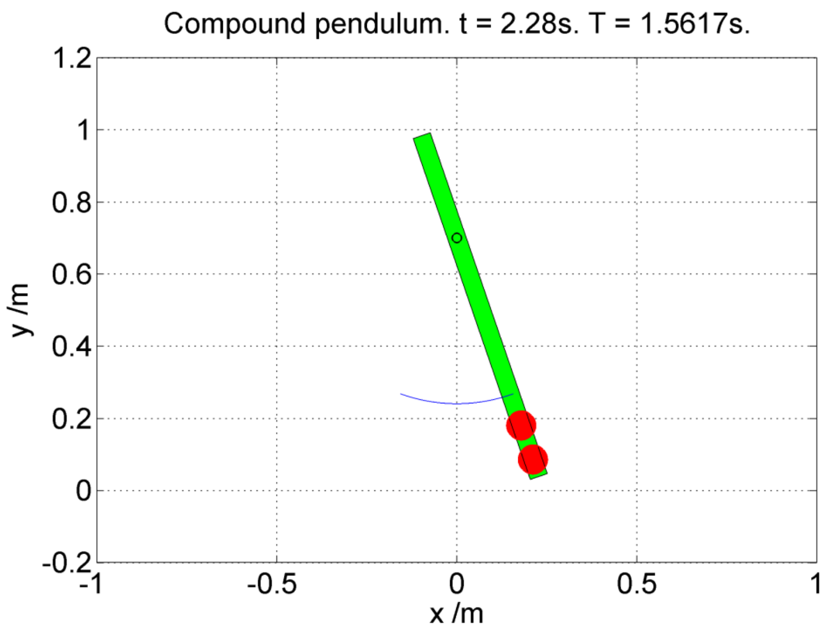
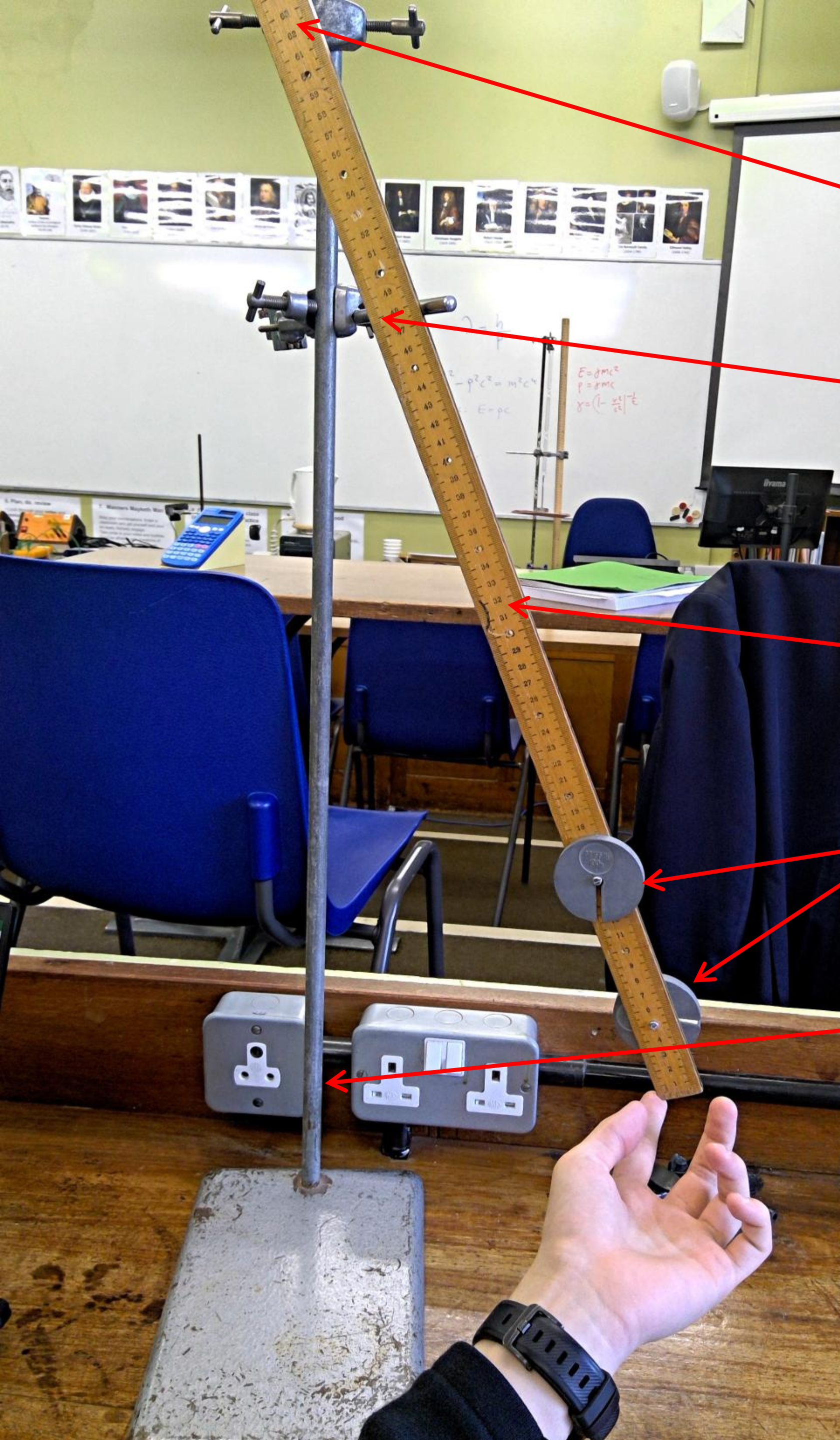
TEACHER NOTES

