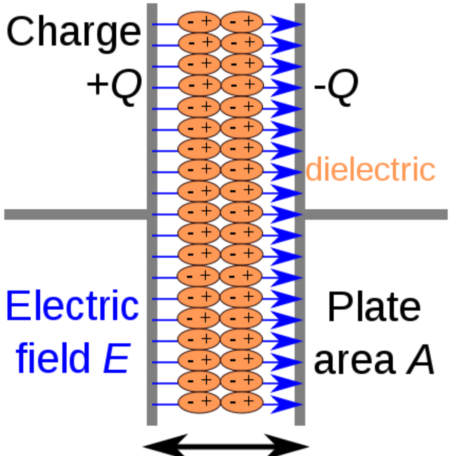
# Capacitors

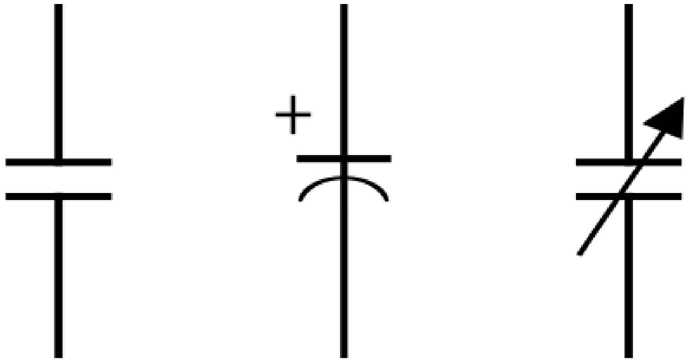
An electrical component which stores charge




$$Q = CV$$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2$$

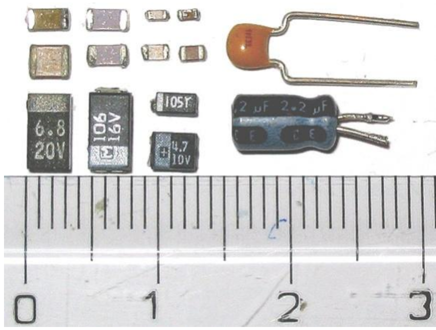
$$C = \frac{\varepsilon\varepsilon_0 A}{d}$$ parallel plate capacitor



