# BPhO 2021 Round 1 Section 1 AF Solutions

m $\quad u_1 \quad$ 2M

a) A railway truck travelling along a level track at $5.0\,\mathrm{m\,s^{-1}}$ collides with a truck of twice the mass moving in the same direction at $2.5\,\mathrm{m\,s^{-1}}$. The trucks couple together and continue moving. Calculate

$u_2 \quad$ ie inelastic

$v$

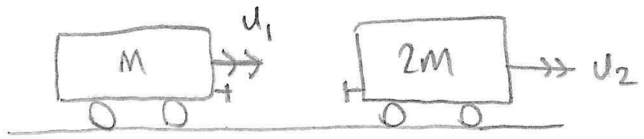
(i) the final speed of the combined trucks, and

(ii) the percentage of the kinetic energy lost in the collision.

↑ $\Delta E$

**[3]**

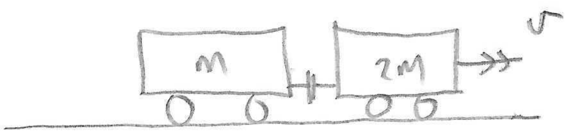
BEFORE



$u_1 = 5.0\,\mathrm{m/s}$

$u_2 = 2.5\,\mathrm{m/s}$

AFTER



(Trucks are coupled together)

(i) By conservation of momentum $\quad \rightarrow +ve$

$$\boxed{mu_1 + 2mu_2 = 3mv}$$

$$\therefore \quad v = \frac{u_1 + 2u_2}{3} = \frac{5.0 + 2(2.5)}{3}\ \mathrm{m/s}$$

$$= \boxed{3.3\ \mathrm{m/s}}$$

$\left(= \tfrac{10}{3}\right.$ for subsequent calcs$\left.\right)$

(ii)
$$\boxed{\underbrace{\tfrac{1}{2}mu_1^2 + \tfrac{1}{2}(2m)u_2^2}_{\text{KE before}} = \underset{\uparrow\ \text{Energy lost in collision}}{\Delta E} + \underbrace{\tfrac{1}{2}(m+2m)v^2}_{\text{KE after}}}$$

Initial KE $= \frac{1}{2}m\left(5^2 + 2\times\left(\frac{5}{2}\right)^2\right) = \frac{25}{2}m\left(1+\frac{1}{2}\right) = \boxed{\frac{75m}{4}}$ (J)

Final KE $= \frac{1}{2}m(1+2)\left(\frac{10}{3}\right)^2 = \boxed{\frac{50m}{3}}$ (J)

$$\therefore \quad \frac{\Delta E}{\text{Initial KE}} = \frac{\frac{75}{4} - \frac{50}{3}}{\frac{75}{4}} = 1 - \left(\frac{50}{3}\right)\left(\frac{4}{75}\right) = \frac{1}{9} = \boxed{11\%}$$

[....] means 'find the units of' eg

| | | |
|---|---|---|
| M | mass | kg |
| L | length | m |
| T | time | s |

SI

b) The fundamental frequency of a drum skin has been shown to be given by $f = \dfrac{0.47hv}{a^2\sqrt{1-\rho^2}}$ where $h$ is the thickness of the skin, $v$ is the speed of sound in the skin, and $\rho$ is a constant of elasticity. Units are in SI units.

(i) What are the units of $\rho$?

(ii) Determine the units of quantity $a$.

**[2]**

(i) $[\rho]$ are no units (ie $\rho$ is a dimensionless number). This must be the case as $\sqrt{1-\rho^2}$ must be a dimensionless number since 1 is.

(ii) $$a^2 = \frac{0.47hv}{f\sqrt{1-\rho^2}}$$

$$\therefore [a^2] = [hv/f] = \frac{(m)(ms^{-1})}{s^{-1}} = m^2$$

So $\boxed{[a] = m}$ ie a has the dimensions of length only.

Could also solve via pure algebra: $x_A = 40t - \frac{1}{2}10t^2 \quad (t \geq 0)$

$x_B = 60(t-T) - \frac{1}{2}10(t-T)^2 \quad (t \geq T)$

BALL A  BALL B

c) A ball is thrown vertically upwards with a velocity of $40\,\mathrm{m\,s^{-1}}$. After 1.0 s, a second ball is thrown upwards with a velocity of $60\,\mathrm{m\,s^{-1}}$.
For this question, you may take $g = 10\,\mathrm{m\,s^{-2}}$

T

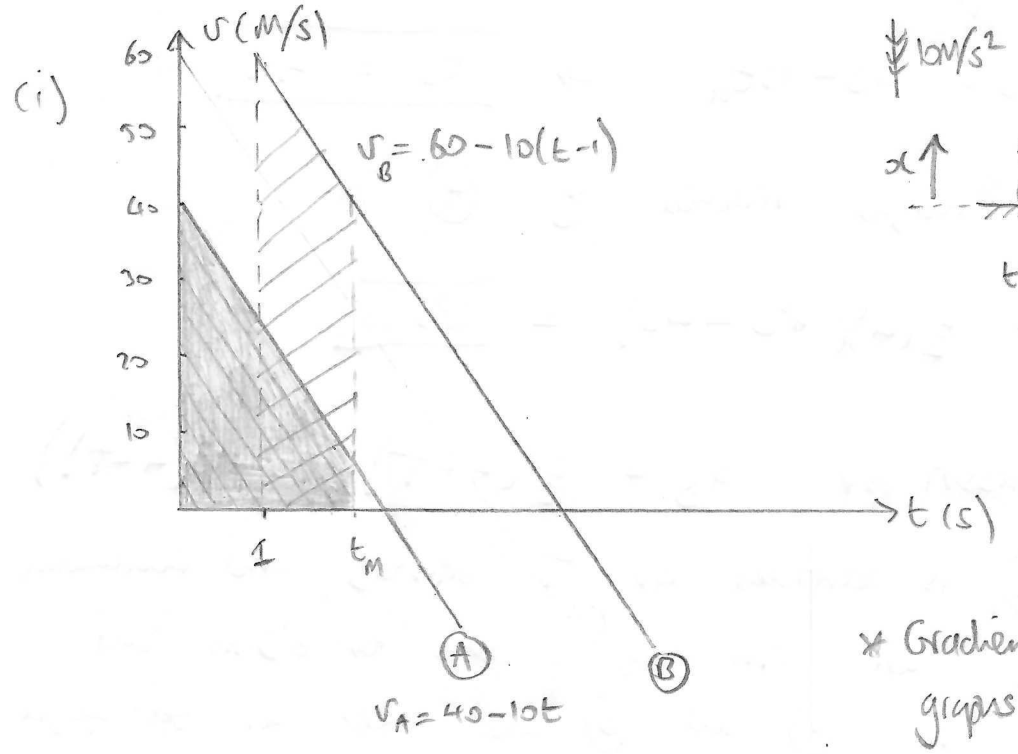
(i) Sketch, on the same axes, velocity-time graphs for each ball. Take the direction upwards as positive.

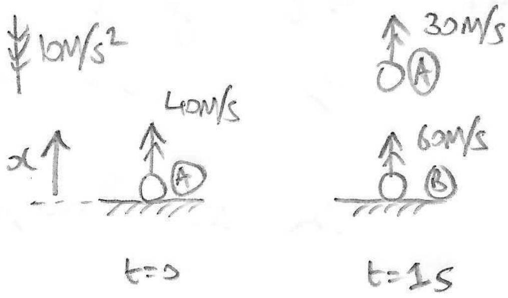
(ii) After what time, and at what height do they meet?

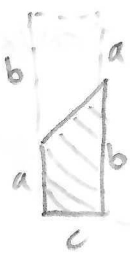
(iii) What would be the required time separation between the two balls being thrown if they were to meet at the moment the first ball reached its maximum height?

**[5]**

(i)




Trapezium area $= \boxed{\frac{1}{2}c(a+b)}$

* Gradient of both (t,v) graphs is $-10\,\mathrm{m/s^2}$.

(ii) Balls meet when the vertical displacement of each ball is the same. Graphically this means the areas under the (t,v) graphs are the same.

Let's assume they meet at $t_m$ seconds.

