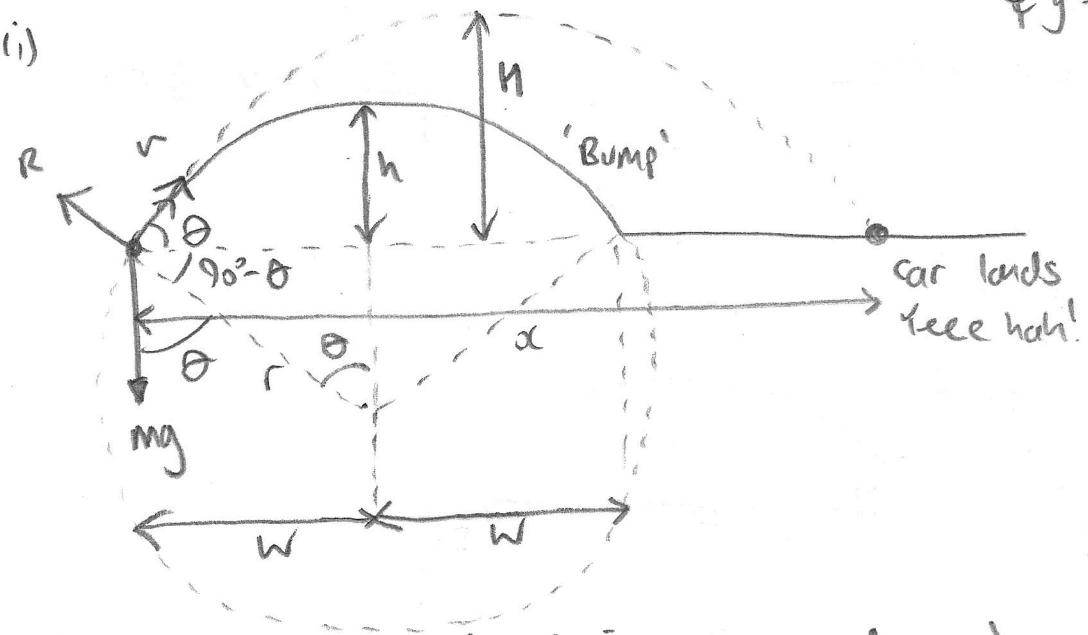
5

$\downarrow g = 9.81\ \text{N/kg}$

(i)



Newton II in radial direction (inwards +ve)

$$\frac{Mv^2}{r} = Mg\cos\theta - R$$

$\therefore$ Normal contact force $\boxed{R = Mg\left(\cos\theta - \frac{v^2}{rg}\right)}$

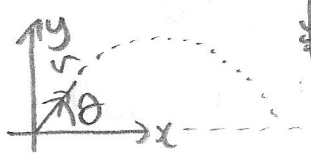
For contact with the hump $R \geq 0$, so to become airborne

$$\cos\theta - \frac{v^2}{rg} < 0$$

$$\Rightarrow \boxed{v > \sqrt{rg\cos\theta}}$$

or $\cos\theta < \frac{v^2}{rg} \Rightarrow \boxed{r < \frac{v^2}{g\cos\theta}}$

(ii) Projectile motion



$\downarrow g$

$x = vt\cos\theta$
$y = vt\sin\theta - \frac{1}{2}gt^2$
$v_x = v\cos\theta$
$v_y = v\sin\theta - gt$

$y=0$ when $t(v\sin\theta - \frac{1}{2}gt) = 0$

So car lands when $v\sin\theta - \frac{1}{2}gt = 0$.

Now $t = \frac{x}{v\cos\theta}$ $\therefore$ $v\sin\theta - \frac{1}{2}g\frac{x}{v\cos\theta} = 0$

so $\boxed{x = \frac{2v^2}{g}\sin\theta\cos\theta}$ is the horizontal range when $y=0$.

