Derivatives and integrals of hyperbolic functions

$\frac{dy}{dx}$	$y = f(x)$	$\int f(x)dx$
$\frac{dy}{dx} = a\sinh ax$	$y = \cosh ax$	$\int \sinh ax dx = \frac{1}{a}\sinh ax + c$
$\frac{dy}{dx} = a\cosh ax$	$y = \sinh ax$	$\int \sinh ax dx = \frac{1}{a}\cosh ax + c$
$\frac{dy}{dx} = a\,\text{sech}^2\, ax$	$y = \tanh ax$	$\int \tanh ax dx = \frac{1}{a}\ln\left(\cosh ax\right) + c$
$\frac{dy}{dx} = -a\,\text{sech}\, ax \times \tanh ax$	$y = \text{sech}\, ax$	$\int \text{sech}\, ax dx = \frac{2}{a}\tan^{-1}\left(e^{ax}\right) + c$
$\frac{dy}{dx} = -a\,\text{cosech}\, ax \times \coth ax$	$y = \text{cosech}\, ax$	$\int \text{cosech}\, ax dx = \frac{1}{a}\ln\left(\tanh \frac{1}{2}ax\right) + c$
$\frac{dy}{dx} = -a\,\text{cosech}^2 ax$	$y = \coth ax$	$\int \coth ax dx = \frac{1}{a}\ln\left(\sinh ax\right) + c$
$\frac{dy}{dx} = \frac{a}{\sqrt{a^2x^2 - 1}}$	$y = \cosh^{-1} ax$	$\int \cosh^{-1} ax dx = x\cosh^{-1} ax - \frac{1}{a}\sqrt{ax-1}\sqrt{ax+1} + c$
$\frac{dy}{dx} = \frac{a}{\sqrt{a^2x^2 + 1}}$	$y = \sinh^{-1} ax$	$\int \sinh^{-1} ax dx = x\sinh^{-1} ax - \frac{1}{a}\sqrt{a^2x^2 + 1} + c$
$\frac{dy}{dx} = \frac{a}{1 - a^2x^2}$	$y = \tanh^{-1} ax$	$\int \tanh^{-1} ax dx = x\tanh^{-1} ax + \frac{1}{2a}\ln\left(1 - a^2x^2\right) + c$

$$\cosh x = \tfrac{1}{2}\left(e^x + e^{-x}\right)$$
$$\sinh x = \tfrac{1}{2}\left(e^x - e^{-x}\right)$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\text{sech}\, x = \frac{1}{\cosh x}$$
$$\text{cosech}\, x = \frac{1}{\sinh x}$$
$$\coth x = \frac{1}{\tanh x}$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$
$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\cosh^2 x - \sinh^2 x = 1$$
$$\text{sech}^2 x + \tanh^2 x = 1$$
$$\coth^2 x - \text{cosech}^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$$
$$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$$
$$\tanh(A \pm B) = \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$$

$$\cosh^2 x = \tfrac{1}{2}\left(\cosh 2x + 1\right)$$
$$\sinh^2 x = \tfrac{1}{2}\left(\cosh 2x - 1\right)$$
$$\cosh^3 x = \tfrac{1}{4}\left(3\cosh x + \cosh 3x\right)$$
$$\sinh^3 x = \tfrac{1}{4}\left(-3\sinh x + \sinh 3x\right)$$

$$y = \cosh^{-1} ax$$
$$\cosh y = ax$$
$$\sinh y = a\frac{dx}{dy}$$
$$\frac{dy}{dx} = \frac{a}{\sinh y}$$
$$\cosh^2 y - \sinh^2 y = 1$$
$$\therefore \sinh y = \sqrt{\cosh^2 y - 1} = \sqrt{a^2x^2 - 1}$$
$$\boxed{\frac{d}{dx}\cosh^{-1} ax = \frac{a}{\sqrt{a^2x^2-1}}}$$

$$\int \cosh^{-1} ax dx = \int \left(\cosh^{-1} ax \times 1\right) dx$$
$$\int \cosh^{-1} ax dx = x\cosh^{-1} ax - \int \left(x \times \frac{d}{dx}\cosh^{-1} ax\right) dx$$
$$\int \cosh^{-1} ax dx = x\cosh^{-1} ax - \int \frac{ax}{\sqrt{a^2x^2-1}} dx$$
$$\frac{d}{dx}\sqrt{a^2x^2-1} = \tfrac{1}{2}\frac{2xa^2}{\sqrt{a^2x^2-1}} = \frac{a^2x}{\sqrt{a^2x^2-1}}$$
$$\therefore \int \frac{ax}{\sqrt{a^2x^2-1}} dx = \frac{1}{a}\sqrt{a^2x^2-1} = \tfrac{1}{a}\sqrt{ax-1}\sqrt{ax+1}$$
$$\boxed{\int \cosh^{-1} ax dx = x\cosh^{-1} ax - \tfrac{1}{a}\sqrt{ax-1}\sqrt{ax+1} + c}$$