So $\underbrace{\frac{1}{2}t_m(40 + 40 - 10t_m)}_{x_A} = \underbrace{\frac{1}{2}(t_m - 1)(60 - 10(t_m - 1) + 60)}_{x_B}$

A  B

$\therefore \quad 80t_m - 10t_m^2 = (t_m - 1)(130 - 10t_m)$

$80t_m - \cancel{10t_m^2} = 130t_m - 130 - \cancel{10t_m^2} + 10t_m$

$130 = 60t_m$

$\frac{13}{6} = t_m \quad \Rightarrow \quad \boxed{t_m = 2.17\,\mathrm{s}}$

Using $x_A = \frac{1}{2}t_M(80 - 10t_M)$ and $t_M = 13/6$ (s)

$$\Rightarrow \quad x_A = \frac{1}{2}\left(\frac{13}{6}\right)\left(80 - \frac{130}{6}\right) = \boxed{63.2} \text{ (m)}$$

($2275/36$ is exact fraction)

(iii) Apogee of first ball is when $v_A = 0$

ie, $0 = 40 - 10t_a \Rightarrow \boxed{t_A = 4s}$

The max height reached by A is

$$x_{A\max} = \frac{1}{2}(4)(80 - 40) = \boxed{80 \text{ m}}$$

∴ using the expression for $x_B = \frac{1}{2}(4-T)(120 - 10(4-T))$ (now assuming B is launched at T seconds, and evaluated at $t = 4s$), we can equate this to 80 m and solve for T. ∴ A and B will meet at the apogee of A.

$$\therefore \quad 80 = \frac{1}{2}(4-T)(120 - 40 + 10T)$$

$$160 = (4-T)(80 + 10T)$$

$$160 = 320 - 80T + 40T - 10T^2$$

$$10T^2 + 40T - 160 = 0$$

$$T^2 + 4T - 16 = 0$$

$$(T+2)^2 - 20 = 0$$

$$T = \pm\sqrt{20} - 2$$

only +ve root has physical meaning

$$\therefore \boxed{T = 2\sqrt{5} - 2}$$

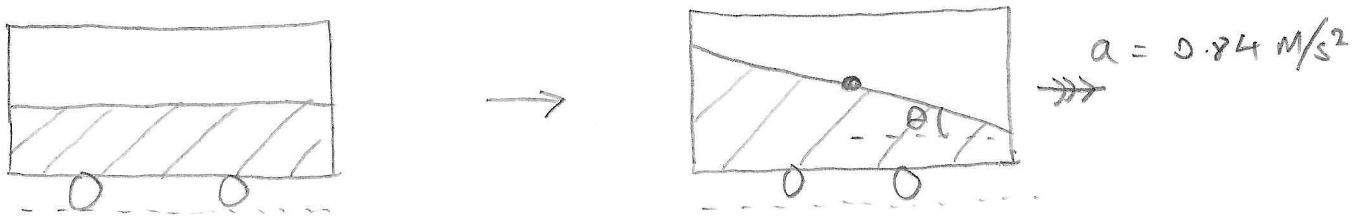
$$\approx \boxed{2.47s}$$

$(\sqrt{20} = \sqrt{2^2 \times 5} = 2\sqrt{5})$

:)

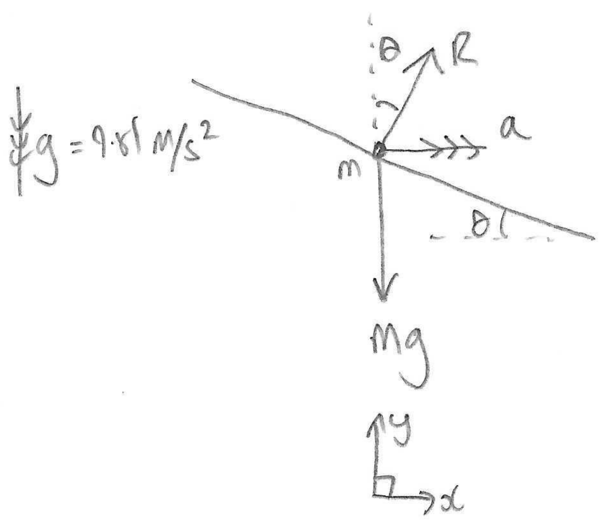
d) A railway carriage for transporting liquids is carrying a viscous liquid and it is only half full. The carriage is attached to an engine which pulls away with a constant acceleration, so that the fluid in the carriage forms a steady sloping surface. If the acceleration of the train is $0.84\,\mathrm{m\,s^{-2}}$, what is the angle of the liquid surface to the horizontal?

**[3]**




Consider a tiny element of the fluid at the surface of mass m

↑ This is the key insight!




Applying NII:

$/\!/x: \quad ma = R\sin\theta$

$/\!/y: \quad 0 = R\cos\theta - mg$

$\therefore$ Normal contact force on fluid element is (from rest of fluid)

$$\boxed{R = mg/\cos\theta}$$

$$\therefore \quad ma = \underbrace{\frac{mg}{\cos\theta}}_{R}\sin\theta \quad \Rightarrow \quad a = g\tan\theta$$

$$\therefore \quad \boxed{\theta = \tan^{-1}\left(\frac{a}{g}\right)}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{0.84}{9.81}\right)$$

$$= \boxed{4.9^\circ} \quad \text{to 2.s.f.}$$


e) Two aeroplanes **A** and **B** travel with velocities $\vec{v}_A = 50\hat{\mathbf{i}} - 125\hat{\mathbf{j}}$ and $\vec{v}_B = -90\hat{\mathbf{i}} + 60\hat{\mathbf{j}}$, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors to the east and north respectively, and the values are in units of m s$^{-1}$.

$\underline{v}_{rel}$

(i) Find the relative velocity of plane **A** as seen from plane **B**.

(ii) When time $t = 0$ s, plane **A** has position $\vec{r}_A = -400\hat{\mathbf{i}} + 1200\hat{\mathbf{j}}$ and plane **B** has position $\vec{r}_B = 800\hat{\mathbf{i}} - 600\hat{\mathbf{j}}$. Find the time and distance of closest approach of the two aeroplanes.

**[4]**

(i) $\underline{v}_{rel} = \underline{v}_A - \underline{v}_B$

$$= 50\hat{\underline{i}} - 125\hat{\underline{j}} - (-90\hat{\underline{i}} + 60\hat{\underline{j}})$$

$$= \hat{\underline{i}}(50+90) + \hat{\underline{j}}(-125-60)$$

$$= \boxed{140\hat{\underline{i}} - 185\hat{\underline{j}}}$$

(ii) $\underline{r}_A = -400\hat{\underline{i}} + 1200\hat{\underline{j}} + (50\hat{\underline{i}} - 125\hat{\underline{j}})t$

$\underline{r}_B = 800\hat{\underline{i}} - 600\hat{\underline{j}} + (-90\hat{\underline{i}} + 60\hat{\underline{j}})t$

Since both moving at constant velocity

Displacement between aeroplanes is $\underline{\Delta} = \underline{r}_B - \underline{r}_A$

$$\Rightarrow \underline{\Delta} = \hat{\underline{i}}(800 - 90t + 400 - 50t) + \hat{\underline{j}}(-600 + 60t - 1200 + 125t)$$

$$= \hat{\underline{i}}(1200 - 140t) + \hat{\underline{j}}(-1800 + 185t)$$

$\therefore$ $\underline{\text{Distance}^2}$ is $|\underline{\Delta}|^2 = (1200 - 140t)^2 + (185t - 1800)^2$

or could complete the square of this quadratic.

Closest approach is when $\dfrac{d|\underline{\Delta}|^2}{dt} = 0$

ie $0 = 2(1200 - 140t)(-140) + 2(185t - 1800)(185)$

↓ PTO

$$\Rightarrow \quad 140(1200 - 140t) = 185(185t - 1800)$$

$$140 \times 1200 + 185 \times 1800 = (185^2 + 140^2)t$$

$$501000 = 53825\,t$$

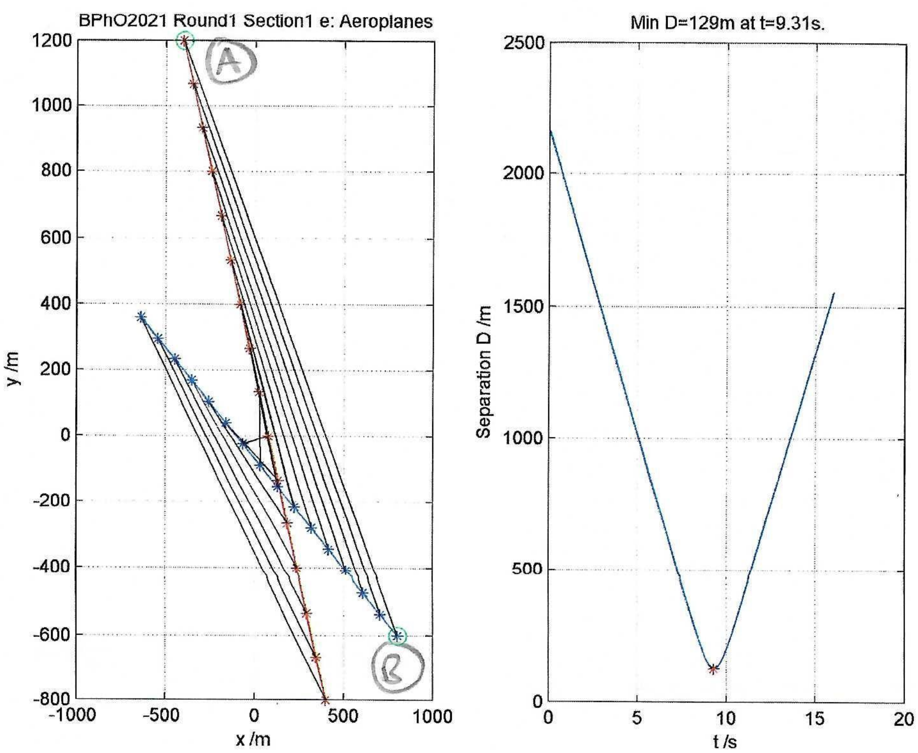
$$\therefore \quad t = \frac{501000}{53825} \quad (s)$$

$$= \boxed{9.31\text{ s}}$$

At this time:
$$\underline{\Delta} = \hat{\underline{i}}\left(1200 - 140 \times \frac{501000}{53825}\right) + \hat{\underline{j}}\left(-1800 + 185 \times \frac{501000}{53825}\right)$$

$$\simeq -103.1\,\hat{\underline{i}} - 78\,\hat{\underline{j}}$$

$$\therefore \quad |\underline{\Delta}| \simeq \sqrt{103^2 + 78^2} = \boxed{129} \quad (m)$$



