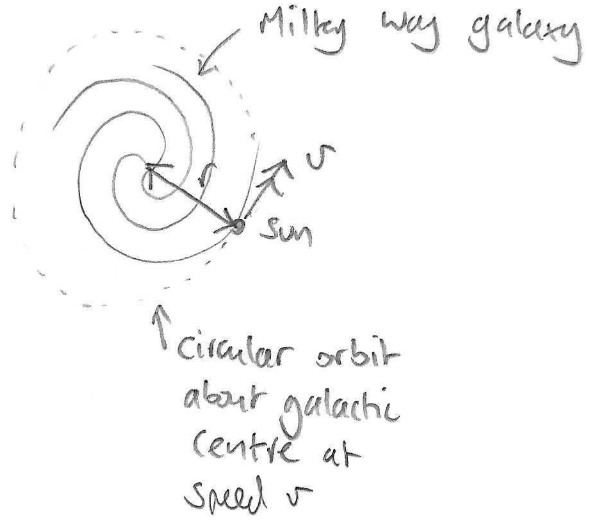
2018 AF SOLUTIONS BPhO Round 1 Section 1

a) The Milky Way galaxy has a period of rotation of 240×10^6 years. The Sun is 26 light years from the centre of the galaxy. How fast is the Sun moving with respect to the centre of the galaxy, given in units of $\mathrm{m\,s^{-1}}$?
A light year is the distance that light travels in one year of 365.25 days. Don't actually need this!

[3]



$$\boxed{v = \frac{2\pi r}{T}}$$ Since constant speed assumed.

$$v = \frac{2\pi \times 26 \times Ly}{240 \times 10^6\ Yr} \quad (m/s)$$

$$1\ Ly = 2.998 \times 10^8\ m/s \times 365.25 \times 24 \times 3600\ s \quad (m)$$

$$Yr = 365.25 \times 24 \times 3600\ s$$

$$\therefore \quad v = \frac{2\pi \times 26 \times 2.998 \times 10^8}{240 \times 10^6} \quad m/s$$

$$= \boxed{204\ m/s}$$

(i.e. seconds in a year cancel in this calculation)

Note 'official answers' use 3×10^8, so $v \approx \frac{2 \times 26 \times 300}{240}\pi$

$$\approx 65\pi$$

$$= \boxed{204\ m/s}$$)

i.e. no difference to 3.s.f.

b) A smooth sphere of radius 6.0 cm is suspended from a thread of length 9.0 cm attached to a smooth wall as shown in shown in **Figure 1**. If the mass of the sphere is 0.5 kg, calculate the tension, T, in the thread.

r $\quad$ l $\quad$ m

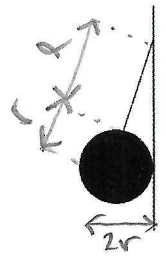
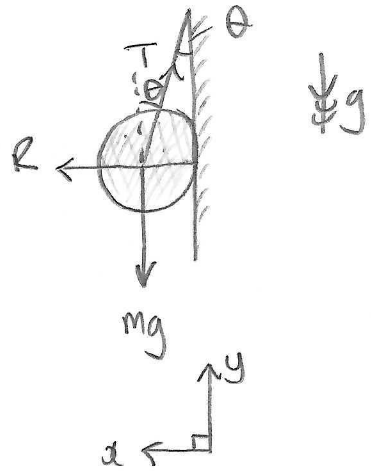
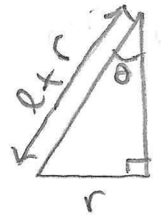


Figure 1

Gotcha! $l+r$ not l

[3]





Geometry:

$$\boxed{(l+r)\sin\theta = r} \quad (1)$$

Resolving forces (assume equilibrium):

$$//x: \quad 0 = R - T\sin\theta \quad (2)$$

$$//y: \quad 0 = T\cos\theta - mg \quad (3)$$

So from (3): $\quad T = \dfrac{mg}{\cos\theta}$

from (1): $\quad \sin\theta = \dfrac{r}{l+r} \Rightarrow \boxed{\theta = \sin^{-1}\left(\dfrac{r}{r+l}\right)}$

$$\sin^2\theta = \frac{r^2}{(l+r)^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{r^2}{(l+r)^2}$$

$$\therefore \boxed{T = mg\left(1 - \frac{r^2}{l^2}\right)^{-\frac{1}{2}}}$$

$$\therefore T = 0.5 \times 9.81\left(1 - \frac{6^2}{15^2}\right)^{-\frac{1}{2}} = \boxed{5.4\,N} \text{ to 2 s.f.}$$

Also: $\theta = \sin^{-1}\left(\dfrac{6}{15}\right) = \boxed{23.6^\circ}$

c) The displacement of an object is determined by the following function:

$$s = 2t^3 - 9t^2 + 12t + 4$$

where s is the displacement in metres, and t the time elapsed in seconds. Determine

(i) the times when the object comes to rest, $v = ds/dt = 0$

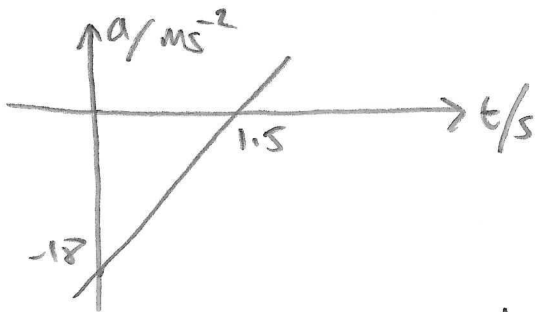
(ii) the time when the acceleration is zero, $dv/dt = 0$

(iii) the object's velocity when its acceleration is zero, $a = 0$

(iv) the object's accelerations when its velocity is zero. $v = 0$

[4]

$$\boxed{\begin{aligned} v = \frac{ds}{dt} &= 6t^2 - 18t + 12 \\ a = \frac{dv}{dt} &= 12t - 18 \end{aligned}}$$

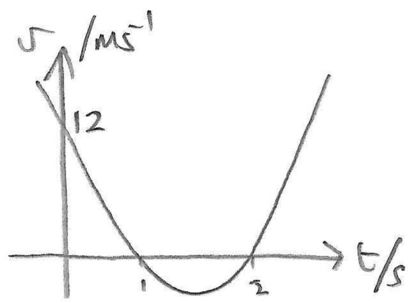


(i) $v = 0$ when: $6t^2 - 18t + 12 = 0$

$t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$

$\therefore \boxed{t = 1, 2\,s}$



(ii) $a = 0$ when: $12t - 18 = 0 \Rightarrow t = \frac{18}{12}$

$t = \frac{3 \times 6}{2 \times 6} = \boxed{1.5\,s}$

(iii) when $a = 0$ at $t = 3/2$

$v = 6\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 12$

$v = \frac{6 \times 9}{4} - \frac{54}{2} + 12$

$v = \frac{54}{4} - \frac{54}{2} + 12$

$v = -\frac{3}{2}$

So $\boxed{v = -1.5\,m/s}$

(iv) $v = 0$ when $t = 1, 2$

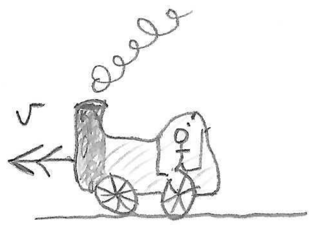
So when $t = 1$:

$a = 12 - 18 = \boxed{-6\,m/s^2}$

when $t = 2$:

$a = 12 \times 2 - 18 = \boxed{6\,m/s^2}$

d) The distance in which a train can be stopped is given by:



$$s = av + bv^2$$

where s is the stopping distance, v the initial velocity, and a and b are constants. When moving at $40\,\mathrm{km\,hr^{-1}}$, the train can be stopped in $100\,\mathrm{m}$, and at $80\,\mathrm{km\,hr^{-1}}$ it can be stopped in $280\,\mathrm{m}$.

