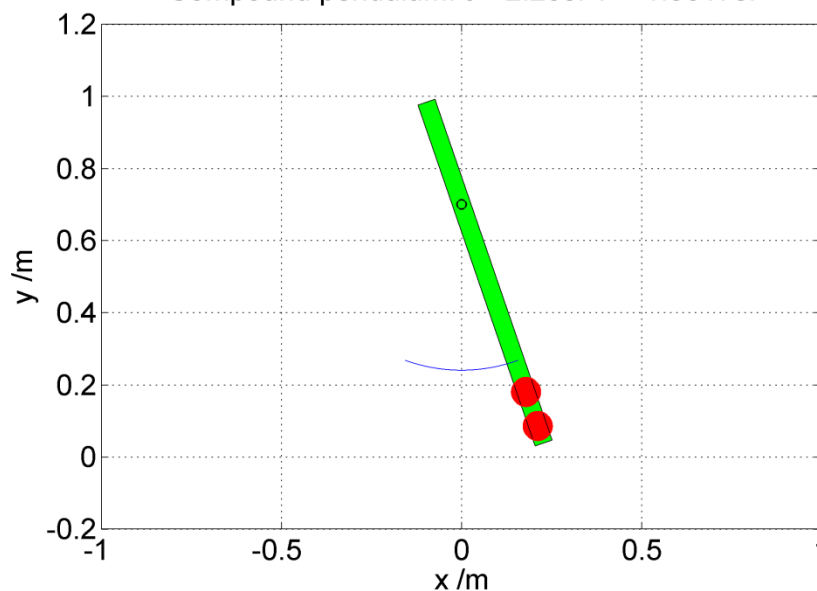


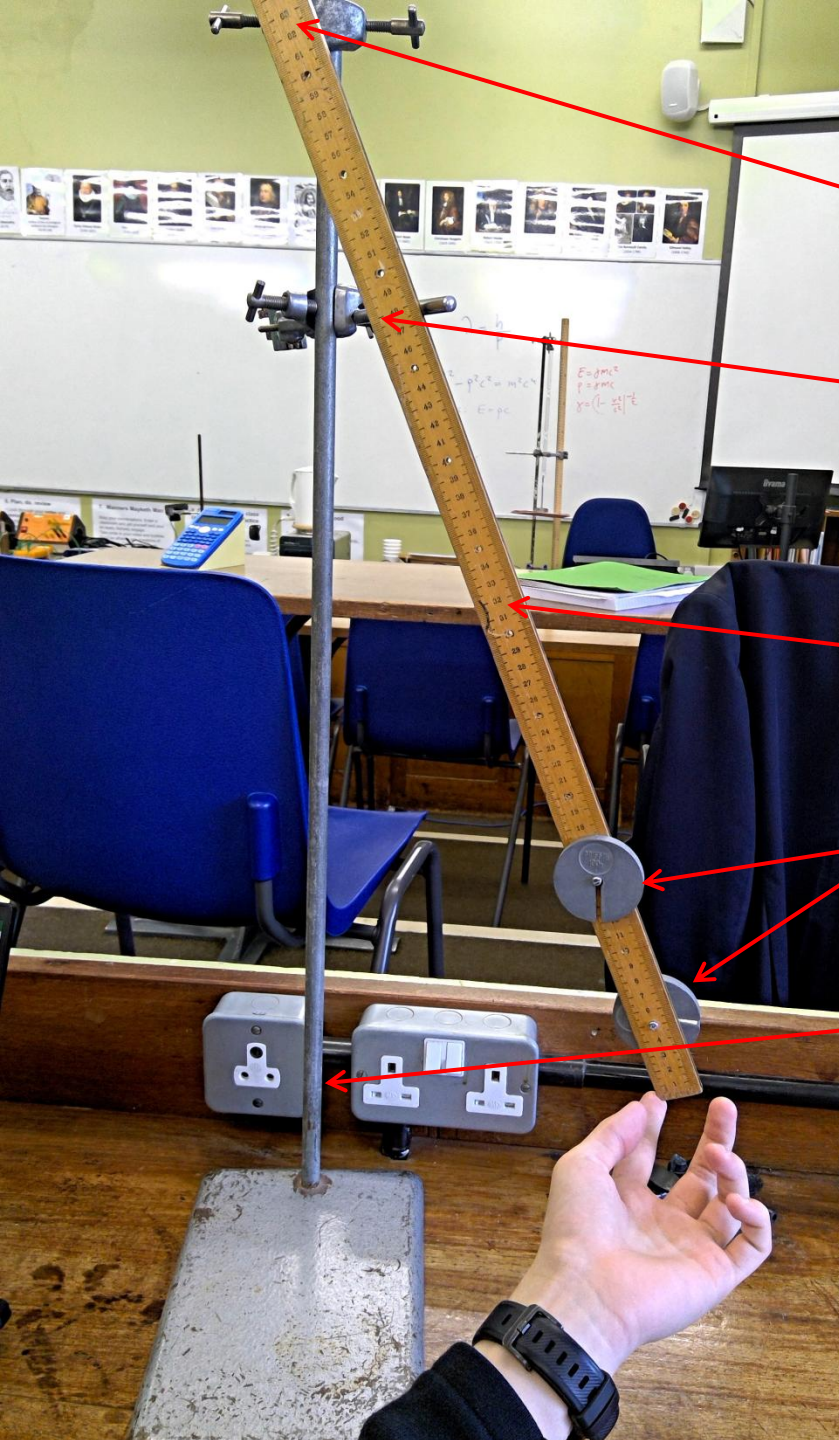
TEACHER NOTES

Compound pendulum



Compound pendulum. $t = 2.28\text{s}$. $T = 1.5617\text{s}$.





Equipment

Boss, clamp and thin metal rod pivot

Boss, clamp and thick metal rod to ensure initial amplitude of all oscillations are the same

