

(a) A chain of resistors, Figure 1.a.i, is composed of n units, each consisting of three resistors, each resistor of resistance R , Figure 1.a.ii. A unit is attached to the left hand end of the chain in order to increase the number of units from n to $(n+1)$.

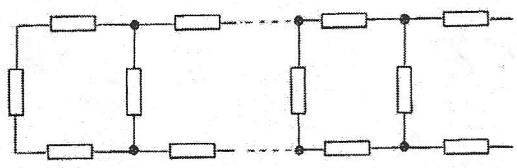


Figure 1.a.i

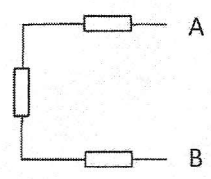
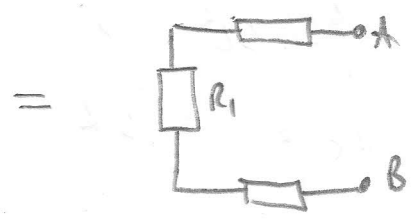
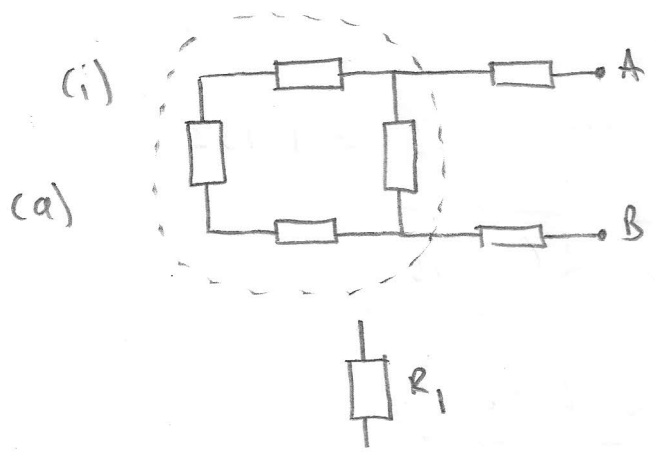


Figure 1.a.ii

- (i) Calculate the resistance (between A and B) across a chain with two units, R_2 , and the resistance R_3 , across a chain with three units.
- (ii) A unit is attached to a long chain. The resistance of the chain, R_T , is not altered by this addition. Determine the resistance of the chain.

(i)

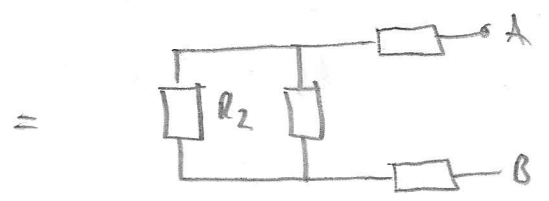
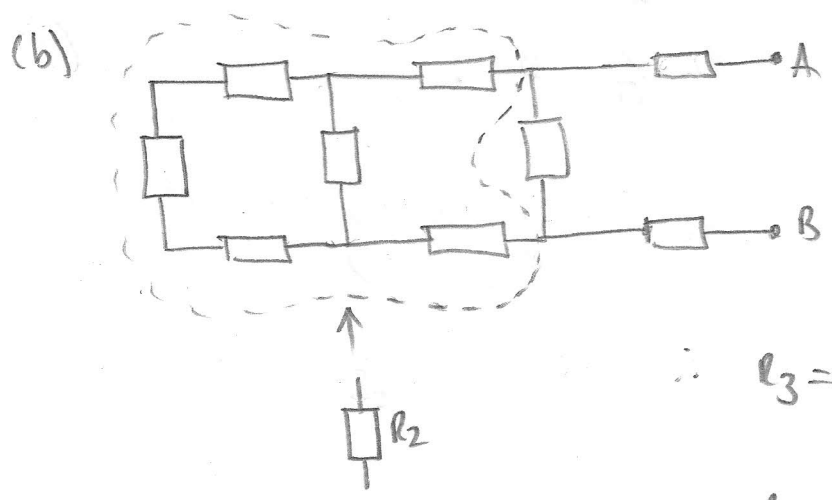
[6]



$$R_2 = 2R + R_1$$

$$R_1 = \frac{1}{\frac{1}{3R} + \frac{1}{R}} = \frac{3}{4}R$$

$$\therefore R_2 = \frac{11}{4}R = \boxed{2\frac{3}{4}R}$$

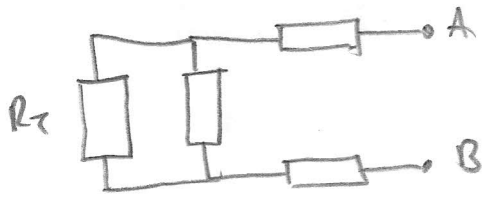


$$\therefore R_3 = 2R + \frac{1}{\frac{1}{R} + \frac{1}{R_2}}$$

$$R_3 = 2R + \frac{R}{1 + \frac{4}{11}} = \boxed{2\frac{11}{15}R}$$

i.e. what we computed in a)(i)

