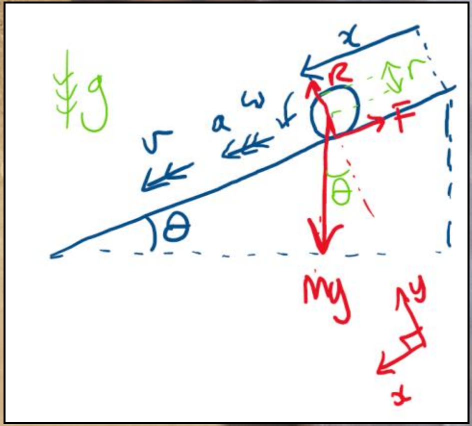
TEACHER NOTES

Ball bearing rolling down a slope

$$x = \frac{1}{2}\frac{5}{7} g \sin\theta \, t^2$$

$$\boxed{t = \sqrt{\frac{14x}{5}\frac{1}{g\sin\theta}}}$$

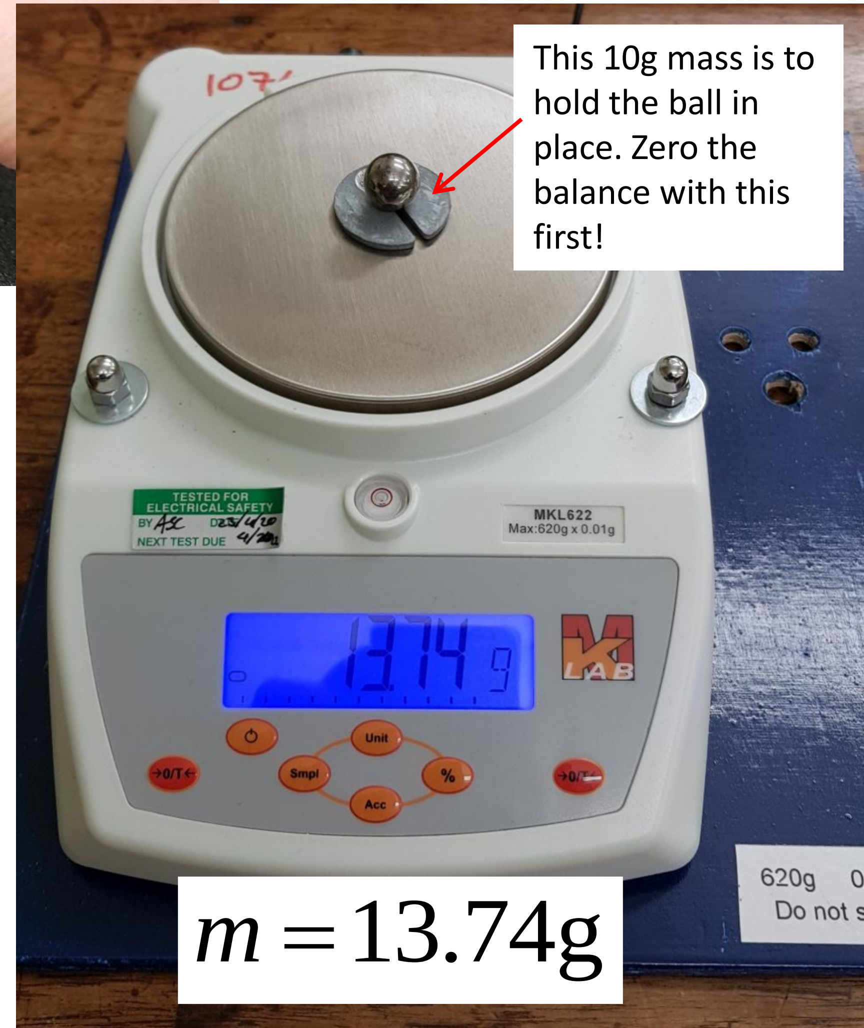


Dr Andrew French. October 2020.

Equipment


Fixed gradient
track with rectangular groove
Laptop
30cm ruler
1kg blocks
15mm diameter
ball bearing
Stopwatch
Metre ruler
1kg blocks
Retort stand,
boss, clamp

$$2r = 15.02\text{mm}$$



This 10g mass is to hold the ball in place. Zero the balance with this first!