Find the greatest speed such that the train can be stopped in $500\,\mathrm{m}$. S

[4]

Choose constants to fit the units of the problem

ie $\left(\dfrac{S}{m}\right) = a\left(\dfrac{v}{\mathrm{km\,hr^{-1}}}\right) + b\left(\dfrac{v}{\mathrm{km\,hr^{-1}}}\right)^2$

Note in this way a, b are dimensionless numbers.

Hence: $100 = 40a + 40^2 b$ ①

$280 = 80a + 80^2 b$ ②

* Need a, b to solve the problem *

② − 2①: $280 - 200 = (80^2 - 2\times 40^2)b$

$\therefore\ b = \dfrac{80}{80^2 - 2\times 40^2} = \dfrac{80}{3200} = \boxed{\dfrac{1}{40}}$

in ①: $a = \dfrac{100 - 40^2\frac{1}{40}}{40} = \dfrac{100-40}{40} = \boxed{\dfrac{3}{2}}$

So, with S in m and v in km/h

$S = \dfrac{3}{2}v + v^2/40$

$\boxed{40S = 60v + v^2}$

$\boxed{v = \sqrt{40S + 900} - 30}$ v in km/h, S in m

$(v+30)^2 - 30^2 - 40S = 0$

$\boxed{v = \pm\sqrt{40S + 900} - 30}$

(Take +ve root since $v > 0$)

So when $S = 500$

$v = -30 + \sqrt{40\times 500 + 900}$

$= \boxed{115}$ (km/h)

$B_A = 180° - 20.34° = \boxed{159.7°}$

$B_B = 360° - (180° - 159.7°) = 360° - 20.34° = \boxed{339.7°}$

e) Two planes set out at the same time from an aerodrome. The first flies north at 360 km h^{-1}, the second south-east at 300 km h^{-1}. After 40 minutes they both turn and fly towards each other. Calculate

Always clockwise from N

(i) the bearing, and ← I think θ of the meeting point!

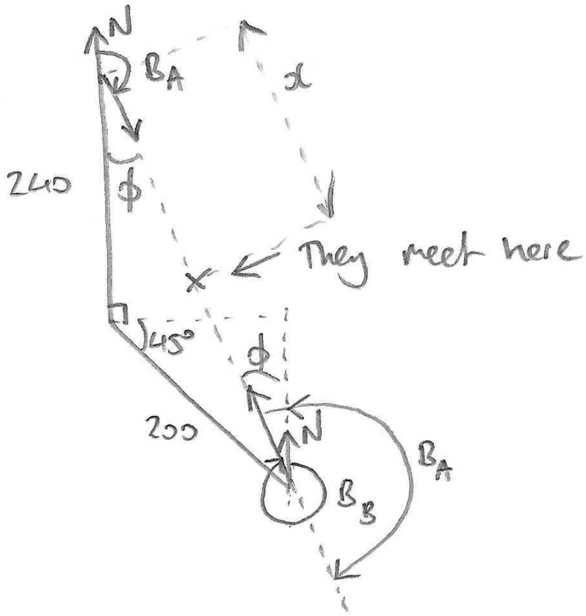
(ii) the distance y

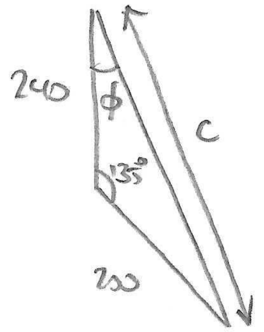
of the meeting point from the aerodrome.

[7]

After 40 mins:

* Plane A flies N for $360\ \text{km/h} \times \frac{40}{60}\ \text{h} = \boxed{240\ \text{km}}$
* Plane B " SE for $300\ \text{km/h} \times \frac{40}{60}\ \text{h} = \boxed{200\ \text{km}}$





Sine rule:

$$\frac{\sin\phi}{200} = \frac{\sin 135}{c}$$

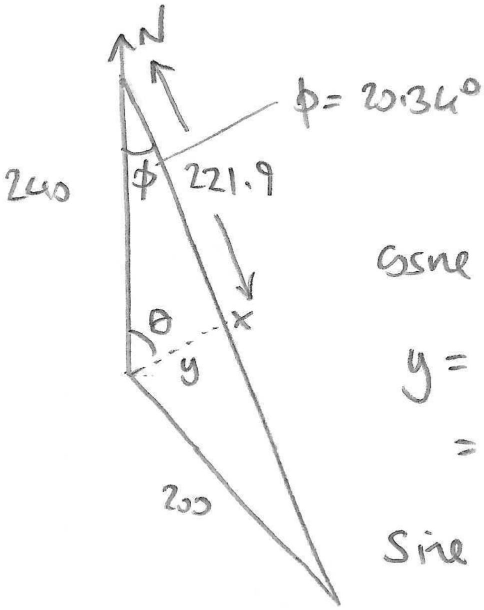
$$\therefore \phi = \sin^{-1}\left(\frac{200 \sin 135}{c}\right) = \boxed{20.34°}$$

Cosine rule: $c = \sqrt{240^2 + 200^2 - 2(240)(200)\cos 135°} = \boxed{406.8\ \text{km}}$

* let $x = 360t$ and $c - x = 300t$ where t (in hrs) is time to meet after turning. *

So $c - 360t = 300t \quad \therefore \frac{c}{660} = t \quad \therefore \boxed{t = 0.616\ \text{hrs}}$

(36.98 mins)



$\therefore x = 360 \times 0.616 = \boxed{221.9\ \text{km}}$

Cosine rule:

$$y = \sqrt{240^2 + 221.9^2 - 2(240)(221.9)\cos(20.34°)} = \boxed{83.5\ \text{km}}$$

Sine rule: $\theta = \sin^{-1}\left(\frac{221.9 \times \sin(20.34°)}{y}\right) = \boxed{67.5°}$

(pleasing question from a Mathematical perspective!)

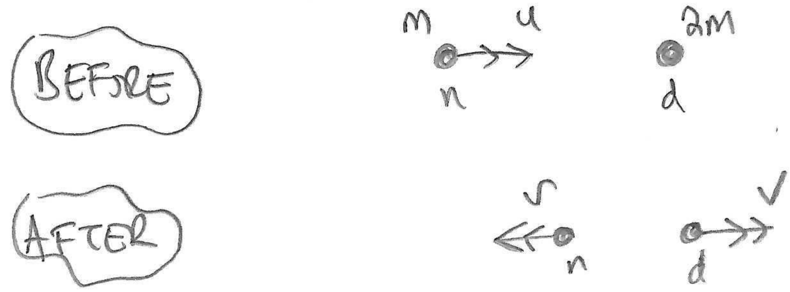
f) A neutron (n) moving through heavy water strikes an isolated and stationary deuteron (d) (the nucleus of an isotope of hydrogen) head-on in an elastic collision.

(i) Assuming the mass of the neutron is equal to half that of the deuteron, find the ratio of the final speed of the deuteron (V) to the initial speed of the neutron (u).

(ii) What percentage of the initial kinetic energy is transferred to the deuteron?

