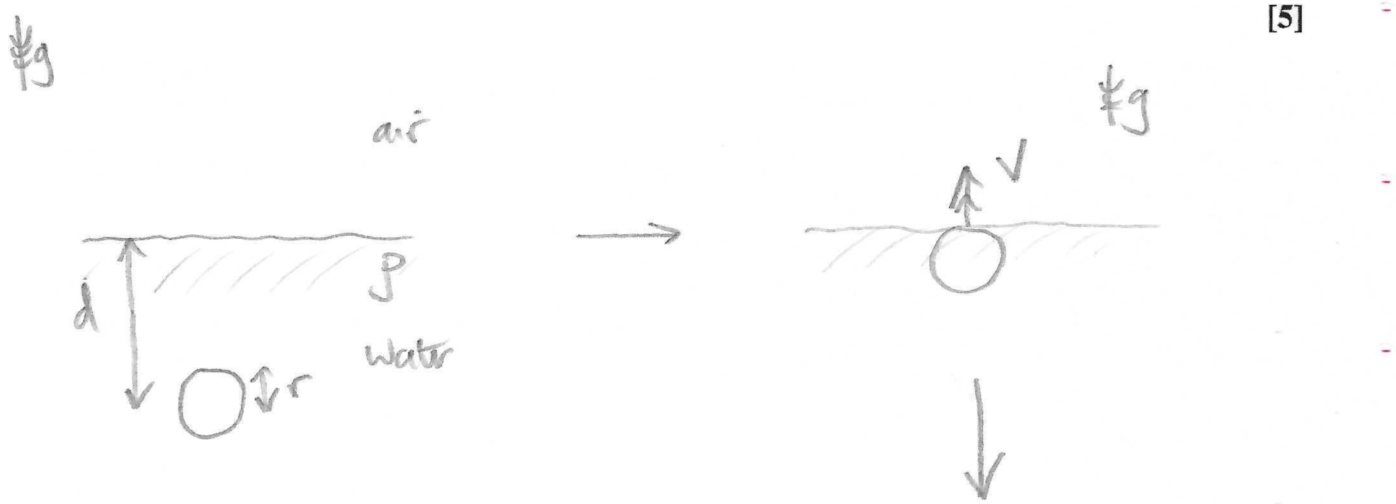
NII: $F = ma$; and $v = \frac{dx}{dt}$

So $ma\frac{dx}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$

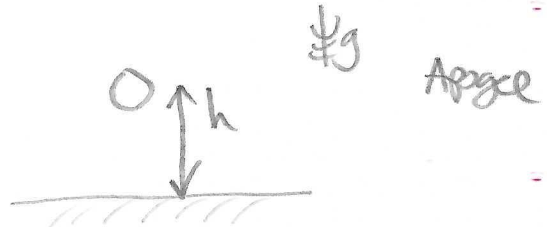
$\therefore\ a\,dx = v\,dv$ $\Rightarrow$ $\boxed{a = \frac{v\,dv}{dx}}$]

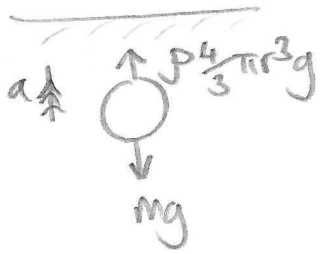
j) An experiment is proposed which involves submerging a ball of mass m and radius r to a depth of $d \gg r$ in a swimming pool. The ball is then released, and emerges from the water and rises to a height $h \gg r$ above the surface. The quantities d and h are measured from the centre of the ball to the water surface. An initial model is proposed, which ignores any resistive effects and the inertia of the water. Determine the prediction this initial model makes for the ratio h/d in terms of m, r and the density of water, ρ.

[5]



Since $d, h \gg r$ treat ball as a particle (ie ignore extension, partial submergence etc).



In water: 

$\rho \frac{4}{3}\pi r^3 g$ ← upthrust "weight of fluid displaced"