1.00m ruler with holes drilled every 5cm

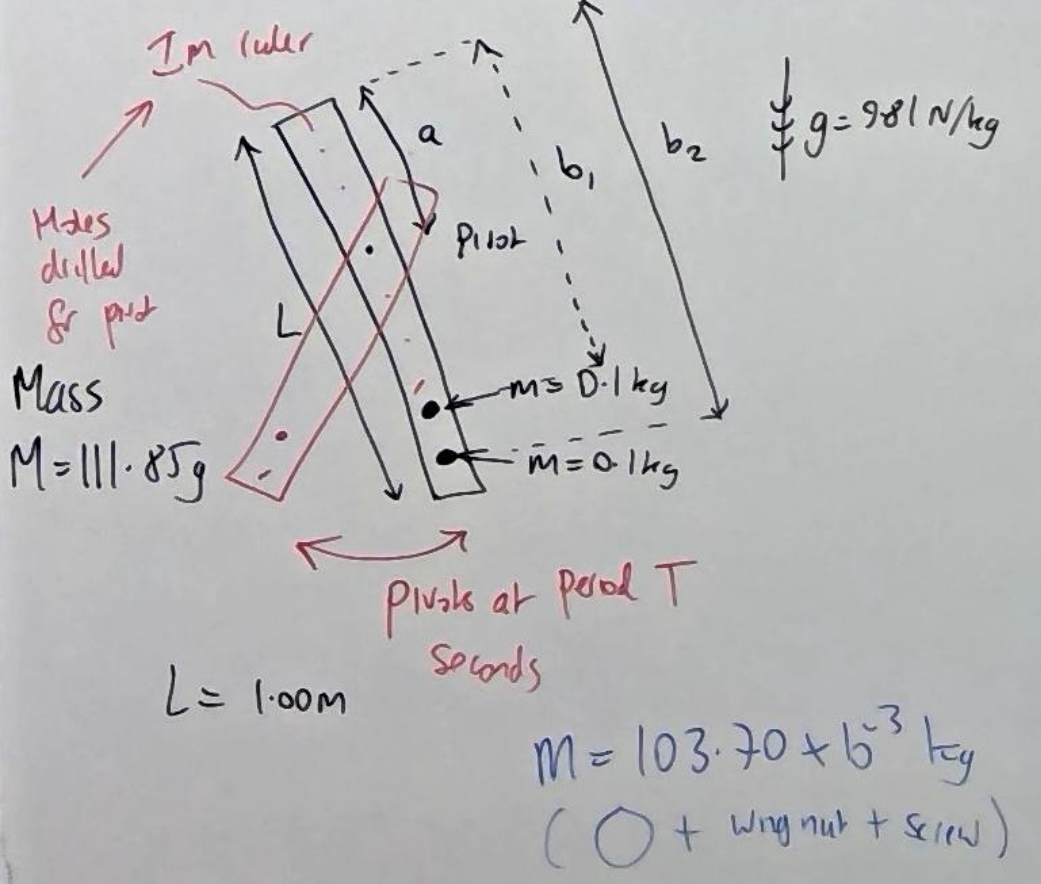
2x 0.1kg masses

Retort stand

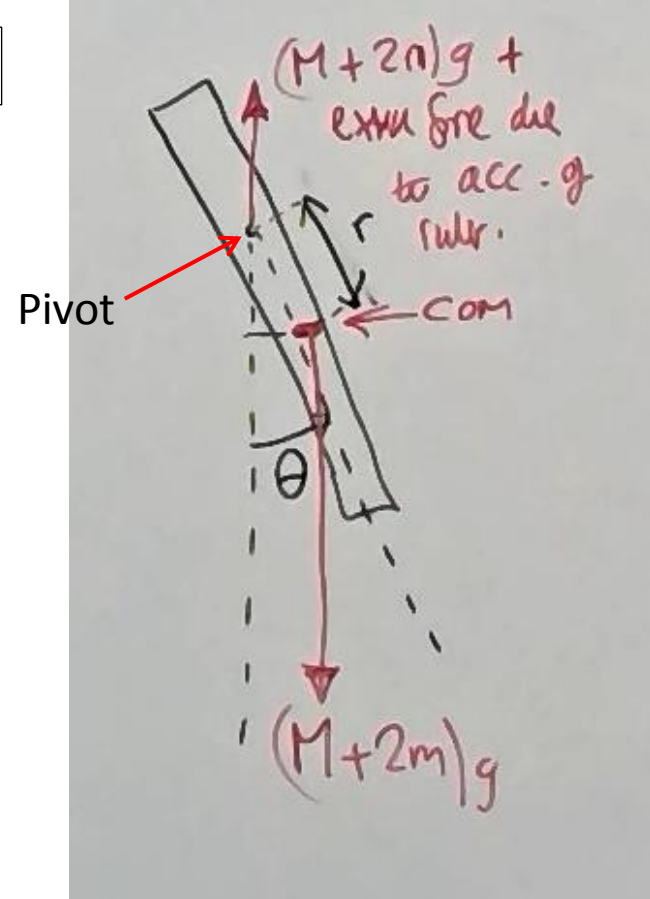
G-clamp for retort stand

Stopwatch

Mass balance



Model



$$I\ddot{\theta} = - \underbrace{(M+2m)g}_{\text{weight}} \times \underbrace{r \sin \theta}_{\perp \text{ distance}}$$

$$\theta \ll 1 \text{ radian} \Rightarrow \sin \theta \approx \theta$$

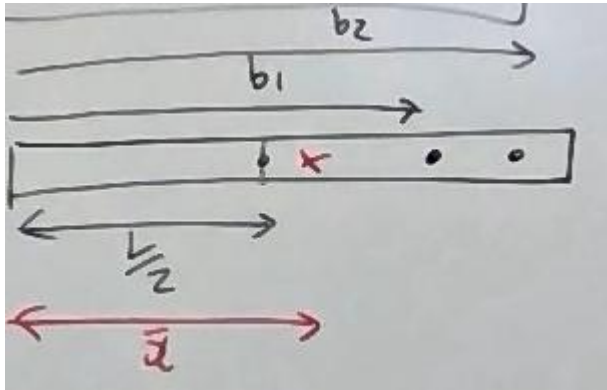
$$\therefore \ddot{\theta} \approx - \frac{(M+2m)g}{I} \theta = - \left(\frac{2\pi}{T} \right)^2 \theta$$