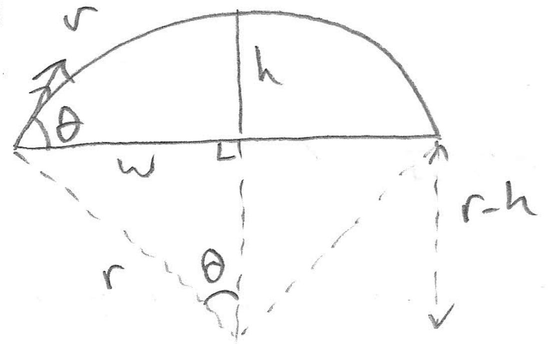
10

Max height attained ("apogee") is when $v_y = 0$

$$\Rightarrow t = \frac{v\sin\theta}{g} \quad \therefore \quad \underset{y=H}{H} = \frac{v\sin\theta}{g}\left(v\sin\theta - \tfrac{1}{2}v\sin\theta\right)$$

$$\Rightarrow \boxed{H = \frac{v^2\sin^2\theta}{2g}}$$

Bump geometry:



Pythagoras:

$$r^2 = w^2 + (r-h)^2$$
$$r^2 = w^2 + r^2 - 2rh + h^2$$
$$2rh = w^2 + h^2$$
$$\boxed{r = \frac{w^2+h^2}{2h}}$$

Now $r\sin\theta = w$

$$\therefore \theta = \sin^{-1}(w/r)$$

$$\therefore \boxed{\theta = \sin^{-1}\left(\frac{2wh}{w^2+h^2}\right)}$$

To get airborne $v > \sqrt{rg\cos\theta}$ (from part (ii))

Now $r\cos\theta = r - h \quad \therefore \quad \cos\theta = 1 - \frac{h}{r}$

$$\cos\theta = 1 - \frac{h \times 2h}{w^2+h^2}$$

$$\cos\theta = \frac{w^2+h^2-2h^2}{w^2+h^2}$$

$$\cos\theta = \frac{w^2-h^2}{w^2+h^2}$$

$$\text{So } v > \sqrt{\underbrace{\frac{w^2+h^2}{2h}}_{r}\, g\, \frac{w^2-h^2}{w^2+h^2}}$$

$$\Rightarrow \boxed{v > \sqrt{\frac{g(w^2-h^2)}{2h}}}$$

if $h = 1.0\text{m}$, $w = 2.0\text{m}$

$\Rightarrow v > \boxed{3.84\text{m/s}}$

$\therefore \quad r = \dfrac{2.0^2 + 1.0^2}{2 \times 1.0} = \boxed{2.5\,m} \qquad \theta = \sin^{-1}\left(\dfrac{2 \times 2.0 \times 1.0}{2.0^2 + 1.0^2}\right)$

$= \boxed{53.1^\circ}$

So if $v > 3.84$ m/s and $x = \dfrac{2v^2}{g} \sin\theta \cos\theta$

$x > \boxed{1.44\,m}$ This is a problem as it is $< w$

ie the General Lee won't clear the bump.

<u>To clear the bump</u>, $x > 2w$ and $H > h$.

So $\dfrac{2v^2}{g} \sin\theta\cos\theta > 2w$ and $\dfrac{v^2 \sin^2\theta}{2g} > h$

and to get airborne $v^2 > rg\cos\theta \Rightarrow \boxed{v_I > \sqrt{\dfrac{g(w^2 - h^2)}{2h}}}$

using $\sin\theta = \dfrac{2wh}{w^2+h^2}$ and $\cos\theta = \dfrac{w^2-h^2}{w^2+h^2}$

$$v^2 > \frac{wg}{\left(\frac{2wh}{w^2+h^2}\right)\left(\frac{w^2-h^2}{w^2+h^2}\right)} \Rightarrow v^2 > \frac{g(w^2+h^2)^2}{2h(w^2-h^2)}$$