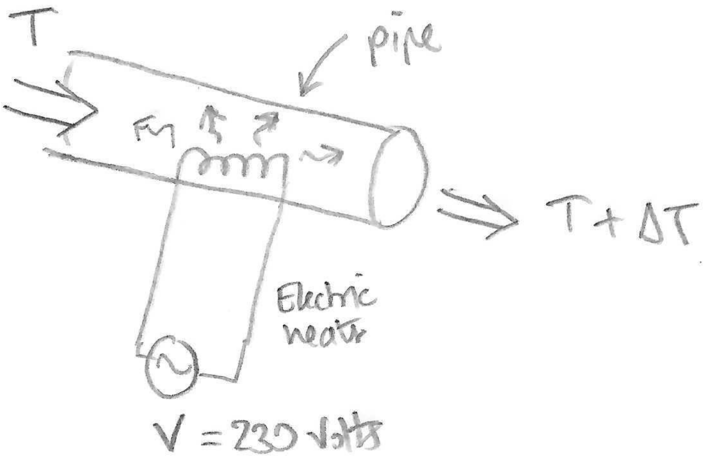
R

f) Water flows at a steady rate of 1.0 litre $\text{min}^{-1}$ through a pipe in which there is an electrical heater connected to a 230 V supply. The rise in temperature of the water after passing through the heater is 60 °C. Calculate the resistance of the heater.
Assume no heat loss to the surroundings.

1 litre = 1000 $\text{cm}^3$
Density of water is 1000 kg $\text{m}^{-3}$ $\rho$

1000 litres = 1 $\text{m}^3$

**[3]**




$\Delta T = 60°C$

$c = 4180\ \text{J/kg/°C}$
specific heat capacity of liquid water.

If heat loss can be ignored, power from heater $V^2/R$ = gain of thermal energy of water per second

resistance

So $\dfrac{V^2}{R} = c\left(\dfrac{R}{60 \times 1000}\right)\rho\,\Delta T$

heater power.

Volume of water/s in $\text{m}^3$

i.e. $\dfrac{R\rho}{60{,}000}$ is mass of water in 1 s.

So $\boxed{R = \dfrac{60{,}000\,V^2}{cR\rho\,\Delta T}}$

$= \dfrac{60{,}000 \times 230^2}{4180 \times 1.0 \times 1000 \times 60}$ $(\Omega)$

$= \boxed{12.7\ \Omega}$

$C_L = 4180\ \mathrm{J\,kg^{-1}\,K^{-1}}$

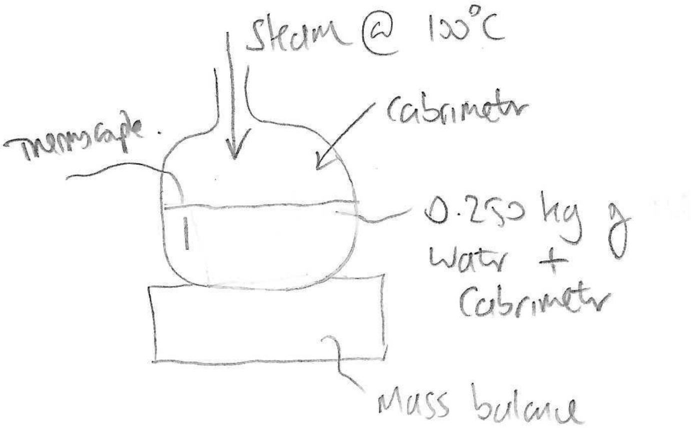
$M_W$

i.e. not ice

g) Dry steam at 100 °C is passed into 0.250 kg of water at 0 °C contained in a calorimeter whose thermal capacity is equivalent to 0.010 kg of water. When the temperature is 30 °C it is found that 0.0128 kg of steam have condensed. Calculate the specific latent heat of steam. $L$ $M_S$ $M$

Specific latent heat of ice = 334 kJ kg$^{-1}$.

Not needed! Water in calorimeter has melted. **[4]**




Energy balance:

Energy loss from steam = Energy gain of water + calorimeter.

i.e. assume no losses.

$$\therefore \quad \underbrace{M_S L}_{\text{steam condensing}} + \underbrace{C_L M_S \Delta T}_{\text{steam cooling}} = \underbrace{C_L (M_W + M)\Delta T_W}_{\text{Gain of water + calorimeter energy}}$$

$\Delta T_S = 70°C$ temp change of steam.

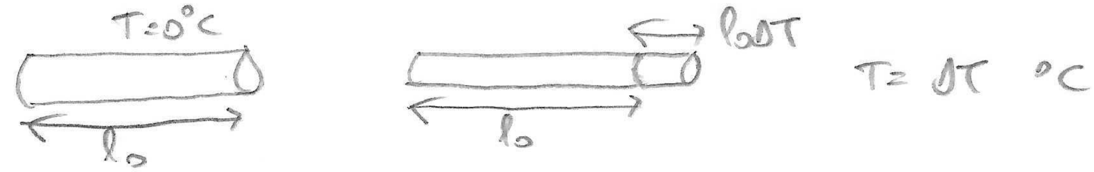
$\Delta T_W = 30°C$ temp change of water.

i.e. Calorimeter energy gain is equivalent to $C_L M \Delta T_W$

$$\text{So} \quad \boxed{L = \frac{C_L (M_W + M)\Delta T_W - C_L M_S \Delta T}{M_S}}$$

$$= \frac{4180(0.250 + 0.010)(30) - 4180(0.0128)(70)}{0.0128} \quad (\mathrm{J/kg})$$

$$= \boxed{2.25 \times 10^6\ \mathrm{J/kg}}$$




h) The expansion of a metal rod varies linearly with temperature, in the form of

$$\ell = \ell_0(1 + \alpha\Delta T)$$

where $\ell_0$ and $\ell$ are the initial and final lengths respectively, $\Delta T$ is the temperature change, and $\alpha$ is the coefficient of linear expansion. $l_{CO}$
An iron rod is 1.00 m at 0 °C. What is the length of a copper rod at 0 °C if the difference between its length and that of the iron rod is not to vary with temperature?

$l_{FeO}$

Coefficient of expansion of copper, $\alpha_{Cu} = 17.0 \times 10^{-6}\,°C^{-1}$
Coefficient of expansion of iron, $\alpha_{Fe} = 11.9 \times 10^{-6}\,°C^{-1}$

**[3]**

$l_c = l_{co}(1+\alpha_c\Delta T)$ Copper

$l_{Fe} = l_{FeO}(1+\alpha_{Fe}\Delta T)$ Iron

Define length difference $\Delta = l_c - l_{Fe} = l_{co}(1+\alpha_c\Delta T) - l_{FeO}(1+\alpha_{Fe}\Delta T)$

If $\Delta$ is independent of temperature, the coefficient of $\Delta T = 0$

$$\boxed{\Delta = l_{co} - l_{FeO} + \Delta T(l_{co}\alpha_c - l_{FeO}\alpha_{Fe})}$$

$\Rightarrow \quad l_{co}\alpha_c - l_{FeO}\alpha_{Fe} = 0$

$$\Rightarrow \boxed{l_{co} = \frac{l_{FeO}\alpha_{Fe}}{\alpha_c}}$$

$\therefore \quad l_{co} = 1.00\,\text{m} \times \frac{11.9}{17.0} = \boxed{0.70\,\text{m}}$

length of copper rod @ 0°C

Tricky.....

i.e. Assume horizontal flight is maintained!

i) An aeroplane files over an observer at speed $v$ and at a fixed height of 3000 m. After some time, the observer sees the plane at an angle of 60° above the horizontal, which is decreasing at a rate of 0.09 rad s$^{-1}$. Calculate