$$\therefore T = 2\pi \sqrt{\frac{I}{(M+2m)gr}}$$

Simple Harmonic Motion (SHM) $\theta = \theta_0 \cos(2\pi t / T)$

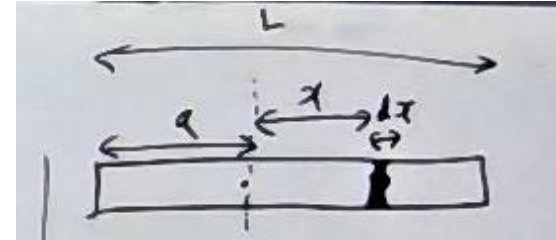
Period /s

Centre of mass



$$\bar{x} = \frac{M \times \frac{1}{2}L + mb_1 + mb_2}{M + 2m}$$

Moment of inertia



$$I = \int x^2 dm$$

$$dm = \frac{dx}{L} M$$

$$I = \frac{M}{L} \int_{-a}^{L-a} x^2 dx = \frac{M}{L} \frac{1}{3} \left[x^3 \right]_{-a}^{L-a}$$

$$I = \frac{M}{3L} \left((L-a)^3 - (-a)^3 \right)$$

$$I = \frac{M}{3L} \left((L-a)^3 + a^3 \right)$$

Compound pendulum model summary

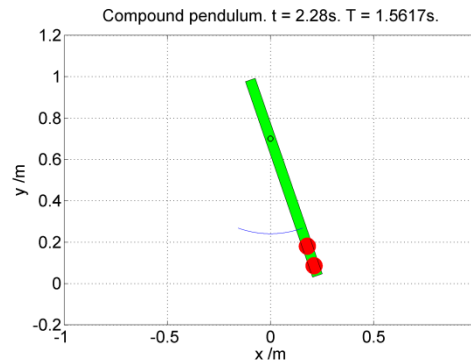
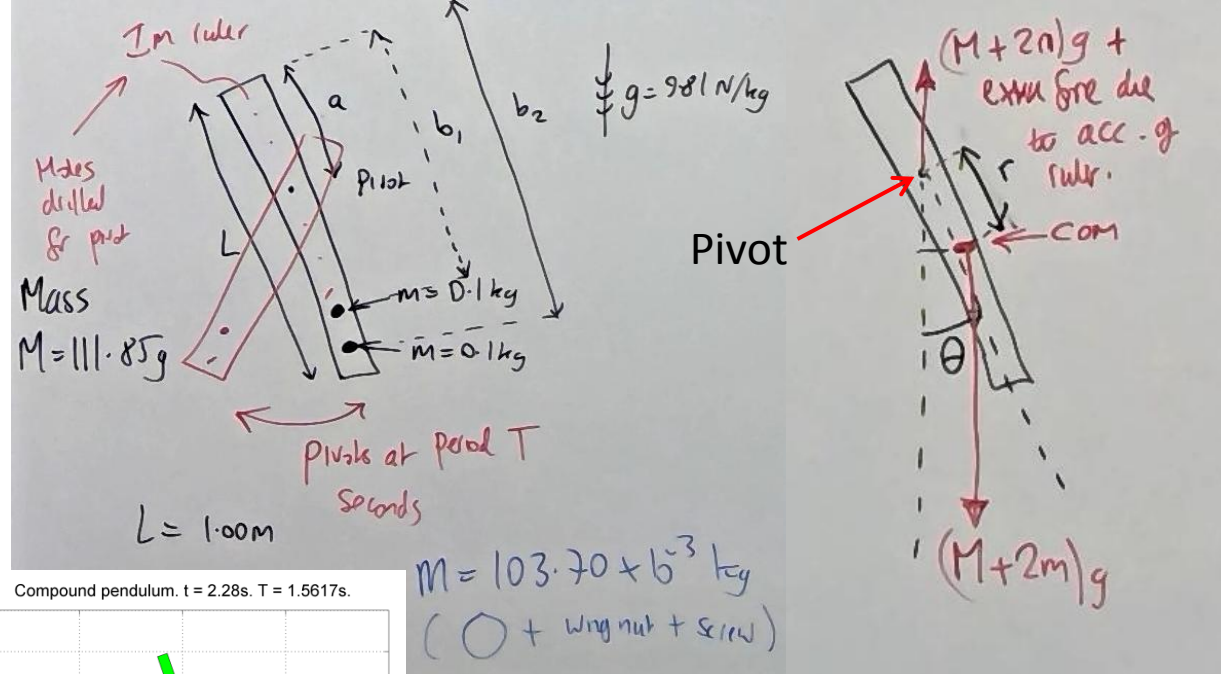
$$\theta = \theta_0 \cos\left(\frac{2\pi t}{T}\right)$$