Fixed Capacitor

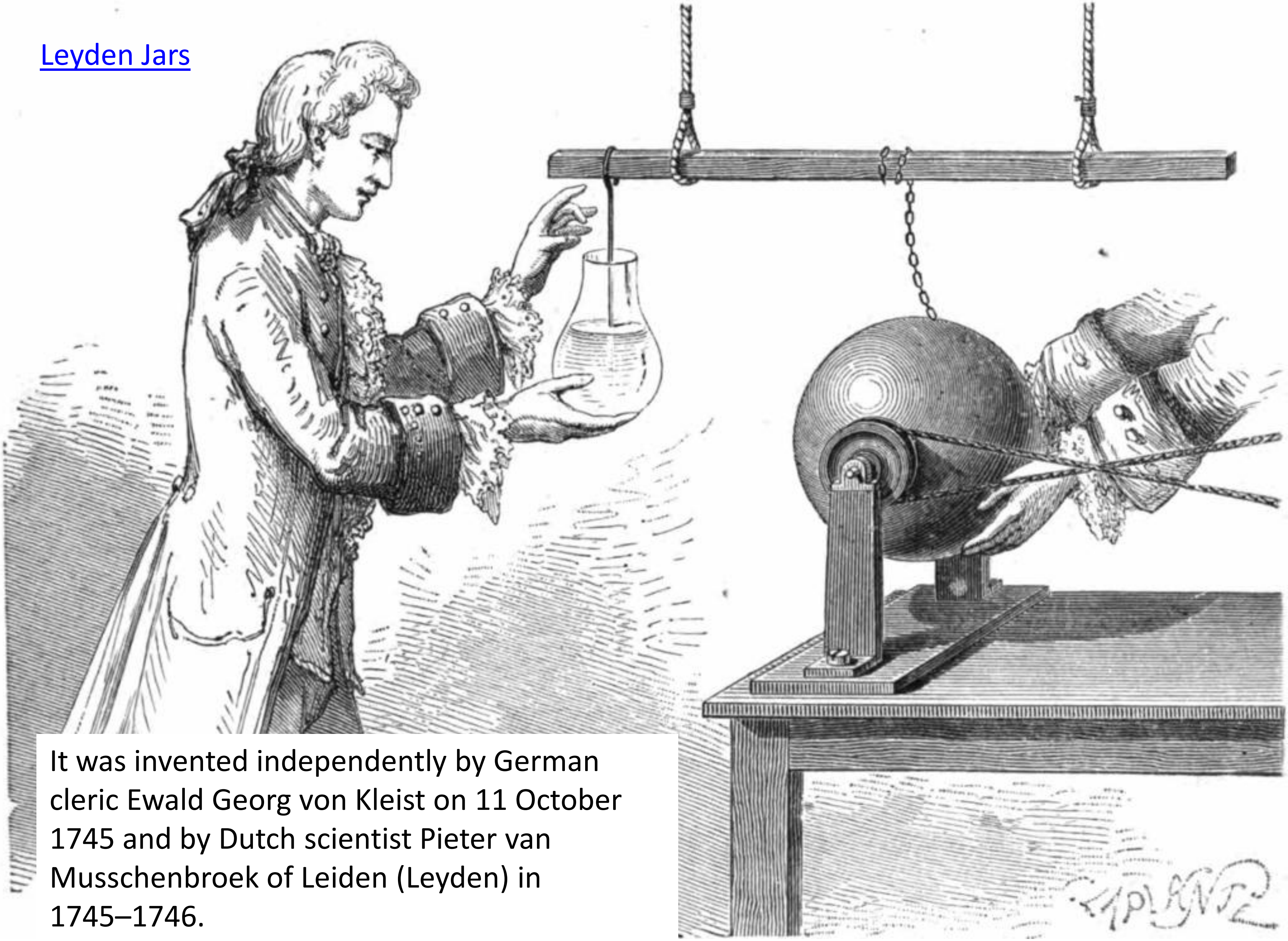
Polarized Capacitor

Variable Capacitor

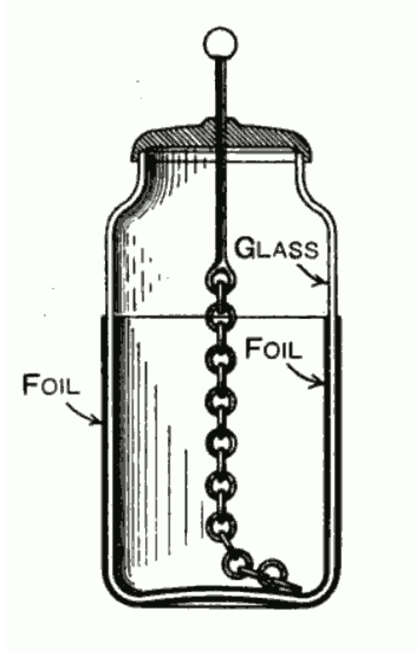


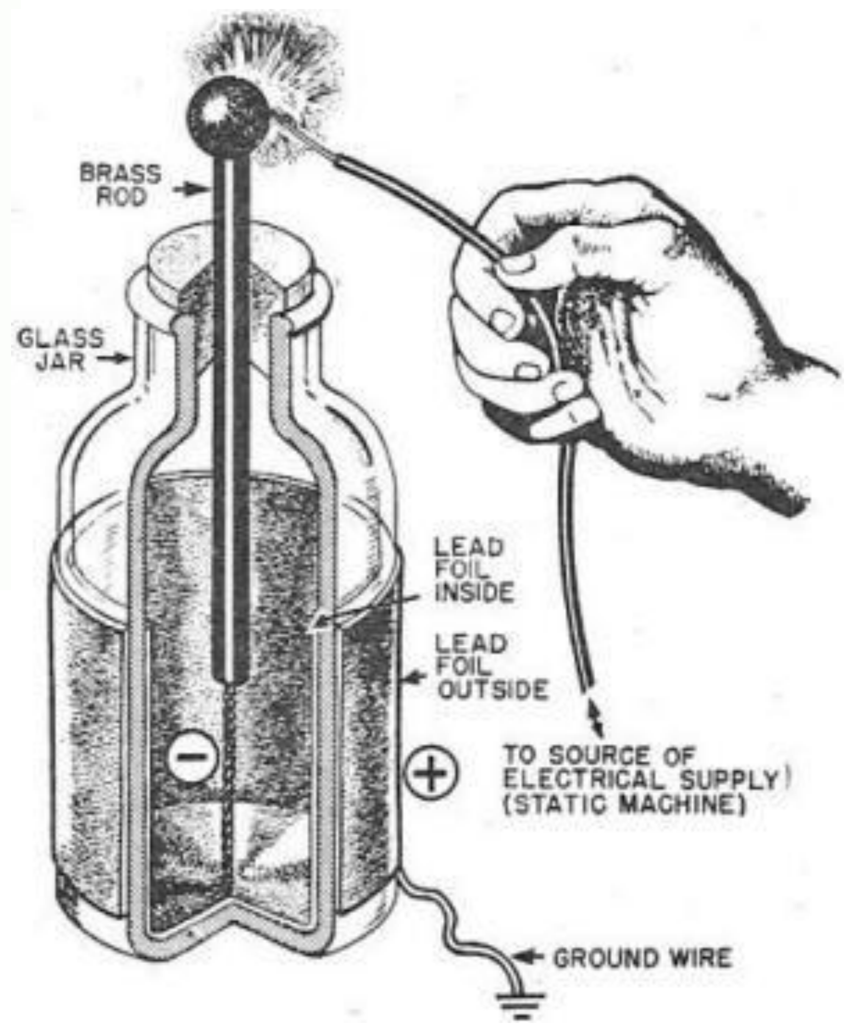
Scale in cm

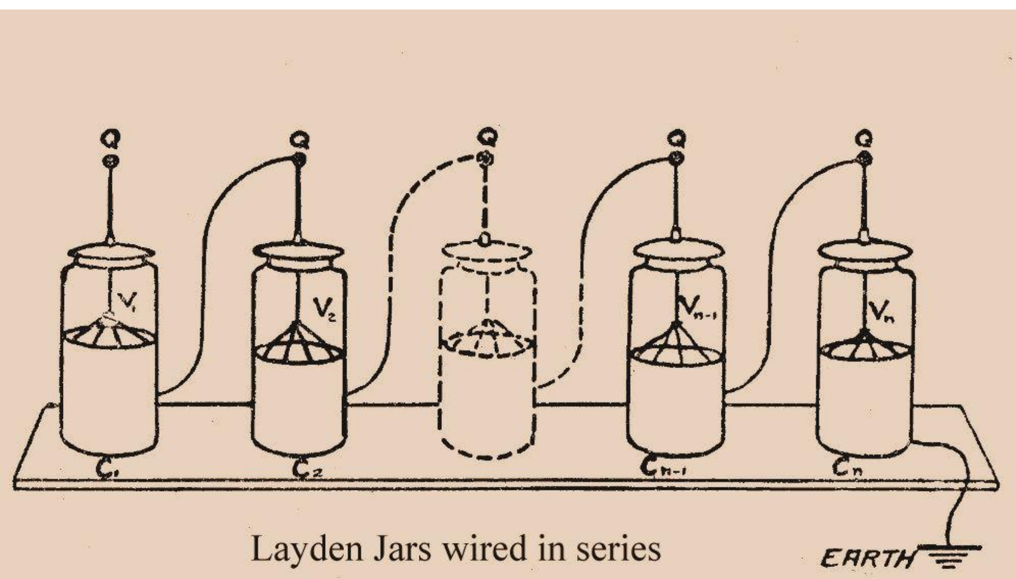
## Leyden Jars



It was invented independently by German cleric Ewald Georg von Kleist on 11 October 1745 and by Dutch scientist Pieter van Musschenbroek of Leiden (Leyden) in 1745–1746.

