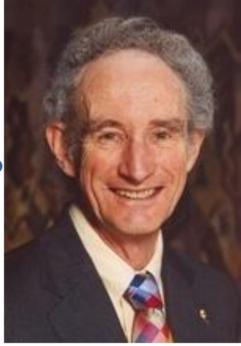
May's Chaotic Bunnies

I published this model in 1976



Robert May
1936-

Assume an ecosystem can support a maximum number of rabbits. Let x be the fraction of this maximum at year n.

To account for **reproduction**, next year's population is proportional to the previous.

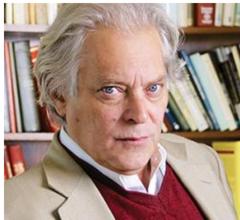
To account for **starvation**, next year's population is *also proportional* to the fraction of the maximum population as yet unfilled.

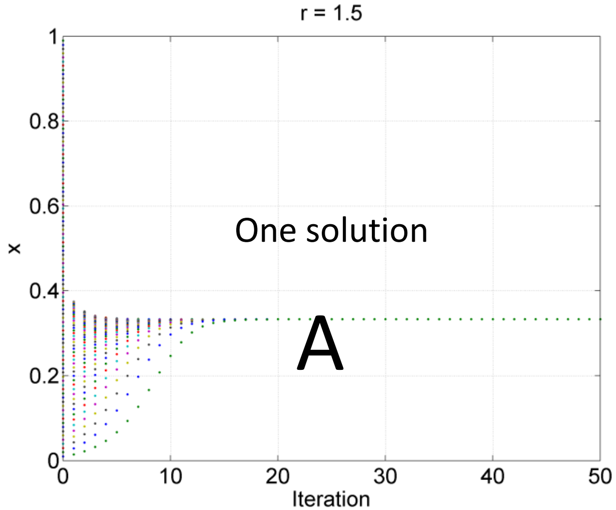
$$\boxed{x_{n+1} = r x_n \left(1 - x_n\right)}$$

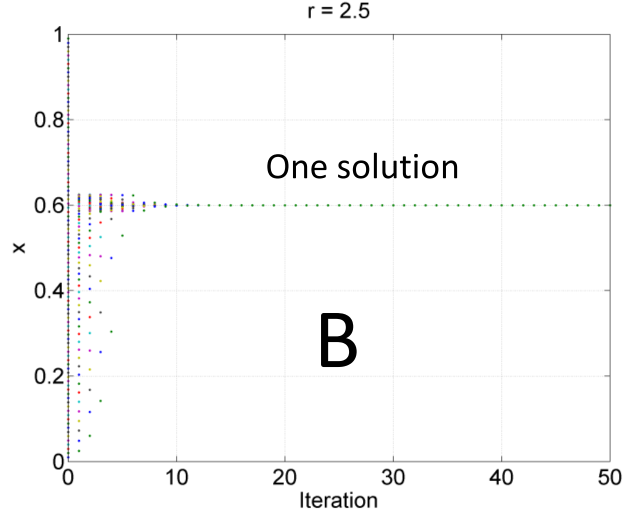
Growth parameter

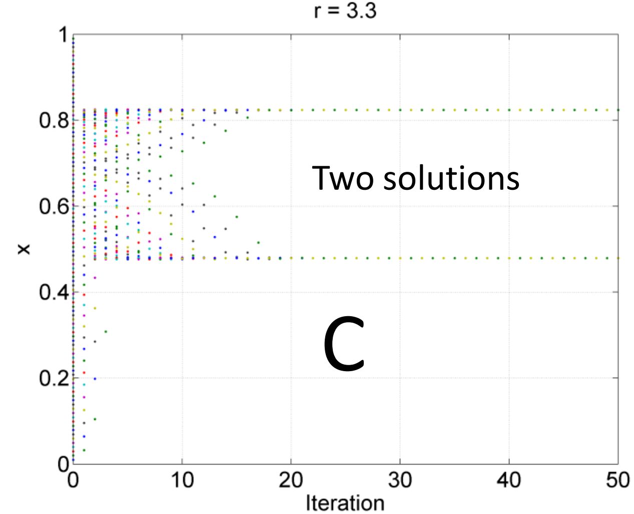
The population next year is predicted using this **iterative equation** called a **logistic map**