$$\Rightarrow \boxed{v_{II} > \frac{w^2+h^2}{\sqrt{2h(w^2-h^2)}}\sqrt{g}}$$

$\left[v^2 > \dfrac{wg}{\sin\theta\cos\theta} \right]$

Also $v^2 > 2gh / \sin^2\theta \Rightarrow v^2 > \dfrac{2gh(w^2+h^2)}{2wh}$

$$\Rightarrow \boxed{v_{III} > \sqrt{\frac{g(w^2+h^2)}{w}}}$$

Airborne at start of bump

clear bump

Apogee $> h$

So $v >$ than max of v_I, v_{II}, v_{III}

In our case: $w = 2.0\,m$, $h = 1.0\,m$

$v_I = 3.84$ m/s $\qquad v_{II} = 6.39$ m/s

$v_{III} = 4.95$ m/s. Opting for $v > v_{II}$ ie $\boxed{v > 6.39\ m/s}$

$\Rightarrow H > 1.33\,m$ and $x > 4.0\,m$.

Now if $v = 25 M/s$, $\theta = 53.1°$

$x = \frac{2v^2}{g} \sin\theta\cos\theta$ $\qquad H = \frac{v^2 \sin^2\theta}{2g}$

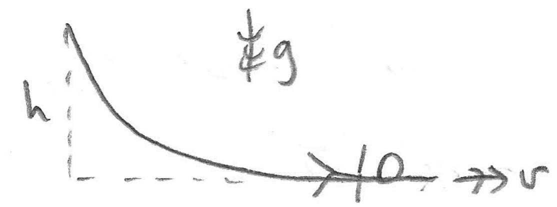
$\therefore$ $\boxed{x = 61.2 \text{ m} \quad (!)}$

$\therefore$ $\boxed{H = 20.4 \text{ m}}$

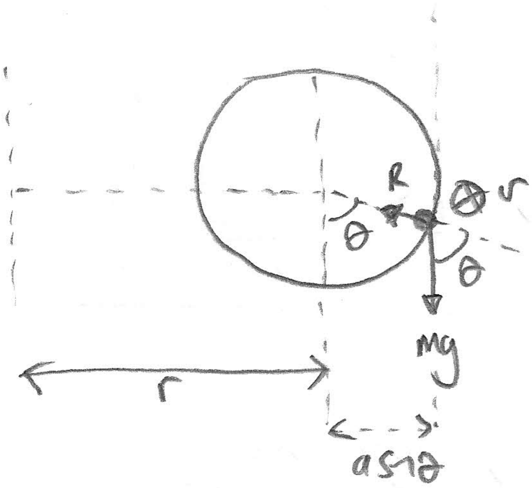
i.e a huge jump!

(θ is perhaps a bit steep to be realistic here, if 30° $x = 41\text{m}$ and $H = 3.7\text{m}$)

6/



$\frac{1}{2}mv^2 = mgh$ $\qquad \therefore \boxed{v = \sqrt{2gh}}$



inside flume tube (horizontal circle of radius $r + a\sin\theta$ speed v)

Newton II: Vertically : $\boxed{0 = R\cos\theta - mg}$

Radially (Horizontally) : $\boxed{\frac{mv^2}{r + a\sin\theta} = R\sin\theta}$

so $\boxed{R = mg/\cos\theta}$

$\therefore \frac{mv^2}{r + a\sin\theta} = \frac{mg\sin\theta}{\cos\theta}$

$\therefore \frac{v^2}{r + a\sin\theta} = g\tan\theta$

$\therefore \frac{v^2}{g} = \tan\theta\,(r + a\sin\theta)$

$\therefore \frac{2gh}{g} = \tan\theta\,(r + a\sin\theta)$

$2h = \frac{\sin\theta}{\cos\theta}(r + a\sin\theta)$

$\therefore 2h\cos\theta = r\sin\theta + a\sin^2\theta$

(12) +