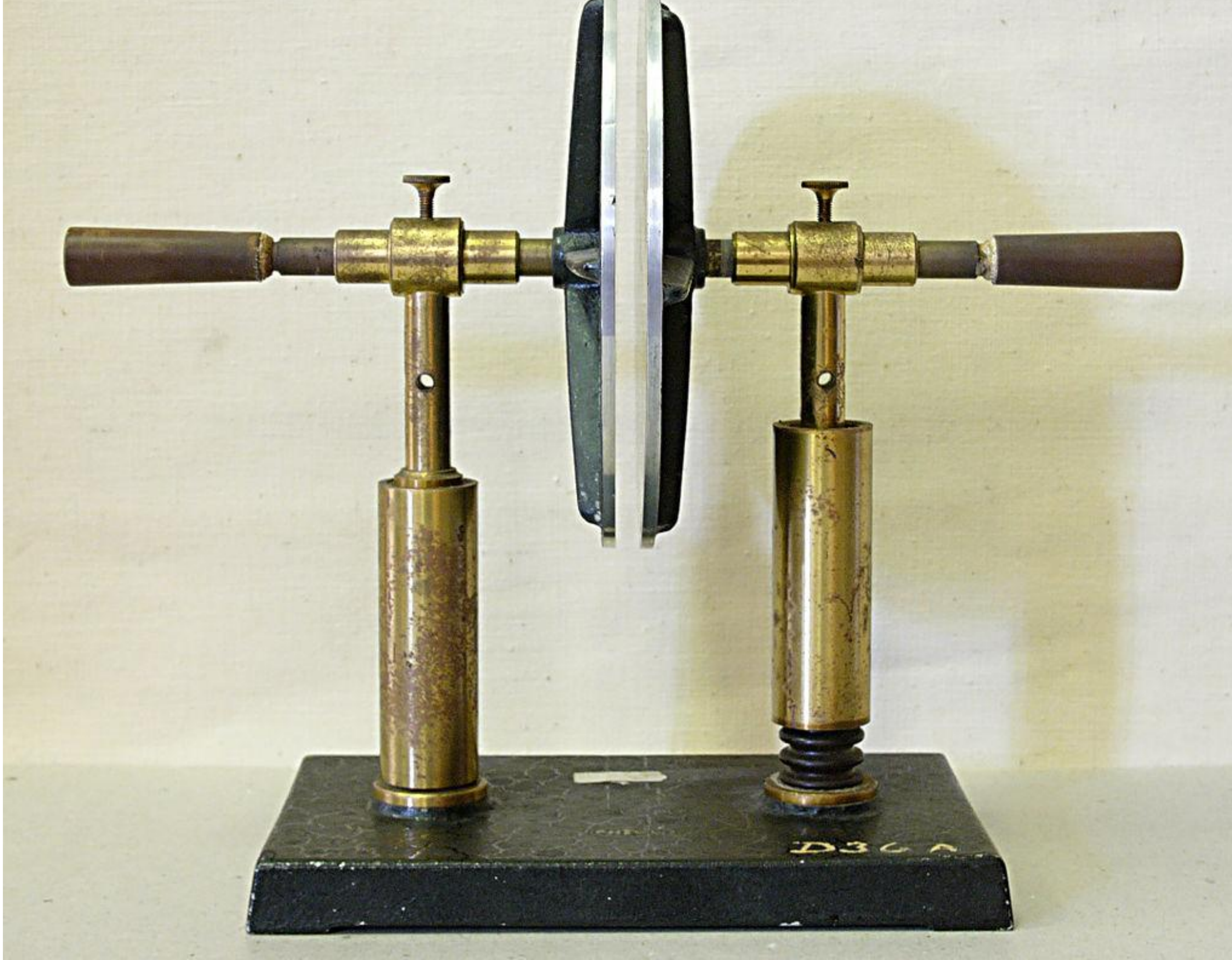


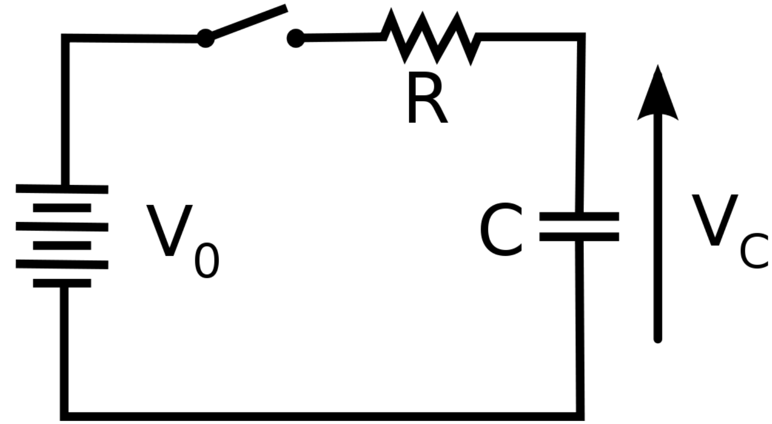

Layden Jars wired in series

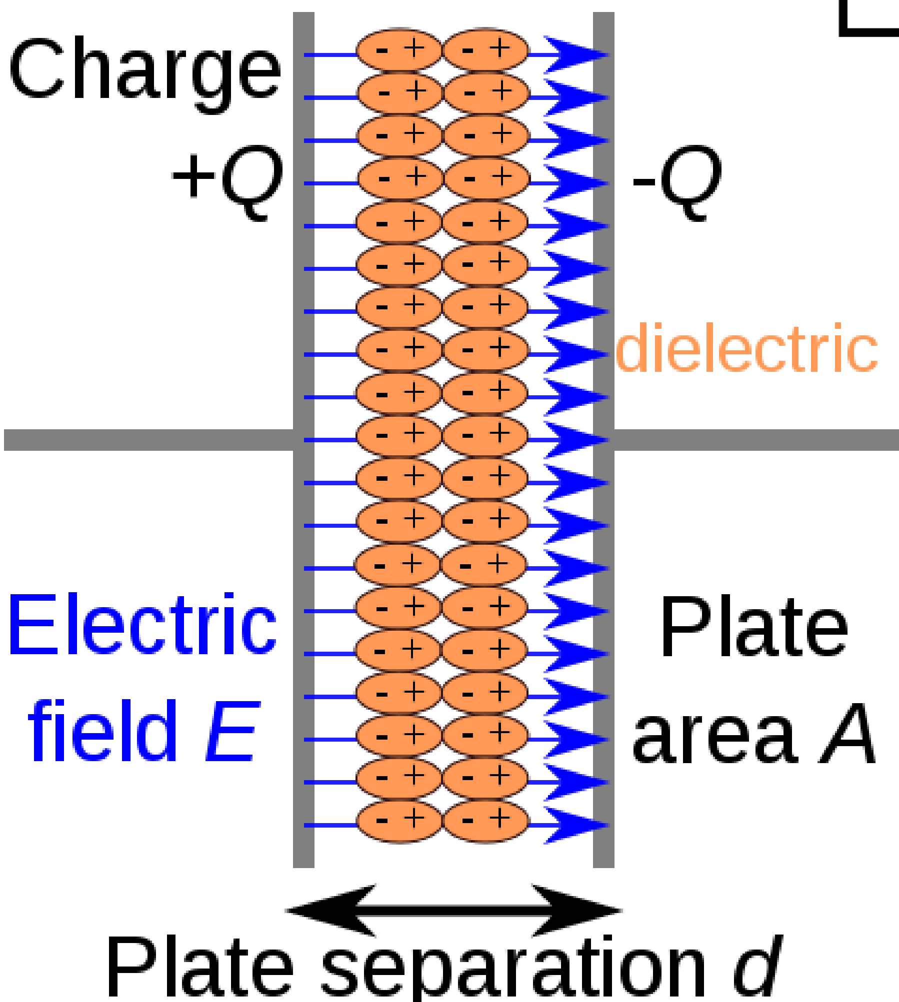
Parallel plate capacitor

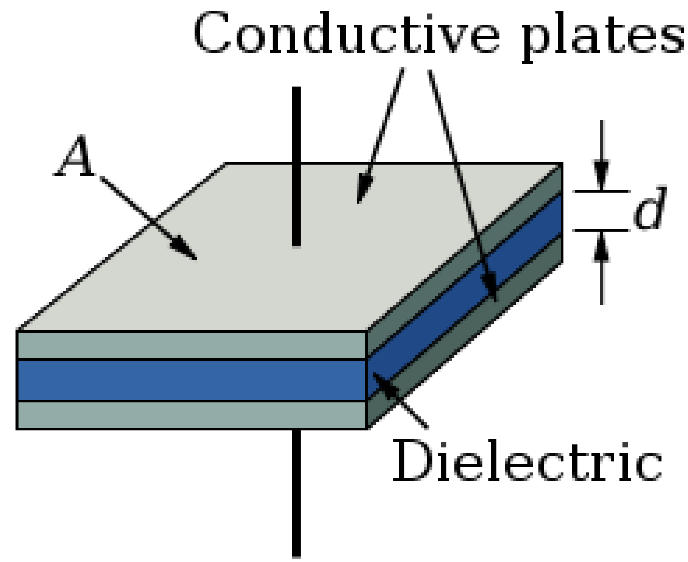


A dielectric *increases* the charge on the plates

$$\boxed{Q = CV}$$



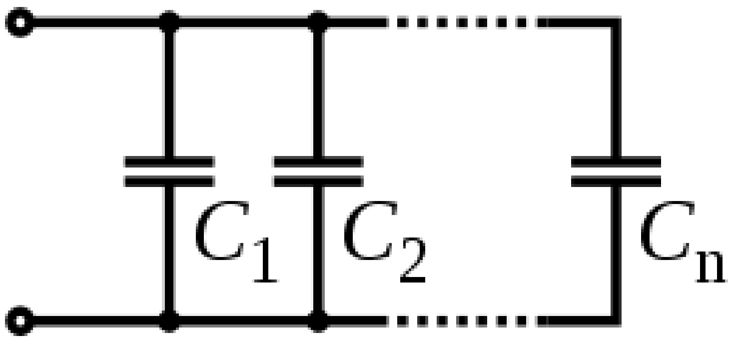




$$\boxed{C = \frac{\varepsilon \varepsilon_0 A}{d}}$$

**Capacitance** is measured in **Farads** $$Q = CV$$ $$1\text{pF} = 10^{-12}\text{F}$$

1F means **one coulomb per unit volt** between the capacitor plates.

A Leyden Jar has a capacitance of around 1 nF. A typical circuit board capacitor will have capacitance from a few pF up to a large number of μF