$$\int \tanh ax dx = \int \frac{\sinh ax}{\cosh ax} dx$$
$$\int \tanh ax dx = \tfrac{1}{a}\int \frac{ae^{ax} - ae^{-ax}}{e^{ax} + e^{-ax}} dx$$
$$\int \tanh ax dx = \tfrac{1}{a}\ln\left|e^{ax} + e^{-ax}\right| + c$$
$$\boxed{\int \tanh ax dx = \tfrac{1}{a}\ln\left|\cosh ax\right| + c}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Use integration by parts

$$\int (uv) dx = u\int v dx - \int\left(\int v dx \times \frac{du}{dx}\right) dx$$
$$u = \tan^{-1} x, \; v = 1$$

$$\int \operatorname{sech} ax dx = \int \frac{1}{\tfrac{1}{2}\left(e^{ax} + e^{-ax}\right)} dx = \int \frac{2e^{ax}}{\left(e^{ax}\right)^2 + 1} dx$$
$$z = e^{ax} \; \therefore \frac{dz}{dx} = ae^{ax} \; \therefore dx = \frac{dz}{ae^{ax}} = \tfrac{1}{a}\frac{dz}{z}$$
$$\int \operatorname{sech} ax dx = \int \frac{2z}{z^2+1}\tfrac{1}{a}\frac{dz}{z} = \tfrac{2}{a}\int \frac{1}{1+z^2} dz = \tfrac{2}{a}\tan^{-1} z$$
$$\boxed{\int \operatorname{sech} ax dx = \tfrac{2}{a}\tan^{-1} e^{ax} + c}$$

$$\int \operatorname{cosech} ax dx = \int \frac{1}{\tfrac{1}{2}\left(e^{ax} - e^{-ax}\right)} dx = \int \frac{2e^{ax}}{\left(e^{ax}\right)^2 - 1} dx$$
$$z = e^{ax} \; \therefore \frac{dz}{dx} = ae^{ax} \; \therefore dx = \frac{dz}{ae^{ax}} = \tfrac{1}{a}\frac{dz}{z}$$
$$\int \operatorname{cosech} ax dx = \int \frac{2z}{z^2-1}\tfrac{1}{a}\frac{dz}{z} = \tfrac{2}{a}\int \frac{1}{z^2-1} dz$$
$$\frac{1}{z^2-1} = \tfrac{1}{2}\frac{1}{z-1} - \tfrac{1}{2}\frac{1}{z+1}$$
$$\tfrac{2}{a}\int \frac{1}{z^2-1} dz = \tfrac{1}{a}\int \frac{1}{z-1} dz - \tfrac{1}{a}\int \frac{1}{z+1} dz = \tfrac{1}{a}\ln\left(\frac{z-1}{z+1}\right)$$
$$\int \operatorname{cosech} ax dx = \tfrac{1}{a}\ln\left(\frac{e^{ax}-1}{e^{ax}+1}\right) = \tfrac{1}{a}\ln\left(\frac{e^{\frac{1}{2}ax}\left(e^{\frac{1}{2}ax} - e^{-\frac{1}{2}ax}\right)}{e^{\frac{1}{2}ax}\left(e^{\frac{1}{2}ax} + e^{-\frac{1}{2}ax}\right)}\right)$$
$$\boxed{\int \operatorname{cosech} ax dx = \tfrac{1}{a}\ln\left(\tanh \tfrac{1}{2}ax\right) + c}$$

$$\int \coth ax dx = \int \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}} dx = \tfrac{1}{a}\int \frac{a\left(e^{ax} - e^{-ax}\right)}{e^{ax} + e^{-ax}} dx$$
$$\frac{d}{dx}\left(e^{ax} + e^{-ax}\right) = a\left(e^{ax} - e^{-ax}\right)$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\boxed{\int \coth ax dx = \tfrac{1}{a}\ln\left(e^{ax} - e^{-ax}\right) = \tfrac{1}{a}\ln(\sinh ax) + \mathrm{c}}$$