(iii) How many such collisions would be needed to slow the neutron down from 10 MeV to 0.01 eV?

[6]



$M_n = m$

$M_d = 2m$

Conservation of momentum: $\longrightarrow$ +ve

$$mu = 2mV - mv \quad (1)$$

$$\therefore \boxed{u = 2V - v}$$

Restitution (since elastic) "speed of separation = speed of approach":

$$\boxed{V + v = u} \quad (2)$$

$$\therefore v = u - V$$

So from (1) and (2):

$$2V - (u - V) = u$$

$$3V = 2u$$

$$\boxed{V = \tfrac{2}{3}u} \longrightarrow \text{So } \boxed{\frac{V}{u} = 2/3} \quad \text{(i)}$$

(ii) Initial n KE is $\frac{1}{2}mu^2$

Deuteron KE is $\frac{1}{2}(2m)V^2 = \frac{1}{2}2m \times \frac{4}{9}u^2$

$$\therefore \frac{E_d}{E_n} = \boxed{8/9} \quad \text{So } \frac{E_d}{E_n} \approx \boxed{89\%}$$

(iii) let each collision result in KE being $\div$ by $\frac{1}{9}$

So $\left(\frac{1}{9}\right)^n = \frac{0.01}{10 \times 10^6}$ $\therefore -n\log_{10} 9 = -9$ $\therefore \boxed{n = \frac{9}{\log_{10} 9}}$

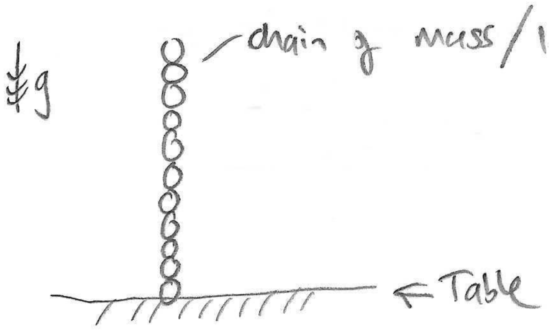
$n = 9.4$, So $\boxed{10 \text{ collisions}}$

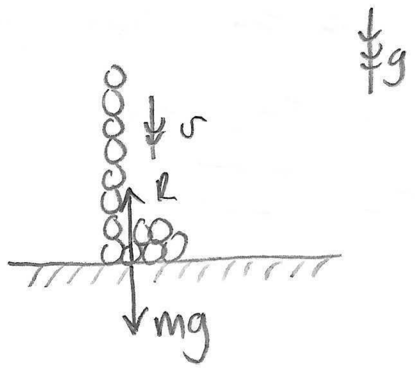
g) A uniform chain of mass per unit length, μ, is suspended from one end above a table, with the lower end just touching the surface. The chain is released, falls and comes to rest on the table without bouncing.

(i) Determine an expression, in terms of μ and the gravitational field strength g, for the reaction force exerted by the table on the chain as a function of time, t.
Hint: you might consider F in the form $F = \frac{\Delta m}{\Delta t}v$.

(ii) In terms of the total weight W of the chain, what is the maximum reaction force exerted by the table, and at what time during the fall does this occur?

[6]





Initially
$t=0$

* m is mass of chain already on the table
* R is the reaction force exerted by the table on the chain

* Now R (upwards on the chain) is the weight of chain on the table + the rate of change of momentum of chain dropping

* In time $t \to t+dt$ chain mass $\boxed{\mu v dt}$ hits the table ($v dt$ is the fall distance in dt). ∴ If chain is brought to rest, momentum change is $\mu v^2 dt$. ∴ Rate of change of momentum is μv^2

* Now chain is in free fall, so after time t, the chain which hits the table has speed (ignoring air resistance) of

$$v = gt$$

∴ $\boxed{R = mg + \mu g^2 t^2}$ Now $\boxed{m(t) = \int_0^t \mu v dt'}$

$= \mu \int_0^t gt' dt' = \boxed{\frac{1}{2}\mu g t^2}$ ∴ $R = \frac{3}{2} g^2 \mu t^2$

↓ PTO

Now to summarise:

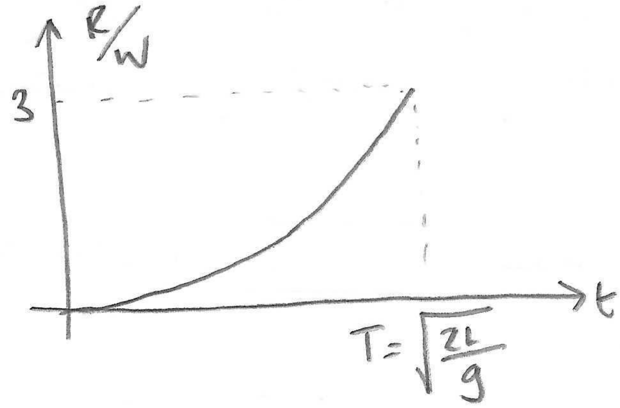
$$M(t) = \frac{1}{2}\mu g t^2$$ mass of chain on table

$$R(t) = \frac{3}{2}\mu g^2 t^2$$ reaction force of table on chain

let chain weight be $W = M(T)g$
where T is the time for the whole chain to fall.
[if L is the chain length, $\boxed{L = \frac{1}{2}gT^2}$

$\therefore T = \sqrt{2L/g}$]

$$\therefore R/W = \frac{\frac{3}{2}\mu g^2 t^2}{\frac{1}{2}\mu g^2 T^2} = \boxed{3\left(\frac{t}{T}\right)^2}$$

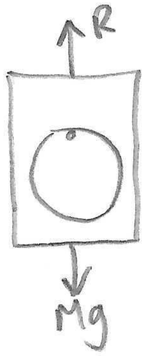


So maximum R/W of 3 occurs right at the end of the fall.

Nice! ☺

h) A small particle of mass m can slide without friction round the inside of a cylindrical hole of radius r, in a rectangular shaped object of mass M. The rectangular object is held between rigid walls by small wheels so that it can slide up and down without friction, as shown in **Figure 2**. If the small particle m is initially at rest at the bottom of the cylindrical hole, and is then given an impulse to give it a speed v, what is the minimum speed v needed to just lift the rectangular mass M off the ground?

+NIII:



Forces on mass M

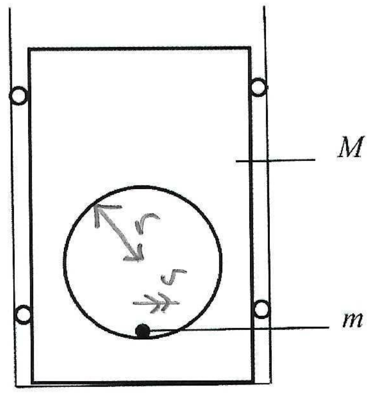


Figure 2

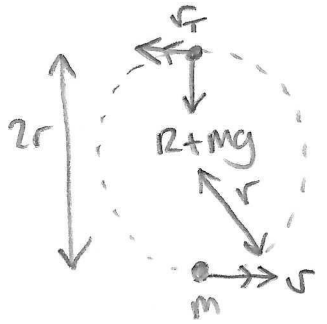
$\downarrow g$

$m \ll M$

[5]

* To find the minimum speed, we want to find the minimum contact force R between mass m and the cylindrical hole surface. When $R > Mg$ then the rectangular mass will lift off the ground, as the effect would be to counter weight Mg, and ∴ the ground to cylinder contact force is zero.