Compound pendulum



Dr Andrew French. September 2020.

Equipment



Boss, clamp and thin metal rod pivot

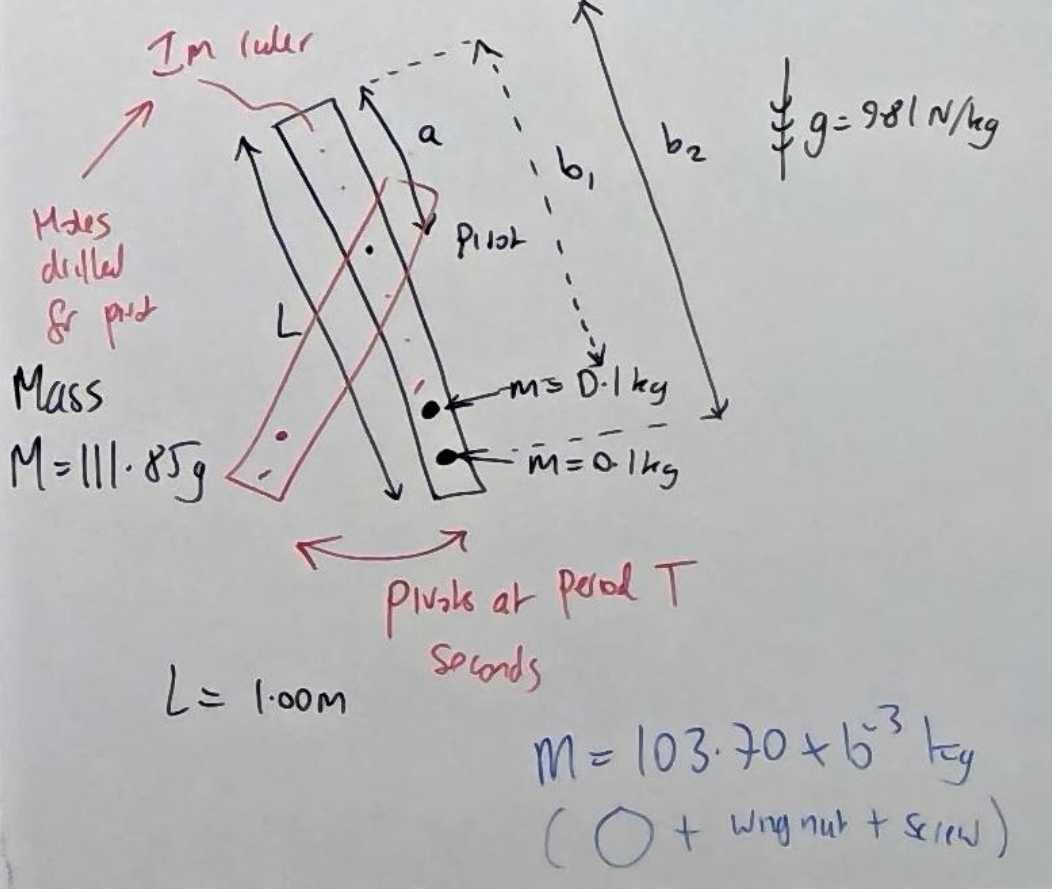
Boss, clamp and thick metal rod to ensure initial amplitude of all oscillations are the same

