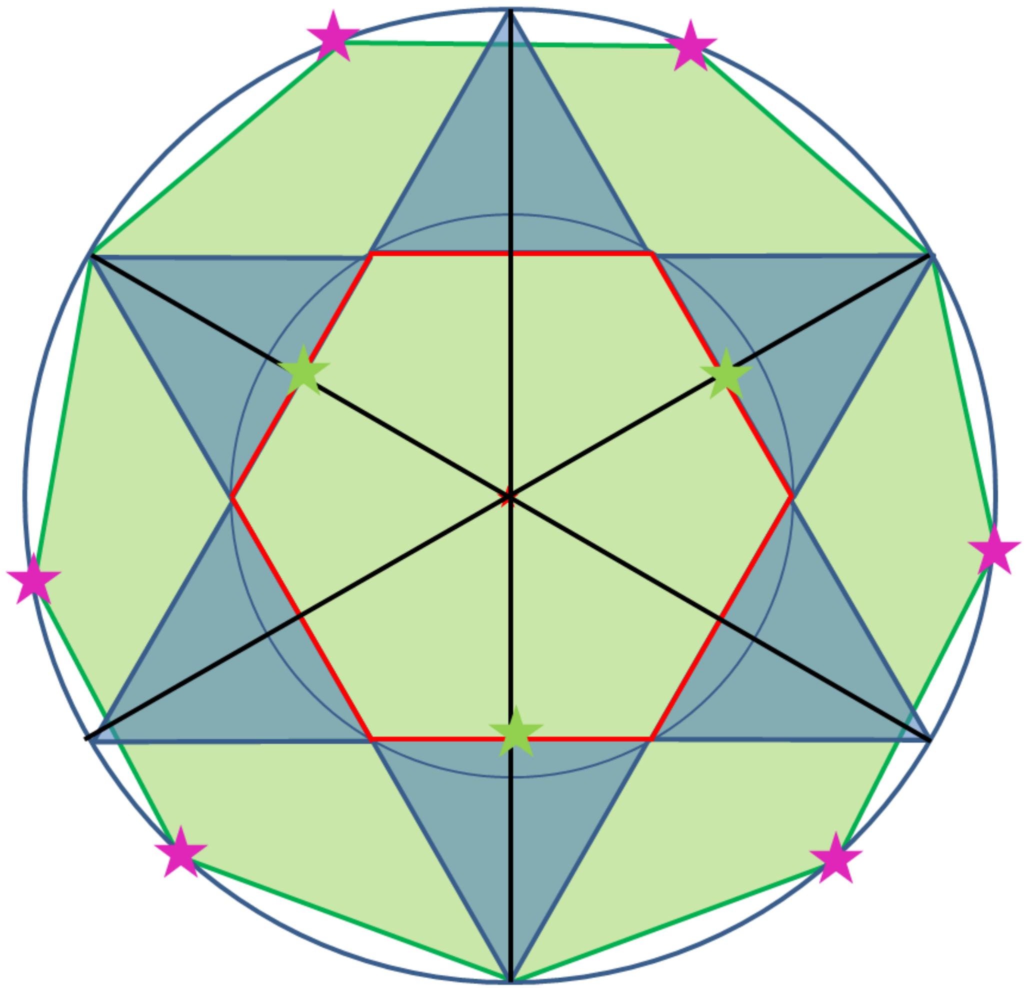
Construction* of a nonagon

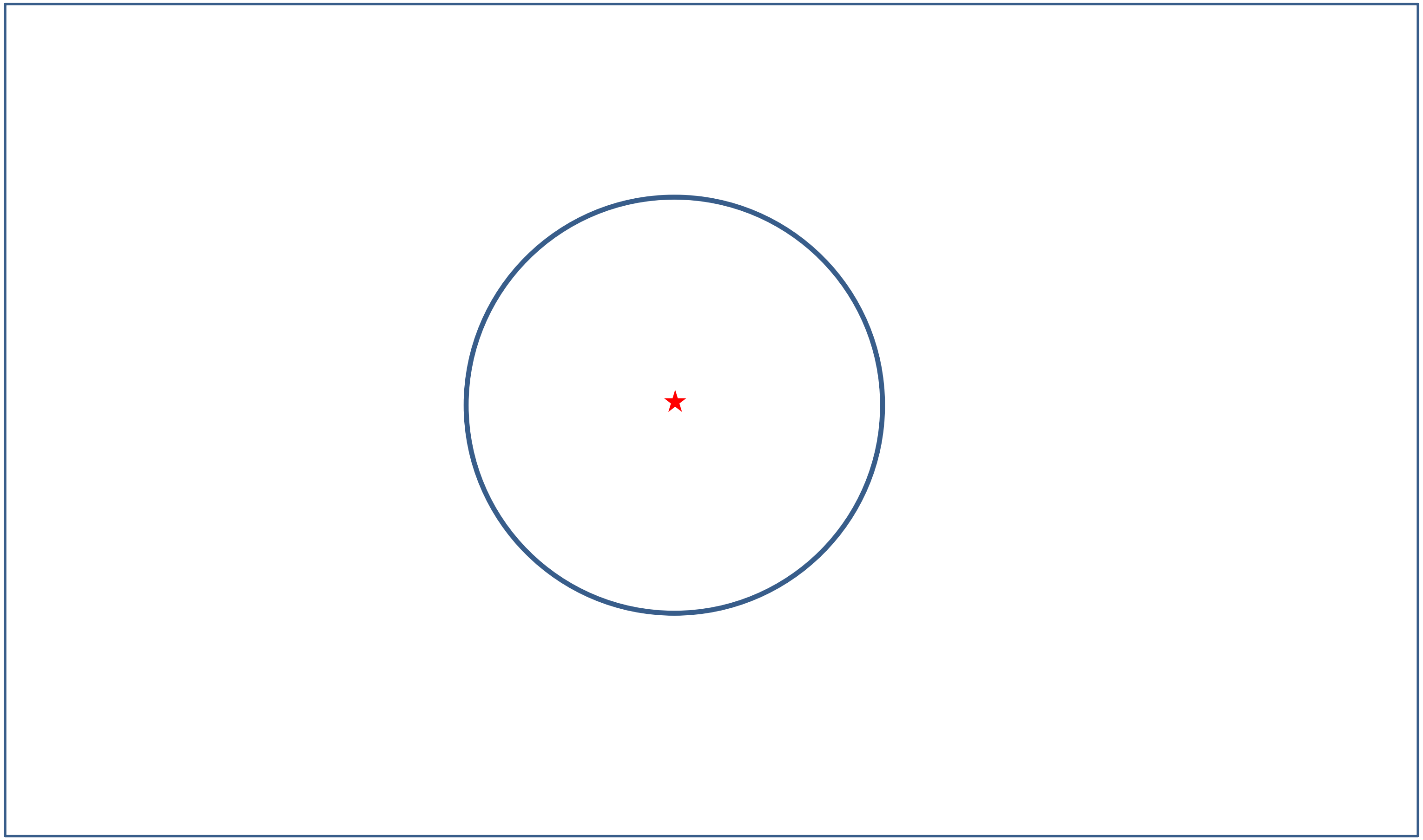
Dr Andrew French



*approximate

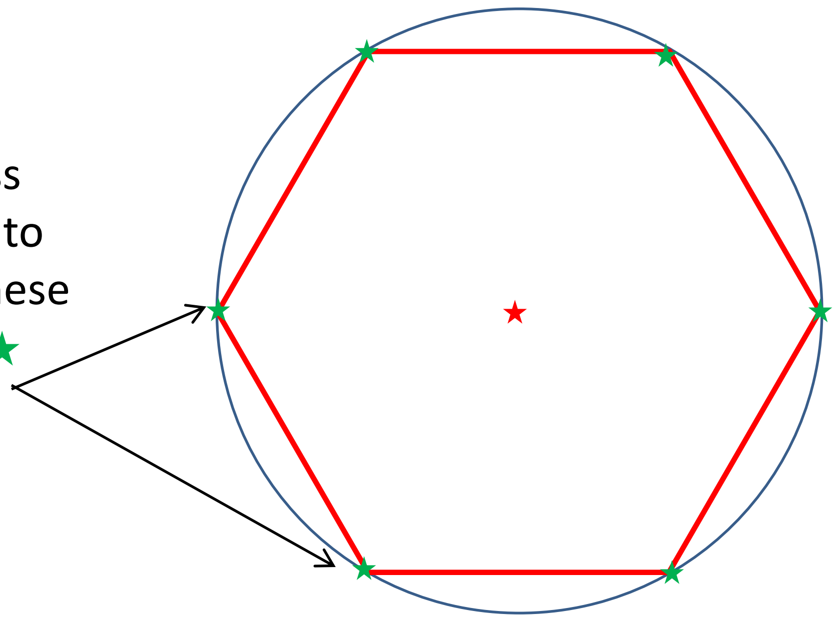
1. Draw a circle using a compass. Make sure the diameter of the circle is about half the length of the smallest dimensions of the paper you are working on.