(ii) Essentially just like (i) (b), but resistance is R_T



$$\therefore R_T = 2R + \frac{1}{\frac{1}{R} + \frac{1}{R_T}}$$

$$R_T = 2R + \frac{R}{1 + \frac{R}{R_T}}$$

$$R_T = 2R + \frac{RR_T}{R_T + R}$$

$$\therefore (R_T + R)R_T = 2R(R_T + R) + RR_T$$

$$\therefore R_T^2 + RR_T - 2RR_T - 2R^2 - RR_T = 0$$

$$\therefore R_T^2 - 2RR_T - 2R^2 = 0$$

$$(R_T - R)^2 - 3R^2 = 0$$

$$R_T = \pm \sqrt{3}R + R$$

$$\sqrt{3} \approx 1.732$$

So taking +ve root,

$$R_T = R(1 + \sqrt{3})$$

$$R_T \approx 2.73R$$

{ \leq the termination of the chain sequence }

$$R_2 = 2\frac{3}{4}R$$

$$\frac{3}{4} = 0.75$$

$$R_3 = 2\frac{1}{15}R$$

$$\frac{1}{15} = 0.0667$$

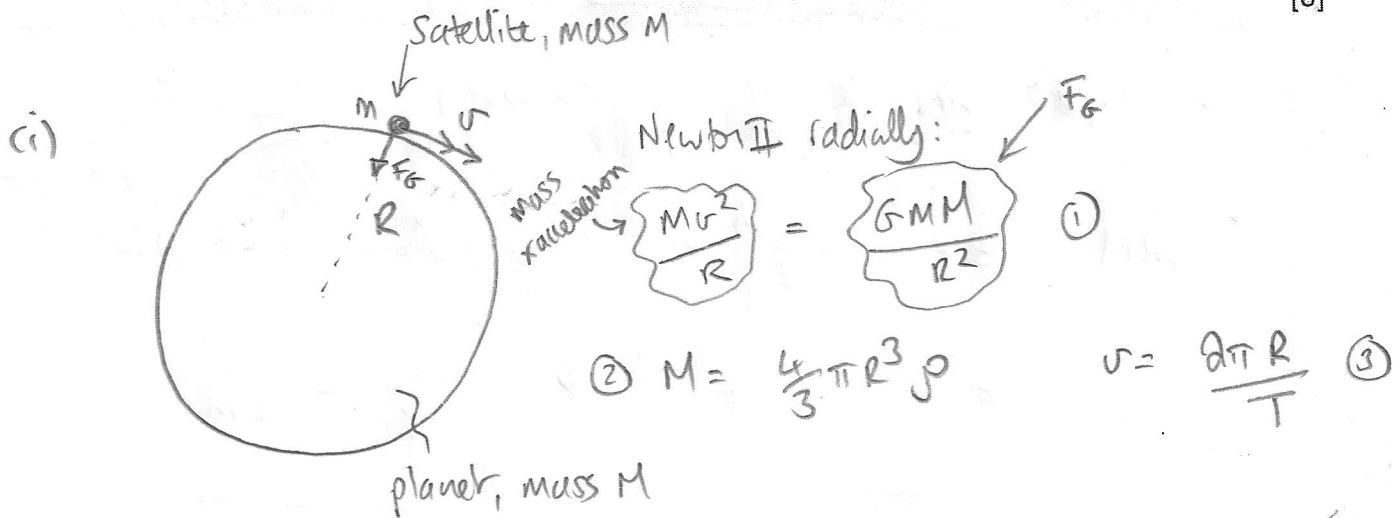
$$R_\infty = 2.73R$$

↑
So probably terminate
very quickly.

(b)

- (i) A satellite is in orbit just above a spherical planet of radius R and uniform density ρ . If the periodic time for each orbit is T , find an expression for ρT^2 . Comment on the result.
- (ii) A man with a mass of 75 kg stands at the end of a diving board, depressing it by 0.30 m. What would be the period of his motion if he was to jump lightly in rhythm with the harmonic motion of the diving board?

[6]



combining equations (1), (2), (3):

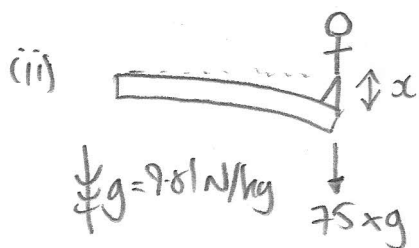
$$\frac{4\pi^2 R^2}{T^2} = \frac{G}{R} \frac{4}{3}\pi R^3 \rho$$

$$\sqrt{\frac{3\pi}{G\rho}} = T$$

or

$$\rho T^2 = \frac{3\pi}{G}$$

So T is independent of planet radius, and ρT^2 is a universal constant (true for all spherical planets).



Assume diving board is like an elastic, Hookean spring for small displacements.

$$\therefore \text{in eq: } 75 \times g = kx$$

$$\Rightarrow \text{Spring constant } k = \frac{75 \times 9.81}{0.3}$$

$$k = \boxed{2453 \text{ Nm}^{-1}}$$

(3)