1.00m ruler with holes drilled every 5cm

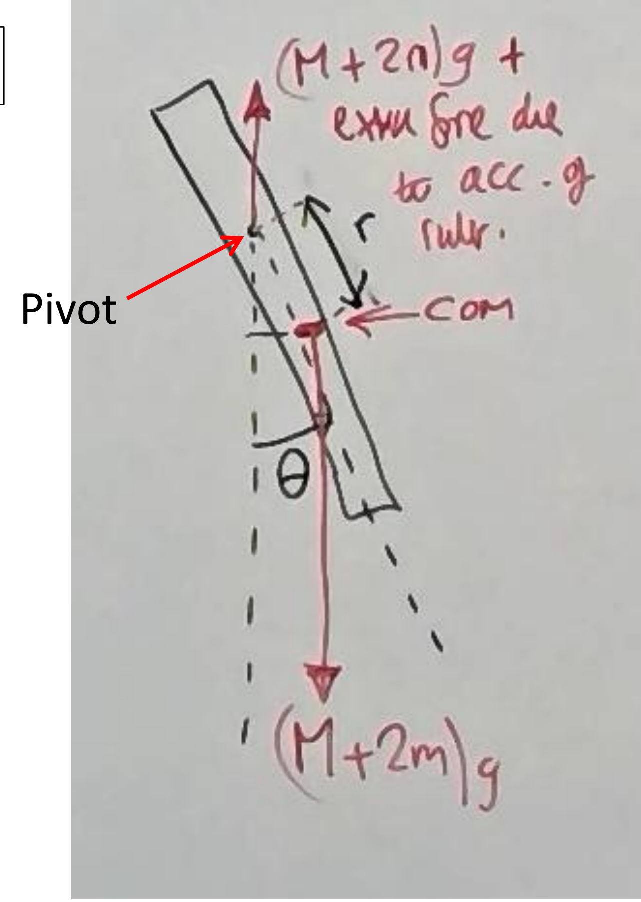
2x 0.1kg masses

Retort stand

G-clamp for retort stand
Stopwatch
Mass balance



Model



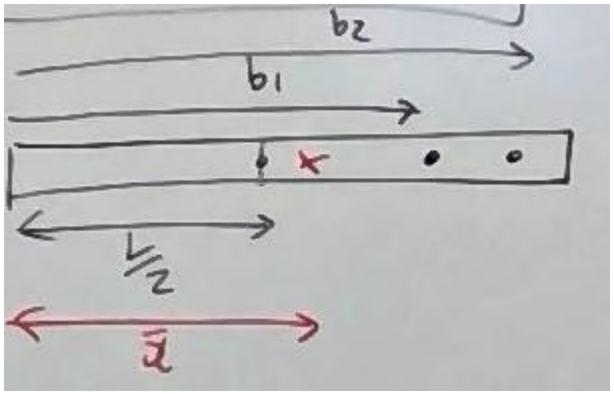
$$I\ddot{\theta} = -\underbrace{(M+2m)g}_{\text{weight}} \times \underbrace{r\sin\theta}_{\perp\text{distance}} \qquad \theta \ll 1 \text{ radian} \Rightarrow \sin\theta \approx \theta$$

$$\therefore \ddot{\theta} \approx -\frac{(M+2m)g}{I}\theta = -\left(\frac{2\pi}{T}\right)^2 \theta \qquad \therefore \boxed{T = 2\pi\sqrt{\frac{I}{(M+2m)gr}}}$$

Period /s

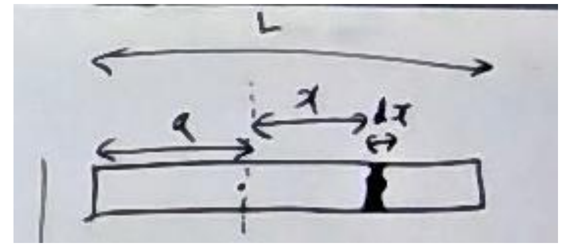
Simple Harmonic Motion (SHM) $\theta = \theta_0 \cos(2\pi t / T)$

