π radians $= 180^{\circ}$	RPM = revolutions per minute $\left(\omega / \text{RPM}\right) = \frac{60}{2\pi}\left(\omega / \text{rads}^{-1}\right)$
$v = r\omega$	Velocity (ms^{-1}) of a particle undergoing constant speed circular motion at angular speed ω (rads^{-1}) at radius r (m). $\omega = 2\pi f$ where f is the rotation frequency (Hz). $f = 1/T$ where T is the rotation period
$a = r\omega^2 = \dfrac{v^2}{r}$	Centripetal acceleration (ms^{-2}) towards the centre of a circular trajectory

Question 1

(i) The Russian ice skater Alexandra Trusova performed a *quadruple lutz* (four complete rotations) in 2019. She was in the air for 0.9s. Calculate her average angular speed ω in rads^{-1}.

(ii) My hand-held gyroscope has a maximum rating of 15,000RPM. What is the corresponding ω in rads^{-1}? What is the maximum frequency in Hz?

(iii) As of 2019, the longest wind turbine blade has a radius of $r = 107\text{m}$. It is very important that the tip of the blade doesn't become supersonic to avoid shock waves. If the speed of the tip must be less than 343ms^{-1}, calculate the maximum rotation rate of the turbine (a) in rads^{-1} and (b) convert this to RPM.

(iv) The radius of lane one of the bend of an athletics track is 36.50m. If an athlete runs the semicircular bend in 12.15s, calculate the centripetal acceleration in m/s^2. Usain Bolt attained a top speed of about 12m/s in about 4.5s when running the 100m at World Record pace. How does this acceleration compare?

(v) The Earth rotates about its axis every 24 hours. If the equatorial radius of the is 6378km, calculate the speed (in km/s) that a point at the equator moves through space.

(vi) The Earth rotates about the Sun on average every 365.25 days. Assuming a circular orbit of radius $1\text{AU} = 1.496 \times 10^{11}\text{m}$, calculate the orbital speed in km/s.

(vii) The Eurofighter Typhoon jet aircraft has a maximum operating limit of about '9g' i.e. an acceleration of $9 \times 9.81\text{ms}^{-2}$. Calculate the airspeed in m/s if it executes a '9g' turn with a radius of 100m.

(viii) In athletics, the Men's hammer is a ball of mass 7.26kg attached to a wire of length 121.3cm. Assuming the wire handle is 70cm from the axis of rotation of the athlete, calculate the wire tension in N if the hammer completes one rotation in 0.99s. Although the hammer will certainly be released at around 45° elevation, assume it follows a horizontal circular path at a particular instant.

(ix) A spacecraft consisting of a hollow hoop rotating at angular speed ω can simulate the effect of gravity on its occupants. If the reaction force per unit mass on an astronaut standing in the spacecraft is 9.81N/kg, calculate the hoop radius r of the spacecraft if it rotates at 3.00RPM.

(x) In the Bohr model of Hydrogen, an electron 'sort-of-orbits' a proton nucleus at a radius of $r = 5.29 \times 10^{-11}\text{m}$ at a speed of about $\frac{1}{137}$ times the speed of light $c = 2.998 \times 10^{8}\text{ms}^{-1}$. Calculate the force (in N) on the electron exerted by the proton. The mass of an electron is $m = 9.109 \times 10^{-31}\text{kg}$.

(xi) In aeronautics, a standard rate turn is accomplished at $3.00^{\circ}/\text{s}$. An Airbus A380 has a top speed of 1,185km/h, and executes a slow turn that corresponds to a centripetal acceleration of '0.1g'. Calculate the radius of the turn in km, and how what % of a 'standard rate turn' this corresponds to. What would be the radius for a '0.1g' standard rate turn, and what would be the airspeed in km/h?

Question2 Calculate the rotational period of the Earth (in hours) if the normal contact force at the equator is half that at the poles. Assume $g = 9.81\text{N/kg}$ at both locations, modeling the Earth as a perfect sphere of radius $r = 6370\text{km}$.