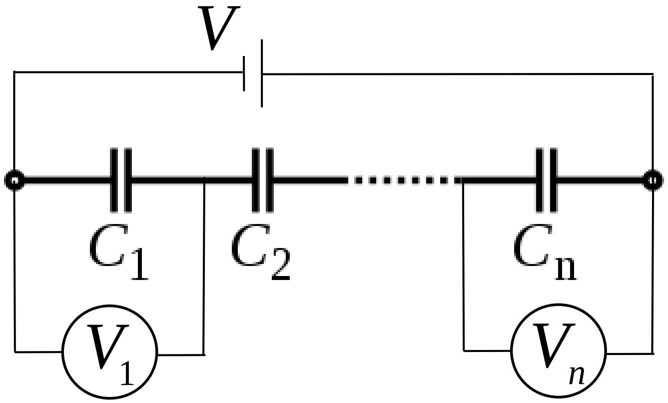

Capacitors in a **parallel** configuration each have the same applied voltage. The total charge stored is therefore

$$Q = C_1 V + C_2 V + \dots\dots + C_n V$$

Since the total capacitance is given by $$C = Q / V$$

$$\boxed{C = C_1 + C_2 + \dots\dots + C_n}$$

Therefore ***parallel* capacitances add**




Sum of voltage drops across capacitors must equate to the applied voltage

$$V = V_1 + V_2 + \ldots + V_n$$

Hence: $$Q_n = C_n V_n$$

$$\therefore \frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \ldots. + \frac{Q_n}{C_n}$$