* Minimum R is when m is at the top of the cylinder



$\downarrow g$

NII: $\boxed{\frac{m v_T^2}{r} = R + mg}$

Conservation of energy: $\boxed{\frac{1}{2} m v^2 = \frac{1}{2} m v_T^2 + 2mrg}$

$\therefore v^2 = v_T^2 + 4rg$

So $\boxed{R = \frac{m}{r}(v^2 - 4rg) - mg}$

Now if $R > Mg$

$v^2 - 4rg > (Mg + mg)\, r/m$

$v^2 - 4rg > \left(\frac{M}{m} + 1\right) rg$

$v^2 > \left(\frac{M}{m} + 5\right) rg$

$\therefore \boxed{v > \sqrt{\frac{M}{m} + 5}\, \sqrt{rg}}$

i) Two resistors and two cells are connected in the circuit shown in **Figure 3**. One cell has an e.m.f. of 2.0 V and an internal resistance of 1.0 Ω, the other an e.m.f. of 1.5 V and an internal resistance of 0.5 Ω. The resistors are connected in series and the point between them is at earth, i.e. zero potential. Calculate

(i) the current through the cells,

(ii) the potential difference across each cell, and

(iii) the potential, relative to earth, at points A and B.

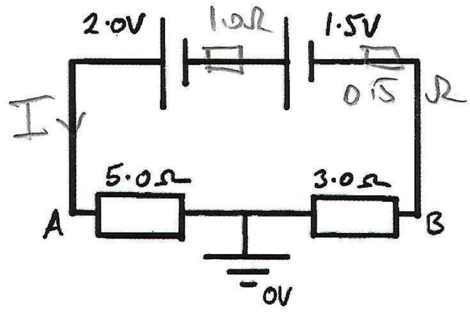


Figure 3

[4]

Kirchoff II: In a loop: $\sum_i \text{EMFs} = \sum_i I_i R_i$

Single loop so $I_i = I \quad \forall i$.

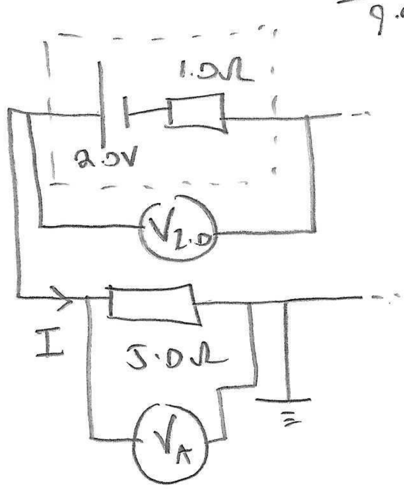
(i) $\therefore \quad 2.0 + 1.5 = (\underbrace{1.0 + 0.5 + 5.0 + 3.0}) I$

ie include all internal resistances in a series loop

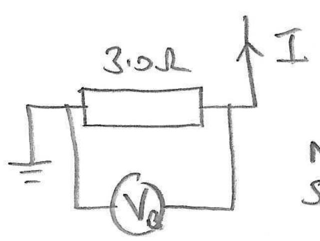
$\therefore \quad I = \dfrac{3.5\,V}{9.5\,\Omega} = \boxed{0.37\,A} \quad (= \tfrac{7}{19}\,A)$

(ii)

(iii)



$V_A = I \times 5.0 = \frac{7}{19} \times 5.0 = \boxed{1.84 \text{ Volts}}$



Note Sign!

$V_B = -I \times 3.0\,\Omega$

$= -\frac{7}{19} \times 3.0$

$= \boxed{-1.11 \text{ Volts}}$

For cells: $V_{2.0} = 2.0 - I \times 1.0$

$= 2.0 - \frac{7}{19} \times 1.0$

$= \boxed{1.63 \text{ Volts}}$

$V_{1.5} = 1.5 - I \times 0.5$

$= \boxed{1.32 \text{ Volts}}$

j) A thick-bottomed, cylindrical glass beaker is placed on a bench. Water and oil are poured into the beaker and form discrete layers, as shown in **Figure 4**. The bottom of the beaker is 1.8cm thick, the water is 1.2 cm deep, and the oil layer is 0.8 cm deep.

(i) Draw a diagram showing the path of a ray at a small angle to the normal, travelling from the underside of the beaker and being refracted through the layers.

(ii) Assuming the angles of deviation of the ray are small, calculate the apparent vertical displacement of the lab bench when viewed from above. h

The refractive indices are 1.5, 1.3 and 1.1 for the glass, water and oil respectively.

$n_1 = 1.5$

$n_2 = 1.3$

$n_3 = 1.1$

$n_4 = 1.0$

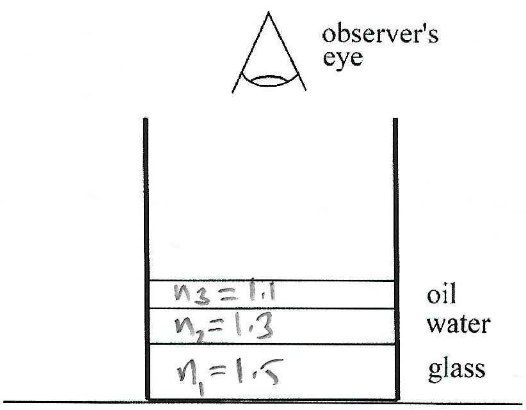
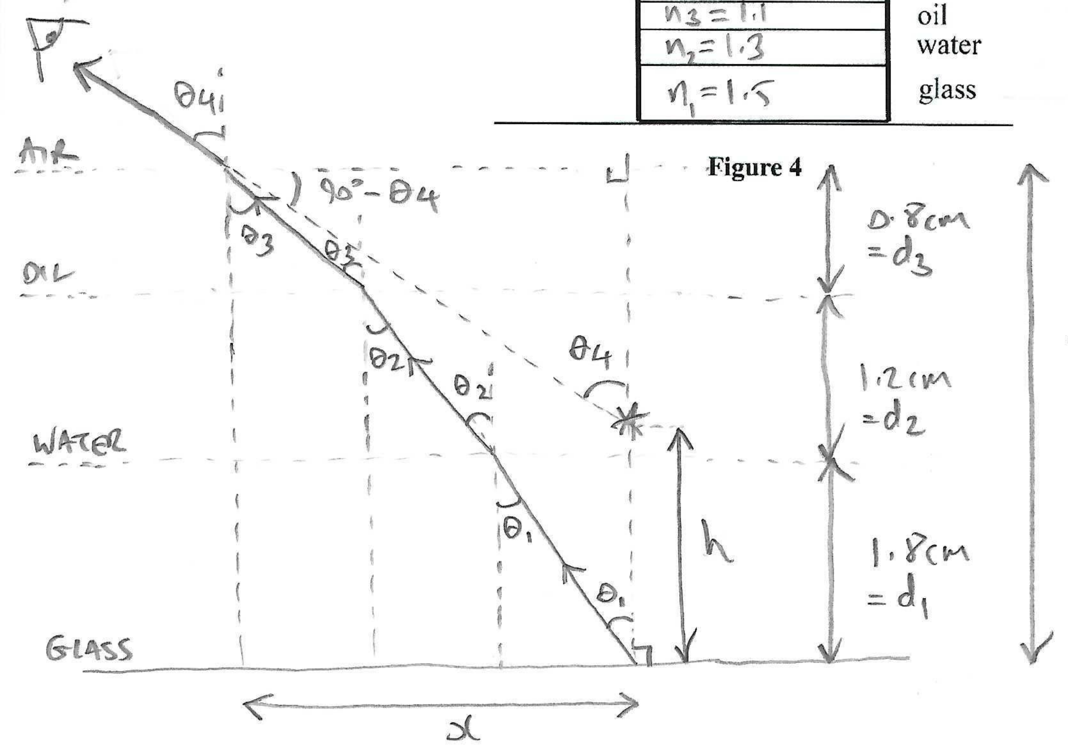