★ circle centre



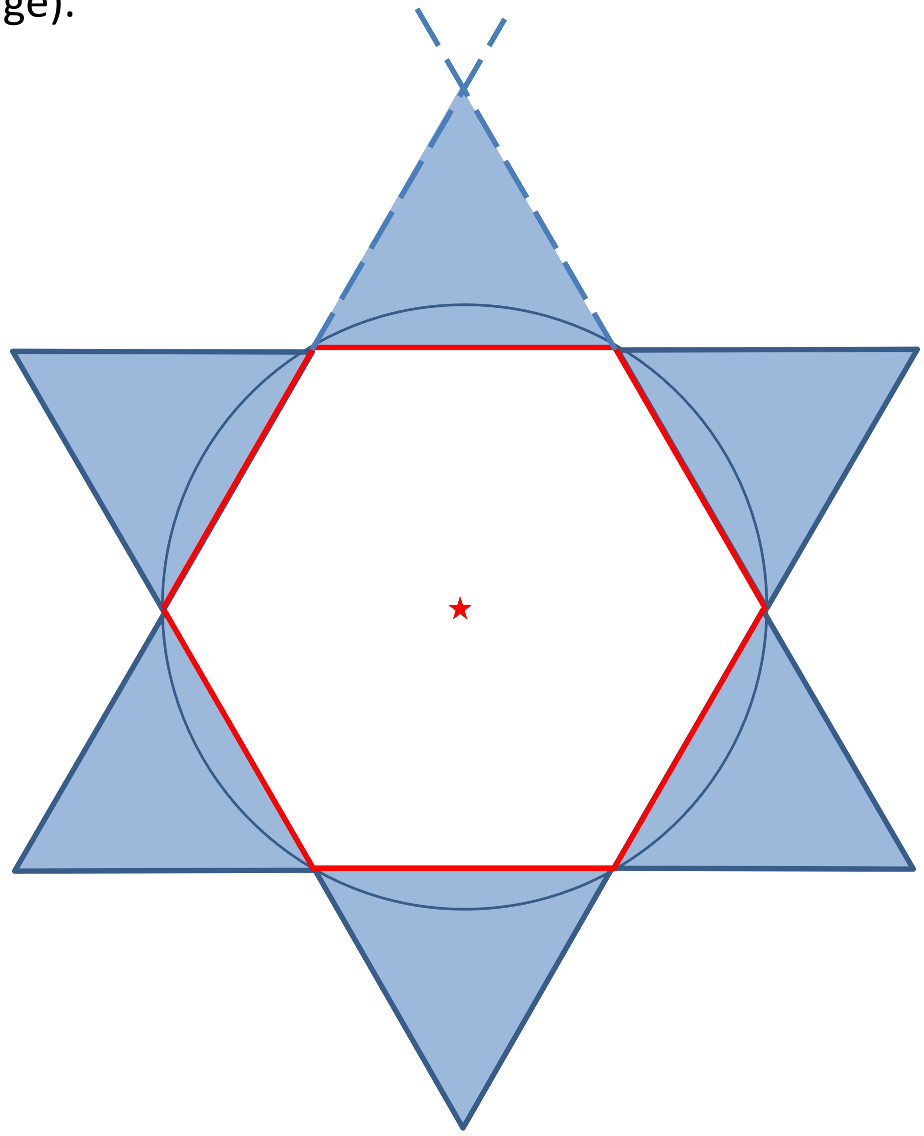
2. Without changing the opening angle of the compass, 'walk' it round the circle and hence construct a regular hexagon.