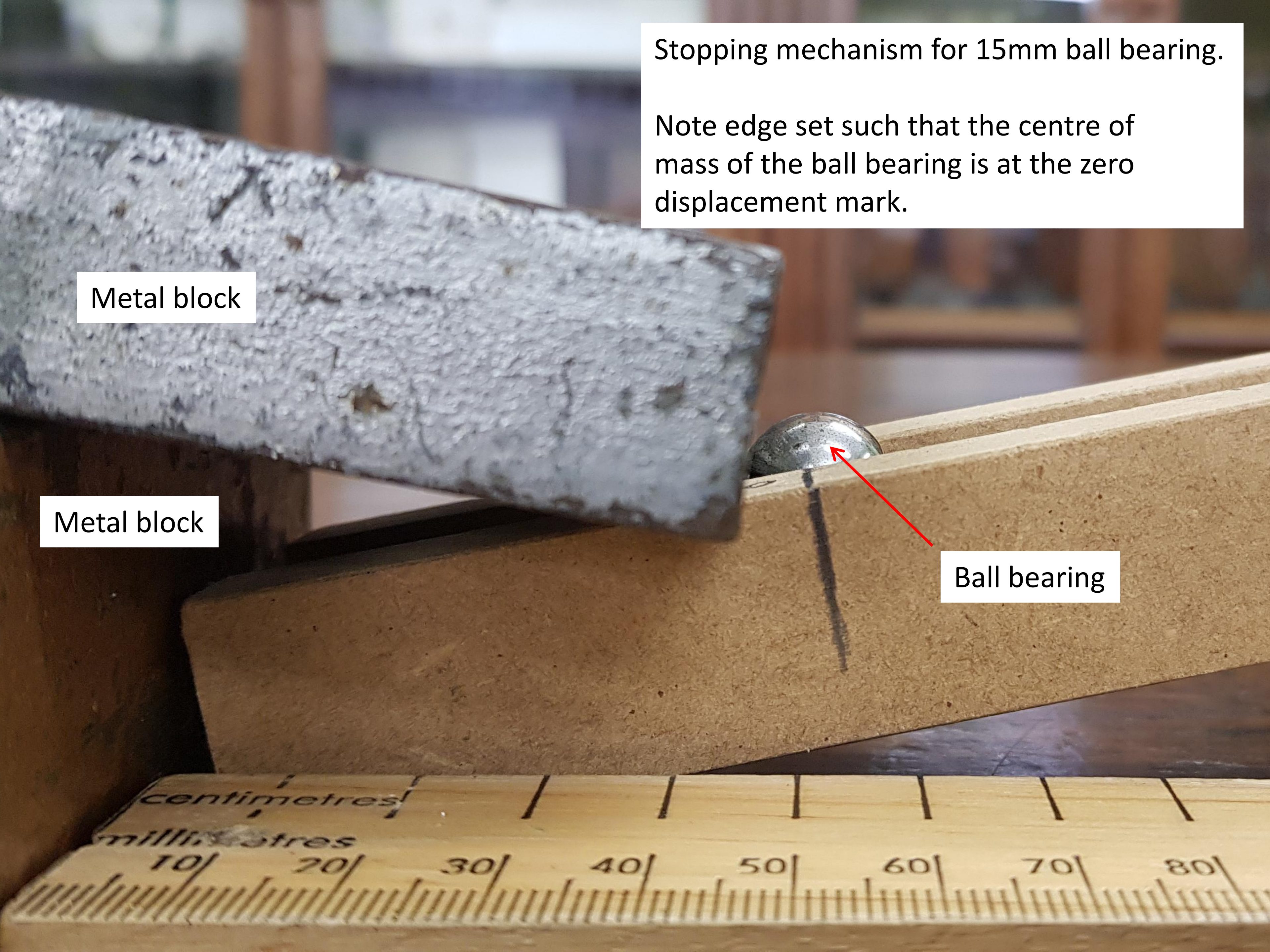
$$m = 13.74\text{g}$$

Although not actually needed for the rolling model, it is good practice to measure the diameter and mass of the ball bearing.