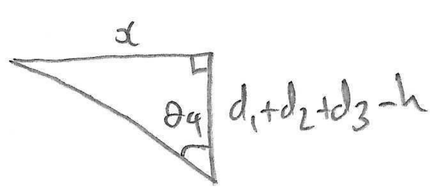
Figure 4

[7]



$3.8\text{cm} = d_1 + d_2 + d_3$

* Apparent position of bottom of beaker is the bench.



$$\tan\theta_4 = \frac{x}{d_1+d_2+d_3-h}$$

$$d_1+d_2+d_3-h = \frac{x}{\tan\theta_4}$$

$$\therefore h = d_1+d_2+d_3 - \frac{x}{\tan\theta_4}$$

$x = d_1\tan\theta_1 + d_2\tan\theta_2 + d_3\tan\theta_3$ from geometry.

$\boxed{\text{Snell}}$ $n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\sin\theta_3 = n_4\sin\theta_4$

Small angles $\tan\theta_i \approx \sin\theta_i \approx \theta_i$ (radians)

$$\therefore \tan\theta_i \approx \frac{n_4}{n_i}\tan\theta_4 \approx \frac{\tan\theta_4}{n_i}$$

↓PTO

$$\therefore \quad x \approx \left(\frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3}\right) \tan\theta_4$$

$$\therefore \quad h \approx d_1+d_2+d_3 - \left(\frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3}\right)$$

$$\boxed{h \approx d_1\left(1-\frac{1}{n_1}\right) + d_2\left(1-\frac{1}{n_2}\right) + d_3\left(1-\frac{1}{n_3}\right)}$$

(You can spot a pattern $h = \sum_i d_i\left(1-\frac{1}{n_i}\right)$)

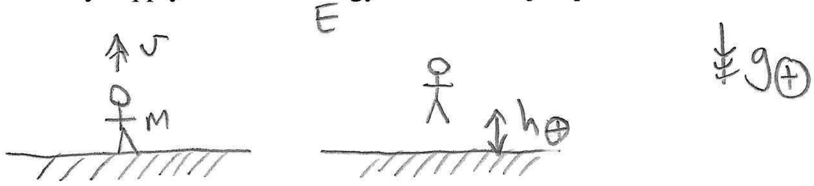
$$\therefore \quad h = 1.8\left(1-\frac{1}{1.5}\right) + 1.2\left(1-\frac{1}{1.3}\right) + 0.8\left(1-\frac{1}{1.1}\right) \text{ (cm)}$$

$$= \boxed{0.95\text{ cm}}$$

k) A person might reasonably expect to jump a height of 1 m on Earth. On a planet with a density two thirds that of Earth, and radius twice that of the Earth, to what height might the person jump? Assume that they supply the same energy to make the jump

[4]

Earth: $\oplus$

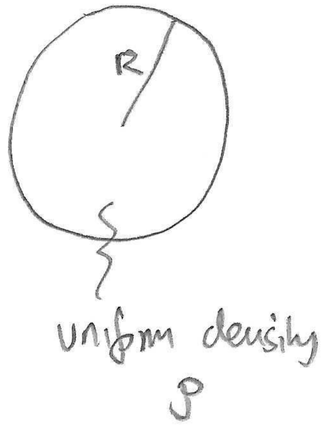


$E = \frac{1}{2}mv^2 = Mg_\oplus h_\oplus$

let's assume this remains the same on Planet X.

Planet X: $E = mg_x h_x$ $\therefore$ $\boxed{g_\oplus h_\oplus = g_x h_x}$ ①

Now for a planet:



Gravitational field strength g at radius R (ie at surface) is:

$$g = \frac{G \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\boxed{g = \frac{4}{3}\pi G \rho R}$$ ②

let $R_x = 2R_\oplus$ and $\rho_x = \frac{2}{3}\rho_\oplus$

From ① $\therefore h_x = \frac{g_\oplus}{g_x} h_\oplus$

$$\therefore \boxed{h_x = \frac{\rho_\oplus R_\oplus}{\rho_x R_x} h_\oplus}$$

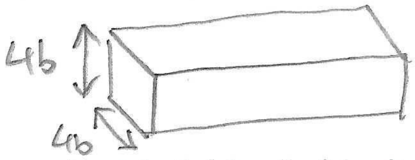
$$h_x = \frac{\rho_\oplus R_\oplus}{\left(\frac{2}{3}\right)(2)\rho_\oplus R_\oplus} h_\oplus$$

$$\therefore \boxed{h_x = \frac{3}{4} h_\oplus}$$

So if $h_\oplus$ is 1.0 m

$$\boxed{h_x = 0.75\,\text{m}}$$

Block density	$\frac{2}{3}\rho$	ρ_b
oil density	$\frac{2}{3}\rho$	ρ_o
Water density	ρ	ρ_w



l) A pond containing water of density ρ is covered to a depth b by oil of density $\frac{2}{3}\rho$. A long wood block of square cross section $4b \times 4b$, with the same density as the oil, floats in the pond, as shown in **Figure 5**. What fraction of the wood block is immersed below the surface oil level?

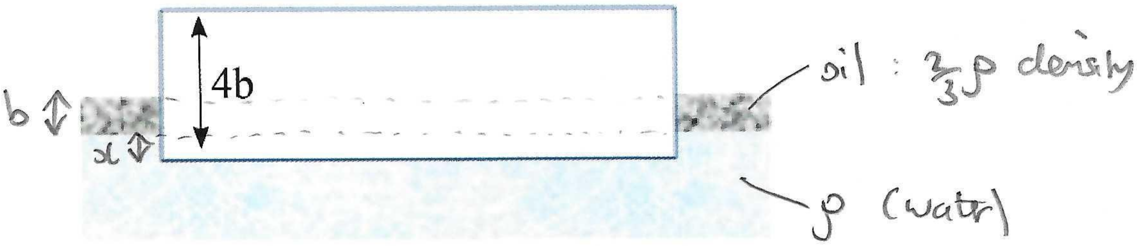


Figure 5

[4]

Proportion of wood block below surface oil level is

$$\boxed{\phi = \frac{b+x}{4b}}$$

N II.

$\boxed{\text{Archimedes : "upthrust = weight of fluid displaced"}}$ = weight in eq. of block

Since in same g-field, ⇒ mass of fluid displaced = mass of block.

∴ per length of block:

$$\underbrace{(4b)^2 \rho_b}_{\text{mass of block}} = \underbrace{4b^2 \rho_o}_{\text{mass of oil displaced}} + \underbrace{4xb\rho_w}_{\text{mass of water displaced}} \Rightarrow x = \frac{4b\rho_b - b\rho_o}{\rho_w}$$

$$\therefore \boxed{\phi = \frac{1}{4}\left[\frac{4\rho_b - \rho_o}{\rho_w} + 1\right]}$$

$$\therefore \phi = \frac{1}{4}\left[\frac{4(\frac{2}{3}) - \frac{2}{3}}{1} + 1\right] = \frac{1}{4}\left[3 \times \tfrac{2}{3} + 1\right] = \boxed{\frac{3}{4}}$$

So block is 75% submerged.

