UNIFORM CIRCULAR MOTION

1/ (i)

$$\omega = \frac{4 \times 2\pi \text{ rad}}{0.9\text{s}} \quad \therefore \quad \omega = \boxed{28 \text{ rad s}^{-1}}$$

(ii) $$15{,}000 \text{ RPM} = \frac{15{,}000 \times 2\pi \text{ rad}}{60\text{s}} = \boxed{1570 \text{ rad s}^{-1}}$$

(which since $\omega = 2\pi f$, is $f = \boxed{250 \text{ Hz}}$)

(iii)

$\boxed{v = r\omega}$ $v < 343 \text{ m/s}$; $r = 107\text{m}$

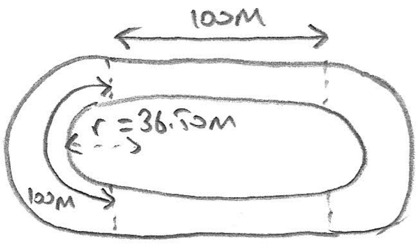
To avoid tips going at supersonic speed

$$\therefore \quad \omega < \frac{343 \text{ m/s}}{107\text{m}}$$

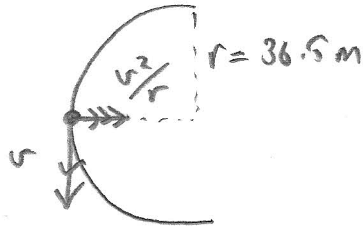
$$\boxed{\omega < 3.21 \text{ rad s}^{-1}}$$

So $\boxed{\omega < 30.6 \text{ RPM}}$ $\left[\left(\frac{\omega}{\text{RPM}}\right) = \frac{60}{2\pi} \times \left(\frac{\omega}{\text{rad s}^{-1}}\right)\right]$

(iv)



Athletics track



$$v = \frac{100\text{m}}{10.6\text{s}} = \boxed{9.43 \text{ m/s}} \qquad a = \frac{v^2}{r} = \frac{(100/10.6)^2}{36.5} \text{ m/s}^2$$

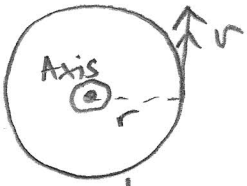
$$= \boxed{2.44 \text{ m/s}^2}$$

Usain Bolt linear acceleration:

$$a = \frac{12 \text{ m/s}}{4.5\text{s}} = \boxed{2.67 \text{ m/s}^2}$$

So very similar ⇒ similar forces applied by the athlete via their muscles.

(v)



Earth

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi \times 6378 \text{ km}}{24 \times 3600\text{s}}$$

$$\boxed{v = 0.46 \text{ km/s}}$$

(vii)

$$\frac{v^2}{r} < 9g$$

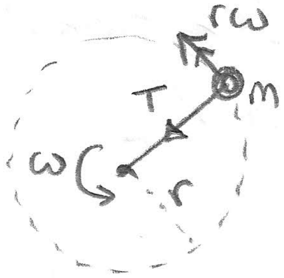
$$\therefore v < \sqrt{9gr}$$

$$\therefore v < \sqrt{9 \times 9.81 \times 100} \qquad (m/s)$$

$$v < \boxed{94.0\ m/s}$$

[This is 338 km/h]

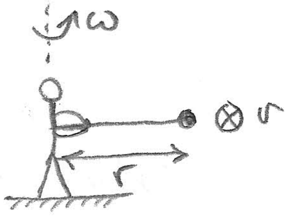
(viii)



Newton II radially inward

$$mr\omega^2 = T$$

$$\omega = \frac{2\pi}{0.99} \quad rad\,s^{-1}$$



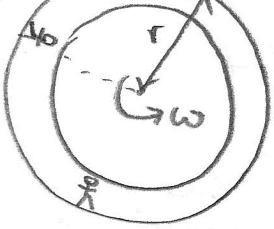
$$r = \frac{121.3 + 70}{100} \quad (m)$$

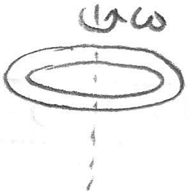
$$\therefore T = 7.26\,kg \times \left(\frac{121.3+70}{100}\right) \times \left(\frac{2\pi}{0.99}\right)^2$$

$$T = \boxed{559\,N}$$

(If Dr F has a mass of 75 kg, this is $\frac{559}{75 \times 9.81}$
$= 0.76$ or 76% of the weight of Dr F).