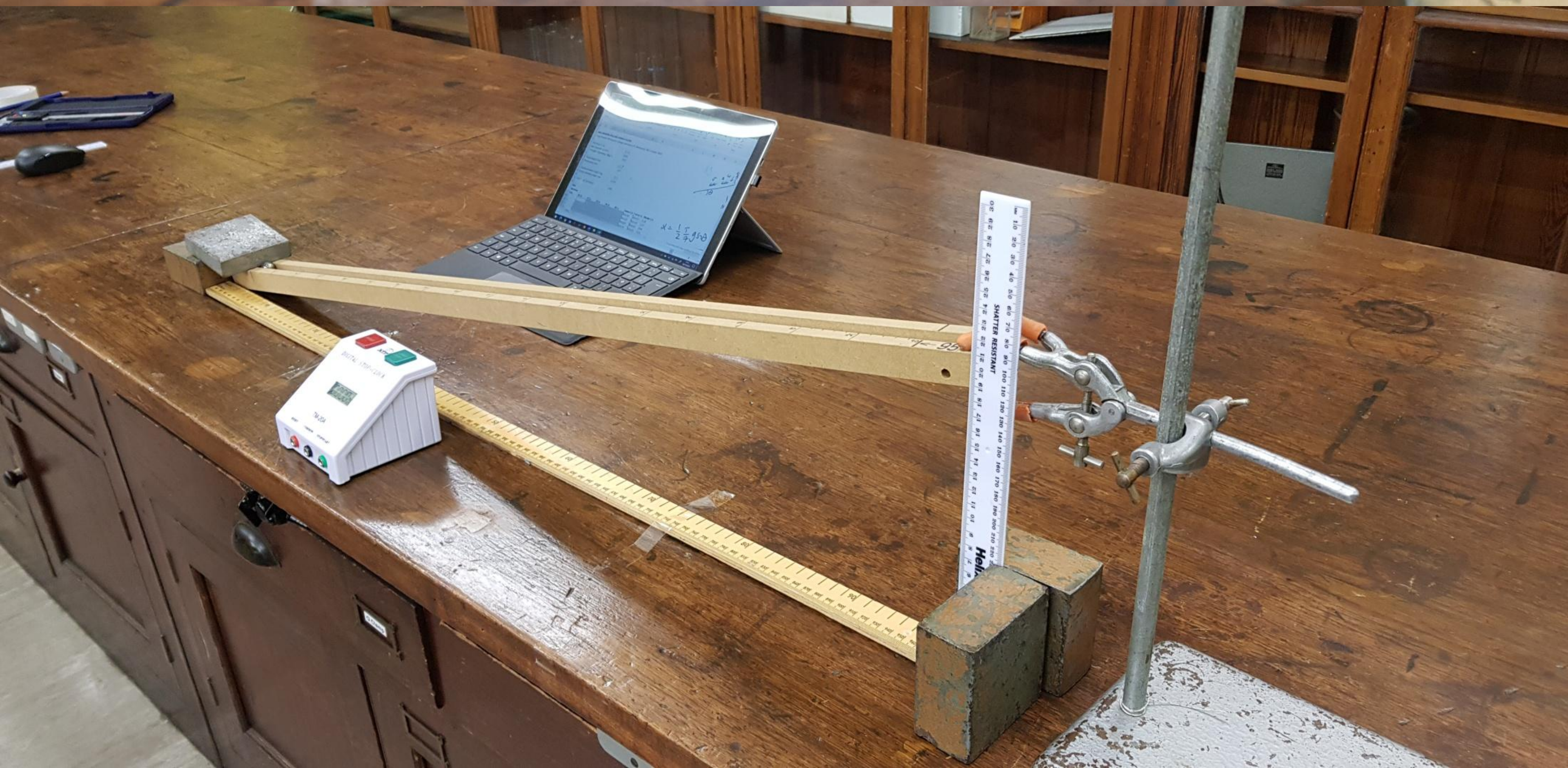
These could be used to calculate its **moment of inertia**.