Question 3 (This assumes knowledge of static friction and inclined planes)

Professor Parker (of mass $m = 80\text{kg}$) has a bet with a climbing friend that he can scale a featureless steel wall. He proposes to win the bet by placing himself against the interior wall of a giant horizontally rotating drum. When the drum attains angular speed ω, Prof. Parker should find himself stuck to the wall even when he pushes himself off the floor of the drum. To enter the drum, Prof. Parker slides down a ramp made from the same material as the drum. He finds that he can just slide when the ramp is at angle $\theta = 42^{\circ}$.

(i) Determine an expression in terms of drum radius r for the minimum angular speed ω that causes Prof. Parker to be stuck to the vertical walls of the drum when he is not in contact with the floor. Plot a graph of ω (in RPM) vs r over the range $3.0\text{m} \le r \le 10.0\text{m}$

(ii) A rigid radial rod connects the outer edge of the drum to the central axle. Prof. Parker hauls himself from $r = 5.0\text{m}$ to $r = 0.5\text{m}$. Assume ω is the minimum angular speed corresponding to $r = 5.0\text{m}$ and remains constant. Calculate how much energy must dissipated in making this journey.

Question 4 The *Sky Rider* attraction at Lightwater Valley in Ripon, Yorkshire involves being seated in a chair dangled from a rotating disc via a chain, which can be considered to be of negligible mass and inextensible. The chain attachment is at radius r from the axis of rotation and the chain has length l. If the ride rotates at angular speed ω the chairs fly out at an angle of θ to the vertical.

(i) Draw a diagram of the system, apply Newton II in vertical and horizontal directions and hence show that:

$$\omega = \sqrt{\frac{g \tan\theta}{r + l\sin\theta}}$$

(ii) If $\theta = 30^{\circ}$, $l = 4.0\text{m}$, and the rotation speed is $v = 5\text{ms}^{-1}$, calculate the disc radius r (in m).

(iii) If a man of mass 100kg is the passenger, use the parameters in (ii) to calculate the tension (in N) in the chain.

Question 5 (This assumes knowledge of projectile motion as well as circular motion)

The Dukes of Hazzard was an American TV series which ran between 1979 to 1985. A signature feature was a series of outrageous jumps performed by an orange Dodge Charger stock car called *The General Lee*. The car is travelling at constant speed v and comes to a bridge of radius of curvature r. The initial gradient of the bridge is $\tan\theta$.

(i) Determine an expression for the normal reaction force R on the car, and hence work out an expression (in terms of v, gravity strength g and θ) for the largest r which results in the car becoming airborne.

(ii) *The General Lee* is often shown to clear rivers, fly through trees, and jump over police cars. If the car hits a bump of height h and horizontal width $2w$, show that $v > \sqrt{\frac{(w^2 - h^2)g}{2h}}$ to get airborne, and the launch angle from horizontal is $\theta = \sin^{-1}\left(\frac{2wh}{w^2 + h^2}\right)$. Also, to clear the bump $v > \sqrt{\frac{(w^2 + h^2)^2 g}{2h(w^2 - h^2)}}$ and $v > \sqrt{\frac{(w^2 + h^2)g}{w}}$. Hence show for $h = 1.0\text{m}$ and $w = 2.0\text{m}$, the lowest speed to 'jump the bump' is 6.39m/s. How far does the car jump at 25m/s (55.9mph)?

Question 6 Dr French rides a water flume consisting of a tube with a cylindrical cross section of radius $a = 0.5\text{m}$. He descends from height h (initially at rest) and then takes a turn of radius $r = 5.0\text{m}$. If the water effectively provides perfect lubrication (i.e. negligible friction) between Dr French and the interior wall of the flume, determine an expression for h as a function of the angle θ he rises up the tube wall from the vertical. Sketch this, and evaluate at $\theta = 60^{\circ}$.