$d\theta/dt$

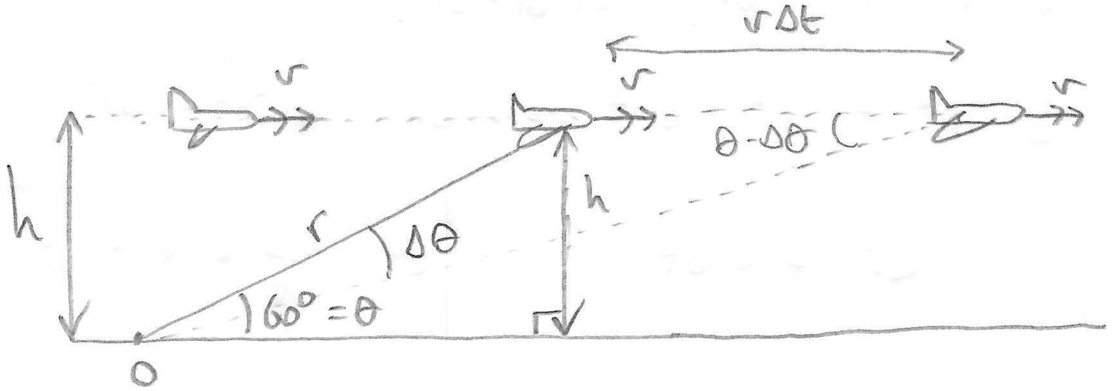
$h = 3000M$

(i) the distance from the observer to the plane

(ii) the speed of the plane

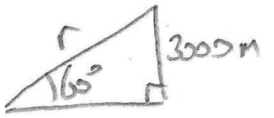
(iii) the speed at which the plane is receding along the observer's line of sight.

[5]

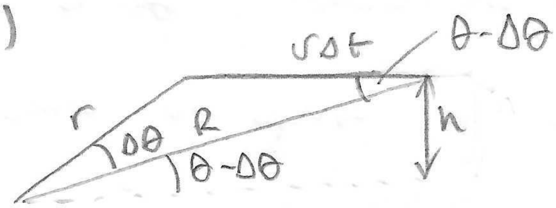



(i) $r\sin\theta = h \quad \therefore \quad r = \frac{h}{\sin\theta} = \frac{3000M}{\sin 60°}$

$= \boxed{3460M}$ to 3sf




(ii)




Sine rule: $\frac{\sin\Delta\theta}{v\Delta t} = \frac{\sin(\theta - \Delta\theta)}{r}$

Assume $\Delta\theta \ll 1$ radian.

$\sin(\theta - \Delta\theta) = \sin\theta\cos\Delta\theta - \cos\theta\sin\Delta\theta$

Small angle approximation: $\cos\Delta\theta \approx 1 \quad \sin\Delta\theta \simeq \Delta\theta$

So $\frac{\Delta\theta}{v\Delta t} \approx \frac{\sin\theta - \Delta\theta\cos\theta}{r} \approx \frac{\sin\theta}{r}$ Since $\theta \gg 0°$

$\therefore \quad v \approx \frac{r}{\sin\theta}\frac{\Delta\theta}{\Delta t} = \boxed{\frac{h}{\sin^2\theta}\frac{\Delta\theta}{\Delta t}} = \frac{3000M}{\sin^2 60°} \times 0.09$

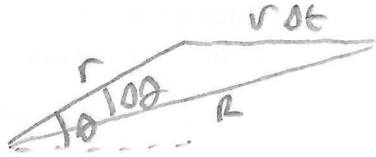
$= \boxed{360 \ M/s}$

PTO

(iii) Cosine rule: $(v\Delta t)^2 = r^2 + R^2 - 2rR\cos\Delta\theta$

let $R = r + \Delta r$

$\cos\Delta\theta \simeq 1$

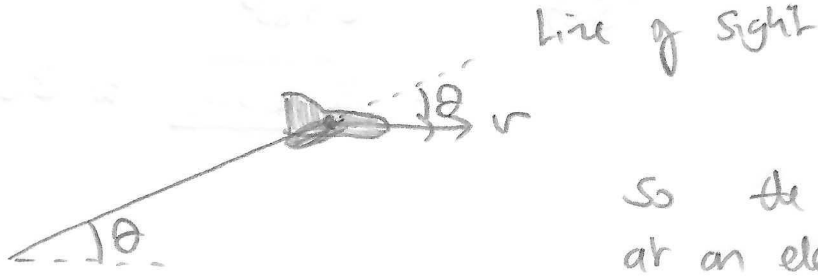



$$\therefore (v\Delta t)^2 \simeq r^2 + (r+\Delta r)^2 - 2r(r+\Delta r)$$

$$\simeq \cancel{r^2} + \cancel{r^2} + \Delta r^2 + \cancel{2r\Delta r} - \cancel{2r^2} - \cancel{2r\Delta r}$$

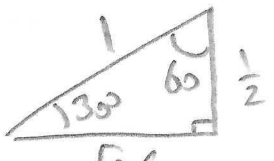
$\therefore \quad \Delta r \simeq v\Delta t \qquad$ so $\dfrac{\Delta r}{\Delta t} = v = \boxed{360\ m/s}$

But MS says 180 m/s i.e. $\boxed{v\cos\theta}$




So the projection of velocity $v$ at an elevation of $\theta$ is $v\cos\theta$

So the answer is perhaps more straightforwardly calculated as $360\ m/s \times \cos 60^\circ = \boxed{180\ m/s}$



Tricky!

½ no friction

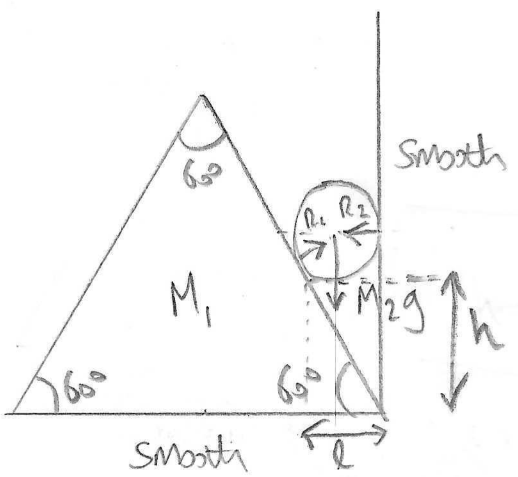
j) A smooth wedge of mass $M_1$ with the cross-section of an equilateral triangular is placed on a smooth horizontal table with its lower edge in contact with a smooth vertical wall. A ~~smooth~~ sphere of mass $M_2$ is placed between the wedge and the wall, so that the sphere falls without rotation.

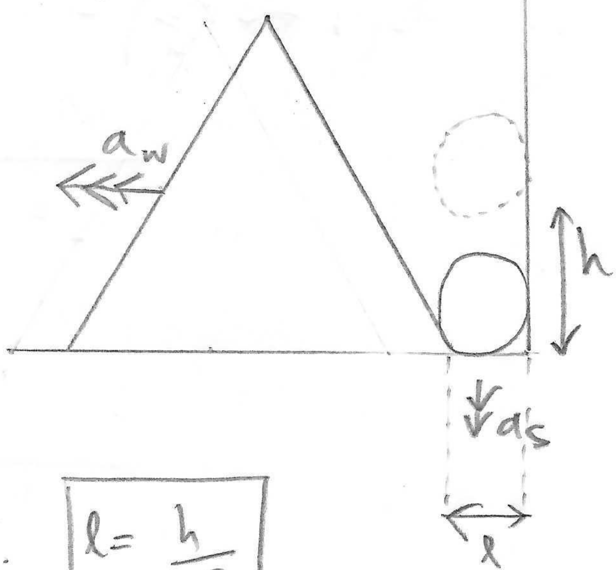
(i) By geometry, find a relation between the height fallen ($h$) by the sphere and the horizontal distance moved ($l$) by the wedge.

(ii) Obtain an expression for $a$ in terms of $M_1$, $M_2$ and $g$.

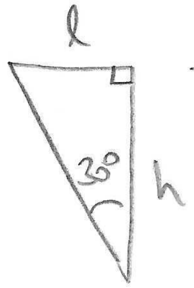
← Acceleration of sphere? $\neq g$

[5]




(i)

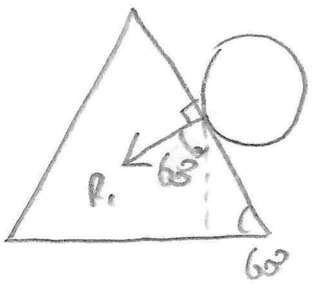



This diagram makes sense!

$\tan 30° = \dfrac{l}{h}$ $\therefore$ $\boxed{l = \dfrac{h}{\sqrt{3}}}$

(ii) So conclude that $\boxed{a_W = \dfrac{a_s}{\sqrt{3}}}$ also.

(since in contact while mutually sliding).