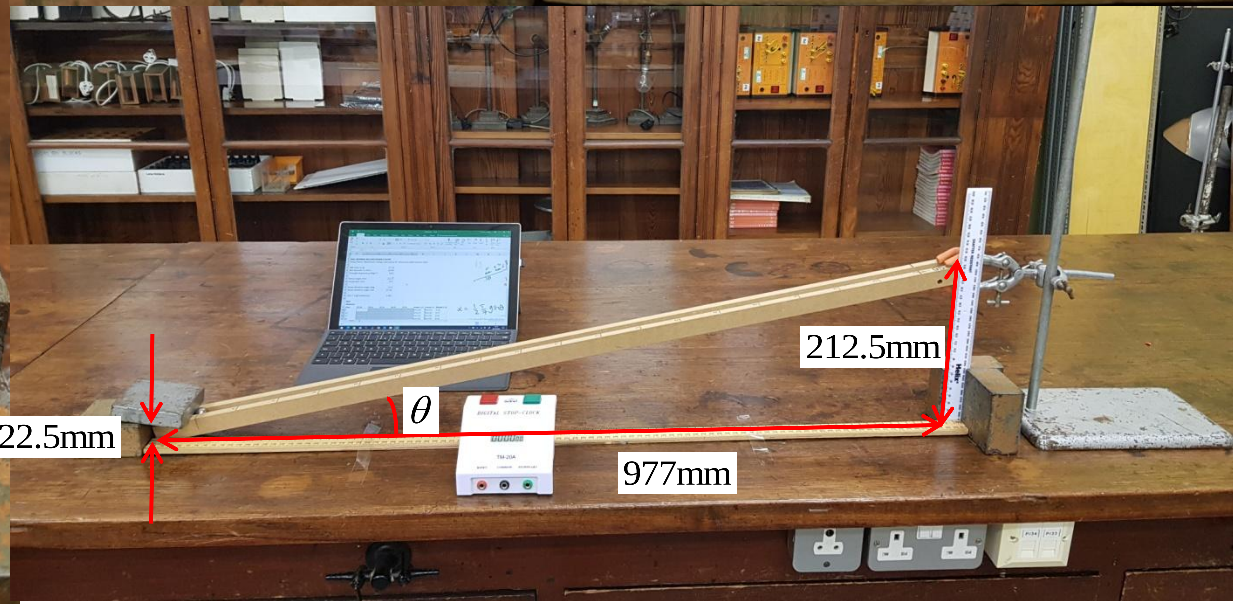
$$I = \tfrac{2}{5} m r^2$$

Moment of inertia of a uniform sphere

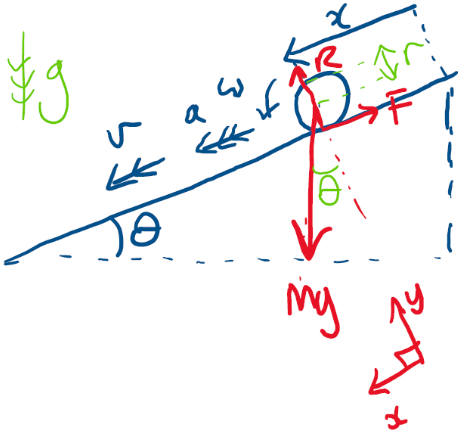

Stopping mechanism for 15mm ball bearing.
Note edge set such that the centre of mass of the ball bearing is at the zero displacement mark.
Metal block
Metal block
Ball bearing
centimetres
10 20 30 40 50 60 70 80

Displacements along the track marked at 5.0cm intervals.