So NII $\quad ma = \rho\frac{4}{3}\pi r^3 g - mg$

$$a = \rho\tfrac{4}{3}\pi r^3 g / m - g \Rightarrow a = g\left(\frac{\rho\frac{4}{3}\pi r^3}{m} - 1\right)$$

ie constant acceleration motion → assume $a > 0$

∴ XUVAT: $\boxed{v^2 = 2ad}$

In air apply XUVAT again, but this time acceleration is downwards at g. ∴ At apogee

$$0 = v^2 - 2gh \quad ∴ \ 2gh = 2ad \Rightarrow h = d\frac{a}{g}$$

∴ $\boxed{h = d\left(\frac{\frac{4}{3}\pi r^3 \rho}{m} - 1\right)}$ So $\boxed{\frac{h}{d} = \frac{\frac{4}{3}\pi r^3 \rho}{m} - 1}$

k) A sand timer is a sealed glass vessel with a narrow section acting as a constraint, so that sand can flow through at a steady rate. A fifteen minute sand timer is shown in **Figure 2** below. Unlike a liquid, the rate of flow of sand grains through the constrained section is independent of the height of the sand above.
Thus the rate of flow of sand through the time can be expressed as a product of powers of the remaining relevant variables:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = k\rho^{\alpha} \times A^{\beta} \times g^{\gamma}$$

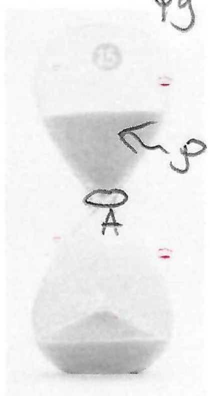
where k is a dimensionless constant, ρ is the density of the sand, A is the cross sectional area at the narrowest point, and g is the gravitational field strength, and α, β, γ are numbers.

(i) By considering the units of the variables on each side of the equation, find the values of α, β and γ.

(ii) On the Moon, the gravitational field strength is $g_{\mathrm{M}} = 1.6\,\mathrm{N\,kg^{-1}}$. How long would the sand timer last on the Moon if it runs for 15 minutes on Earth?

[4]

(i) $\left[\frac{dm}{dt}\right] = kg\,s^{-1}$ ∴ by dimensional analysis:



∴ $kg\,s^{-1} = (kg\,m^{-3})^{\alpha} (m^{2})^{\beta} (m s^{-2})^{\gamma}$

By comparing powers of kg, m, s:

kg: $\boxed{1 = \alpha}$

m: $0 = -3\alpha + 2\beta + \gamma$ ∴ $\boxed{2\beta + \gamma = 3}$ (1)

s: $-1 = -2\gamma \Rightarrow \boxed{\gamma = \frac{1}{2}}$

∴ $\beta = \frac{3 - \gamma}{2} = \boxed{\frac{5}{4}}$ (from (1))

So $\boxed{\frac{dm}{dt} = k\rho A^{\frac{5}{4}} g^{\frac{1}{2}}}$

(ii) $\frac{dm}{dt}$ is a constant ∴ for mass of sand M in both timers: $\boxed{M = k\rho A^{\frac{5}{4}} g^{\frac{1}{2}} t}$ i.e. t is time for sand to run out.

∴ $g_{\oplus}^{\frac{1}{2}} t_{\oplus} = g_M^{\frac{1}{2}} t_M$ Since M, ρ, k, A same on Earth and Moon.

∴ $\boxed{t_M = \sqrt{\frac{g_{\oplus}}{g_M}}\, t_{\oplus}} = \sqrt{\frac{9.81}{1.6}} \times 15 \text{ mins} = \boxed{37.1 \text{ minutes}}$

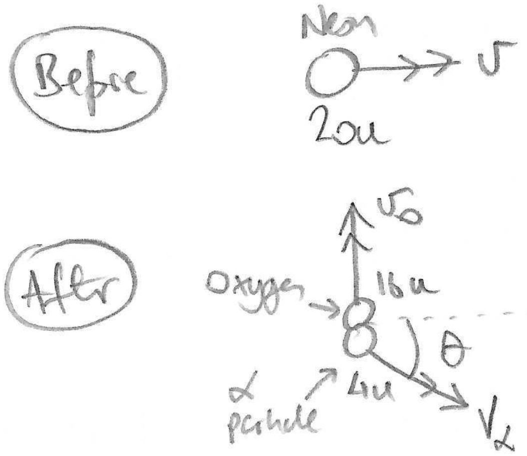
u = unified atomic mass unit $= 1.66 \times 10^{-27}$ kg

1) An excited neon-20 isotope travels with a velocity of $\overset{v}{3.0 \times 10^{6}}\ \mathrm{m\,s^{-1}}$ into a detector ~~and disintegrates~~ into an alpha particle and oxygen-16. The event produces an additional $\Delta E \rightarrow$ 6.25 MeV of kinetic energy. The oxygen nucleus leaves the event at right angles to the ~~path of the~~ original neon nucleus.