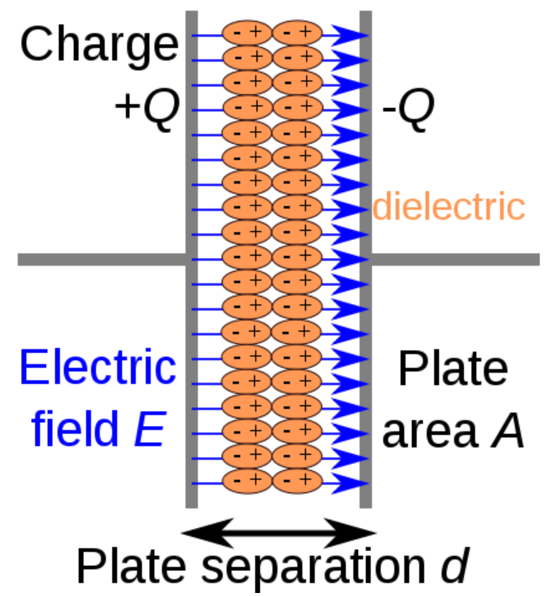
Now the charge on each adjacent capacitor plate *must be the same*, otherwise current would flow between the capacitors, re-apportioning charge

$$Q = Q_1 = Q_2 = \ldots = Q_n$$

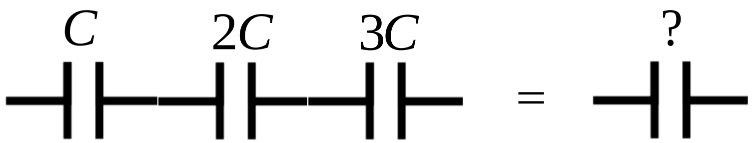
Hence:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots. + \frac{1}{C_n}}$$

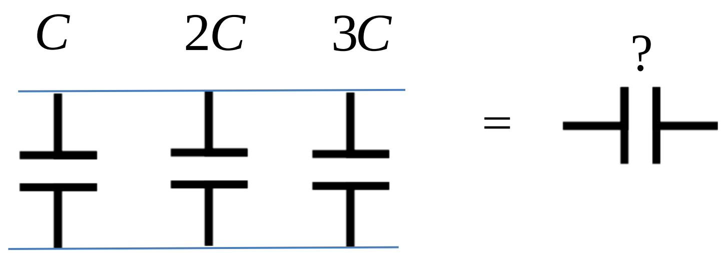
Therefore the **reciprocals** of ***series*** **capacitances add**



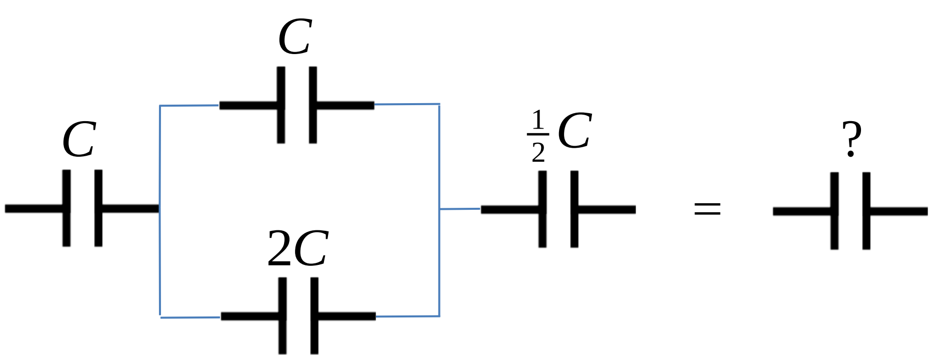
## Capacitance examples




$$? = \frac{1}{1+\frac{1}{2}+\frac{1}{3}}C = \frac{6}{11}C$$




$$? = C + 2C + 3C = 6C$$




$$? = \frac{1}{1+\dfrac{1}{1+\frac{1}{2}}+2} = \frac{3}{11}C$$

**Energy stored in a capacitor**

The instantaneous power required to establish voltage V across the plates of a capacitor is