Slope angle: $$\theta = \tan^{-1}\left(\frac{212.5 - 22.5}{977}\right) = 11.0^{\circ}$$



NII:

$/\!/x$: $Ma = Mg\sin\theta - F$

$/\!/y$: $0 = R - Mg\cos\theta$

Rotational motion:

($I\dot{\omega}$ = torque)

sphere: → $\boxed{I = \frac{2}{5}Mr^2}$

$I\dot{\omega} = Fr$

Now if no slip:

$F \leq \mu R$

$v = r\omega \Rightarrow a = r\dot{\omega}$

$\therefore \quad \dot{\omega} = a/r$

$\therefore \quad I\dot{\omega} = Fr$

$\Rightarrow \quad Ia/r = Fr$

$\Rightarrow \quad a = Fr^2/I$

$\Rightarrow \quad \boxed{F = Ia/r^2}$

$\therefore \quad Ma = Mg\sin\theta - Ia/r^2$

$\therefore \quad a = g\sin\theta - Ia/Mr^2$

$\therefore \quad a\left(1 + I/Mr^2\right) = g\sin\theta$

$$\therefore \quad \boxed{a = \frac{g\sin\theta}{1 + I/Mr^2}}$$

Rolling model

So since $I = \frac{2}{5} m r^2$

$\Rightarrow I/mr^2 = 2/5$

$\therefore 1 + I/mr^2 = 7/5$

$\therefore \boxed{a = \frac{5}{7} g \sin\theta}$

kinematics:

$x = \frac{1}{2} a t^2$

(roll from rest)

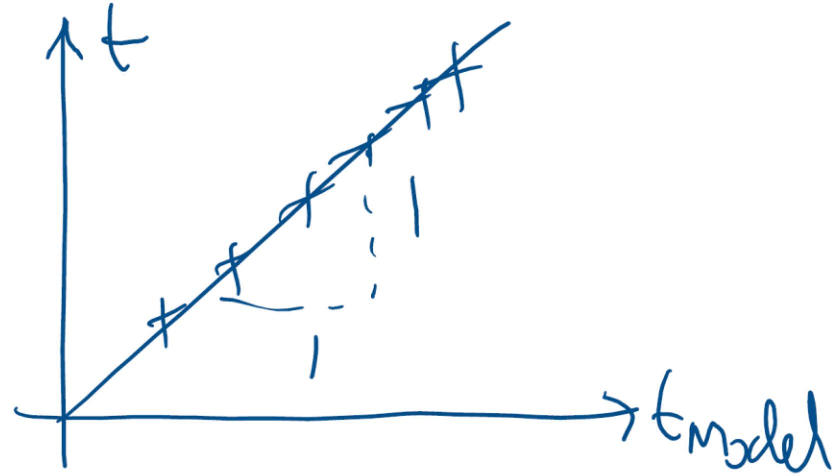
$\therefore x = \frac{1}{2} \frac{5}{7} g \sin\theta \, t^2$

$\therefore \boxed{t = \sqrt{\frac{14x}{5} \frac{1}{g \sin\theta}}}$

MODEL.

For various x in 5cm intervals, record t (Fix θ).

Then plot t vs t_{model}



Hopefully a 1:1 correlation.

Model

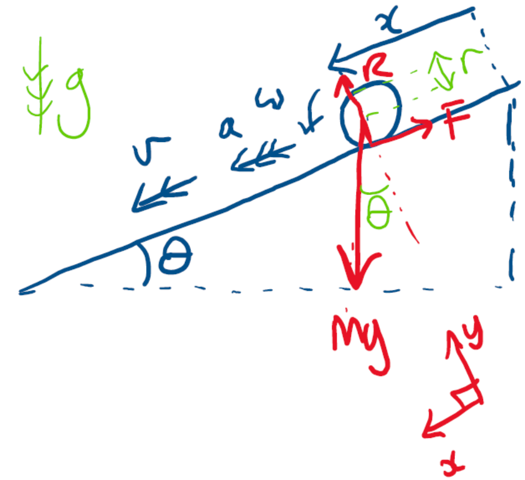
BALL BEARING ROLLING DOWN A SLOPE

Andy French. Winchester College Laboratory P5. Wenesday 28th October 2020.