Determine the velocity of the alpha particle. Relativistic effects may be neglected.
$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

[5]

$\Rightarrow$ $KE = \frac{1}{2}mv^2$ not $(\gamma - 1)mc^2$
and momentum is mv not $\gamma m v$
$\left(\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right)$



$\boxed{\text{Conservation of momentum}}$

$/\!/x$: $\quad 20uv = 4u v_\alpha \cos\theta \quad \Rightarrow \quad \boxed{5v = v_\alpha \cos\theta}$ ①

$/\!/y$: $\quad 0 = 16u v_0 - 4u v_\alpha \sin\theta \quad \Rightarrow \quad \boxed{4v_0 = v_\alpha \sin\theta}$ ②

$\boxed{\text{Conservation of energy}}$ $\quad \frac{1}{2}20uv^2 + \Delta E = \frac{1}{2}16u v_0^2 + \frac{1}{2}4u v_\alpha^2$

$$10v^2 + \Delta E/u = 8v_0^2 + 2v_\alpha^2$$

$$\boxed{5v^2 + \frac{\Delta E}{2u} = 4v_0^2 + v_\alpha^2}$$ ③

①^2 + ②^2: $\quad 25v^2 + 16v_0^2 = v_\alpha^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1})$ ④

So: $\quad 20v^2 + \frac{2\Delta E}{u} = 16v_0^2 + 4v_\alpha^2 \qquad 4③ = ⑤$

$\therefore$ since from ④: $16v_0^2 = v_\alpha^2 - 25v^2$

⑤: $\quad 20v^2 + 2\Delta E/u = v_\alpha^2 - 25v^2 + 4v_\alpha^2$

$\therefore \quad 45v^2 + 2\Delta E/u = 5v_\alpha^2$

$$\therefore \boxed{\sqrt{9v^2 + \frac{2}{5}\frac{\Delta E}{u}} = v_\alpha}$$

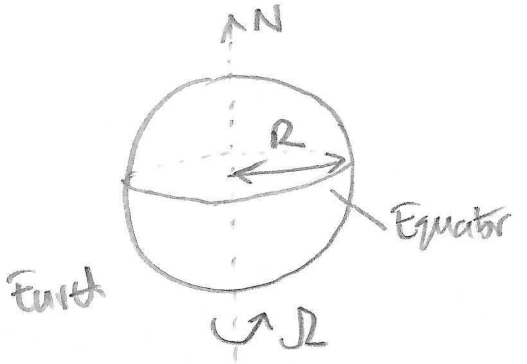
$$\therefore v_\alpha = \sqrt{9\times(3.0\times10^6)^2 + \frac{2}{5}\times\frac{6.25\times10^6\times1.6\times10^{-19}}{1.66\times10^{-27}}}$$

$$= \boxed{1.8\times10^7\ \mathrm{m/s}}$$

m) An aeroplane flies due East along the equator at a constant low altitude and constant speed relative to the ground. On the aeroplane, a one kilogram mass is suspended on a spring balance and records a weight W_1. The aeroplane then flies due West along the equator, at the same altitude and speed, and measures a balance reading of W_2.
If the speed of the plane relative to the ground is $250\,\mathrm{m\,s^{-1}}$, calculate the difference in apparent weights.

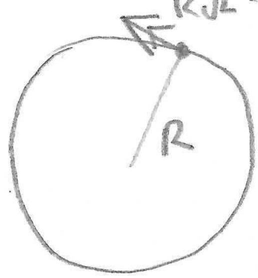
v

[5]



West to East:

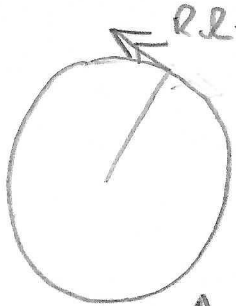
$R\Omega + v = V_1$



Circular motion relative to centre of Earth

$R\Omega - v = V_2$

East to West:



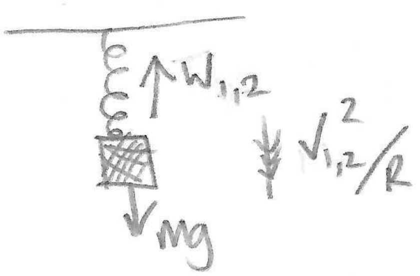
$R\Omega = 6.37\times10^6 \times \dfrac{2\pi}{24\times3600}$ m/s

$= \boxed{463\ \mathrm{m/s}}$

(rotation speed of Earth's surface at the equator)

So $R\Omega - v$ is +ve so still ↖ not ↘

$\boxed{\text{Inside aircraft}}$



Since $V_1 > V_2$
$\Rightarrow W_2 > W_1$

MS $\Rightarrow W_1 > W_2$
Error?