This big assumption should have been stated in the question I think.

m) A volume of 80 cm^3 of water in a copper calorimeter of mass 150 g takes 12 minutes to cool from 40 °C to 15 °C in a cold room. The same volume of ethanol of density 0.8 g cm^{-3} takes 8 minutes to cool also from 40 °C to 15 °C in the same calorimeter in the same circumstances. Calculate the specific heat capacity of ethanol.
The specific heat capacity of copper = 400 J kg^{-1} °C^{-1} and of water = 4200 J kg^{-1} °C^{-1}.
The density of water, $\rho_w = 1.0\,\mathrm{g\,cm^{-3}}$.

[5]

* Assume average rate of energy loss is a constant for both systems. { is this consistent with say Fourier's law, or Newtonian cooling ie $\frac{dT}{dt} \propto \Delta T$?? }

Water: $$\frac{\Delta E}{\Delta t} \approx \frac{C_W M_W \Delta T_W + C_C M_C \Delta T_W}{\Delta t_W}$$

$M_W = 0.08\,\mathrm{kg}$ $\quad M_C = 0.150\,\mathrm{kg}$ $\quad C_W = 4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$

$\Delta T_W = 25\,\mathrm{K}$ $\quad \Delta t_W = 12 \times 60\,\mathrm{s}$ $\quad C_C = 400\,\mathrm{J\,kg^{-1}\,K^{-1}}$

Ethanol: $$\frac{\Delta E}{\Delta t} = \text{Same as above} = \frac{(C_E M_E + C_C M_C)\Delta T_E}{\Delta t_E}$$

$$M_E = \frac{80\,\mathrm{cm^3} \times 0.8\,\mathrm{g\,cm^{-3}}}{1000}\,\mathrm{kg} = \boxed{0.064\,\mathrm{kg}}$$

$\Delta T_E = 25\,\mathrm{K}$ $\quad \Delta t_E = 8 \times 60\,\mathrm{s}$

so balancing $\Delta E / \Delta t$: $$\frac{(C_E M_E + C_C M_C)\Delta T_E}{\Delta t_E} = \frac{(C_W M_W + C_C M_C)\Delta T_W}{\Delta t_W}$$

$$\Rightarrow \boxed{C_E = \frac{(C_W M_W + C_C M_C)\left(\frac{\Delta T_W}{\Delta T_E}\right)\left(\frac{\Delta t_E}{\Delta t_W}\right) - C_C M_C}{M_E}} = \text{*}$$

PTO

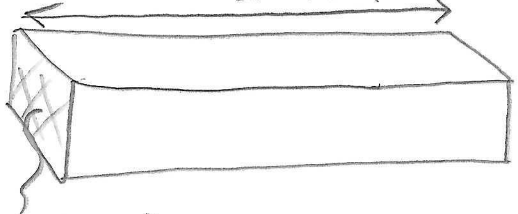
$$\therefore \quad C_E = \frac{(4200 \times 0.08 + 400 \times 0.150)\left(\frac{25}{25}\right)\left(\frac{8 \times 60}{12 \times 60}\right) - 400 \times 0.150}{0.064}$$

$$= \boxed{3{,}190 \text{ Jkg}^{-1}\text{K}^{-1}}$$

n) A steel girder is planted securely between two sides of a ravine in order to provide a bridge. The total cross-sectional area of the girder is $30\,\mathrm{cm}^2$, and the length of the girder is $4.0\,\mathrm{m}$. Installed at a temperature of $5\,°\mathrm{C}$, the temperature now rises to $20\,°\mathrm{C}$. Calculate the force exerted by the girder due to the change in temperature, assuming the ends do not move. F
Young modulus of steel = 2.0×10^{11} Pa $= Y$
Linear expansivity of steel (fractional expansion per unit temperature rise) = $1.2 \times 10^{-7}\,°\mathrm{C}^{-1}$ at $5\,°\mathrm{C}$. α

[4]

Girder $L = 4.0\,\mathrm{m}$ @ $5°C$.



$\Delta T = 15°C$.

$A = 30\,\mathrm{cm}^2$
$= 30 \times 10^{-4}\,\mathrm{m}^2$
$= \boxed{3.0 \times 10^{-3}\,\mathrm{m}^2}$

Extension: $x = L\alpha\Delta T$

$\therefore$ strain: $\frac{x}{L} = \alpha\Delta T$

Now stress: $\sigma = Y\frac{x}{L} = Y\alpha\Delta T$

Force on girder (if it doesn't move) is $\therefore F = \sigma A$

$\therefore \boxed{F = Y\alpha\Delta T A}$

$F = 2.0 \times 10^{11} \times 1.2 \times 10^{-7} \times 15 \times 3.0 \times 10^{-3}$ (N)
$= \boxed{1080\,\mathrm{N}}$

So $\boxed{F = 1.1\,\mathrm{kN}}$ to 2 s.f.

o) A narrow beam of monochromatic light falls on a diffraction grating of 1200 lines mm^{-1}, and two diffracted beams of successive orders are observed at 14° and 73° to the normal, both of them on the same side of the normal. The incident beam of light is not along the normal to the grating.

(i) Sketch a diagram to show the path difference between rays passing through adjacent slits, for a ray incident on the diffraction grating at angle θ_1, and for the corresponding ray emerging from the grating at angle θ_2, with respect to the normal.

(ii) Derive an equation relating the angles θ_1 and θ_2 to the order of diffraction, n, and the wavelength, λ.

Determine:

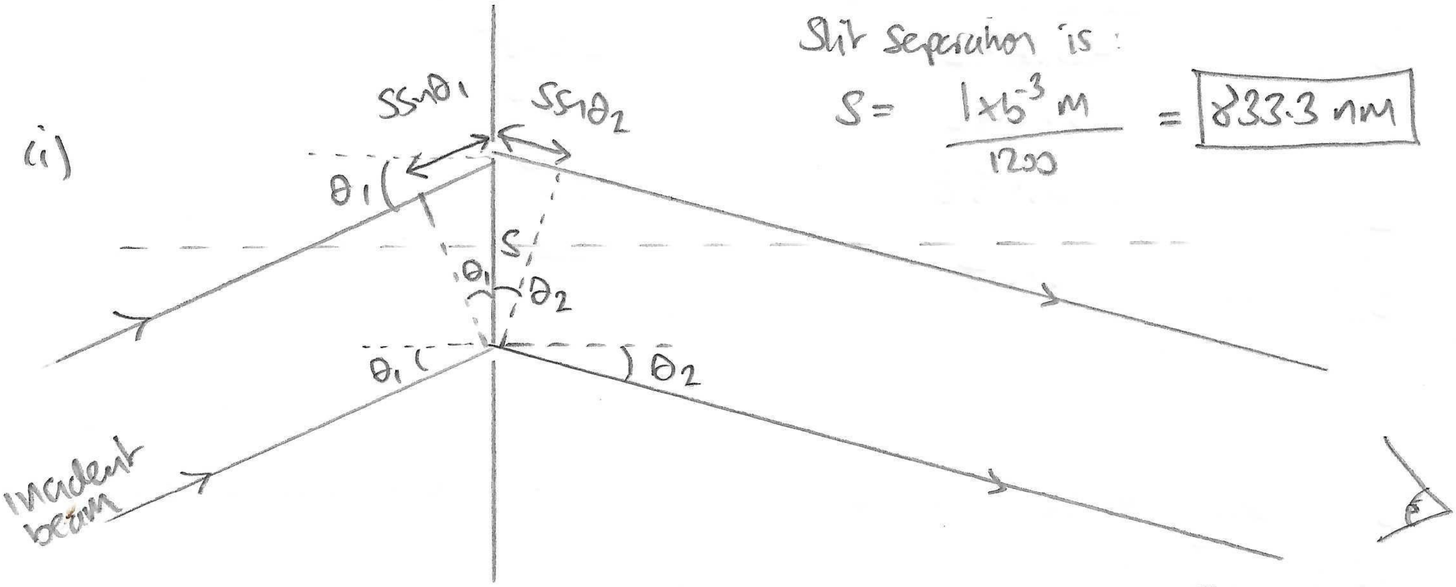
(iii) The wavelength of the light used.