Forces on wedge. Horizontally, only force is $\boxed{R_1 \sin 60°}$

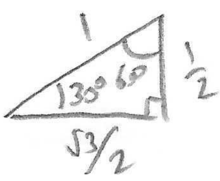
So by NII: $\boxed{M_1 a_W = R_1 \sin 60°}$

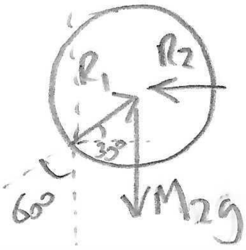
$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\therefore$ $a_s = \dfrac{R_1 \sin 60° \sqrt{3}}{M_1} = \dfrac{R_1 \times 3}{2M_1}$

Forces on sphere:




Vertically, NII is

$\boxed{M_2 a_s = M_2 g - R_1 \sin 30°}$

↓

So $M_2 a_s = M_2 g - R_1/2$

$\therefore \quad R_1 = 2(M_2 g - M_2 a_s)$

$\therefore \quad a_s = \dfrac{3R_1}{2M_1} \quad \Rightarrow \quad a_s = \dfrac{3}{\cancel{2}M_1} \times \cancel{2}(M_2 g - M_2 a_s)$

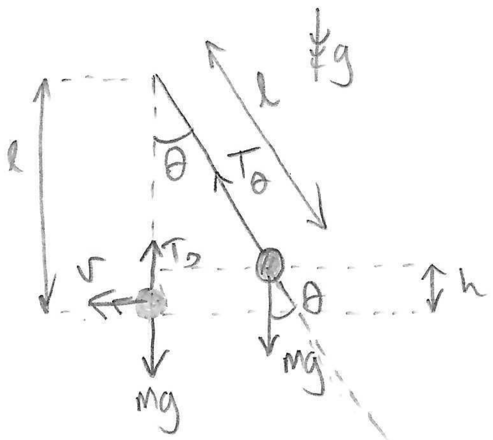
$\therefore \quad a_s\left(1 + 3\dfrac{M_2}{M_1}\right) = \dfrac{3M_2 g}{M_1}$

$$\therefore \quad \boxed{a_s = \frac{\dfrac{3M_2 g}{M_1}}{1 + 3M_2/M_1}}$$

$$\left(\text{or} \quad a_s = \frac{3M_2 g}{M_1 + 3M_2}\right)$$

k) An oscillating pendulum bob has a maximum angle of swing of $\theta$. In its lowest position, the tension in the string is $n$ times the weight of the bob. Obtain an expression for $\cos\theta$ in terms of $n$.

[4]




Conservation of energy

$$\underbrace{\tfrac{1}{2}mv^2}_{\text{KE gain}} = \underbrace{mgh}_{\text{GPE loss}}$$

$$l\cos\theta + h = l$$

$$\therefore \quad h = l(1-\cos\theta)$$

So $\boxed{v^2 = 2gl(1-\cos\theta)}$

ie $T_0 = nmg$

Now by NII (radially inwards, and vertically upwards at lowest point): $\boxed{\frac{mv^2}{l} = T_0 - mg} = (n-1)mg$

so $v^2 = l(n-1)g$

$$\therefore \quad 2g\cancel{l}(1-\cos\theta) = \cancel{l}(n-1)\cancel{g}$$

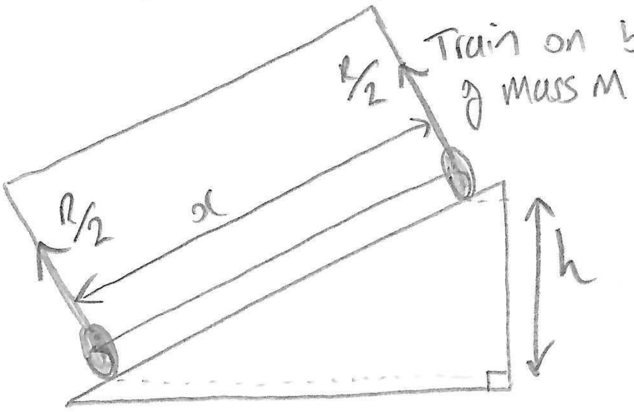
$$\therefore \quad 1-\cos\theta = \frac{n-1}{2}$$

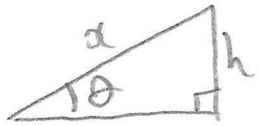
$$1 - \frac{n}{2} + \frac{1}{2} = \cos\theta$$

$$\therefore \quad \boxed{\cos\theta = \frac{3-n}{2}}$$

1) The standard railway gauge has tracks separated by 1435 mm. To travel around a curve the track is banked. Calculate the vertical displacement between the two tracks such that a train travelling at 200 km h$^{-1}$ along a curve of radius 1500 m will experience a normal reaction force on the wheels only.

**[5]**

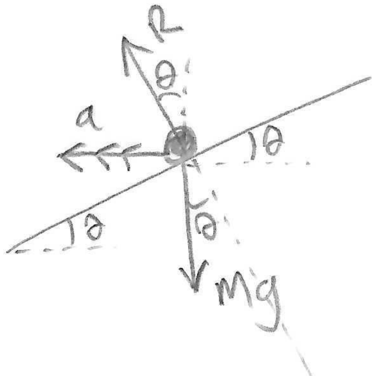


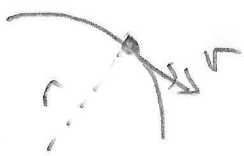

So $\boxed{h = x \sin\theta}$

particle model to keep things simple

Side view








Top view (if circular motion)

Slightly confusing wording – implication is that there is no net vertical acceleration, which means no extra force is required // banked slope to maintain contact of the train with the slope.

NII ↑: $R\cos\theta = Mg$ (2)

NII ←: $Ma = \frac{Mv^2}{r} = R\sin\theta$ (1)

So $\frac{v^2}{rg} = \tan\theta$ $\therefore$ $h = x\sin\left(\tan^{-1}\left(\frac{v^2}{rg}\right)\right)$

$= 1435\,\text{mm}\,\sin\left(\tan^{-1}\left(\frac{\left(\frac{200\times10^3}{3600}\right)^2}{1500\times9.81}\right)\right)$

$= 1435\,\text{mm}\,\sin 11.85^\circ = \boxed{295\,\text{mm}}$

m) An isosceles glass prism is shown in **Fig. 1**. A ray of light in the plane of the paper is incident from air on the face AB.

(i) Calculate the critical angle for light in the prism.

(ii) Sketch the path of the ray incident on face AB such that the refracted ray strikes the face BC at the critical angle.

(iii) Calculate the angle of incidence on face AB for this same condition. $\theta_i$

Refractive index of glass, $n = 1.5$

**[3]**

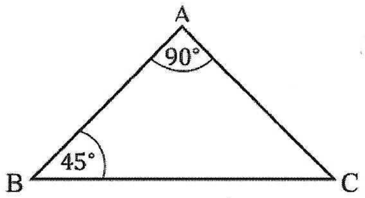
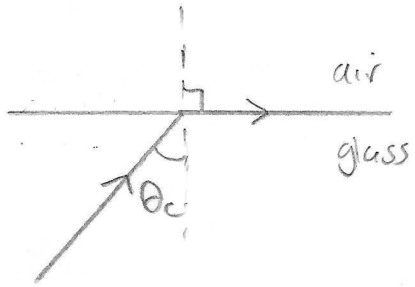



**Figure 1:** Isosceles glass prism with an apex angle of 90°.

(i)



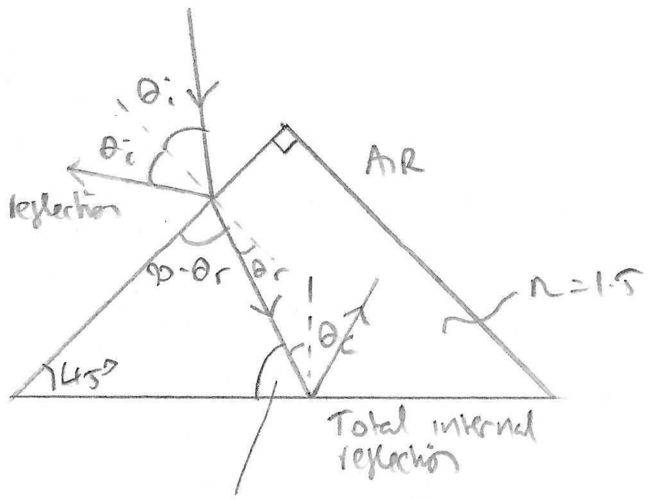

Snell, given refractive index of air ≈ 1.00

$$n \sin\theta_c = 1.00 \sin 90^\circ$$

$$\therefore \theta_c = \sin^{-1}\left(\frac{1.00}{n}\right)$$

$$= \sin^{-1}\left(\frac{1.00}{1.5}\right) = \boxed{41.8^\circ}$$

(ii)




(iii)

$$180^\circ - 45^\circ - (90^\circ - \theta_r) = 90^\circ - \theta_c$$

$$\Rightarrow \cancel{180^\circ} - 45^\circ - \cancel{90^\circ} - \cancel{90^\circ} + \theta_r = -\theta_c$$

$$\boxed{\theta_r = 45^\circ - \theta_c}$$

Snell: $1.00 \sin\theta_i = n \sin\theta_r$

So $\theta_i = \sin^{-1}\left(1.5 \times \sin(45^\circ - 41.8^\circ)\right) = \boxed{4.8^\circ}$ (So much closer to normal than my illustration!)