ie ↑ centripetal acc, feel more weightless

NII: West to East $\quad \dfrac{mV_1^2}{R} = mg - W_1$

East to West $\quad \dfrac{mV_2^2}{R} = mg - W_2$

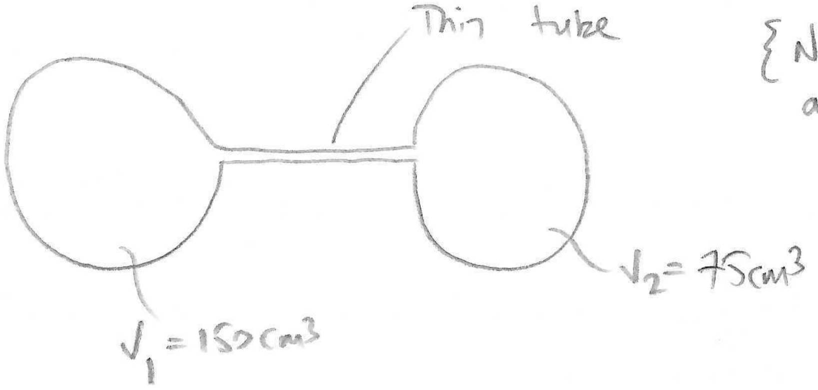
$\therefore\ W_2 - W_1 = \dfrac{m}{R}\left(V_1^2 - V_2^2\right) = \dfrac{1}{6.37\times10^6}\left((463+250)^2 - (463-250)^2\right)$

$= \boxed{7.3\times10^{-2}\ \mathrm{N}}$

$(m = 1.00\,\mathrm{kg})$

n) Two glass bulbs are connected by a thin tube. One glass bulb has a volume of 75 cm^3, the other 150 cm^3, and gas can move freely between them. Initially the system contains nitrogen at −12 °C and 0.91×10^5 Pa. The smaller bulb is then warmed to 24 °C, whilst the larger bulb is maintained at −12 °C. p
Calculate the new pressure in the system. Assume the thermal expansion of the bulbs and the volume of the connecting tube are negligible.

[5]



{Nitrogen gas in bulbs and tube}

* idea is that pressure in system is uniform in equilibrium, otherwise a pressure gradient would imply a force acting on gas molecules, which would then redistribute.

$\boxed{\text{Initially:}}$ $T_1 = T_0 = (273 - 12)$ K
$T_2 = T_0$
$P_0 = 0.91 \times 10^5$ Pa

Assuming ideal gas:

$$\boxed{\begin{aligned} P_0 V_1 &= n_1 R T_0 \quad (1) \\ P_0 V_2 &= n_2 R T_0 \quad (2) \end{aligned}}$$

(ignore molecules in thin tube)

After bulb 2 is heated to $T_2 = (273 + 24)$ K
assuming no molecules are lost: $\boxed{n_1' + n_2' = n_1 + n_2}$ (3)

$$\boxed{\begin{aligned} p V_1 &= n_1' R T_0 \quad (4) \\ p V_2 &= n_2' R T_2 \quad (5) \end{aligned}}$$

Substituting for # of moles in (3)

$$\frac{pV_1}{RT_0} + \frac{pV_2}{RT_2} = \frac{P_0 V_1}{RT_0} + \frac{P_0 V_2}{RT_0}$$

$$\therefore \quad p = \frac{\frac{P_0}{RT_0}(V_1 + V_2)}{\frac{V_1}{RT_0} + \frac{V_2}{RT_2}} \Rightarrow \boxed{p = P_0 \frac{V_1 + V_2}{V_1 + \frac{T_0}{T_2} V_2}}$$

$$\therefore \quad p = 0.91 \times 10^5\,\text{Pa} \times \frac{150 + 75}{150 + \frac{261}{297} \times 75}$$

99/95

$$= \boxed{0.95 \times 10^5\,\text{Pa}}$$

o) Determine the current in the 6.0 Ω resistor shown in the **Fig 3**. The cells have no internal resistance.

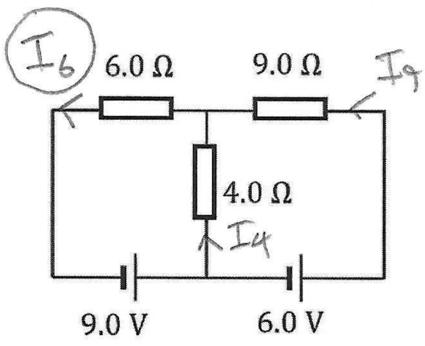


Figure 3

conserve currents at each junction

[5]