(iv) The angle of incidence of the beam on the grating.

(v) The angle of diffraction of a third transmitted beam.

[6]

(i)



Slit separation is:

$S = \frac{1\times10^{-3}\,m}{1200} = \boxed{833.3\,nm}$

Far-field of diffracted beam on same side as the normal.

(ii) Path difference is $S(\sin\theta_1 + \sin\theta_2)$

$\therefore$ for constructive interference

$$\boxed{n\lambda = S(\sin\theta_1 + \sin\theta_2)}$$ (n integer)

(iii) $(n+1)\lambda = S(\sin\theta_1 + \sin 73^\circ)$
$n\lambda = S(\sin\theta_1 + \sin 14^\circ)$
$\Big\} \therefore \lambda = S(\sin 73^\circ - \sin 14^\circ) = \boxed{595\,nm}$

$\lambda/S \simeq 5/7$

(iv) $\therefore \sin\theta_1 = \frac{n\lambda}{S} - \sin 14^\circ$

$n=1$: $\theta_1 = \sin^{-1}\left(\frac{5}{7} - \sin 14^\circ\right) = \boxed{28.2^\circ}$

$n=2$: $\theta_1 = \sin^{-1}(1.19)$ No real value

$\Big\}$ So $n=1$ only possible solution.

(v) let $n = 3$. Is this possible?

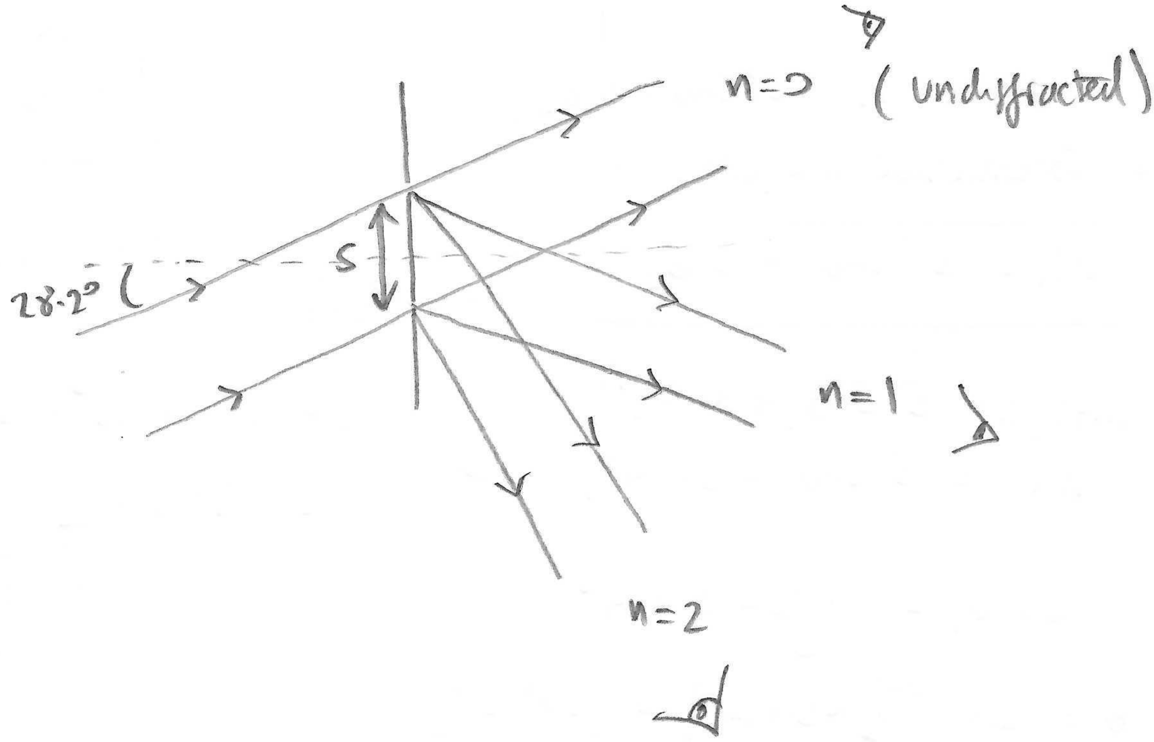
$$\sin\theta_2 = \frac{n\lambda}{s} - \sin\theta_1$$

If $\sin\theta_1 = \frac{s}{7} - \sin 14°$

$$\therefore \sin\theta_2 = (n-1)\frac{s}{7} + \sin 14°$$

n	$\sin\theta_2$	θ_2
0	-0.472	$-28.2°$
1	0.242	$14°$
2	0.956	$73°$
3	1.67	—

So only three rays are possible, but not $n=3$.
$n = 0, 1, 2$.



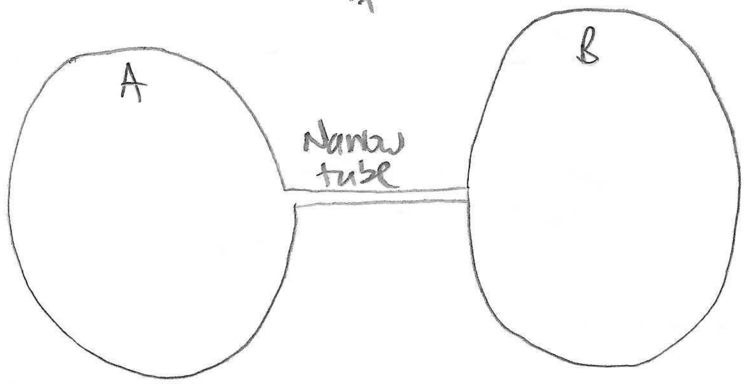
T_0

p) Two identical spherical glass containers are joined by a narrow tube, whose volume is negligible compared to the spheres. The spheres contain air at 100 °C. One of the spheres is then heated by 50 °C whilst the other is cooled by 50 °C. This produces a small change in pressure, from P_{initial} to P_{final}, of the air in the system. What common temperature of the two spheres could produce the same final pressure P_{final}? $\leftarrow P_0$

ΔT $\qquad P_f$

↑ This last bit needs explaining better!

[4]



let sphere volumes be V.

$T_A : 373 \rightarrow 423$ (K) $\qquad T_B : 373 \rightarrow 323$ (K)

and P_0 the initial pressure

let T_0 be the initial temperature of both spheres

So $T_A = T_0 + \Delta T$ and $T_B = T_0 - \Delta T$.

