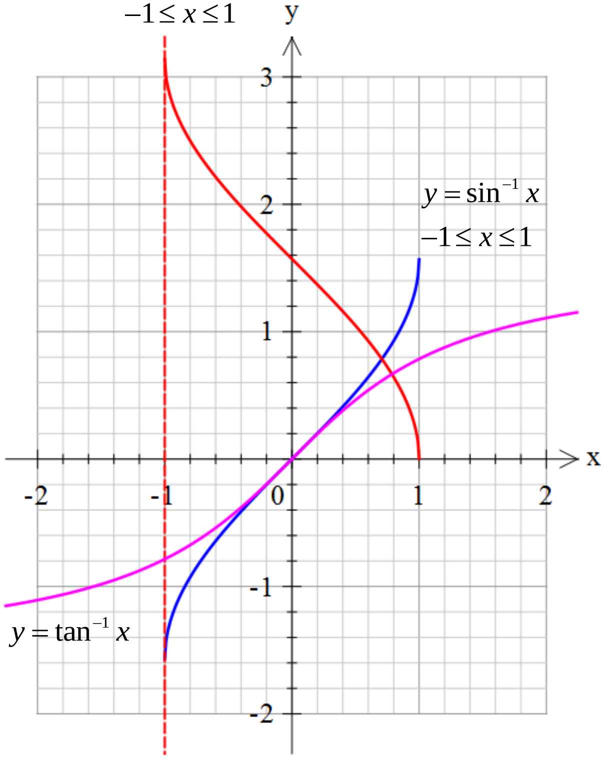
## Calculus with inverse trigonometric functions

$y = \cos^{-1} x$

$-1 \le x \le 1$




### Complement rules

$$\cos^{-1} x + \sin^{-1} x = \tfrac{1}{2}\pi$$

$$\sec^{-1} + \operatorname{cosec}^{-1} = \tfrac{1}{2}\pi$$

$$\tan^{-1} x + \cot^{-1} x = \begin{cases} \tfrac{1}{2}\pi & x \ge 0 \\ -\tfrac{1}{2}\pi & x < 0 \end{cases}$$

$$\sec^{-1} x = \cos^{-1} \tfrac{1}{x}$$

$$\operatorname{cosec}^{-1} x = \sin^{-1} \tfrac{1}{x}$$

$$\cot^{-1} x = \tan^{-1} \tfrac{1}{x}$$

$$y = \tan^{-1} x \quad \therefore x = \tan y$$

$$\frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\int \frac{1}{a^2 + x^2} = \tfrac{1}{a}\tan^{-1}\tfrac{1}{a} x + c$$

$$\int \tan^{-1} x dx = \int \left(\tan^{-1} x \times 1\right) dx$$

$$\int \tan^{-1} x dx = x\tan^{-1} x - \int x \times \frac{1}{1+x^2} dx$$

$$\int \tan^{-1} x dx = x\tan^{-1} x - \tfrac{1}{2}\int \frac{2x}{1+x^2} dx$$

$$\int \tan^{-1} x dx = x\tan^{-1} x - \tfrac{1}{2}\ln(1+x^2) + c$$

Use integration by parts

$$\int (uv) dx = u \int v dx - \int \left( \int v dx \times \frac{du}{dx} \right) dx$$

$u = \tan^{-1} x, \; v = 1$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$y = \sin^{-1} x \quad \therefore x = \sin y$$

$$\frac{dx}{dy} = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\tfrac{1}{a} x + c$$

$$\int \sin^{-1} x dx = \int \left(\sin^{-1} x \times 1\right) dx$$

$$\int \sin^{-1} x dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$\frac{d}{dx}\left(1-x^2\right)^{\frac{1}{2}} = \tfrac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{1-x^2}}$$

$$\therefore -\int \frac{x}{\sqrt{1-x^2}} = \sqrt{1-x^2} + c$$

$$\int \sin^{-1} x dx = x\sin^{-1} x + \sqrt{1-x^2} + c$$

$$y = \cos^{-1} x \quad \therefore x = \cos y$$

$$\frac{dx}{dy} = -\sin y = -\sqrt{1 - \cos^2 y} = -\sqrt{1 - x^2}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\int \cos^{-1} x dx = \int \left(\cos^{-1} x \times 1\right) dx$$

$$\int \cos^{-1} x dx = x\cos^{-1} x - \int -\frac{x}{\sqrt{1-x^2}} dx$$

$$\int \cos^{-1} x dx = x\cos^{-1} x - \sqrt{1-x^2} + c$$

Find derivatives of inverse reciprocal trig functions using the chain rule

$$\frac{d}{dx}\cot^{-1} x = \frac{d}{dx}\tan^{-1}\tfrac{1}{x}$$

$$\frac{d}{dx}\cot^{-1} x = \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\tfrac{1}{x^2}\right)$$

$$\frac{d}{dx}\cot^{-1} x = -\frac{1}{x^2+1}$$

Similarly:

$$\frac{d}{dx}\sec^{-1} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\operatorname{cosec}^{-1} = -\frac{1}{|x|\sqrt{x^2-1}}$$

using $\sqrt{x^2} = |x|$