(ix)





Newton II radially inward

$$mr\omega^2 = R$$

R is the reaction force of outer wall of hoop on astronaut

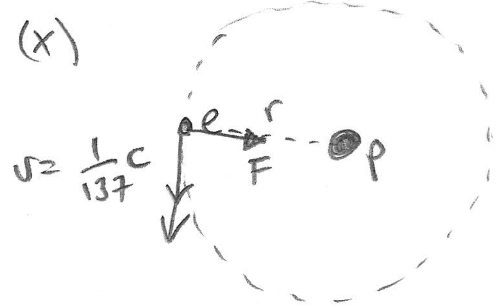
let $R = mg$ where $g = 9.81$ N/kg

$$\therefore \boxed{r = \frac{g}{\omega^2}}$$

if $\omega = 3.00 \times 2\pi/60$ rad s^{-1}

$$\Rightarrow r = \frac{9.81}{(3.00 \times 2\pi/60)^2} = \boxed{99.4\ m}$$

(x)



$$F = \frac{mv^2}{r}$$ (Newton II radially inward)

$$F = \frac{9.109\times10^{-31} \times \left(\frac{2.998\times10^{8}}{137}\right)^2}{5.29\times10^{-11}} \quad (N)$$

e electron

p proton

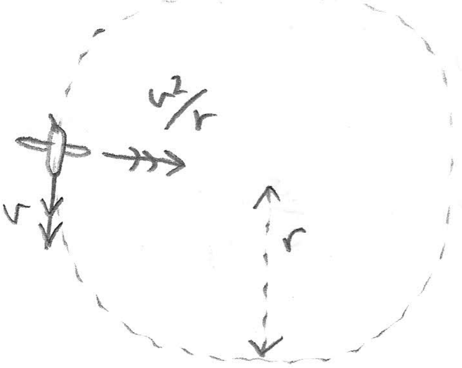
$$\boxed{F = 8.25\times10^{-8}\,N}$$

[check $F = \frac{e^2}{4\pi\varepsilon_0 r^2}$ ← Coulomb's law

$$= \frac{(1.6\times10^{-19})^2}{4\pi\times8.85\times10^{-12}\times(5.29\times10^{-11})^2}$$

$$= 8.2\times10^{-8}\,N \checkmark$$]

(xi)



$$v = 1185\times10^3/3600 \quad m/s$$

$$= \boxed{329.2\ m/s}$$

$$\frac{v^2}{r} = 0.1g \qquad g = 9.81\ m/s^2$$

$$\therefore\ r = \frac{v^2}{0.1g}$$

$$\therefore\ r = \frac{\left(1185\times10^3/3600\right)^2}{0.1\times9.81} \quad (m)$$

$$\boxed{r = 110.4\ km}$$

Now $v = r\omega$ so $\omega = \frac{1185\times10^3/3600}{\text{ANS above}\times1000}$

$$= 2.98\times10^{-3}\ rad\,s^{-1}$$

$$= 0.17°/s \quad \times\frac{180}{\pi}$$

So $\boxed{5.7\%}$ of a 3°/s standard rate turn

If the aircraft executes a 3°/s standard rate turn

$\omega = \dfrac{3 \times \pi}{180}$ rads^{-1}

$\therefore \quad r\omega^2 = 0{\cdot}1g \Rightarrow r = 0{\cdot}1g/\omega^2$

$$r = \frac{0{\cdot}1 \times 9{\cdot}81}{(3\pi/180)^2} = \boxed{358\text{M}}$$

$$v = r\omega = \frac{0{\cdot}1 \times 9{\cdot}81}{(3\pi/180)^2} \times \left(\frac{3\pi}{180}\right)$$

$$= 18{\cdot}7 \text{ M/S} = \boxed{67{\cdot}4 \text{ km/h}}$$

This is far too slow for the aircraft – at this speed it won't generate sufficient lift and will crash.