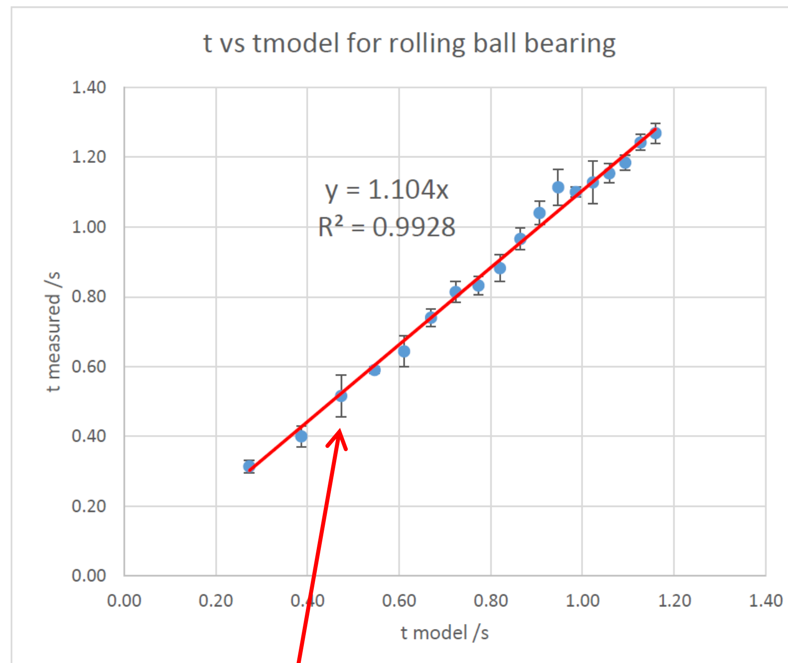
Ball mass m /g	13.74
Ball diameter 2r /mm	15.02
Strength of gravity g /Nkg^-1	9.81
Ramp height /mm	190
Ramp base /mm	977
Ramp elevation angle /deg	11.0
Ramp elevation angle /rad	0.192
14/5 * 1/(g*sin(theta))	1.495

$$x = \frac{1}{2}\frac{5}{7} g \sin\theta \, t^2$$

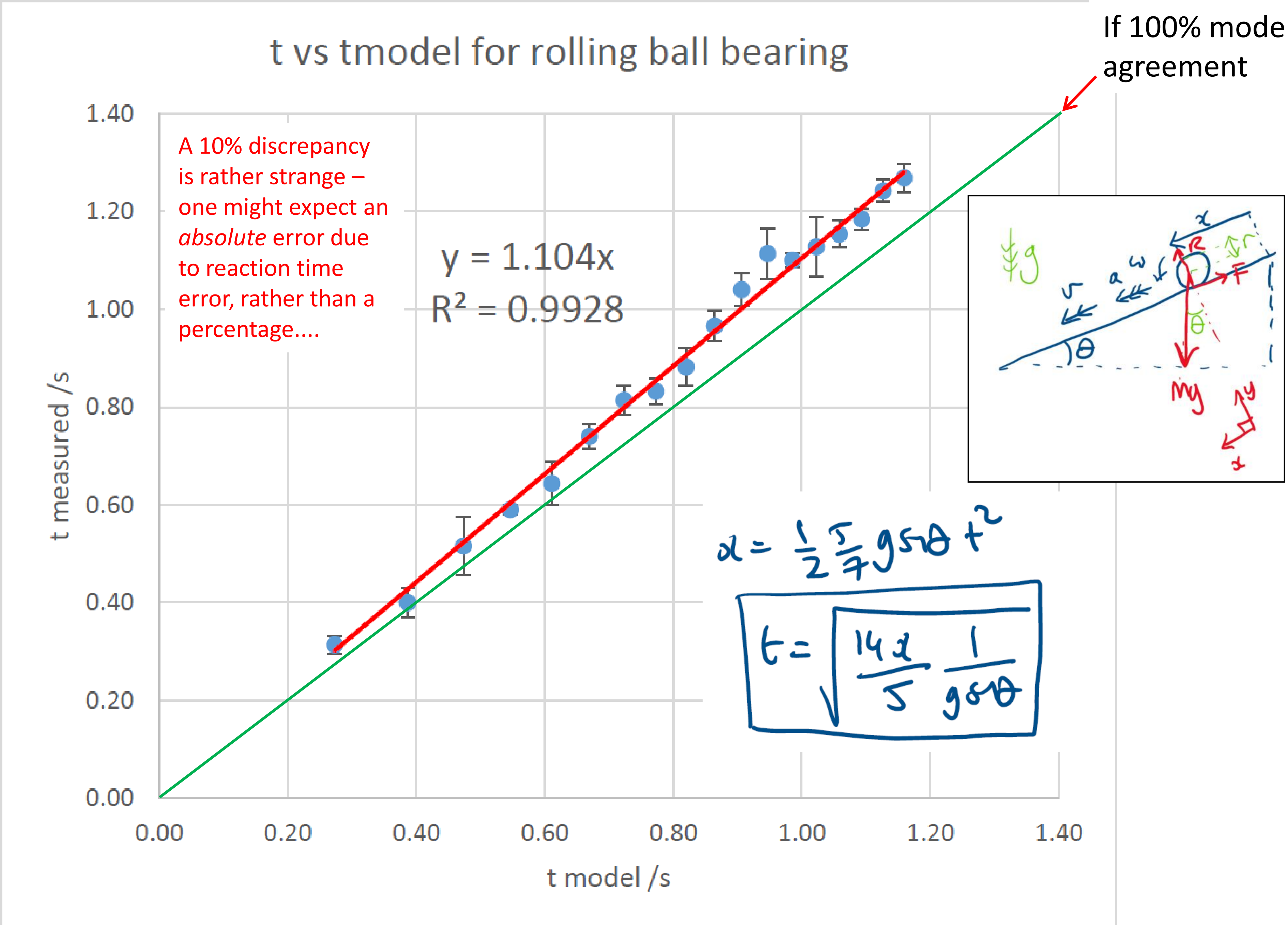
$$\boxed{t = \sqrt{\frac{14x}{5}\frac{1}{g\sin\theta}}}$$