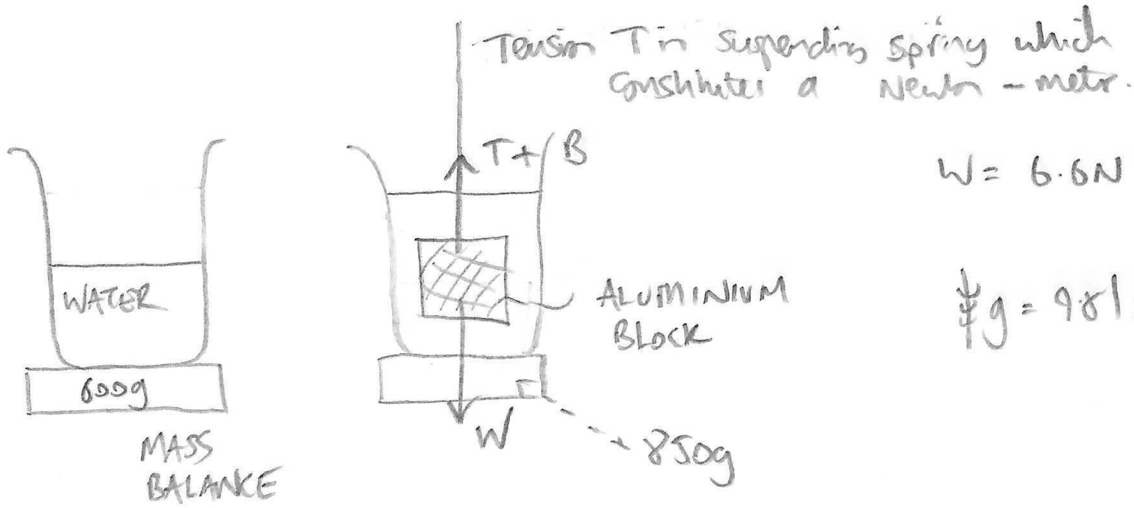
n) An aluminium block 5.0 cm × 5.0 cm × 10 cm is attached to a newton-meter that records a weight of 6.6 N. A beaker of water sits on a mass balance and records a mass of 600 g. The aluminium block is then lowered into the water and completely submerged without touching the sides of the beaker. What is

(i) the new reading on the newton-meter, and

(ii) the new reading on the mass balance?

Density of water is $1000\,\mathrm{kg\,m^{-3}}$.

[4]




$W = 6.6\,N$

$g = 9.81\,N/kg$

U is the upthrust on the Al. block $\quad U = \rho_w V g$

[ Assume block is in eq so $T + U = W$ ].

By NIII, the force on the water by the block is $U$ ↓
So reaction force at the mass balance is now $W_{water} + U$.

So reading in g would be $600 + \frac{U}{g} \times 1000$
where U is in N.

$U = 1000\,kg/m^3 \times 5 \times 5 \times 10 / 10^6 \times 9.81 \quad (N)$
$= \boxed{2.45\,N}$

(i) So $T = 6.6 - 2.45 = \boxed{4.15\,N}$ ie reading on the Newton Meter

(ii) $600 + \frac{2.45}{9.81} \times 1000 = \boxed{850\,g}$ ie reading on the mass balance.

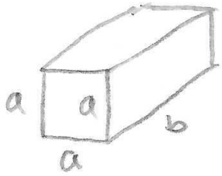
A useful paradigm question

$r$ is 'radius of curvature'

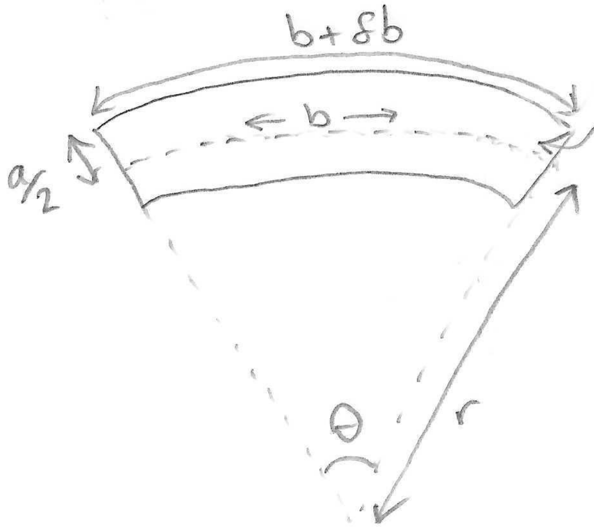
o) A solid square cross-section mild steel bar, of side 2.0 cm ($a$) is to be bent on the arc of a circle. What is the smallest radius ($r$) to which it can be bent, if the breaking stress of the steel is 840 MPa, and Young's Modulus is 210 GPa ($Y$). Assume that the radius of curvature ($r$) is much larger than the thickness ($a$) of the bar.

**[4]**

Might be easier to draw a cuboid and then set $b = a$ at the end




Doesn't Matter!




"Neutral line where is no compression or tension"

$$Y = \frac{\sigma}{\varepsilon} \quad \text{(stress } \sigma\text{)}$$

$$\varepsilon = \frac{\delta b}{b} \quad \text{(strain } \varepsilon\text{)}$$

$b = r\theta$

$b + \delta b = \left(r + \frac{a}{2}\right)\theta$

So $\delta b = \left(r + \frac{a}{2}\right)\frac{b}{r} - b$

$\therefore \frac{\delta b}{b} = \frac{r + \frac{a}{2}}{r} - 1$

$= \boxed{\frac{a}{2r}}$

$$\therefore \text{ Stress } \sigma = \varepsilon Y = Y\left(\frac{r + a/2}{r} - 1\right)$$

$$= Y\left(\frac{a}{2r}\right)$$

So $\boxed{r = \frac{aY}{2\sigma}}$

If $\sigma_{max} = 840\text{ MPa}$

and $Y = 210\text{ GPa}$

and $a = 2.0\text{ cm}$

$$\therefore r_{min} = \frac{2.0 \times 10^{-2} \times 210 \times 10^{9}}{2 \times 840 \times 10^{6}} \quad (m)$$

$$= \boxed{2.5\text{m}}$$

Tip: work out speeds ... store into a calculator.

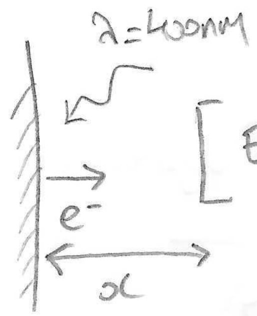
p) In the photoelectric effect, an electrode is placed 0.10 m from a clean sodium surface, which has a work function of 2.28 eV. Light of wavelength 400 nm is shone on the surface and electrons are emitted.

(i) What is the shortest time it takes the most energetic photoelectrons to reach the electrode after illumination is started?

(ii) If a stopping potential of 0.50 V is applied between a plane electrode parallel to the photoemissive surface to produce a uniform electric field, what is now the shortest time it takes the most energetic photoelectrons to reach the electrode after illumination is started?

**[6]**

(i)




Sodium Surface

Maximum KE of photoelectrons is

$$\frac{1}{2}m_e v_{max}^2 = \frac{hc}{\lambda} - \phi$$

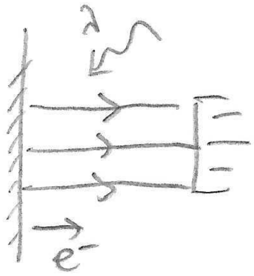
if $v_{max}$ is maximum speed of $e^-$ emission towards electrode.

$$\therefore t_{min} = \frac{x}{v_{max}} = \boxed{\frac{x}{\sqrt{\dfrac{\frac{2hc}{\lambda} - 2\phi}{m_e}}}}$$

$$= \frac{0.1}{\sqrt{\dfrac{\frac{2\times 6.63\times 10^{-34}\times 3.00\times 10^{8}}{400\times 10^{-9}} - 2\times 2.28\times 1.6\times 10^{-19}}{9.11\times 10^{-31}}}} \quad (s)$$

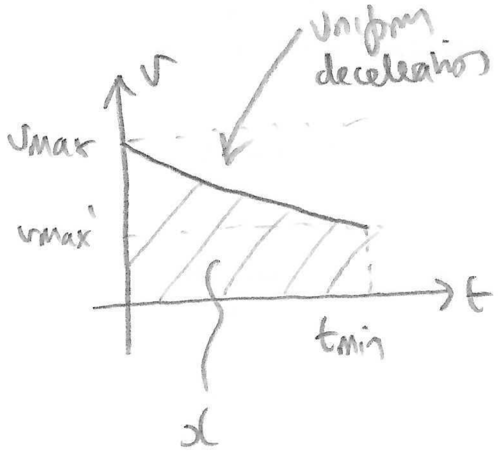
$$= \boxed{1.85\times 10^{-7}\,s}$$

(ii)