$$T = 2\pi \sqrt{\frac{I}{(M + 2m)gr}}$$

$$I = \frac{M}{3L} \left((L - a)^3 + a^3 \right)$$

$$\bar{x} = \frac{M \times \frac{1}{2}L + mb_1 + mb_2}{M + 2m}$$

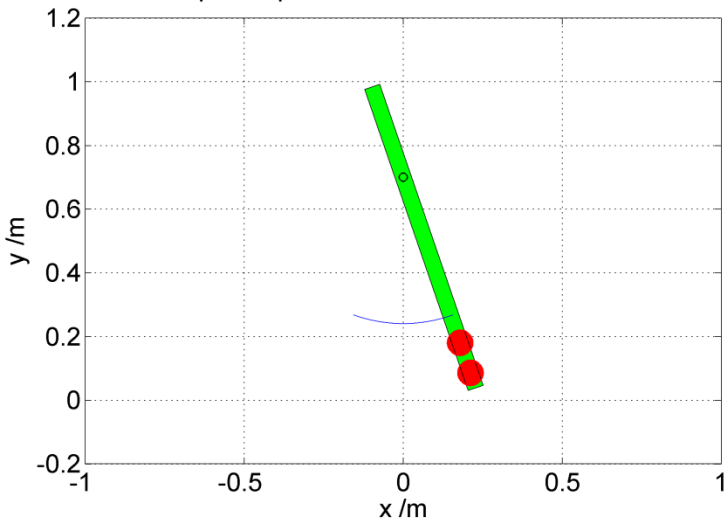
$$r = \bar{x} - a$$



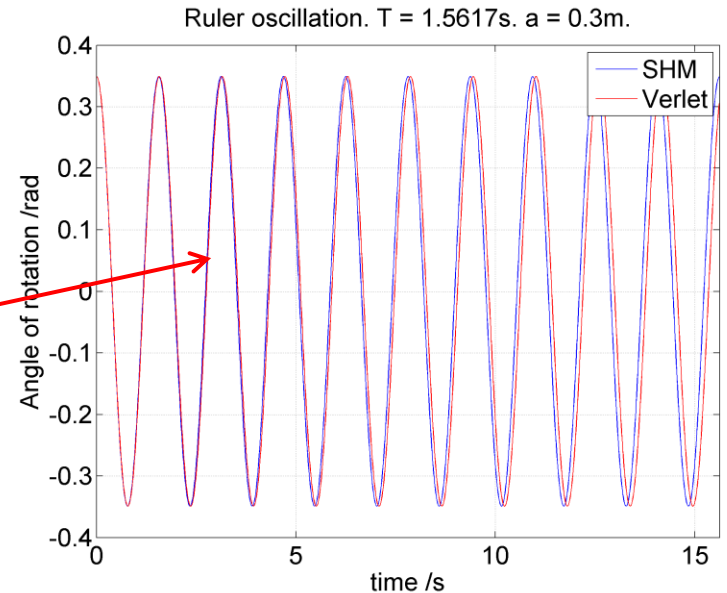
Period measurement					Period	Pivot to COM / m	Moment of inertia	Model period / s
Ruler pivot position / cm	a / m	10T / s			T / s	r / m	I / Nm	$2\pi \sqrt{I / ((M+2m)gr)}$
99	0.01	18.39	18.28	18.36	1.83	0.750	2.010E-01	1.8381
95	0.05	17.72	17.84	17.98	1.78	0.710	1.823E-01	1.7994
90	0.1	17.27	17.57	17.34	1.74	0.660	1.605E-01	1.7509
85	0.15	16.91	16.99	16.67	1.69	0.610	1.402E-01	1.7023
80	0.2	16.59	16.61	16.52	1.66	0.560	1.215E-01	1.6542
75	0.25	16.01	16.11	16.13	1.608333	0.510	1.045E-01	1.6070
70	0.3	15.63	15.73	15.59	1.565	0.460	8.898E-02	1.5617
65	0.35	15.26	15.28	15.24	1.526	0.410	7.509E-02	1.5197
60	0.4	14.78	14.85	14.82	1.481667	0.360	6.281E-02	1.4833
55	0.45	14.3	14.5	14.5	1.443333	0.310	5.212E-02	1.4561
50	0.5	14.11	14.26	14.28	1.421667	0.260	4.302E-02	1.4447