Alternatively, if $v = 1185$ km/h and $\omega = 3\pi/180$ rads^{-1}

$a = \dfrac{v^2}{r}$ $\quad v = r\omega \Rightarrow r = v/\omega$ $\quad$ so $a = \dfrac{v^2}{v/\omega} = \omega v$

so $\boxed{\dfrac{a}{g} = \dfrac{\omega v}{g}}$

$$\therefore \frac{a}{g} = \left(\frac{3\pi}{180}\right)\left(\frac{1185 \times 1000}{3600}\right)\Big/9{\cdot}81$$

$$= \boxed{1{\cdot}76} \quad \text{or} \quad \text{"}1{\cdot}76g\text{"}$$

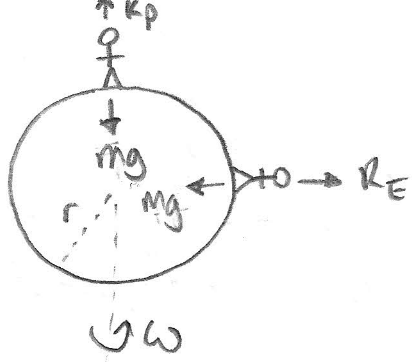
This acceleration is rather more than passengers in a commercial airline will tolerate!

[Also, $r = \dfrac{1185 \times 1000/3600}{3\pi/180} = 6287\text{M}$]

ie $\approx 6{\cdot}3$km radius.

* Project * Build a spreadsheet that performs these calculations automatically, ie work out turn rate and r given airspeed and $a = v^2/r$ (in terms of g)

2/



$\omega = 2\pi / T$ T is rotational period

Newton II radially inward:

Pole: $0 = mg - R_p$

Equator: $mr\omega^2 = mg - R_E$

let $R_E = \frac{1}{2} R_p$

So $R_p = mg$ $\therefore$ $R_E = \frac{1}{2} mg$

$\therefore$ $mr\omega^2 = mg - \frac{1}{2} mg$

$r\omega^2 = \frac{1}{2} g$

$\omega = \sqrt{g/2r}$

$2\pi / T = \sqrt{g / 2r}$

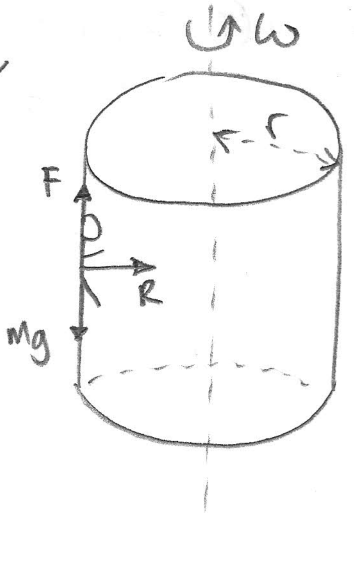
$\therefore$ $T = 2\pi \sqrt{\frac{2r}{g}}$ ("looks like the pendulum period equation")

$\therefore$ $T = 2\pi \sqrt{\frac{2 \times 6370 \times 1000}{9.81}}$

$T = 7160.3s$ $\div 24 \times 3600$

$\boxed{T \approx 0.08 \text{ days}}$ $\simeq 1.99$ hours

3/



Newton II vertically: $0 = F - mg$

radially: $mr\omega^2 = R$

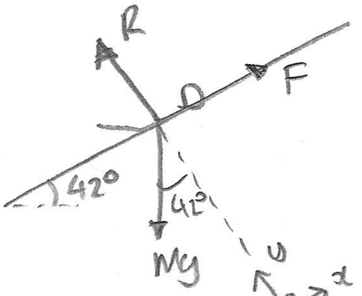
Now for no slip $F \le \mu R$

So since $F = mg$ and $R = mr\omega^2$

$mg \le \mu m r \omega^2$

$\therefore$ $\boxed{\omega \ge \sqrt{g / \mu r}}$

For slide:



Newton II 'on the point of sliding'

$//x: \quad 0 = F - Mg\sin 42°$

$//y: \quad 0 = R - Mg\cos 42°$

$\therefore \quad F = Mg\sin 42°$

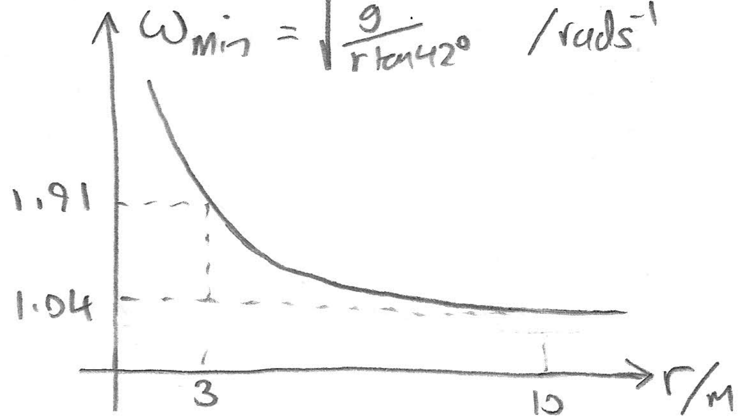
$R = Mg\cos 42°$

Now on the point of sliding: $F = \mu R$

$\therefore \quad Mg\sin 42° = \mu Mg\cos 42°$

$\therefore \quad \boxed{\tan 42° = \mu}$

So $\boxed{\omega \geq \sqrt{\dfrac{g}{r\tan 42°}}}$



when $r = 3.0\,m$

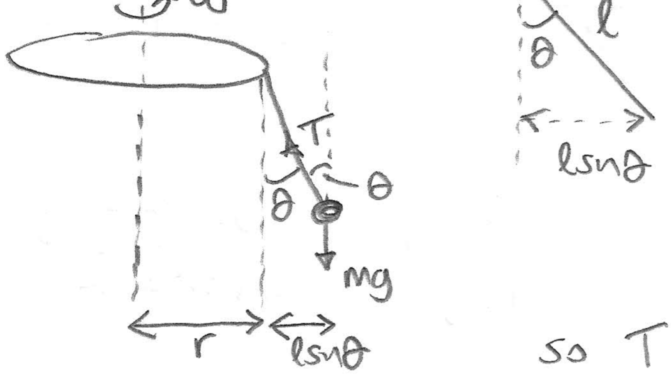
$\omega_{min} = 1.91$ rads^{-1}

$r = 10.0\,m$

$\omega_{min} = 1.04$ rads^{-1}

[part (ii) on next page!]

4/



Newton II:

Vertically: $0 = T\cos\theta - mg$

radially inward: $m(r + l\sin\theta)\omega^2 = T\sin\theta$

so $T = mg/\cos\theta$

$\therefore \quad m(r + l\sin\theta)\omega^2 = mg\sin\theta/\cos\theta$

$\therefore \quad \boxed{\omega = \sqrt{\dfrac{g\tan\theta}{r + l\sin\theta}}}$

(ii) $r\omega = v$ so $\omega = \frac{v}{r}$

$v = 5\,m/s$
$l = 4.0\,m$
$\theta = 30^\circ$

$$\therefore \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r + l\sin\theta}}$$