For a stopping potential of 0.50V, the max KE of $e^-$ is now $0.50\times 1.6\times 10^{-19}$ J less at the electrode. (Uniform deceleration to this point).

so in this case $\boxed{\frac{1}{2} m_e v_{max}'^2 = \frac{hc}{\lambda} - \phi - 0.5e}$




$x = \frac{1}{2} t_{min} ( v_{max} + v_{max}' )$

(ie $t_{min} \times$ average speed)

$\therefore \boxed{t_{min} = \frac{2x}{v_{max} + v_{max}'}}$

Annoyingly ill have to work out the speeds again....

$$v_{max} = \sqrt{\frac{2\left(\frac{6.63\times10^{-34}\times3.00\times10^{8}}{400\times10^{-9}} - 2.28\times1.6\times10^{-19}\right)}{9.11\times10^{-31}}} \quad (m/s)$$

$$= \boxed{5.395\times10^{5}\ m/s}$$

$$v_{max}' = \sqrt{\frac{2\left(\frac{6.63\times10^{-34}\times3.00\times10^{8}}{400\times10^{-9}} - (2.28+0.5)\times1.6\times10^{-19}\right)}{9.11\times10^{-31}}} \quad (m/s)$$

$$= \boxed{3.395\times10^{5}\ m/s}$$

$$\therefore \quad t_{min} = \frac{2\times0.1}{(5.395+3.395)\times10^{5}} \quad (s)$$

$$= \boxed{2.28\times10^{-7} s}$$

q) For the circuit shown in **Fig. 2**, what should be the value of the variable resistor $R$ in order to minimise the power converted in the $10\,\Omega$ resistor?

**[3]**

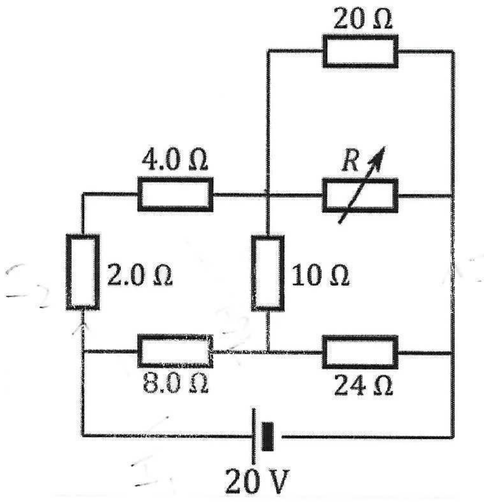
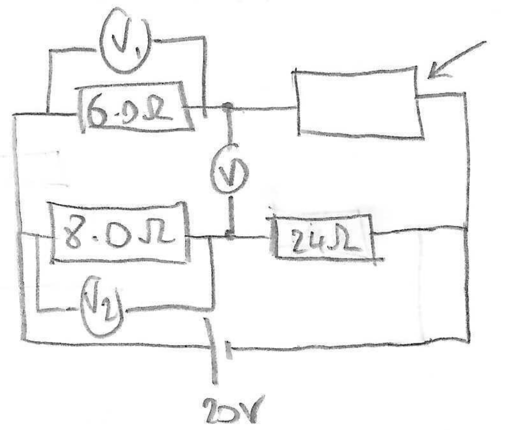



**Figure 2:** A circuit of a cell and seven resistors.

Trick is to Simplify first!

* Minimum power converted in $10\,\Omega$ resistor ⇒ PD across it is zero
So no current flows through the $10\,\Omega$ resistor.

Simplify:




$$\frac{1}{\frac{1}{R}+\frac{1}{20}} = \frac{20R}{20+R}$$

So (V) reads zero if: $\dfrac{6}{6+\frac{20R}{20+R}} = \dfrac{8}{8+24}$

(i.e. $V_1 = V_2 = 20 \times \frac{8}{32}$
$= \boxed{5V}$)

These are the fractions of 20V dropped across the 6.0Ω and 8.0Ω resistors – which must be the same.

$\therefore \dfrac{6 \times 32}{8} = 6 + \dfrac{20R}{20+R}$

$18 = \dfrac{20R}{20+R}$

$\therefore 20 \times 18 + 18R = 20R$

$360 = 2R$

$\boxed{180\,\Omega = R}$ (2)

r) In the arrangement of capacitors in **Fig. 3**, calculate the charge stored on the 4.0 μF capacitor.

**[3]**

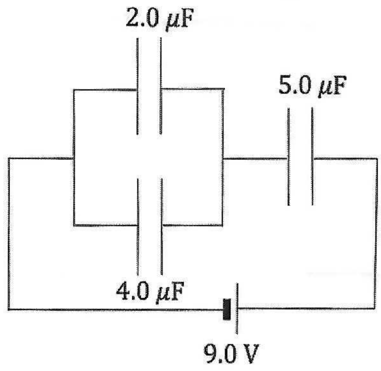
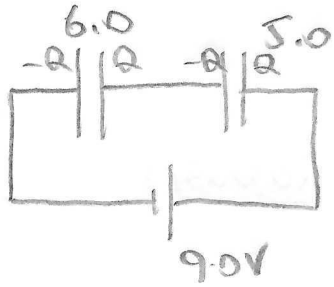
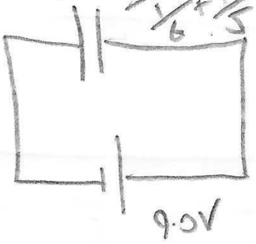



**Figure 3:** A circuit with capacitors and a cell.

Equivalent circuits (Capacitances in μF)




$-Q\ +Q$ $\quad \frac{1}{\frac{1}{6}+\frac{1}{5}} = \frac{30}{11}$ (μF)




* So charge Q stored on equivalent single capacitor plates is $Q = \frac{30}{11} \times 9.0$ (μC) $= \boxed{\frac{270}{11}\ \mu C}$

* If no charge is to flow (i.e. equilibrium is assumed) each capacitor in series has the same charge.

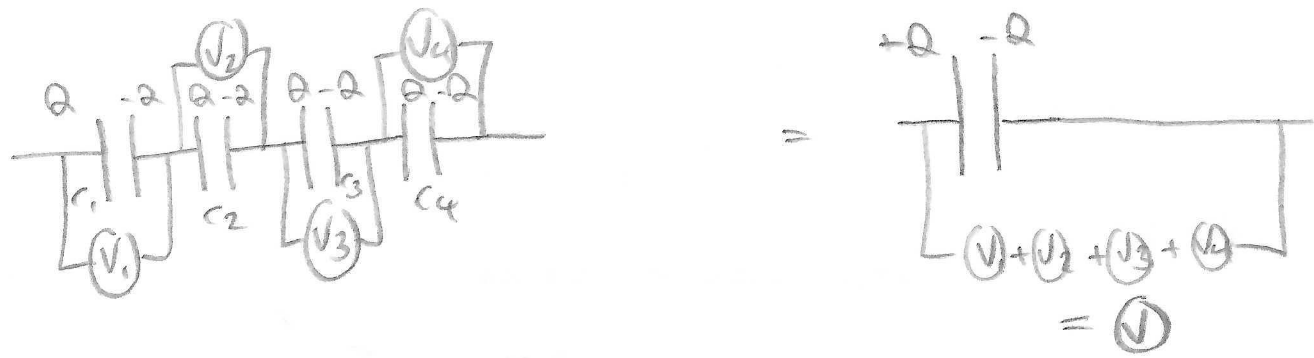
* Capacitances add in parallel so $Q_{4.0\mu F} = \frac{4}{4+2} \times \frac{270}{11}\ \mu C$

i.e. charge on 4.0μF capacitor is ∝ fraction of 6.0μF capacitance of // arrangement.

$\therefore \quad Q_{4\mu F} = \frac{4}{6} \times \frac{270}{11} \quad (\mu C)$

$= \boxed{16.4 \mu F}$

[




Equivalent Capacitor: $\boxed{Q = CV}$

$\frac{Q}{C} = V_1 + V_2 + V_3 + V_4$

ie


$\boxed{Q = C_1 V_1}$

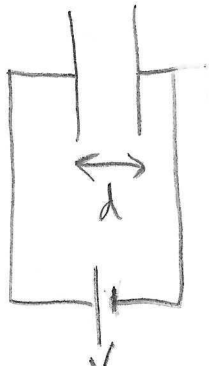
$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \frac{Q}{C_4}$

$\therefore \quad \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}}$

ie capacitances in series add in <u>reciprocals</u> ]

s) A capacitor is made of two parallel conducting plates of area $A$ and initial separation $d$. It is attached to a constant voltage supply, $V$. The energy stored in the capacitor is $E_1$. The separation of the plates is gradually reduced to $d/3$. The supply is then disconnected and the separation of the plates is gradually restored to the value $d$, where the energy stored is now $E_3$.
Calculate the difference in the energy stored in the capacitor between the final ($E_3$) and initial ($E_1$) states.