Centre of mass



$$\bar{x} = \frac{M \times \frac{1}{2} L + m b_1 + m b_2}{M + 2m}$$

Moment of inertia



$$I = \int x^2 dm$$

$$dm = \frac{dx}{L} M$$

$$I = \frac{M}{L} \int_{-a}^{L-a} x^2 dx = \frac{M}{L} \tfrac{1}{3} \left[x^3 \right]_{-a}^{L-a}$$

$$I = \frac{M}{3L} \left((L-a)^3 - (-a)^3 \right)$$

$$I = \frac{M}{3L} \left((L-a)^3 + a^3 \right)$$

Compound pendulum model summary

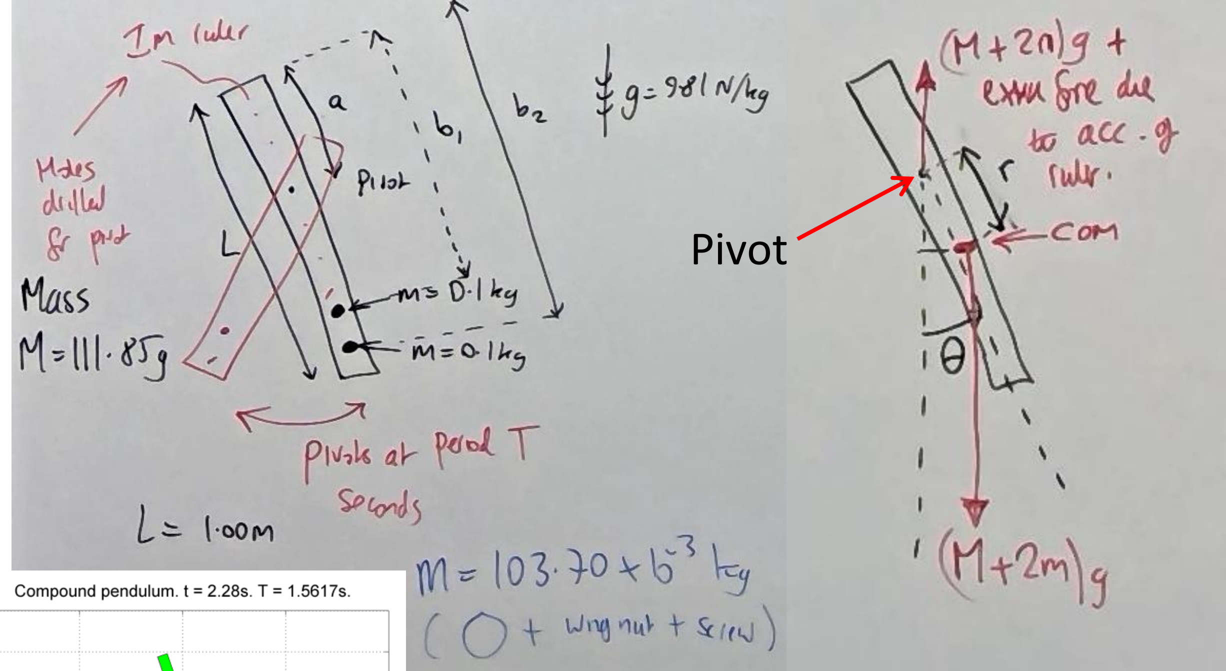
$$\theta = \theta_0 \cos\left(\frac{2\pi t}{T}\right)$$