$$\frac{v^2}{r^2} = \frac{g\tan\theta}{r + l\sin\theta}$$

$$v^2(r + l\sin\theta) = r^2 g\tan\theta$$

$$\therefore r^2 g\tan\theta - rv^2 - v^2 l\sin\theta = 0$$

$$r^2 - \frac{rv^2}{g\tan\theta} - \frac{v^2 l\sin\theta}{g\tan\theta} = 0$$

$$\left(r - \frac{v^2}{2g\tan\theta}\right)^2 - \frac{v^4}{4g^2\tan^2\theta} - \frac{v^2 l\sin\theta}{g\tan\theta} = 0$$

$$r = \pm\sqrt{\frac{v^4}{4g^2\tan^2\theta} + \frac{v^2 l\sin\theta}{g\tan\theta}} + \frac{v^2}{2g\tan\theta}$$

$$r = \pm\frac{v^2}{2g\tan\theta}\sqrt{1 + \frac{v^2 l\sin\theta}{g\tan\theta}\,\frac{4g^2\tan^2\theta}{v^4}} + \frac{v^2}{2g\tan\theta}$$

$$r = \frac{v^2}{2g\tan\theta}\left(\pm\underbrace{\sqrt{1 + \frac{4gl\sin\theta\tan\theta}{v^2}}}_{\text{clearly larger than 1}} + 1\right)$$

So $$\boxed{r = \frac{v^2}{2g\tan\theta}\left(1 + \sqrt{1 + \frac{4gl}{v^2}\sin\theta\tan\theta}\right)}$$

Since $\theta = 30°$ $\tan\theta = \frac{1}{\sqrt{3}}$

$\sin 30° = \frac{1}{2}$

$$\therefore \quad r = \frac{v^2}{2g}\sqrt{3}\left(1 + \sqrt{1 + \frac{4gl}{v^2}\frac{1}{2\sqrt{3}}}\right)$$

$$r = \frac{5^2}{2\times 9.81}\sqrt{3}\left(1 + \sqrt{1 + \frac{4\times 9.81\times 4.0}{5^2}\frac{1}{2\sqrt{3}}}\right)$$

$$\boxed{r = 5.91\ \text{m}}$$

Check: $\omega = \frac{v}{r} = \frac{5}{5.91} = 0.846\ \text{rads}^{-1}$

$$\sqrt{\frac{g\tan\theta}{r + l\sin\theta}} = \sqrt{\frac{9.81 \cdot \frac{1}{\sqrt{3}}}{5.91 + 4/2}} = 0.846\ \text{rads}^{-1}\ ✓$$

(iii) Now from above: $T = mg/\cos\theta$

$$\text{so } T = \frac{100\,\text{kg} \times 9.81\,\text{N/kg}}{\sqrt{3}/2} = \boxed{1133\,\text{N}}$$

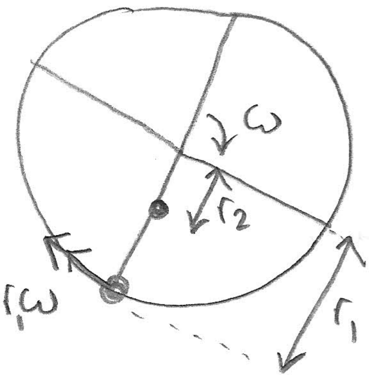
$$\text{Check:} \quad T = \frac{m(r + l\sin\theta)\omega^2}{\sin\theta}$$

$$= \frac{100\left(5.91 + 4\tfrac{1}{2}\right)\left(5/5.91\right)^2}{\frac{1}{2}}$$

$$= \boxed{1133\,\text{N}}$$

Q3 (ii) $\omega \geq \sqrt{\dfrac{g}{\tan 42^\circ r}}$ for Prof. Parker to stick to wall.

let $\boxed{\omega = \sqrt{\dfrac{g}{r \tan 42^\circ}}}$ ie minimum ω



KE at r_1 is $\frac{1}{2}m(r_1\omega)^2$

KE at r_2 is $\frac{1}{2}m(r_2\omega)^2$

So $\boxed{\Delta E = \frac{1}{2}m\omega^2(r_1^2 - r_2^2)}$

let $\omega^2 = \dfrac{g}{r_1 \tan 42^\circ}$ $\quad$ $r_1 = 5.0\text{m}$, $r_2 = 0.5\text{m}$

so $\Delta E = \frac{1}{2}mg\left(\dfrac{r_1^2 - r_2^2}{r_1 \tan 42^\circ}\right)$