$$P = VI$$

In time interval *dt*, the amount of charge added to the capacitor is

$$dQ = CdV$$

Hence current I is

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}$$

$$\therefore P = VI$$

$$\Rightarrow P = CV\frac{dV}{dt}$$

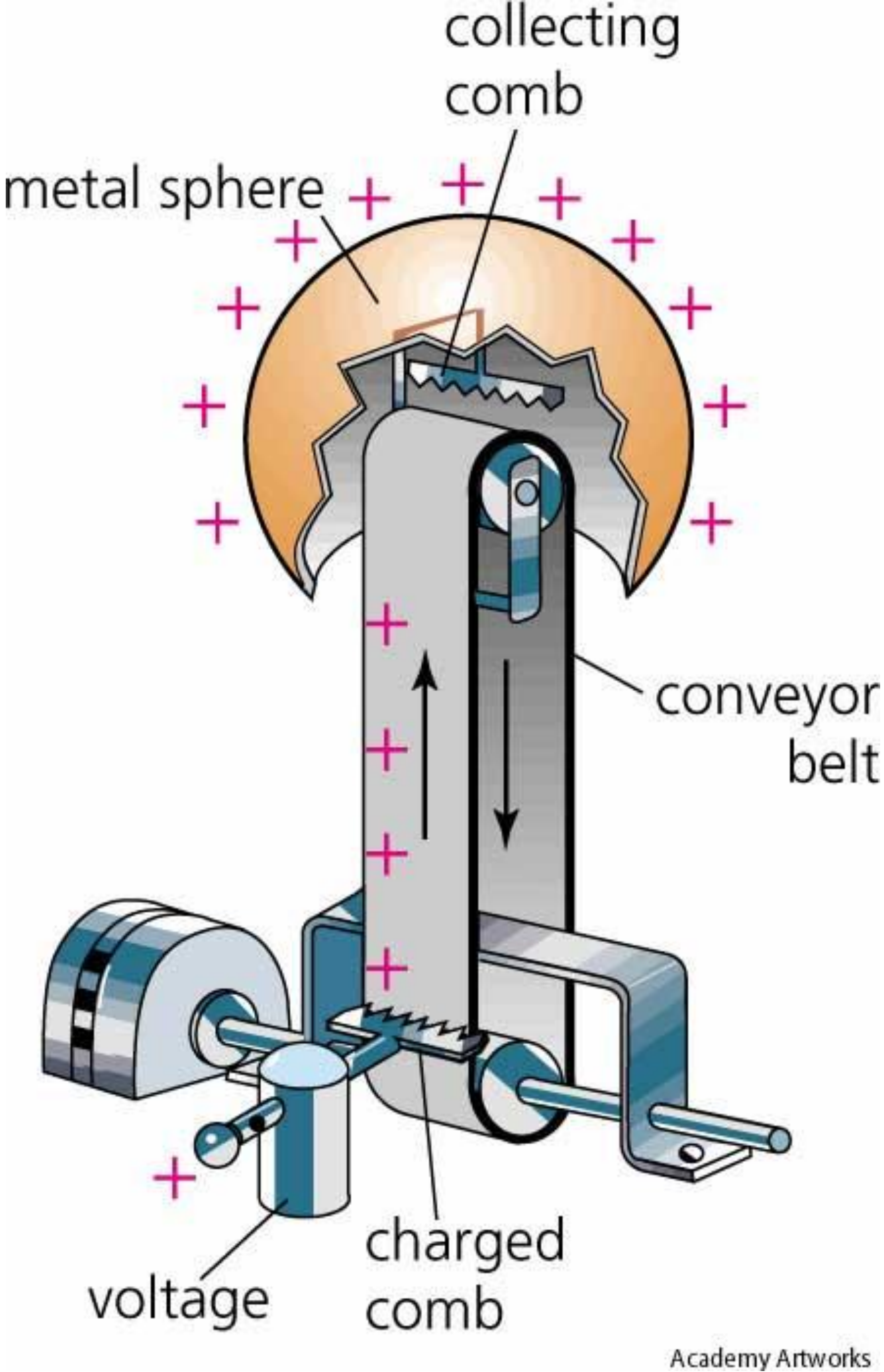
Total energy required to establish voltage *V* across the capacitor plates (in time *t*) is therefore

$$E = \int_0^t Pdt = \int_0^t CV\frac{dV}{dt}dt$$

$$E = C\int_0^V VdV$$

$$E = \tfrac{1}{2}CV^2$$

$$\boxed{E = \tfrac{1}{2}CV^2}$$




Reference

The capacitance of a metal sphere of radius *R* is given by

$$\boxed{C = 4\pi\varepsilon_0 R}$$

The permittivity of free space is

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{Fm}^{-1}}$$

So the Capacitance of a 0.2m radius Van der Graaf generator is about 22pF

The energy stored in a capacitor is

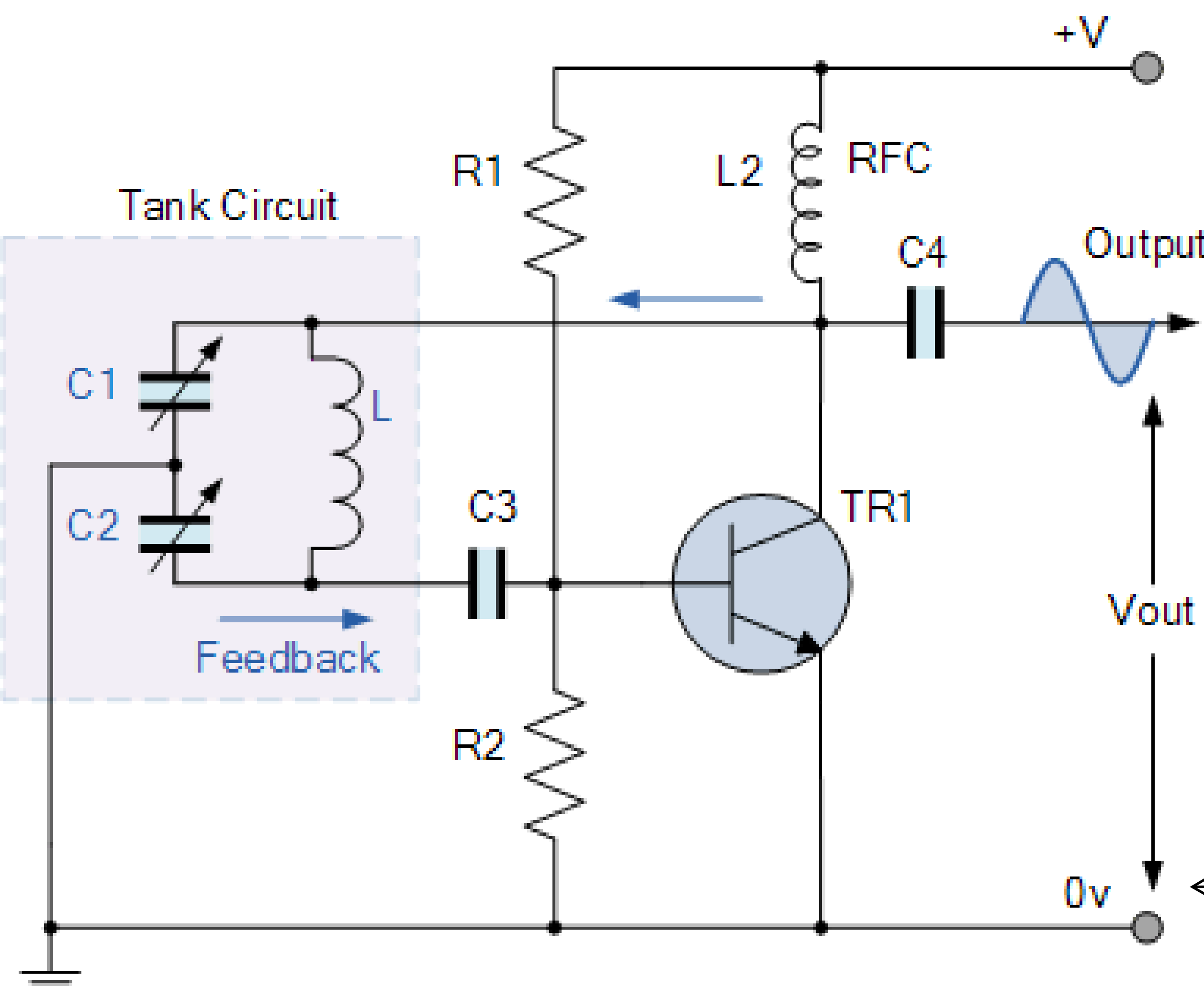
$$\boxed{E = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2}$$