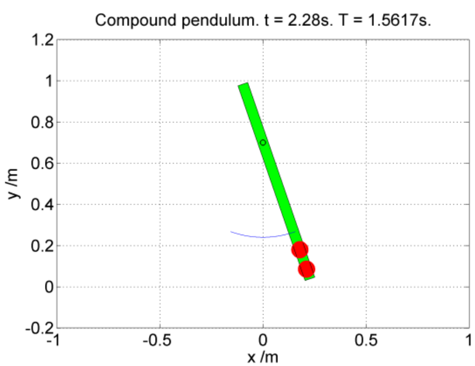
$$T = 2\pi\sqrt{\frac{I}{(M+2m)gr}}$$

$$I = \frac{M}{3L}\left((L-a)^3 + a^3\right)$$

$$\bar{x} = \frac{M \times \frac{1}{2}L + mb_1 + mb_2}{M+2m}$$

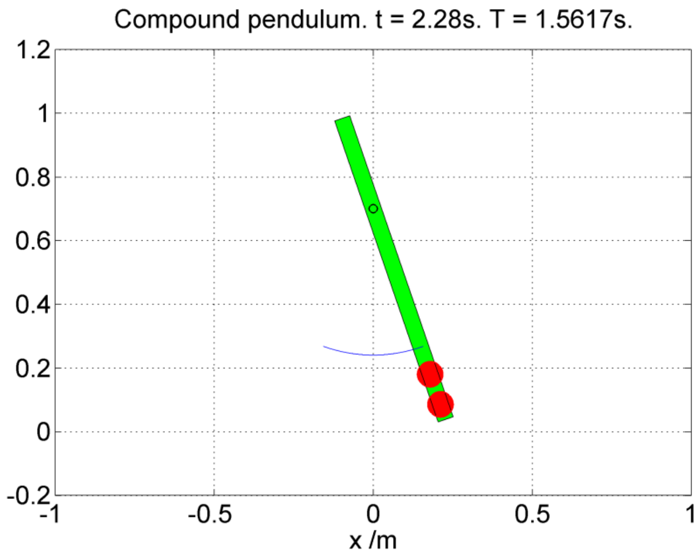
$$r = \bar{x} - a$$



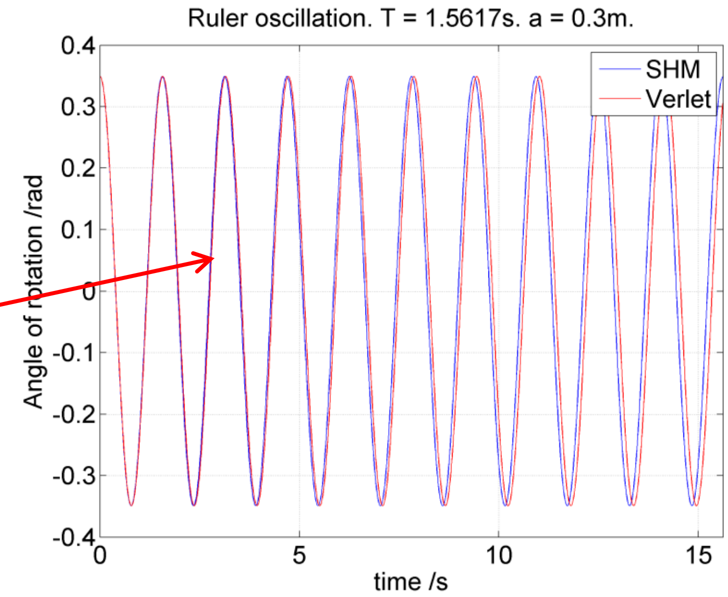


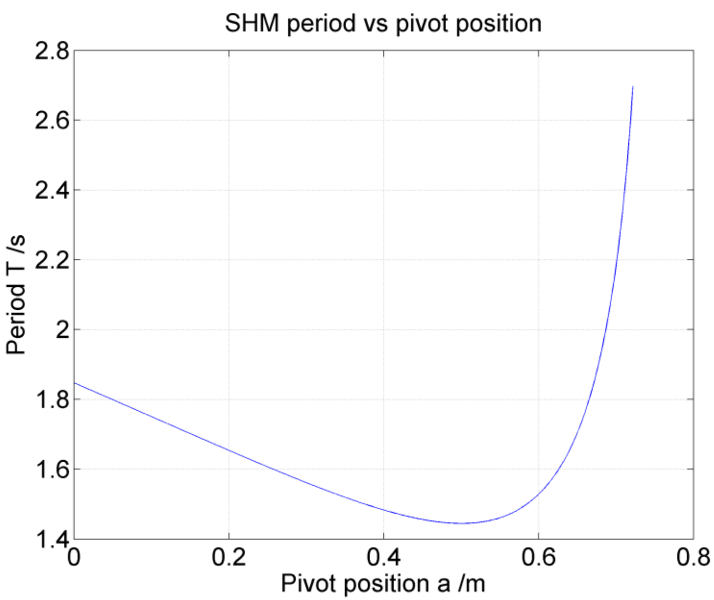
Ruler pivot position /cm	a /m	Period measurement 10T /s			Period T /s	Pivot to COM /m r /m	Moment of inertia I /Nm	Model period /s 2*pi()*sqrt(I /((M+2m)gr))
99	0.01	18.39	18.28	18.36	1.83	0.750	2.010E-01	1.8381
95	0.05	17.72	17.84	17.98	1.78	0.710	1.823E-01	1.7994
90	0.1	17.27	17.57	17.34	1.74	0.660	1.605E-01	1.7509
85	0.15	16.91	16.99	16.67	1.69	0.610	1.402E-01	1.7023
80	0.2	16.59	16.61	16.52	1.66	0.560	1.215E-01	1.6542
75	0.25	16.01	16.11	16.13	1.608333	0.510	1.045E-01	1.6070
70	0.3	15.63	15.73	15.59	1.565	0.460	8.898E-02	1.5617
65	0.35	15.26	15.28	15.24	1.526	0.410	7.509E-02	1.5197
60	0.4	14.78	14.85	14.82	1.481667	0.360	6.281E-02	1.4833
55	0.45	14.3	14.5	14.5	1.443333	0.310	5.212E-02	1.4561
50	0.5	14.11	14.26	14.28	1.421667	0.260	4.302E-02	1.4447