$\Delta E = \frac{1}{2}\, 80 \times 9.81 \left(\dfrac{5.0^2 - 0.5^2}{5.0 \tan 42^\circ}\right)$

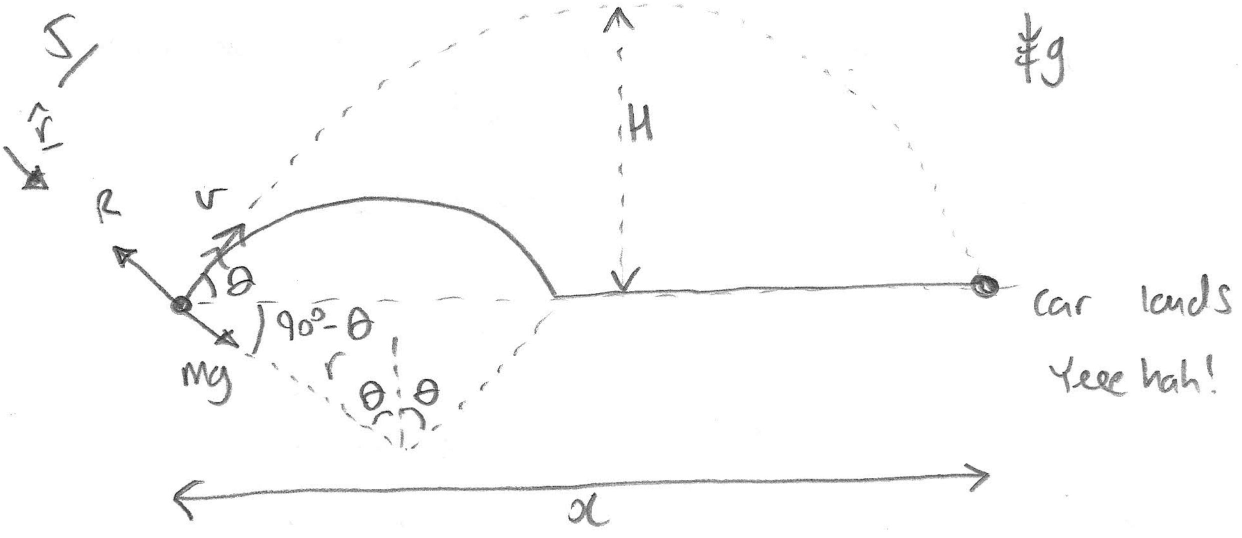
$= \boxed{2157\text{ J}}$ ie quite hard work.

[comparing to mgh $\Rightarrow$ $h = \dfrac{2157}{80 \times 9.81}$ (m)

ie a GPE gain $= \boxed{2.75\text{m}}$

So like Prof. Parker climbing a 2.75m vertical rope]

5/ #g



Newton II in radial direction $\hat{r}$ (inwards)

$$\frac{Mv^2}{r} = Mg - R$$

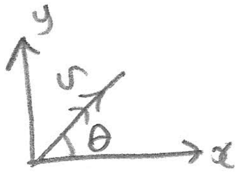
$$\therefore \quad \boxed{R = Mg - \frac{Mv^2}{r}}$$

For contact with the hump, $R \geq 0$

so to become airborne, $\frac{v^2}{r} > g$

$$\Rightarrow \quad \boxed{v > \sqrt{rg}} \quad \text{or} \quad \boxed{r < \frac{v^2}{g}}$$

(i) So $\boxed{r_{max} = \frac{v^2}{g}}$



(ii) Projectile motion!