Roll distance /cm	t1 /s	t2 /s	t3 /s	t4 /s	t5 /s	mean t /s	t error /s	Model t /s
5	0.29	0.30	0.32	0.34	0.32	0.31	0.02	0.27
10	0.42	0.38	0.42	0.43	0.35	0.40	0.03	0.39
15	0.44	0.58	0.50	0.59	0.47	0.52	0.06	0.47
20	0.58	0.60	0.58	0.59	0.60	0.59	0.01	0.55
25	0.65	0.65	0.56	0.69	0.67	0.64	0.04	0.61
30	0.77	0.71	0.72	0.73	0.77	0.74	0.03	0.67
35	0.85	0.85	0.78	0.80	0.79	0.81	0.03	0.72
40	0.82	0.81	0.84	0.88	0.81	0.83	0.03	0.77
45	0.91	0.93	0.88	0.87	0.82	0.88	0.04	0.82
50	1.00	0.99	0.92	0.94	0.98	0.97	0.03	0.86
55	1.04	1.10	1.02	1.04	1.00	1.04	0.03	0.91
60	1.03	1.17	1.12	1.16	1.09	1.11	0.05	0.95
65	1.10	1.08	1.11	1.12	1.09	1.10	0.01	0.99
70	1.09	1.18	1.06	1.22	1.09	1.13	0.06	1.02
75	1.18	1.15	1.12	1.19	1.13	1.15	0.03	1.06
80	1.16	1.16	1.19	1.22	1.19	1.18	0.02	1.09
85	1.28	1.25	1.23	1.21	1.24	1.24	0.02	1.13
90	1.27	1.25	1.23	1.32	1.27	1.27	0.03	1.16



Error bars (in measured time) calculated from the standard deviation of the five timings.



t vs tmodel for rolling ball bearing
If 100% model agreement
A 10% discrepancy is rather strange – one might expect an *absolute* error due to reaction time error, rather than a percentage....
y = 1.104x
$R^2 = 0.9928$
t measured /s
t model /s
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
$x = \frac{1}{2}\frac{5}{7} g \sin\theta \, t^2$
$\boxed{t = \sqrt{\frac{14x}{5}\frac{1}{g\sin\theta}}}$
g
x
R
r
a
ω
v
F
θ
mg
y