Q5/ # ways of arranging M quanta of energy in N identical molecular "containers" is

$$\boxed{W = \frac{(N+M-1)!}{M!\,(N-1)!}}$$

Container |

quanta x

ie | (x x | x x x | . . . . |) |

$N+1$ |

M x

$N+1-2+M$ symbols with M repeats of x and $N-1$ repeats of |.

So $$W = \frac{(N+1-2+M)!}{M!\,(N-1)!}$$

Before:

System A: $M = 10$, $N = 5$

System B: $M = 5$, $N = 10$

$$\therefore\ S = S_A + S_B = k_B \ln\left(\frac{(5+10-1)!}{10!\,(5-1)!}\right) + k_B \ln\left(\frac{(10+5-1)!}{5!\,(10-1)!}\right)$$

$$= k_B \ln\left(\frac{14!}{10!\,4!}\right) + k_B \ln\left(\frac{14!}{5!\,9!}\right)$$

$$= k_B \ln\left(\frac{14\times13\times12\times11}{4\times3\times2}\right) + k_B \ln\left(\frac{14\times13\times12\times11\times10}{5\times4\times3\times2}\right)$$

$$= \boxed{k_B \ln 1001 + k_B \ln 2002}$$

After: (combined system) $M = 15$, $N = 15$

$$\therefore\ S = k_B \ln\left(\frac{(15+15-1)!}{15!\,14!}\right) = k_B \ln\left(\frac{29!}{15!\,14!}\right)$$

$$= \boxed{k_B \ln(77558760)}$$

$\therefore \quad \Delta S = k_B \ln(77558760) - k_B \ln(1001) - k_B \ln(2002)$

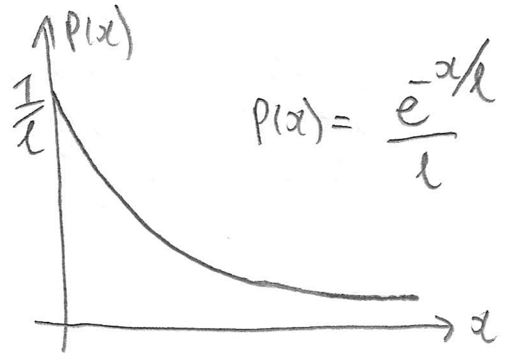
$\therefore \quad \Delta S = k_B \ln\left(\dfrac{77558760}{1001 \times 2002}\right)$

$= k_B \ln(38.70)$

$= \boxed{3.66\, k_B}$

Q6/

(i)



$P(x) = \dfrac{e^{-x/l}}{l}$

$\boxed{\text{Beer-lambert law}}$

ie probability of a molecule travelling distance between $a < x < b$ is:

$P = \int_a^b p(x)\, dx.$

l is the mean free path.

(ii) $\displaystyle \int_0^\infty p(x)\,dx = \frac{1}{l}\int_0^\infty e^{-x/l}\,dx$

$\displaystyle = \frac{1}{l}\left[\frac{1}{-\frac{1}{l}} e^{-x/l}\right]_0^\infty$

$\displaystyle = -\left[e^{-x/l}\right]_0^\infty$

$= -\left[0 - 1\right]$

$= 1$ ✓

so $p(x)$ is a valid continuous probability distribution.

$\displaystyle E[x] = \int_0^\infty x\, p(x)\,dx = \frac{1}{l}\int_0^\infty x e^{-x/l}\,dx$

$$= \frac{1}{l}\left\{ \left[x \frac{1}{-\frac{1}{l}} e^{-\frac{x}{l}} \right]_0^\infty - \int_0^\infty \frac{1}{-\frac{1}{l}} e^{-\frac{x}{l}}\, dx \right\}$$

$$= \frac{1}{l}\left\{ [0 + 0] + l \int_0^\infty e^{-\frac{x}{l}}\, dx \right\}$$

$$= l \times \underbrace{\frac{1}{l}\int_0^\infty e^{-x/l}\, dx}_{=1 \text{ from above}}$$

$\therefore$ $\boxed{E[x] = l}$ as required.

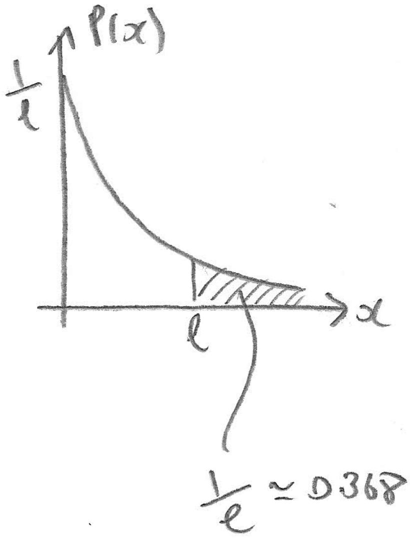
(iii) $$P(x \geq l) = \int_l^\infty \frac{e^{-x/l}}{l}\, dx$$

$$= \frac{1}{l}\left[\frac{1}{-1/l} e^{-x/l} \right]_l^\infty$$

$$= \left[-e^{-x/l} \right]_l^\infty$$

$$= \left[(0) - (-e^{-1}) \right]$$

$$= \boxed{1/e}$$



So $P(x < l) = 1 - \frac{1}{e} \simeq 0.632$

$P(x \geq l) = \frac{1}{e} = 0.368.$