$x = vt\cos\theta$ $\qquad v_x = v\cos\theta$

$y = vt\sin\theta - \frac{1}{2}gt^2$ $\qquad v_y = v\sin\theta - gt$

$y = 0$ when $t(v\sin\theta - \frac{1}{2}gt) = 0$

so car lands when $v\sin\theta - \frac{1}{2}gt = 0$

Now $t = \frac{x}{v\cos\theta}$ $\quad \therefore \quad v\sin\theta - \frac{1}{2}g\frac{x}{v\cos\theta} = 0$

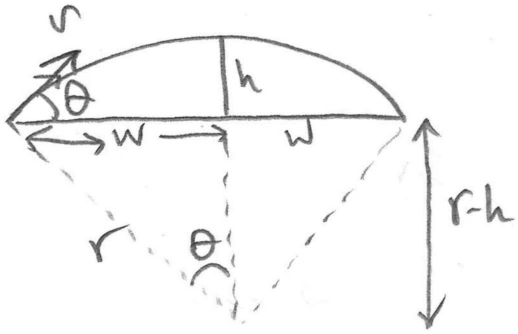
So $\boxed{\dfrac{2v^2 \sin\theta \cos\theta}{g} = x}$

Maximum height H attained is when $v_y = 0$

$$\Rightarrow \quad t = \frac{v\sin\theta}{g} \qquad \therefore \quad H = \frac{v\sin\theta}{g}\left(v\sin\theta - \tfrac{1}{2}v\sin\theta\right)$$

$$\boxed{H = \frac{v^2\sin^2\theta}{2g}}$$

Geometry of bump



Pythagoras: $r^2 = w^2 + (r-h)^2$

$$r^2 = w^2 + r^2 - 2rh + h^2$$

$$2rh = w^2 + h^2$$

$$\boxed{r = \frac{w^2+h^2}{2h}}$$

Now $r\sin\theta = w \quad \therefore \quad \theta = \sin^{-1}\left(\frac{w}{r}\right)$

$$\boxed{\theta = \sin^{-1}\left(\frac{2wh}{w^2+h^2}\right)}$$

So $v > \sqrt{rg}$ to get airborne

$$\Rightarrow \quad v > \sqrt{\frac{(w^2+h^2)g}{2h}}$$

If $h = 1.0\,m$
$w = 2.0\,m$

$$\Rightarrow \quad \boxed{v > 4.95\ m/s}$$

$$\therefore \boxed{\begin{aligned} r &= 2.5\text{ m} \\ \theta &= 53.1^\circ \end{aligned}}$$

$$x = \frac{2v^2}{g}\sin\theta\cos\theta$$

so if $v > 4.95$ m/s

$$\Rightarrow \boxed{x > 12.48\text{ m}}$$

Aside: $H = \dfrac{v^2 \sin^2\theta}{2g}$

$$x = \frac{2v^2}{g}\sin\theta\cos\theta$$

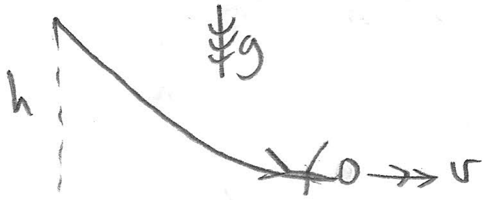
$$\text{so } \frac{v^2}{g} = \frac{x}{2\sin\theta\cos\theta}$$

$$\therefore H = \frac{x}{2\sin\theta\cos\theta}\cdot\frac{\sin^2\theta}{2}$$

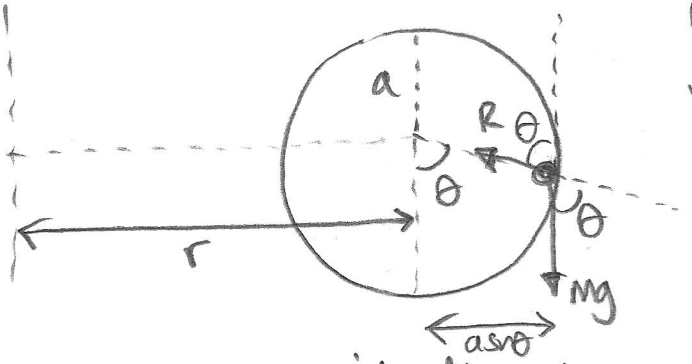
$$\Rightarrow \boxed{H = \tfrac{1}{4}x\tan\theta}$$

