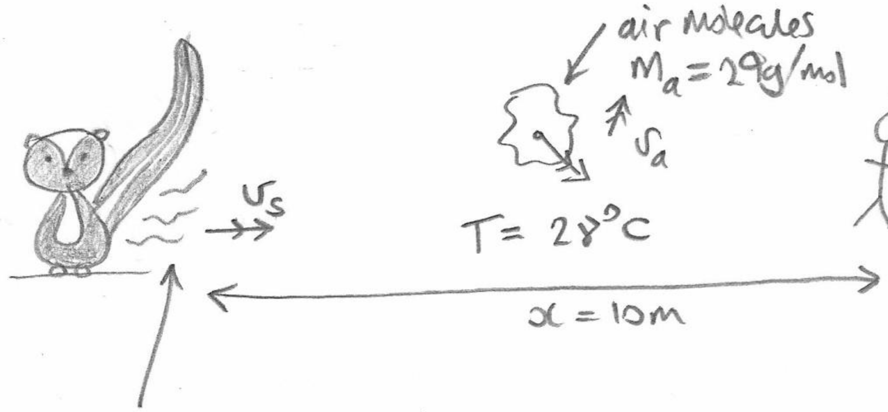
Q3



2-butene-1-thiol
Skunk musk molecules
$m_s = 88.2$ g/mol

v_s and v_a are RMS speeds of skunk, air molecules

Thermal equilibrium: $\frac{1}{2} m_s v_s^2 = \frac{1}{2} m_a v_a^2$

(ie mean KE of molecules is the same)

$\therefore \boxed{v_s = \sqrt{\frac{m_a}{m_s}}\, v_a}$

Now $\frac{1}{2} m_a v_a^2 = \frac{3}{2} RT \qquad \therefore \boxed{v_a = \sqrt{\frac{3RT}{m_a}}}$

$\therefore v_a = \sqrt{\frac{3 \times 8.314 \times (28 + 273)}{29 \times 10^{-3}}} = \boxed{509 \text{ m/s}}$

$\therefore \quad v_s = \sqrt{\frac{29}{88.2}} \times 509 = \boxed{292 \text{ m/s}}$

$\therefore$ Since $x = v_s t$ it should take

$$t \simeq \frac{10\text{m}}{292\text{ m/s}} \simeq \boxed{0.03\text{s}}$$

Too quick!

Note this is pretty quick. To perceive a smell, many molecules will have to diffuse the 10m from the skunk to the tourist. Note random collisions between the skunk musk and air molecules.

So if the diffusion is a RANDOM WALK we might expect $\boxed{x \simeq \sqrt{l v_s t}}$ or $\boxed{t = x^2/l v_s}$

l is the mean free path of the skunk musk.

From our kinetic theory notes: $\boxed{l = \dfrac{k_B T}{\pi \sqrt{2} d^2 p}}$

let $d \simeq 0.5 \times 10^{-9}$m
$T = (28 + 273)$ K
$p = 10^5$ Pa

so $l \simeq \dfrac{1.38 \times 10^{-23} \times (28 + 273)}{\pi \sqrt{2} \times (0.5 \times 10^{-9})^2 \times 10^5}$

$\simeq \boxed{3.7 \times 10^{-8}\text{ m}}$

[i.e. about $75d$]

$\therefore \quad t \simeq \dfrac{10^2}{3.7 \times 10^{-8} \times 292}$ (s)

$\simeq \boxed{9.2 \times 10^6 \text{ s}}$ – which is clearly nonsense!

However, that assumes all the skunk molecules diffuse 10m. In reality, only a few need to make it for the smell to be detected.

Q4

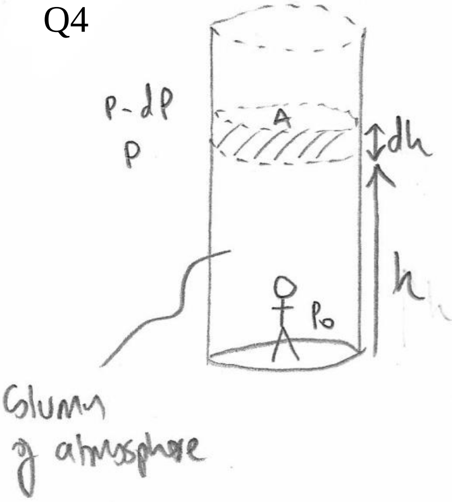


Fig (i) Pressure change between altitudes h and h + dh is

$$dP = - \frac{\text{weight of air}}{A}$$

$$dP = - \frac{\rho A dh \times g}{A}$$

$$\boxed{dP = - \rho g dh}$$

(ii) if ideal gas: $\boxed{pV = nRT}$

Now if M = mass of a gas molecule

$$\rho = \frac{n N_A M}{V} \qquad \text{So} \qquad \frac{n}{V} = \rho / N_A M$$

$$\therefore \quad p = \frac{n}{V} RT$$

$$p = \frac{\rho}{N_A M} RT$$

$$\boxed{p = \rho k_B T / M}$$

$$\Rightarrow \rho = mp / k_B T$$

$R = 8.314$ J/mol/K

$k_B = R / N_A$

Boltzmann's constant k_B 1.38×10^{-23} J/K

Avogadro's number $N_A = 6.02 \times 10^{23}$

$$\therefore \quad dP = - \frac{mp}{k_B T} g dh$$

$$\therefore \quad \frac{dP}{P} = - \frac{mg dh}{k_B T}$$

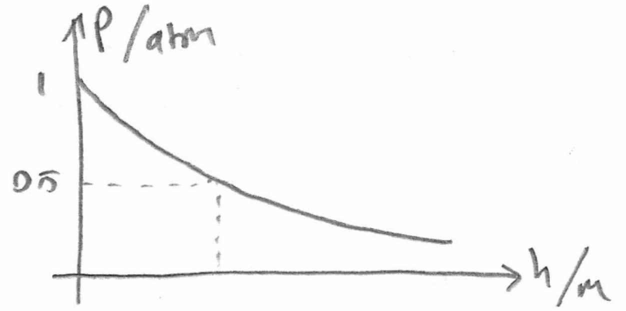
$$\therefore \quad \int_{P_0}^{P} \frac{dP}{P} = - \frac{mg}{k_B T} \int_0^h dh$$

$$\therefore \quad \ln P / P_0 = - \frac{mgh}{k_B T}$$

$$\therefore \quad \boxed{P = P_0 e^{-mgh / k_B T}}$$

A Boltzmann factor!

So for our isothermal atmosphere:



when $P = 0.5P_0$ $\therefore$ $\frac{1}{2} = e^{-mgh/k_BT}$

$\therefore$ $2 = e^{mgh/k_BT}$

"A half-height" à la half-life in radioactive decay or RC ln2 capacitor decay

$\therefore$ $\boxed{h = \frac{k_BT \ln 2}{mg}}$

$\therefore$ $h = \dfrac{1.38\times10^{-23}\times(10+273)\ln 2}{\dfrac{28.97\times10^{-3}}{6.02\times10^{23}}\times 9.81}$

↑ m for air molecule (see Q4)

$= \boxed{7560\,\text{m}}$

[Actually the isothermal model is not very realistic for the Troposphere. There is a lapse rate of between 0.5°C and 1.0°C per 100m. If you factor in a linear drop in T then $P \to P_0/2$ after about 5500m]