The voltage of a typical Van der Graaf might be as high as 300kV

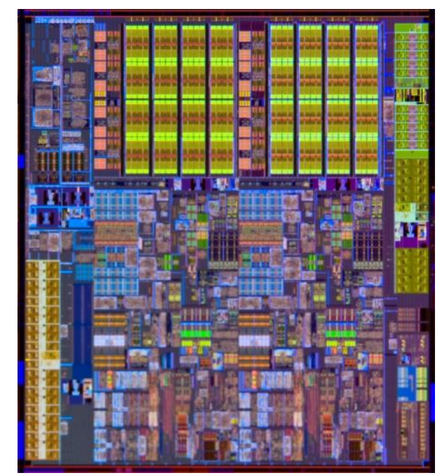
Hence the energy discharged is only about 1J and the charge about 6.6μC

**Capacitors**, combined with other basic components such as **resistors** and **inductors** can form a huge variety of circuits, each with a different application.

An integrated circuit (on a 'chip') is the basis of modern electronics. A computer microprocessor may contain *billions* of circuits!







This particular circuit generates a *sine wave oscillating signal* at a frequency dependent upon the parameters of the components i.e. capacitance, inductance etc

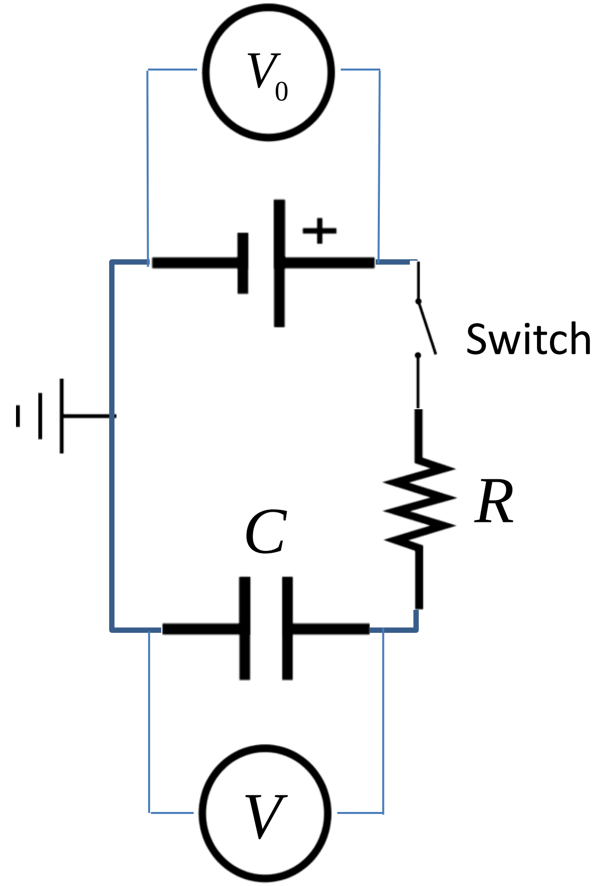
## Charging a capacitor using a DC source

1. Switch closed. Current flows through resistor and positive charge builds up on right capacitor plate. An equal amount of negative charge builds up on left plate.

2. *Electrical field* set up between capacitor plates as no current can flow. Voltage $V$ between the plates is $V = Q/C$ where $Q$ is the total charge deposited and $C$ is the *capacitance* ('charge per unit volt')

3. As charge builds up on right plate, potential difference between capacitor and source reduces. This reduces the current flowing onto the plate. Eventually the voltage $V$ becomes $V_0$ and hence no more current can flow.