Kirchoff I: $I_6 = I_4 + I_9$ ①

Kirchoff II (outer loop) $9.0 + 6.0 = I_9 \times 9.0 + I_6 \times 6.0$ ②

" (left loop) $9.0 = I_4 \times 4.0 + I_6 \times 6.0$ ③

i.e. Σ of EMFs $= \Sigma I_i R_i$ round a loop

②: $I_9 = \dfrac{15 - 6I_6}{9}$

③: $I_4 = \dfrac{9 - 6I_6}{4}$

$\therefore$ ①: $I_6 = \frac{9}{4} - \frac{3}{2}I_6 + \frac{5}{3} - \frac{2}{3}I_6$

$I_6\left(1 + \frac{3}{2} + \frac{2}{3}\right) = \frac{9}{4} + \frac{5}{3}$

$I_6 \dfrac{(6 + 9 + 4)}{6} = \dfrac{27 + 20}{12}$

$I_6 = \dfrac{47}{2} \dfrac{1}{19}$ (A)

$= \dfrac{47}{38}$ (A)

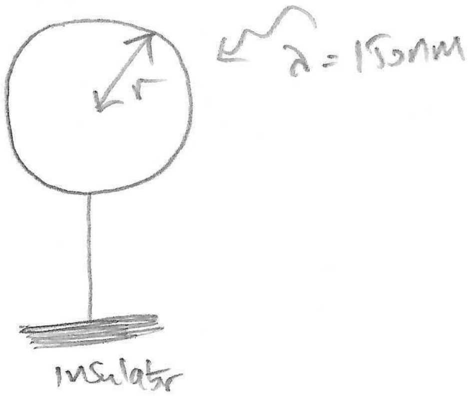
$= \boxed{1.24\ \text{A}}$ (close to 1.23 A but not quite!)

p) An electrically isolated copper sphere of radius 2 mm is illuminated by light of wavelength 150 nm. Determine

(i) the maximum electric potential that the copper sphere can reach

(ii) the number of electrons lost reaching the maximum potential
(Work function of copper = 4.5 eV)

[4]



Electric potential = energy per unit charge, so max potential is the max KE (E_{max}) of electrons that are liberated by the PHOTOELECTRIC EFFECT, ÷ charge on the electron.

(i) So since $\boxed{E_{max} = \frac{hc}{\lambda} - \phi}$

$$\therefore \quad V_{max} = \frac{E_{max}}{e} = \frac{hc}{\lambda e} - 4.5 \quad \text{(Volts)}$$

$$= \frac{6.63 \times 10^{-34} \times 2.998 \times 10^{8}}{150 \times 10^{-9} \times 1.60 \times 10^{-19}} - 4.5$$

$$= 8.28 - 4.5$$

$$= \boxed{3.8 \text{ Volts}}$$

Electric potential of a sphere

(ii) $\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$ $\quad \therefore \quad \#\text{ electrons} = \frac{Q}{e} = \frac{4\pi\varepsilon_0 r V}{e}$

unrounded!

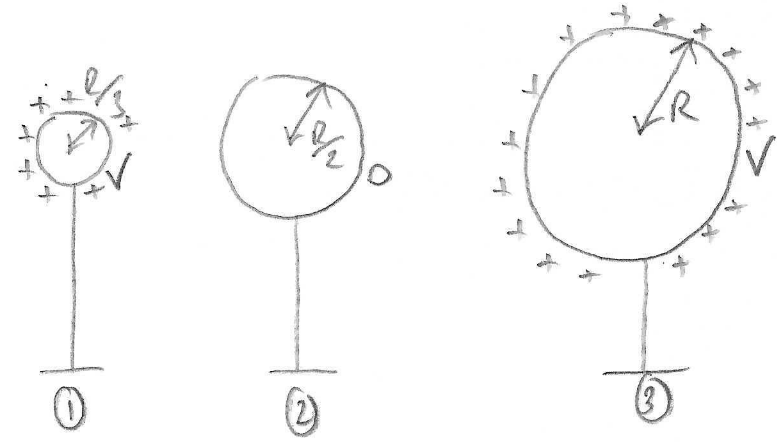
$$= \frac{4\pi \times 8.85 \times 10^{-12} \times 2 \times 10^{-3} \times 3.8\ldots}{1.60 \times 10^{-19}}$$