Ruler mass	M / kg	0.11185
Added mass	m / kg	0.1037
Added mass position	b1 / m	0.85
	b2 / m	0.95
Strength of gravity	g / N/kg ⁻¹	9.81
Ruler length	L / m	1
Centre of mass	xbar / m	0.760

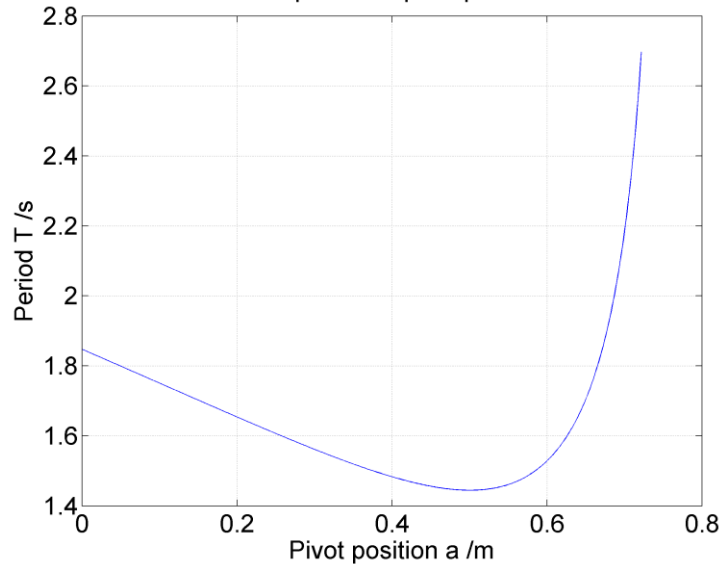
Compound pendulum. $t = 2.28\text{s}$. $T = 1.5617\text{s}$.



Verlet simulation
does not use the
small angle
approximation
(but a fixed
timestep)



SHM period vs pivot position



$g = 9.81$; %Strength of gravity /Nkg⁻¹

$L = 1.00$; %Length of ruler /m

$W = L/20$; %Width of ruler in m

$M = 0.11185$; %Ruler mass /kg

$m = 0.1037$; %Added mass (there are two of these added)

%Added mass positions from end of ruler

$b1 = 0.85$; $b2 = 0.95$;

$dt = 0.01$; %Timestep /s for simulation

$N = 10$; %Number of periods for simulation

$fsz = 18$; %Fontsize for plotting

%Initial clockwise angular deviation of ruler

$\theta_0 = 20 \cdot \pi / 180$;