'Walk' compass around to mark these points ★

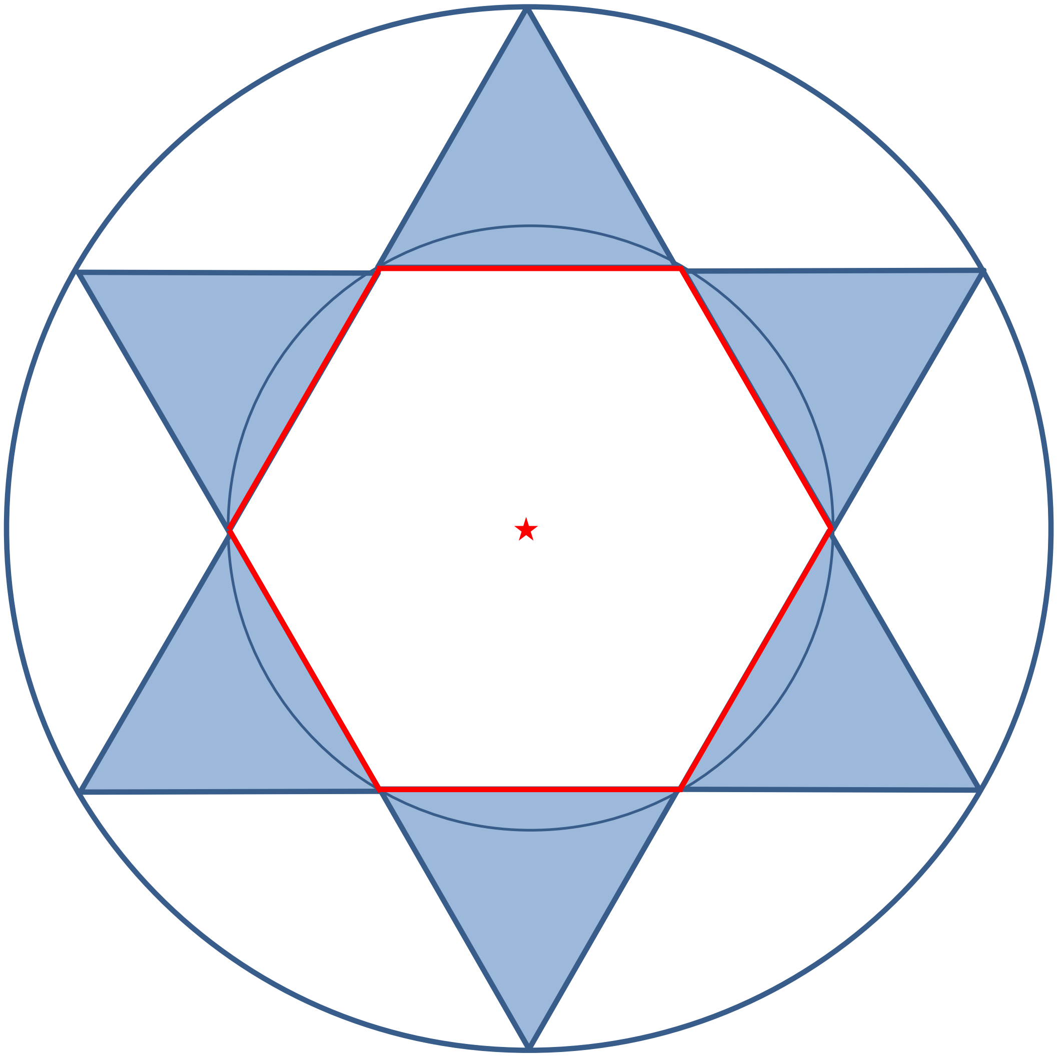


Keep the angle the same as used to draw the original circle

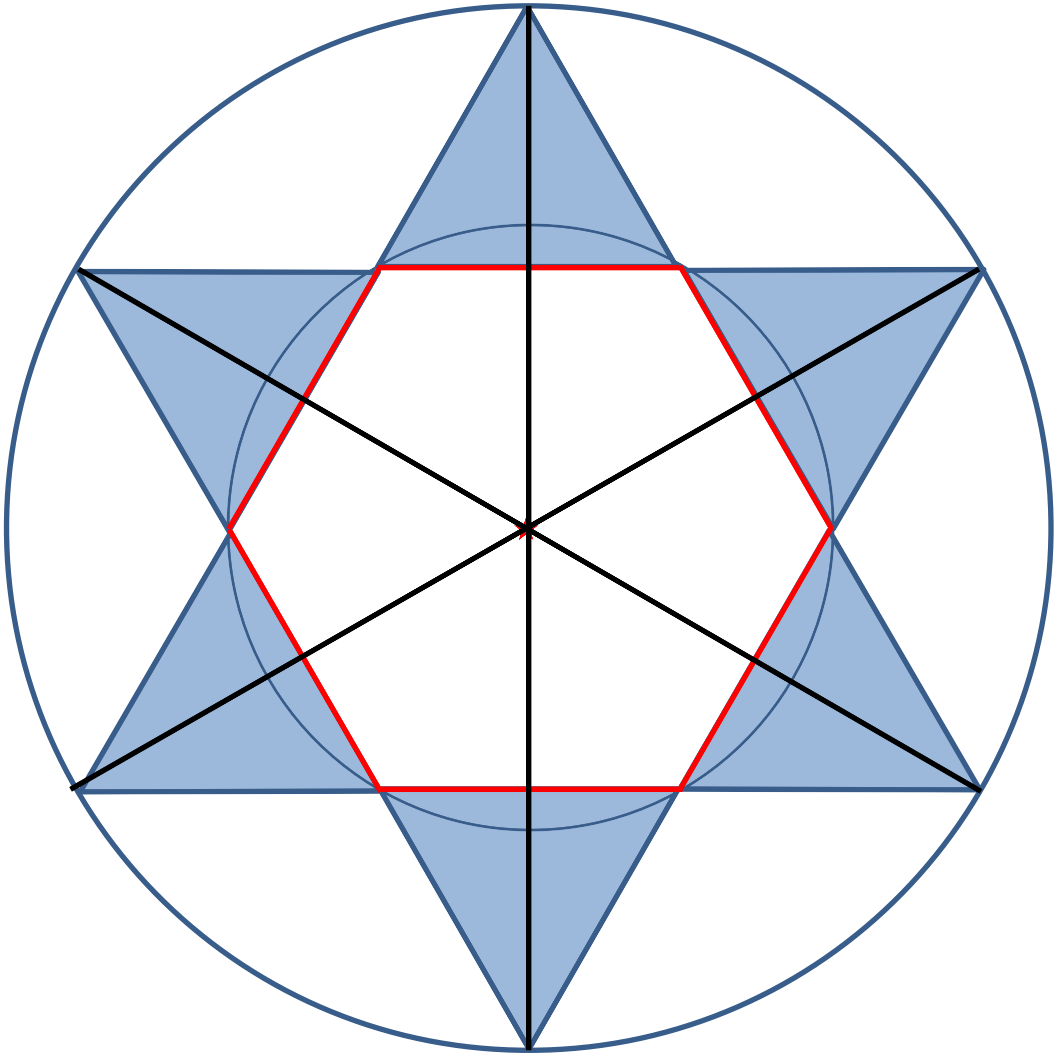
3. Extend all sides of the hexagon until they meet. Connect up these lines to form a *hexagram*. (Alternatively use the compass to construct equilateral triangles on top of each edge).