$$= \boxed{5.3 \times 10^{6} \text{ electrons}}$$

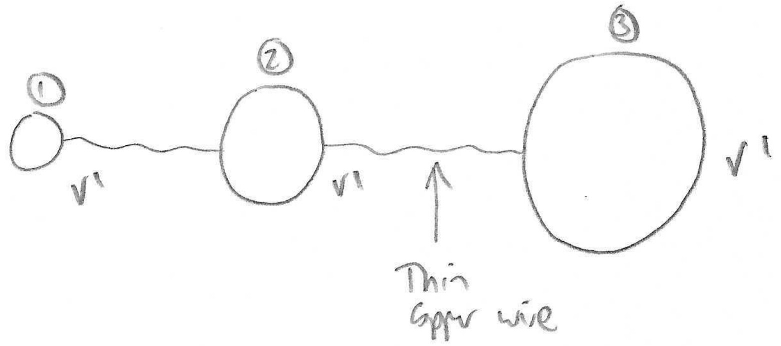
q) Three conducting spheres of radii $\frac{1}{3}R$, $\frac{1}{2}R$ and R are mounted on insulating rods, and are well separated from each other. The $\frac{1}{3}R$ and R spheres are each charged to a potential V, whilst the $\frac{1}{2}R$ sphere is uncharged. Then a thin copper wire is used to briefly connect all three spheres. What fraction of the original charge on the two spheres is now on the $\frac{1}{2}R$ sphere?

[5]

Before:



After



V'

Wire connection means all spheres are now at the same potential, otherwise a potential difference and current will flow. So 'equipotential' must be the equilibrium position, that we assume is rapidly attained.

Capacitance of a charged sphere is $\boxed{C = 4\pi\varepsilon_0 r}$ and $\boxed{Q = CV}$. So conserving charge:

$$4\pi\varepsilon_0\left(\frac{R}{3}V + RV\right) = 4\pi\varepsilon_0\left(\frac{R}{3}V' + \frac{R}{2}V' + RV'\right)$$

$$\Rightarrow V' = \frac{V\left(\frac{4}{3}\right)}{\frac{1}{3}+\frac{1}{2}+1} = \frac{\frac{4}{3}}{\frac{11}{6}}V = \boxed{\frac{8}{11}V}$$

So $$\frac{Q_2}{\text{original charge}} = \frac{4\pi\varepsilon_0 \frac{R}{2} V'}{4\pi\varepsilon_0\left(\frac{R}{3}V + RV\right)} = \frac{\frac{1}{2}}{\frac{4}{3}}\,\frac{8}{11} = \boxed{\frac{3}{11}}$$

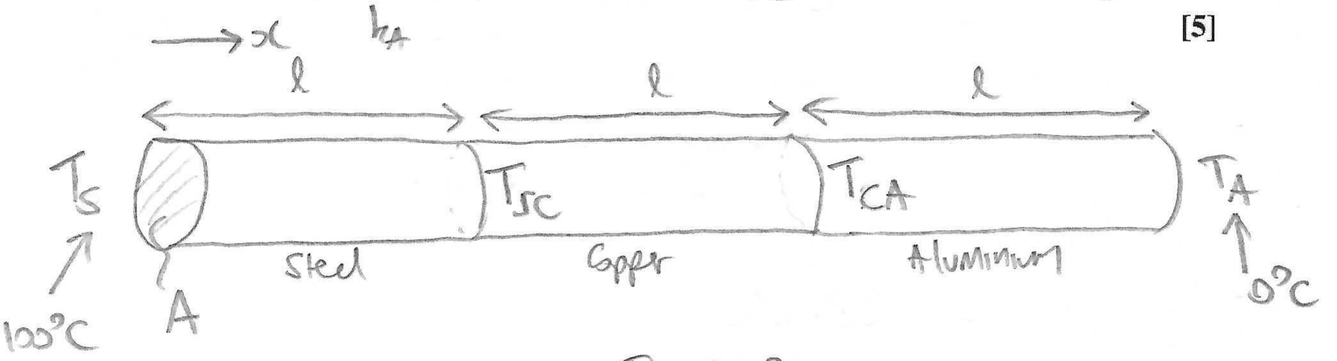
r) The thermal power flowing by conduction through a surface is proportional to the temperature difference across the surface, $\Delta\theta$, the area of the surface, A and inversely proportional to the thickness Δx. The constant of proportionality is known as the *thermal conductivity.* k

$l = 0.20\,\mathrm{m}$

A 60 cm composite rod, of constant cross section, is made of 20 cm lengths of steel, copper and aluminium joined together. The rod is well insulated. The tip of the steel end of the rod is maintained at 100 °C and the tip of the aluminium end, at 0 °C. What are the temperatures at each of the two junctions of dissimilar metals?

Thermal conductivities are as follows: steel 60 W m^{-1} K^{-1} (k_S); copper 400 W m^{-1} K^{-1} (k_C); aluminium 240 W m^{-1} K^{-1} (k_A).