**[4]**

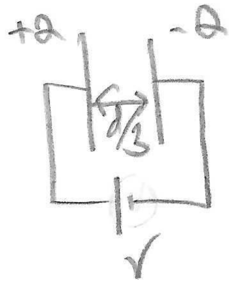



* $E_1 = \frac{1}{2} C_1 V^2$ $\quad C_1 = \frac{\varepsilon_0 A}{d}$

* Reduce $d \to d/3$ with same power supply connection. Energy stored is now $E_2 = \frac{1}{2} C_2 V^2$

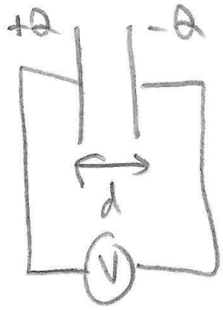
where $C_2 = \frac{3\varepsilon_0 A}{d}$ $\quad$ (ie $C_2 = \frac{\varepsilon_0 A}{d/3}$)

* Charge




$Q = C_2 V$
$= \frac{3\varepsilon_0 A}{d} V$

* Removing the power supply means charge remains fixed. So




$Q = C_1 V_1$
$C_1 = \varepsilon_0 A / d$

So equating charges: $C_1 V_1 = C_2 V$ $\therefore$ $V_1 = \frac{C_2}{C_1} V$

* New stored energy is $E_3 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} C_1 \frac{C_2^2}{C_1^2} V^2$
$= \frac{1}{2} \frac{C_2^2}{C_1} V^2$

↓

So $E_3 - E_1 = \frac{1}{2} V^2 \left( \frac{C_2^2}{C_1} - C_1 \right)$

$$= \frac{1}{2} V^2 \frac{\varepsilon_0 A}{d} \left( 3^2 - 1 \right)$$

$$= \boxed{\frac{4 \varepsilon_0 A V^2}{d}}$$

which is the work done to separate the capacitors $\frac{d}{3} \to d$.

t) An isotope of polonium, $^{210}_{84}Po$ decays by alpha particle emission with a half-life of 138 days. A mass of 5 mg of this isotope is in the form of a thin film in a very thin-walled glass container such that the alpha particles can escape. By how much will the mass of the thin film of polonium be reduced after 100 days?

**[6]**

$$^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + {}^{4}_{2}\alpha$$

# Po atoms after t days is $\frac{N_0}{2^{t/t_{\frac{1}{2}}}}$

$N_0 = \frac{5\,mg}{210\,u}$ # Po atoms in 5 mg.

$\therefore$ reduction of $\Delta N = N_0 - \frac{N_0}{2^{t/t_{\frac{1}{2}}}}$ α particles (or Po atoms converted to Pb).

$\therefore$ reduction by mass of $\boxed{4u\,\Delta N = \Delta m}$

$$\therefore \Delta m = \underbrace{\frac{5\,mg}{210\,\cancel{u}}}_{N_0}\left(1 - \frac{1}{2^{100/138}}\right) \times 4\cancel{u}$$

$$= 5\,mg \times \frac{4}{210}\left(1 - \frac{1}{2^{100/138}}\right)$$

$$= 3.8 \times 10^{-2}\ mg$$

$$= \boxed{3.8 \times 10^{-5}\ g}$$

Since the α particles can escape.

Sneaky! Requires a systematic combination of pairs of equations.

u) Beats are variations in sound intensity, produced by interference between two sources of sound very close in frequency.
The *beat frequency* is simply the difference in the frequencies of the two sources.
Three tuning forks in the audible range produce beats: **B** and **C** produce beats at 7 Hz, **A** and **C** produce beats at 8 Hz. The frequency of **B** is 5.9% higher than that of **A**. Find the frequencies of the three tuning forks.

**[4]**

$f_A, f_B, f_C$

Don't know up front that $f_C > f_B > f_A$

But $\boxed{f_B > f_A}$

So $f_A - f_C = 8$ ①  |  $f_B - f_C = 7$

or $f_C - f_A = 8$ ②  |  or $f_C - f_B = 7$

Now $\boxed{f_B = 1.059 f_A}$

So $1.059 f_A - f_C = 7$ ③

or $f_C - 1.059 f_A = 7$ ④

→ These could work....

| | | |
|---|---|---|
| ②, ④ | inaudible | ✗ |
| ①, ③ | impossible | ✗ |
| ①, ④ | unphysical. | ✗ |
| ②, ③ ✓ | Try these. | |

SUMMARY

So $f_C - f_A = 8$ ②

$f_C - 1.059 f_A = 7$ ④

② - ④ $\quad 0.059 f_A = 1 \quad \Rightarrow \quad f_A = \dfrac{1}{0.059}$ (Hz

$= \boxed{16.9 \text{ Hz}}$

But this is not audible! ☹

So since $\begin{cases} 1.059 f_A - f_C = 7 \\ f_A - f_C = 8 \end{cases}$ is impossible, only other option is ①, ④ or ②, ③

$\begin{cases} f_A - f_C = 8 \\ f_C - 1.059 f_A = 7 \end{cases}$

↓

(2) $\quad f_c - f_A = 8$

(3) $\quad 1.059 f_A - f_c = 7$

(2) + (3) $\quad 0.059 f_A = 15$

$$\therefore \quad f_A = \frac{15}{0.059} \quad (\mathrm{Hz})$$

$$= \boxed{254.2\ \mathrm{Hz}}$$

$$\Rightarrow \quad f_c = 8 + f_A = \boxed{262.2\ \mathrm{Hz}}$$

and $\quad f_B = 1.059 f_A = \boxed{269.2\ \mathrm{Hz}}$

---

check $\quad$ (1) $\quad f_A - f_c = 8$

(4) $\quad f_c - 1.059 f_A = 7$

(1) + (4): $\quad -0.059 f_A = 15. \quad \Rightarrow f_A < 0$

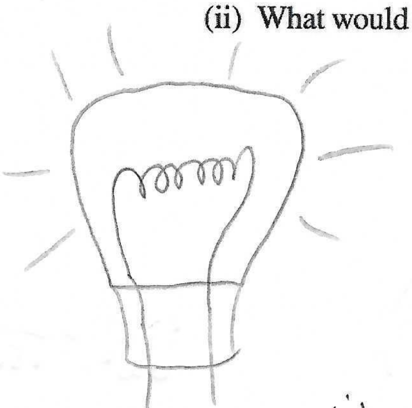
which is unphysical.

v) Resistance of a filament light bulb is given by $R = A+BP$ where $A$ and $B$ are constants, and $P$ is the power emitted by the bulb. When operating at 230 V the power emitted is 100 W. When switched on from cold, the filament has a resistance value 1/5 of its operating resistance, and consequently for the same mains voltage, the power is instantaneously five times greater than when at its normal operating power.

(i) Determine the values of A and B (if you wish, you may leave the answers as products of integer values given).

(ii) What would be the steady emitted power of the bulb if connected to a 210 V supply?

**[5]**



$P = V^2/R$ so if $\boxed{R = A + BP}$

$\Rightarrow \boxed{PA + BP^2 = V^2}$ (1)

(i) Normal operating power: $100A + B\times 100^2 = 230^2$ (2)

Cold: $A + B\times 500 = \frac{1}{5}(A + B\times 100)$ (3)

ie $R$ @ 500W is $\frac{1}{5}$ $R$ @ 100W.

(3): $5A + 2500B = A + 100B$

$4A + 2400B = 0$

$A + 600B = 0$

$\therefore \boxed{B = -A/600}$

In (1): $100A + \left(-\frac{A}{600}\right)\times 100^2 = 230^2$

$\therefore A\left(100 - \frac{100^2}{600}\right) = 230^2$

$A = \frac{230^2}{100 - 100/6} = \frac{6\times 230^2}{500} = \boxed{\frac{3174}{5}}$ ($\Omega$)

So $\boxed{A = 634.8\,\Omega}$ and $\boxed{B = -1.058\ \Omega/W}$

(ii) $PA + Bp^2 = v^2$ $\boxed{B = -A/600}$

$\therefore \quad PA - \frac{A}{600} p^2 - v^2 = 0$ let $\boxed{V = 210}$

$\boxed{A = 3174/5}$

$\therefore \quad -600P + P^2 + \frac{600v^2}{A} = 0$

$$\therefore \quad P = \frac{600 \pm \sqrt{600^2 - 2400v^2/A}}{2}$$

$$P = 300\left(1 \pm \sqrt{1 - \frac{v^2}{150A}}\right)$$ $\frac{2400}{600^2} = \frac{1}{150}$

$$\sqrt{1 - v^2/150A} = \sqrt{1 - \frac{210^2}{150 \times 3174/5}} = 0.7327$$

So $P_+ = 300(1 + 0.7327) = 519.8W$

$P_- = 300(1 - 0.7327) = \boxed{80.2W}$

P @ V = 230volts was low, so P must be lower at V = 210 volts.

So answer is $\boxed{80.2W}$