3. Note the amount of charge which can be deposited depends on the resulting *electrical field strength* between the plates. Above the breakdown field strength, current will flow between the plates




| Dielectric | Breakdown field strength /$Vm^{-1}$ |
|---|---|
| Air | $3 \times 10^6$ |
| Mineral oil | $15 \times 10^6$ |
| Neoprene | $16 \times 10^6$ |
| Water | $65 \times 10^6$ |
| Mica | $118 \times 10^6$ |

## Charging a capacitor using a DC source

$$Q = CV$$

capacitor charge, voltage relationship

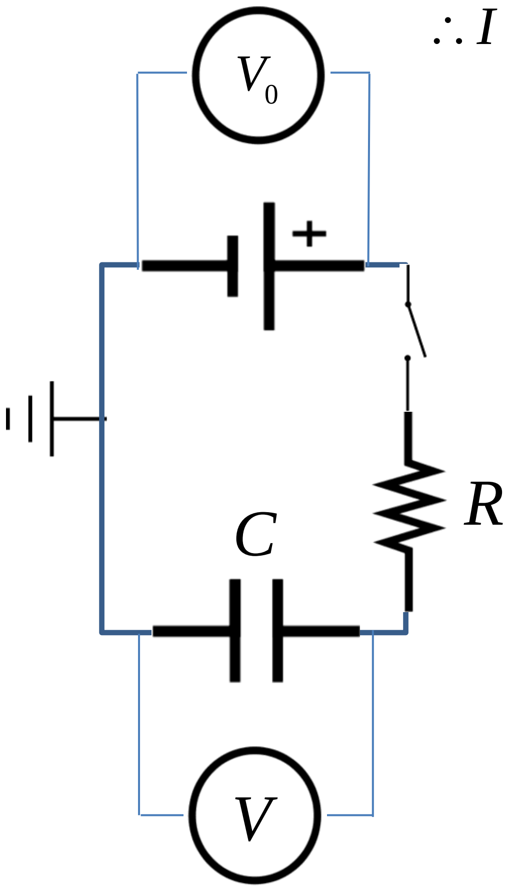
$$V_0 - V = IR$$

Ohm's law

$$I = \frac{dQ}{dt}$$

Definition of current

$$\therefore I = \frac{V_0 - V}{R} = C\frac{dV}{dt}$$

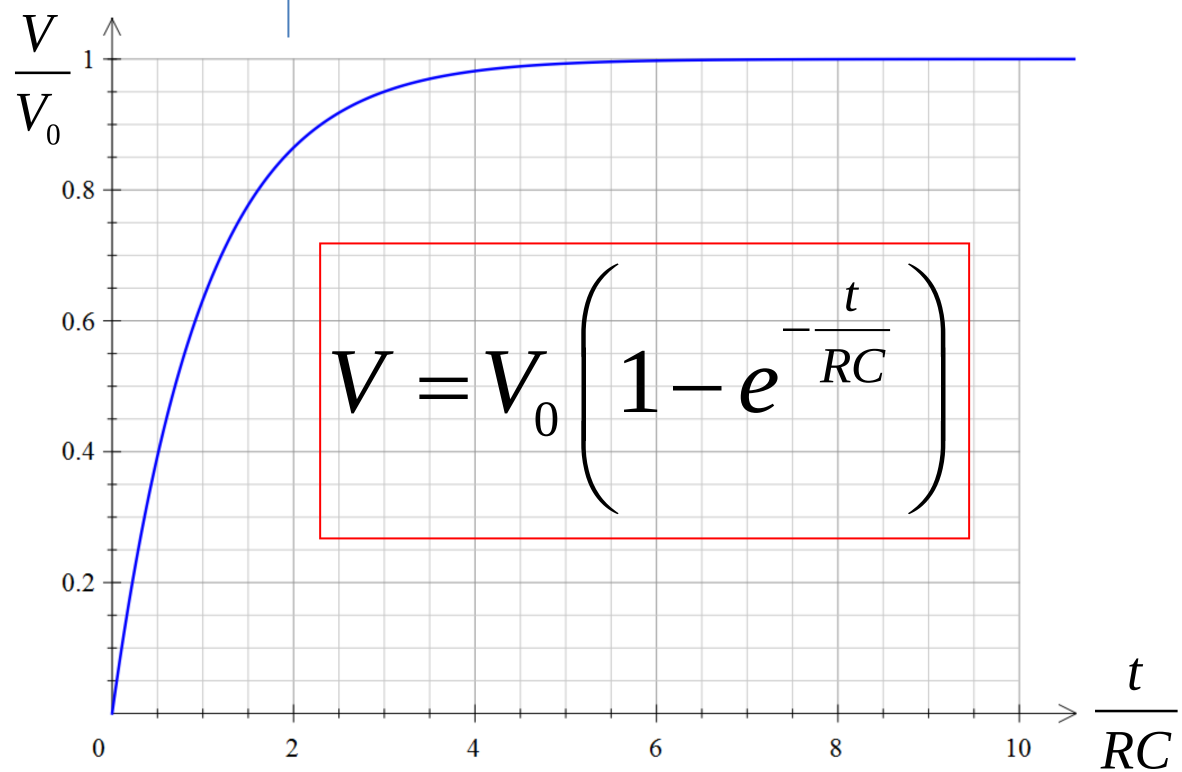



$$\frac{1}{RC}\int_0^t dt = \int_0^V \frac{dV}{V_0 - V} = -\int_0^V \frac{-dV}{V_0 - V}$$

$$\frac{t}{RC} = -\left[\ln\left|V_0 - V\right|\right]_0^V$$

$$-\frac{t}{RC} = \ln\left(V_0 - V\right) - \ln\left(V_0\right) = \ln\left(\frac{V_0 - V}{V_0}\right)$$

$$\frac{V_0 - V}{V_0} = e^{-\frac{t}{RC}}$$




$$\boxed{V = V_0\left(1 - e^{-\frac{t}{RC}}\right)}$$

## Discharging a capacitor

$$Q = CV$$

capacitor charge, voltage relationship

$$V = IR$$

Ohm's law

$$I = -\frac{dQ}{dt}$$

Definition of current, and negative since charge is discharged from plates

$$\therefore I = \frac{V}{R} = -C\frac{dV}{dt}$$

Note $V=V_0$ when $t = 0$

$$\frac{1}{RC}\int_0^t dt = -\int_{V_0}^{V}\frac{dV}{V}$$

$$\frac{t}{RC} = -\left[\ln|V|\right]_{V_0}^{V}$$

$$\frac{t}{RC} = -\ln\left(\frac{V}{V_0}\right)$$

$$V = V_0 e^{-\frac{t}{RC}}$$