Ruler mass	M /kg	0.11185
Added mass	m /kg	0.1037
Added mass position	b1 /m	0.85
	b2 /m	0.95
Strength of gravity	g /Nkg^-1	9.81
Ruler length	L /m	1
Centre of mass	xbar /m	0.760



Verlet simulation does not use the small angle approximation (but a fixed timestep)






```
g = 9.81; %Strength of gravity /Nkg^-1
L = 1.00;  %Length of ruler /m
W = L/20;  %Width of ruler in m
M = 0.11185;    %Ruler mass /kg
m = 0.1037; %Added mass (there are two of these added)
%Added mass positions from end of ruler
b1 = 0.85; b2 = 0.95;
dt = 0.01; %Timestep /s for simulation
N = 10;  %Number of periods for simulation
fsize = 18;  %Fontsize for plotting

%Initial clockwise angular deviation of ruler
theta0 = 20*pi/180;
a = 0.3;  %Position of pivot from top of ruler
```

MATLAB model evaluation

Compound pendulum

06/06/2019

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	b2 /m	0.95
Strength of gravity	g /Nkg^-1	9.81
Ruler length	L /m	1
Centre of mass	xbar /m	0.760

Analysis

Three repeats of ten period timings

Plot measured period against predicted period

Ruler pivot position /cm	a /m	Period measurement 10T /s			Period T /s	Pivot to COM /m r /m	Moment of inertia I /Nm	Model period /s 2*pi()*sqrt(I /((M+2m)gr))
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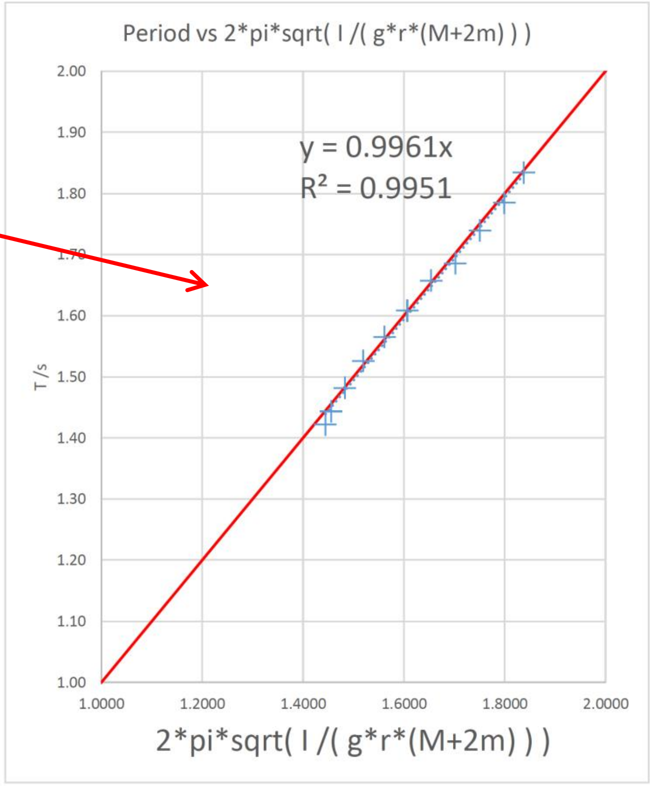

Model

T /s	2*pi*sqrt(I /((M+2m)gr))
0.00	0
0.20	0.2
0.80	0.8
1.40	1.4
2.00	2
2.60	2.6
3.20	3.2
3.80	3.8

a is ruler end to pivot (in metres)