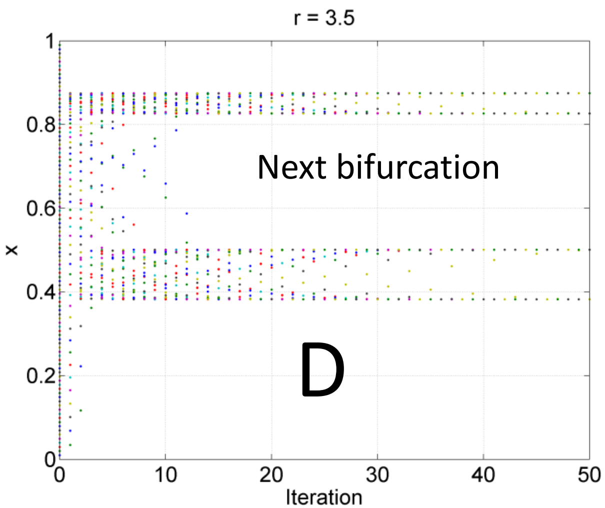
The pattern of x values with n is not always simple .....

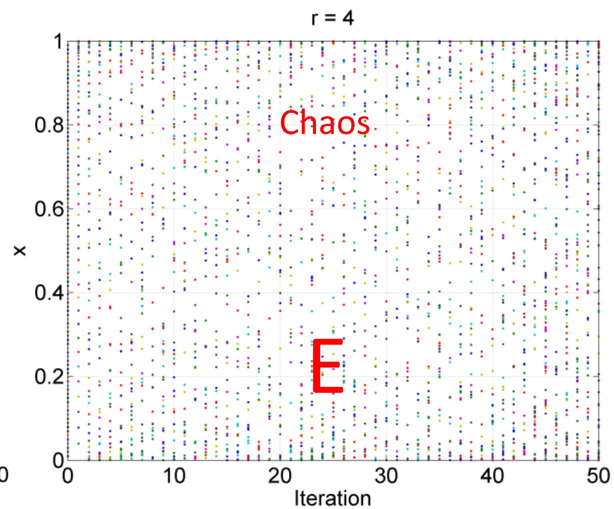




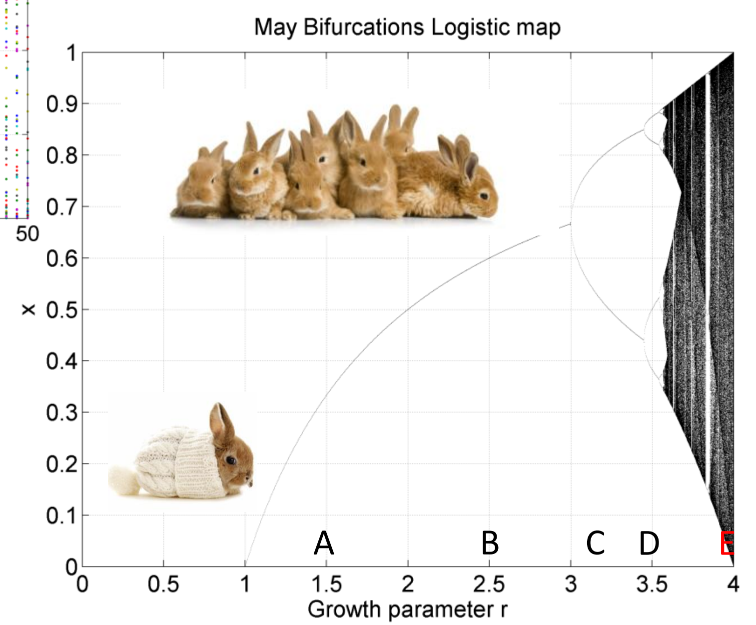






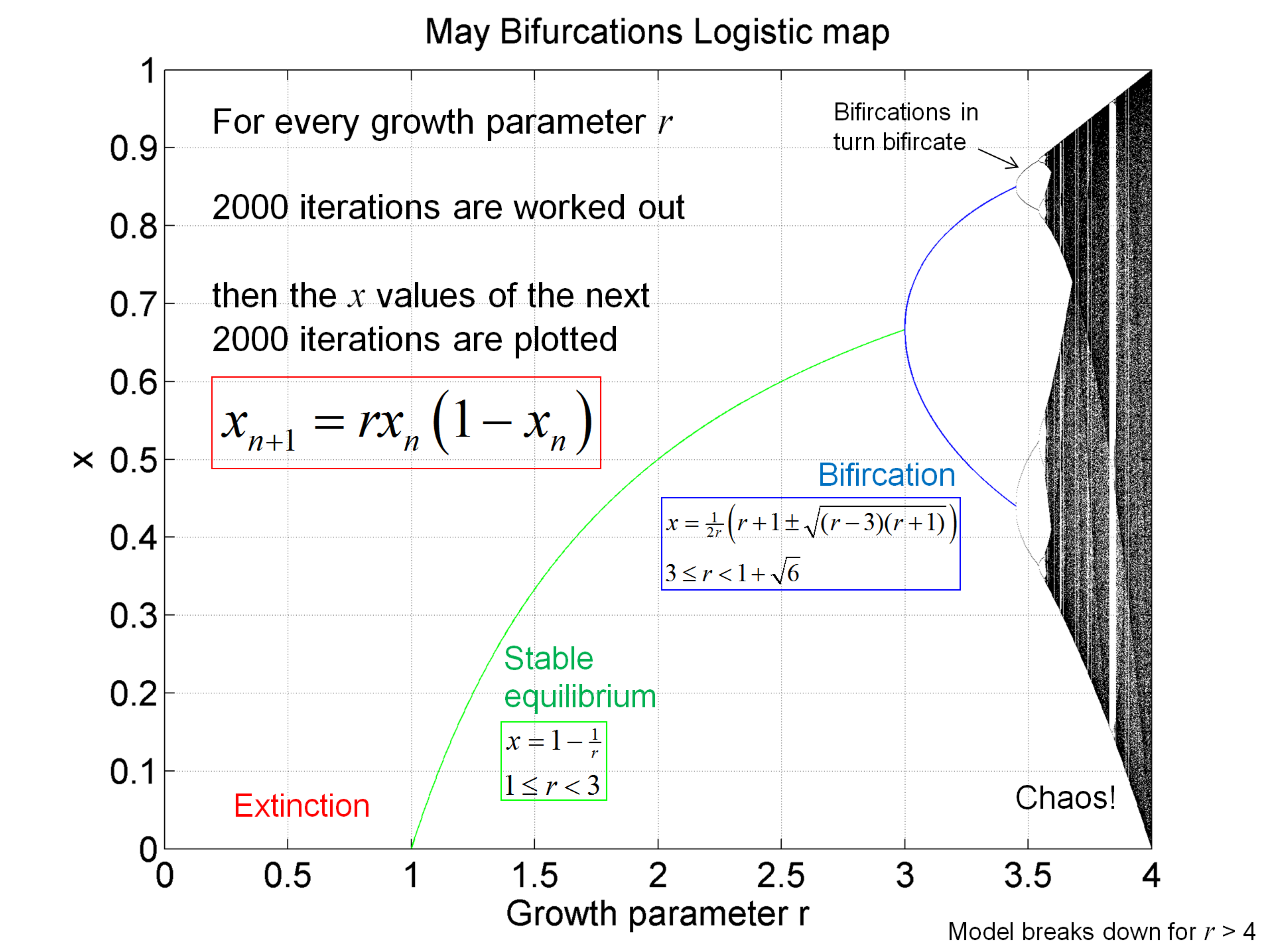


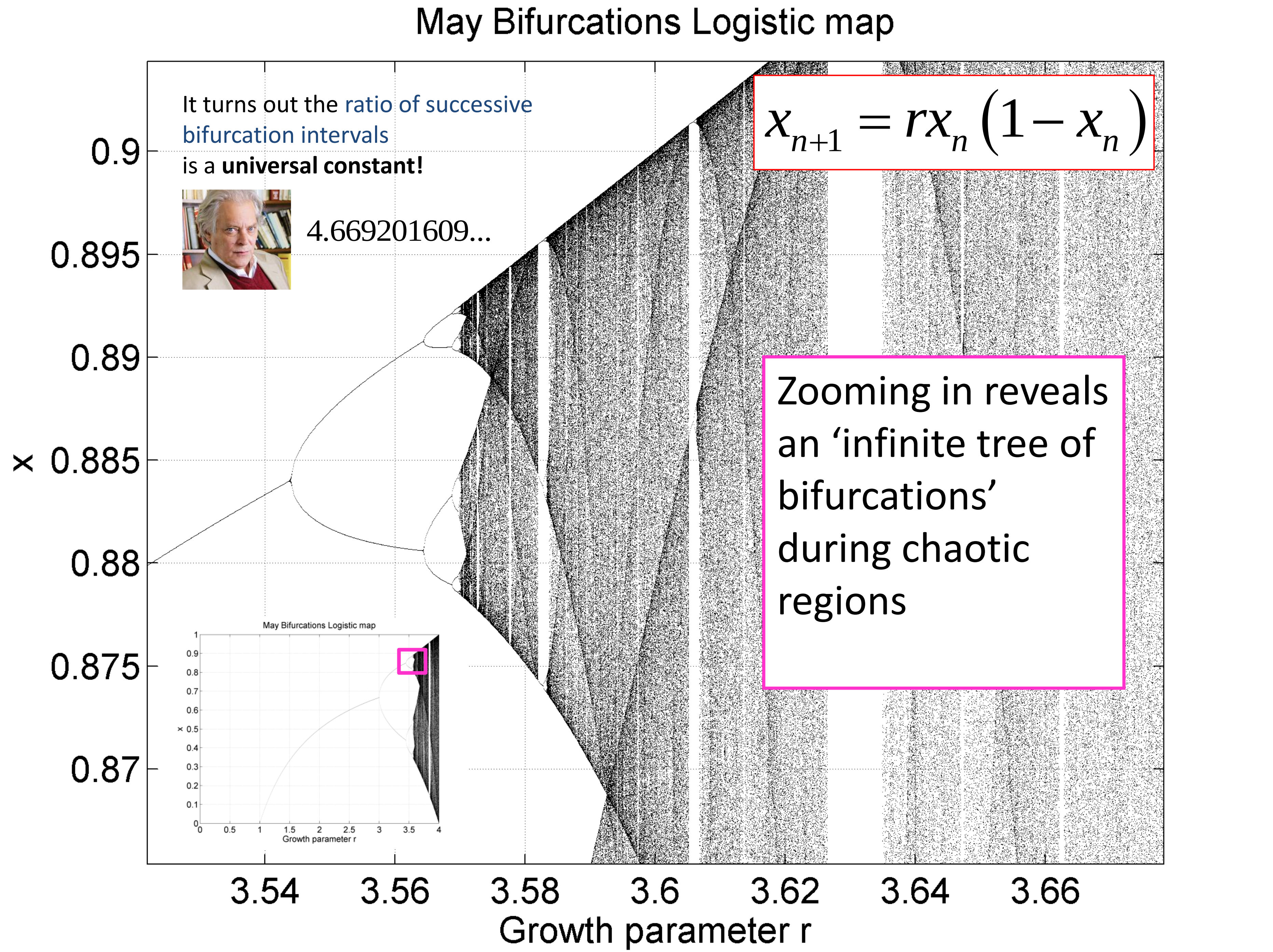
$$\boxed{x_{n+1} = rx_n\left(1 - x_n\right)}$$



Tracking the bifurcations maps the 'road to chaos'. The ratio of successive bifurcation intervals is a **universal constant!**

4.669201609...


May Bifurcations Logistic map
For every growth parameter r
2000 iterations are worked out
then the x values of the next
2000 iterations are plotted
$x_{n+1} = rx_n(1 - x_n)$
Bifircations in
turn bifircate
Bifircation
$x = \frac{1}{2r}\left(r + 1 \pm \sqrt{(r-3)(r+1)}\right)$
$3 \le r < 1 + \sqrt{6}$
Stable
equilibrium
$x = 1 - \frac{1}{r}$
$1 \le r < 3$
Extinction
Chaos!
x
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
Growth parameter r
Model breaks down for $r > 4$



May Bifurcations Logistic map
It turns out the ratio of successive bifurcation intervals is a **universal constant!**
4.669201609...
$x_{n+1} = rx_n(1 - x_n)$
Zooming in reveals an 'infinite tree of bifurcations' during chaotic regions
x
0.9
0.895
0.89
0.885
0.88
0.875
0.87
3.54
3.56
3.58
3.6
3.62
3.64
3.66
Growth parameter r
May Bifurcations Logistic map
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x
0
0.5
1
1.5
2
2.5
3
3.5
4
Growth parameter r