$a = 0.3$; %Position of pivot from top of ruler

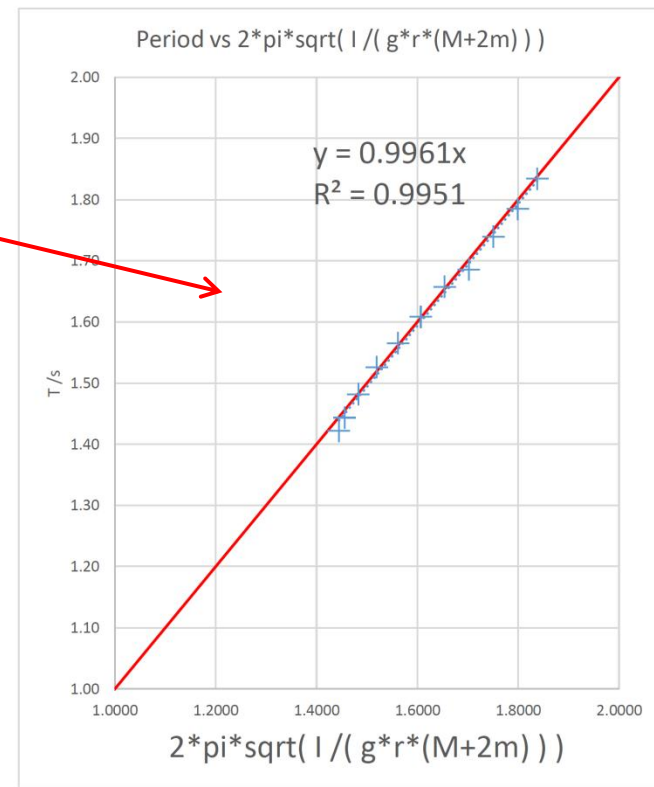
MATLAB model evaluation

Three repeats
of ten period
timings

Ruler mass	M /kg	0.11185
Added mass	m /kg	0.1037
Added mass position	b1 /m	0.85
	b2 /m	0.95
Strength of gravity	g /Nkg ⁻¹	9.81
Ruler length	L /m	1
Centre of mass	xbar /m	0.760

Analysis

Plot measured period
against predicted
period



Ruler pivot position /cm	a /m	Period measurement			Period T /s	Pivot to COM /m	Moment of inertia I /Nm	Model period /s
		10T /s						
99	0.01	18.39	18.28	18.36	1.83	0.750	2.010E-01	1.8381
95	0.05	17.72	17.84	17.98	1.78	0.710	1.823E-01	1.7994
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50	0.5	14.11	14.26	14.28	1.421667	0.260	4.302E-02	1.4447

Model

T /s	$2\pi\sqrt{I/(M+2m)gr}$
0.00	0
0.20	0.2
0.80	0.8
1.40	1.4
2.00	2
2.60	2.6
3.20	3.2
3.80	3.8

a is ruler
end to pivot (in
metres)

Handwritten notes and diagrams illustrating the analysis of a compound pendulum.

Diagram 1: Shows a ruler of length L pivoted at a distance a from the end. The center of mass is at distance x-bar from the pivot. The added mass m is at distance b1 and b2 from the pivot. The ruler mass is M. The diagram shows the ruler at an angle theta from the vertical.

Equations:

- $\ddot{\theta} = -\omega^2 \theta$
- $\ddot{\theta} = -\frac{(M+2m)gr}{I} \theta$
- $I = I_{\text{ruler}} + (M+2m)r^2$
- $T = 2\pi\sqrt{\frac{I}{(M+2m)gr}}$
- $\ddot{x} = \frac{M \times \frac{L}{2} + mb_1 + mb_2}{M+2m}$
- $r = \ddot{x} - a$

Diagram 2: Shows the ruler pivoted at the end (a=0). The center of mass is at distance L/2 from the pivot. The added mass m is at distance b1 and b2 from the pivot. The ruler mass is M. The diagram shows the ruler at an angle theta from the vertical.

Equations:

- $I = I_{\text{ruler}} + (M+2m)r^2$
- $T = 2\pi\sqrt{\frac{I}{(M+2m)gr}}$
- $\ddot{x} = \frac{M \times \frac{L}{2} + mb_1 + mb_2}{M+2m}$
- $r = \ddot{x} - a$