[5]



$T_0 = 100°C$

* The thermal power (ie in Watts) is $\boxed{\dfrac{k\Delta\theta A}{\Delta x}}$ (ie kA × temperature gradient). Assume this is uniform through the rod in 'dynamic equilibrium' ie $T(x)$ is time independent.

$$\therefore \underset{①}{(T_S - T_{SC})k_S} = \underset{②}{(T_{SC} - T_{CA})k_C} = \underset{③}{(T_{CA} - T_A)k_A}$$

Since $\Delta x = l$, and A, are the same for each rod section.

$$\therefore \boxed{T_{CA} = \frac{k_C T_{SC} - k_S T_S + k_S T_{SC}}{k_C}} \qquad ① = ② \qquad ④$$

$$\therefore T_{CA}(k_A + k_C) = T_{SC}k_C + T_A k_A \qquad ② = ③$$

$$\therefore \boxed{T_{CA} = \frac{T_{SC}k_C + T_A k_A}{k_A + k_C}} \qquad ⑤$$

we know this.

Equating T_{CA}: ④ = ⑤

$$T_{SC}\left(1 + \frac{k_S}{k_C} - \frac{k_C}{k_A + k_C}\right) = \frac{T_A k_A}{k_A + k_C} + \frac{k_S}{k_C}T_S$$

collecting terms

i.e. can now solve for T_{SC}

$$\therefore \quad \boxed{T_{SC} = \frac{\dfrac{T_A k_A}{k_A + k_C} + T_S k_S / k_C}{1 + k_S/k_C - \dfrac{k_C}{k_A + k_C}}}$$

$k_S = 60$
$k_C = 400$
$k_A = 240$
(units $W m^{-1} K^{-1}$)

$$= \frac{0 + 100 \times 60/400}{1 + 60/400 - \dfrac{400}{240 + 400}} \quad (°C)$$

$$= \boxed{28.6°C}$$

From (5):
$$T_{CA} = \frac{T_{SC} k_C + T_A k_A}{k_A + k_C}$$

$$= \frac{28.6\ldots \times 400 + 0}{240 + 400}$$

$$= \boxed{17.9°C}$$

Perhaps more elegant to define: $\boxed{\begin{aligned} \Delta\theta_1 &= T_S - T_{SC} \\ \Delta\theta_2 &= T_{SC} - T_{CA} \\ \Delta\theta_3 &= T_{CA} - T_A \end{aligned}}$

$$\therefore \quad k_S \Delta\theta_1 = k_C \Delta\theta_2 = k_A \Delta\theta_3$$

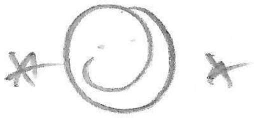
ie solve for $\Delta\theta_{1,2,3}$ instead.
then find $\boxed{\begin{aligned} T_{SC} &= T_S - \Delta\theta_1 \\ T_{CA} &= T_{SC} - \Delta\theta_2 \\ &= \Delta\theta_3 \end{aligned}}$

Note: $\boxed{\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 = T_S - T_A}$
$= 100°C$

$$\Rightarrow \left(\frac{k_A}{k_S} + \frac{k_A}{k_C} + 1\right)\Delta\theta_3 = T_S - T_A$$

$$\therefore \quad T_{CA} = \Delta\theta_3 = \frac{100}{\frac{240}{60} + \frac{240}{400} + 1} = \boxed{17.9°C}$$

$$\Delta\theta_1 = \frac{k_A \Delta\theta_3}{\ldots} = \frac{240}{\ldots} 17.9 = 71.4°$$