$$\Rightarrow \boxed{H > 4.16\text{ m}}$$

6/



$$\tfrac{1}{2}mv^2 = mgh \quad \therefore \boxed{v = \sqrt{2gh}}$$



Inside flume tube

Newton II

Vertically: $0 = R\cos\theta - mg$

Radially: $\dfrac{mv^2}{r + a\sin\theta} = R\sin\theta$

So $R = mg/\cos\theta$

$$\therefore \frac{mv^2}{r + a\sin\theta} = \frac{mg\sin\theta}{\cos\theta}$$

$$\frac{v^2}{r + a\sin\theta} = g\tan\theta$$

$$\frac{v^2}{g} = \tan\theta\,(r + a\sin\theta)$$

$$\frac{2gh}{g} = \tan\theta\,(r + a\sin\theta)$$

$$2h = \frac{\sin\theta}{\cos\theta}(r + a\sin\theta)$$

$$2h\cos\theta = r\sin\theta + a\sin^2\theta$$

$$4h^2\cos^2\theta = 2ra\sin^3\theta + r^2\sin^2\theta + a^2\sin^4\theta$$

$$4h^2(1-\sin^2\theta) = a^2\sin^4\theta + 2ra\sin^3\theta + r^2\sin^2\theta$$

$$a^2\sin^4\theta + 2ra\sin^3\theta + (r^2+4h^2)\sin^2\theta - 4h^2 = 0$$

..... Can't solve easily! So change question ($h(\theta)$ instead).

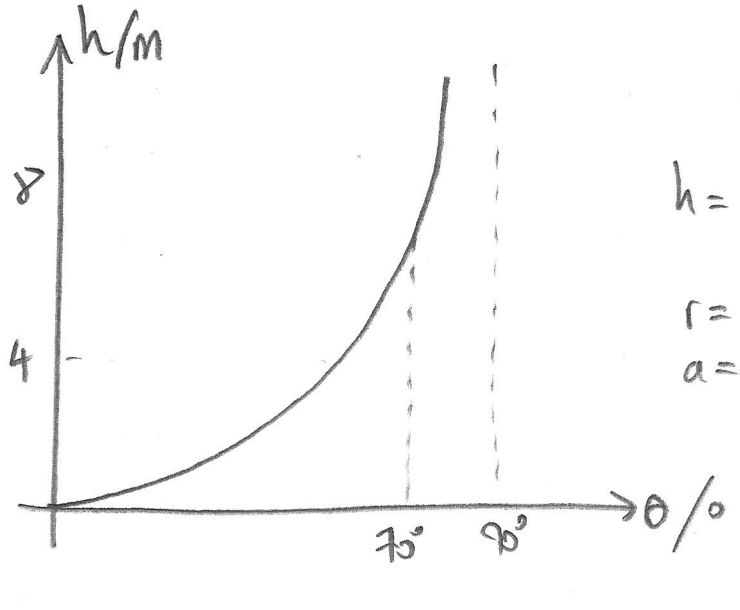
Compute h s.t. $\boxed{\theta = 60^\circ}$ and plot h vs θ.

$$\boxed{h = \tfrac{1}{2}\tan\theta(r + a\sin\theta)}$$

let $r = 5.0\,m$ $a = 0.5\,m$

$$\therefore h = \tfrac{1}{2}\tan 60^\circ(5.0 + 0.5\sin 60^\circ)$$

$$\boxed{h = 4.71\,m}$$



$h = \frac{1}{2}\tan\theta(r + a\sin\theta)$

$r = 5.0\,m$
$a = 0.5\,m$