Using the ideal gas equation: $pV = nRT$

Total # moles of gas is:

$n = 2P_0V/RT_0$ $\quad$ ∴ Since $n = n_A + n_B$ if no leaks at T_A and T_B

$$2P_0V/RT_0 = \frac{P_fV}{RT_A} + \frac{P_fV}{RT_B}$$

ie assume same pressure in both spheres (if not molecules will move from one to the other until pressure equates)

So $\boxed{P_f = P_0\left(\frac{2/T_0}{\frac{1}{T_0+\Delta T} + \frac{1}{T_0-\Delta T}}\right)}$

PTO ↓

Alternatively just heat (or cool) both spheres to temperature T_c

Such that $P = P_F$

$\therefore \quad n = \dfrac{2P_F V}{RT_c}$ $\qquad \therefore \quad P_0/T_0 = P_f/T_c$

$$\therefore \quad T_c = T_0 \left(\frac{P_f}{P_0}\right)$$

$$\therefore \quad \boxed{T_c = \frac{2}{\frac{1}{T_0+\Delta T} + \frac{1}{T_0-\Delta T}}}$$

$$= \frac{2}{\frac{1}{423} + \frac{1}{323}}$$

$$= \boxed{366\ k} \qquad (93.3°C)$$

q) A simple pendulum consists of a small mass on the end of a light, inextensible string, as shown in **Figure 6**. It swings from an initial angle $\theta = 14°$, for which it would have a period T_0, but it hits a wall elastically, which is at angle $\phi = 7°$ to the vertical. What is the new period of oscillation in terms of T_0?

(θ, ϕ are small angles such that $\sin\theta \approx \theta$ and $\sin\phi \approx \phi$).

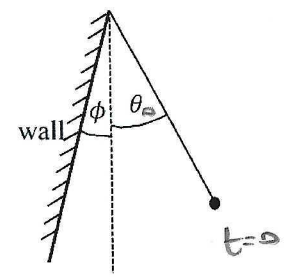


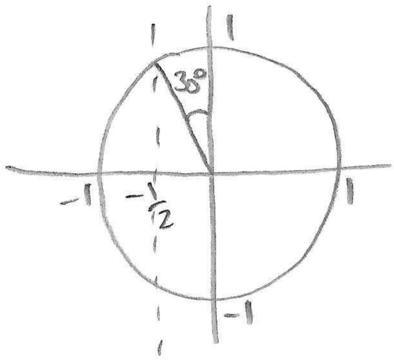
Figure 6

[4]

Without the wall, SHM so $\boxed{\theta(t) = \theta_0 \cos\left(2\pi t / T_0\right)}$

$\theta_0 = 14°$.

when $\theta = -7°$:



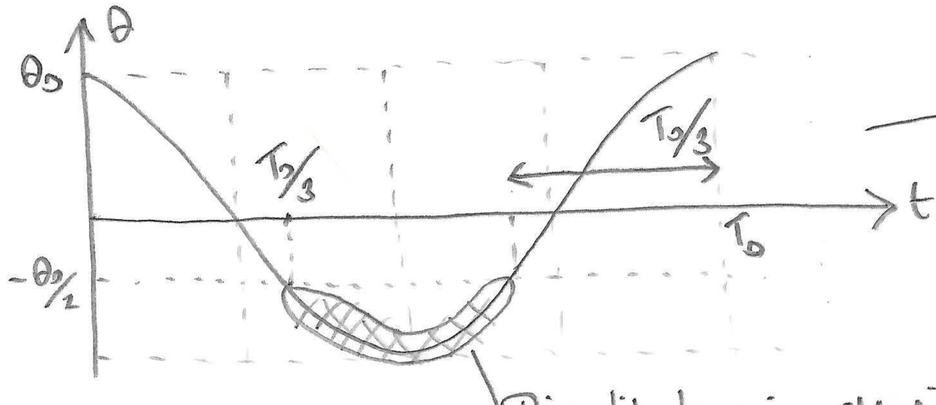
$$-7/14 = \cos\left(2\pi t / T_0\right)$$

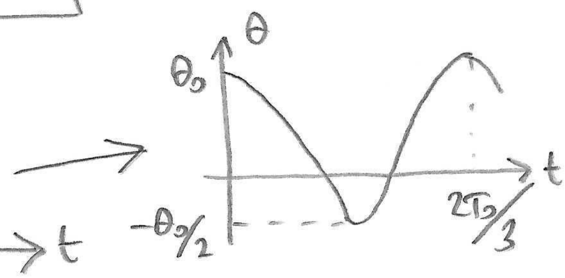
So $2\pi t / T_0 = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2}{3}\pi$ radians

$\therefore \boxed{t = \frac{1}{3} T_0}$

So since an elastic collision, trajectory reverses and it should take another $\frac{1}{3} T_0$ to rise to $\theta = \theta_0$

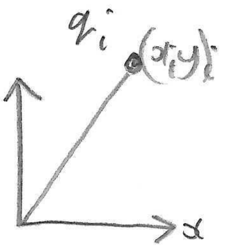
$\therefore$ New period is $\boxed{\frac{2}{3} T_0}$





This bit here is effectively ignored

At origin: $\underline{E} = \sum_i \frac{-q_i}{4\pi\varepsilon_0 (x_i^2+y_i^2)^{3/2}} \begin{pmatrix} x_i \\ y_i \end{pmatrix}$



r) Four charges are placed at the corners of a square of side 10 cm, as shown in **Figure 7**.

$A = +10 \times 10^{-9}$ C

$B = +8 \times 10^{-9}$ C

$C = -12 \times 10^{-9}$ C

$D = -6 \times 10^{-9}$ C

(i) Calculate the magnitude and direction of the electric field strength at the centre of the square.

(ii) Calculate the work done taking an electron from the centre to the mid-point of side CD.

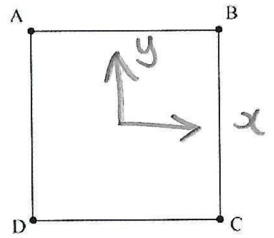
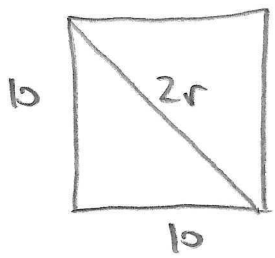


Figure 7



[7]

Coulomb: $\boxed{\underline{E} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{\underline{r}}}$

$(2r)^2 = 2 \times 10^2$

$4r^2 = 200$

$r^2 = 50$

$\boxed{r = \sqrt{50}}$ (cm)

$r = \sqrt{50}/100$ m

$= \boxed{\sqrt{2}/20}$ (m)

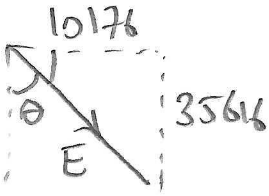
Field strength at $(0,0)$ is

$$\underline{E} = \frac{10^{-9}}{4\pi\varepsilon_0 \left(\frac{\sqrt{2}}{20}\right)^2} \frac{1}{\sqrt{2}} \left\{ 10\begin{pmatrix} +1 \\ -1 \end{pmatrix} + 8\begin{pmatrix} -1 \\ -1 \end{pmatrix} - 12\begin{pmatrix} -1 \\ 1 \end{pmatrix} - 6\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

$$\underline{E} = \underbrace{\frac{10^{-9}}{4\sqrt{2}\pi \times 8.85 \times 10^{-12} \times \frac{2}{400}}}_{1272\ N/C} \begin{pmatrix} 10 - 8 + 12 - 6 \\ -10 - 8 - 12 - 6 \end{pmatrix}$$

$$= 1272\ N/C \begin{pmatrix} 8 \\ -36 \end{pmatrix} = \begin{pmatrix} 10176 \\ -45792 \end{pmatrix} N/C$$

i.e. a bearing of 167°



$\theta = \tan^{-1}\left(\frac{10176}{45792}\right) = \boxed{12.5°}$, $\boxed{|\underline{E}| = 4.7 \times 10^4\ N/C}$