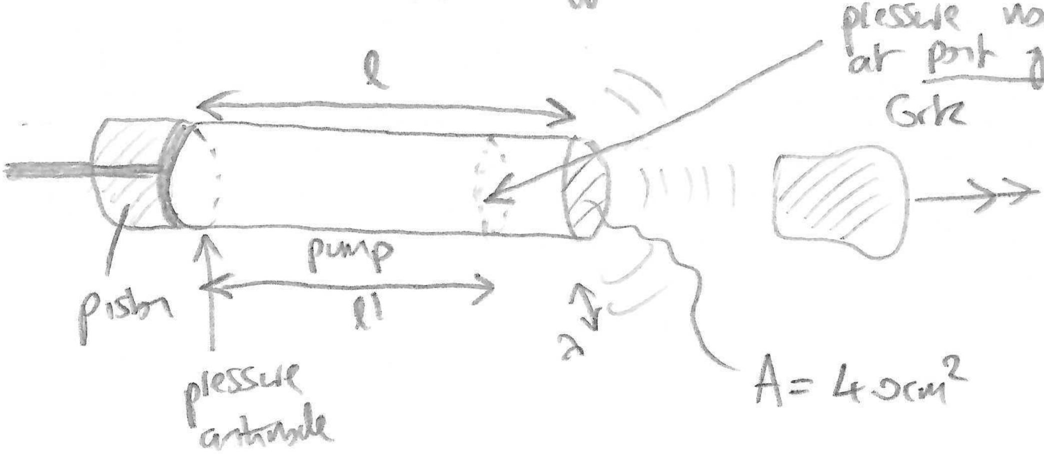


Although l', l should be better defined $\boxed{l = 0.25\,m}$

s) A bicycle pump of cross-sectional area $4.0\,\mathrm{cm}^2$ has one end sawn off and a cork is fitted into the end. The piston is pushed slowly inwards and the cork is fired out with a popping sound which has a frequency of 512 Hz. The initial distance between the cork and the piston is 25 cm, with atmospheric pressure equal to 1.0×10^5 Pa and the speed of sound in air being $330\,\mathrm{m\,s}^{-1}$. C
Calculate the force required to eject the cork. **[4]**

I think this means between piston and open end of tube

Assume $\boxed{f = 512\,Hz}$ is the fundamental frequency of a standing wave in the open ended tube once the cork has popped off.

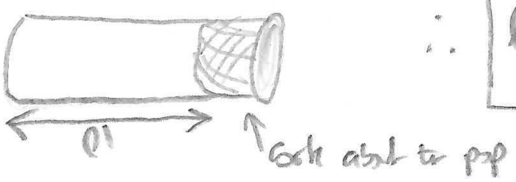


$$\lambda = \frac{330}{512} \text{ (m)}$$

$$= \boxed{0.645\,m}$$

Pressure wave in tube at fundamental frequency has a pressure antinode at piston end and node (atmospheric pressure) at the open end ← when the cork is released! So $l' \neq l$.

$\therefore \boxed{l' = \lambda/4}$ $\quad \therefore \boxed{l' = \frac{c}{4f}}$ $\quad$ ($l' = l -$ cork length)



Now assume piston moves slow enough not to heat the air inside ie isothermal compression. If the gas inside is ideal $\Rightarrow$

$$pAl' = P_a A l \quad \text{(Boyle's law)}$$

[$pV = nRT$
if $T =$ const
$n =$ "
$\therefore pV =$ const]

$$\therefore \boxed{p = P_a \frac{l}{c} \times 4f}$$

$$\therefore p = 1.0 \times 10^5\,Pa \times \frac{25}{100} \times \frac{4 \times 512}{330}$$

$$p = \boxed{1.55 \times 10^5\,Pa}$$

So force to pop the cork is:

$$\boxed{F = (p - p_a) A}$$

$$= 0.551 \times 10^5\,\text{Pa} \times 4 \times (10^{-2}\,\text{m})^2 \quad (\text{N})$$

$$= \boxed{22\,\text{N}}$$

t) A cup of tea cools from 30.2 °C to 29.7 °C in 1 minute, in an ambient temperature of 20.0 °C. Assuming the tea cools at a rate directly proportional to the temperature difference between the tea and the surroundings, calculate how long it will take for the tea to cool from 24.0 °C to 23.0 °C.

[5]

At time t:

T_a

Assume Newtonian cooling

i.e.:

$$\boxed{\frac{dT}{dt} = -k(T - T_a)}$$

$(T = T_0 \text{ when } t = 0)$

$$\therefore \int_{T_0}^{T} \frac{dT}{T - T_a} = -kt$$

$$\Big[\ln(T - T_a)\Big]_{T_0}^{T} = -kt$$

$$\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = -kt$$

$$\therefore \boxed{k = \frac{1}{t}\ln\left(\frac{T_0 - T_a}{T - T_a}\right)}$$

Using $t = 1 \text{ min}$, $T_0 = 30.2\,°C$, $T_a = 20\,°C$, $T = 29.7\,°C$

$$\Rightarrow \quad k = \frac{1}{1}\ln\left(\frac{30.2 - 20}{29.7 - 20}\right) = 0.050 \text{ min}^{-1} = \boxed{\frac{1}{19.9 \text{ min}}}$$

$\therefore$ If $T = 23.0\,°C$, $T_0 = 24.0\,°C$, $T_a = 20\,°C$, and same k.

$$\boxed{t = \frac{1}{k}\ln\left(\frac{T_0 - T_a}{T - T_a}\right)} = 19.9 \text{ mins} \times \ln\left(\frac{24.0 - 20}{23 - 20}\right)$$

$$= \boxed{5.7 \text{ mins}}$$