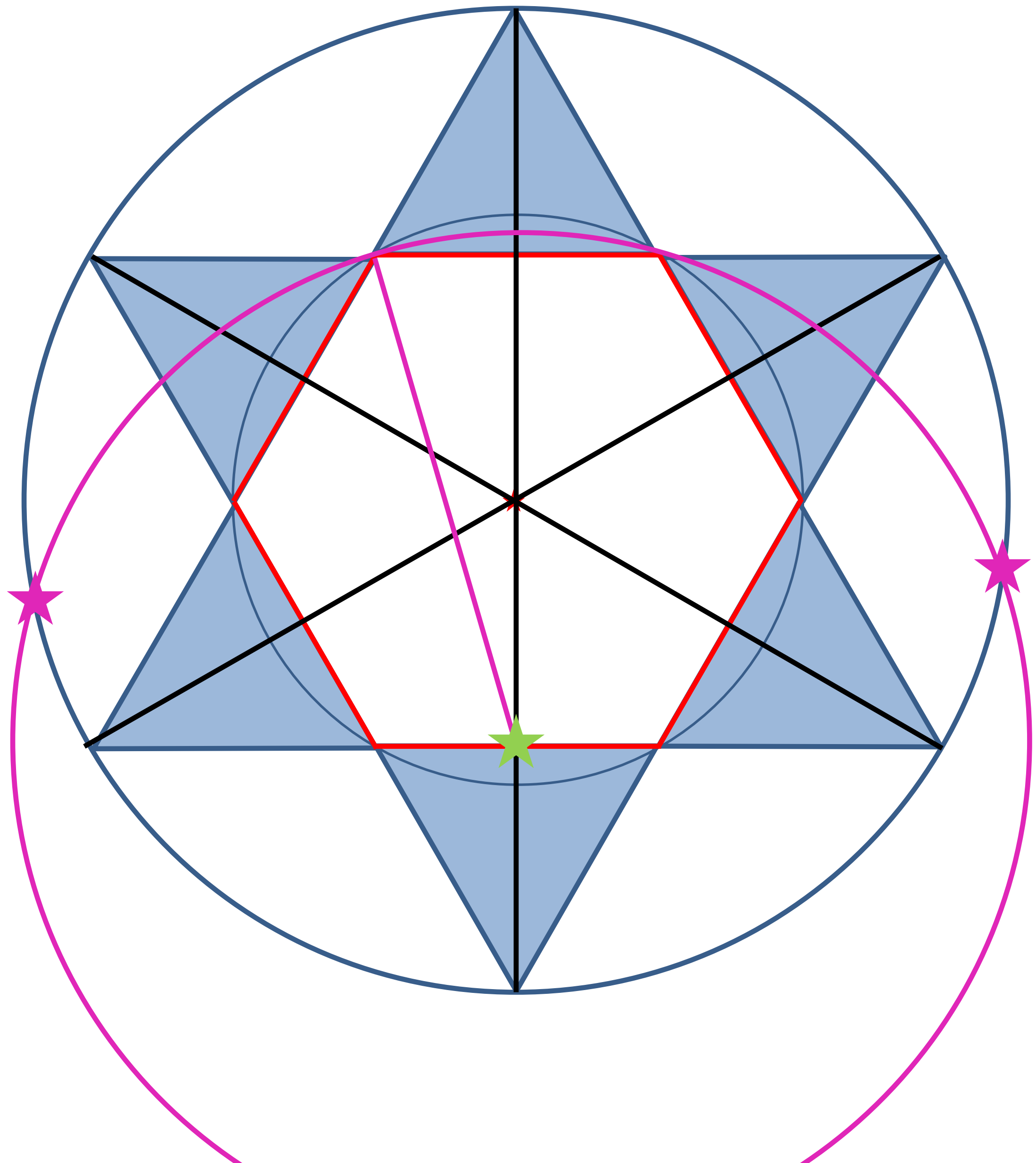
4. Circumscribe the hexagram using a compass

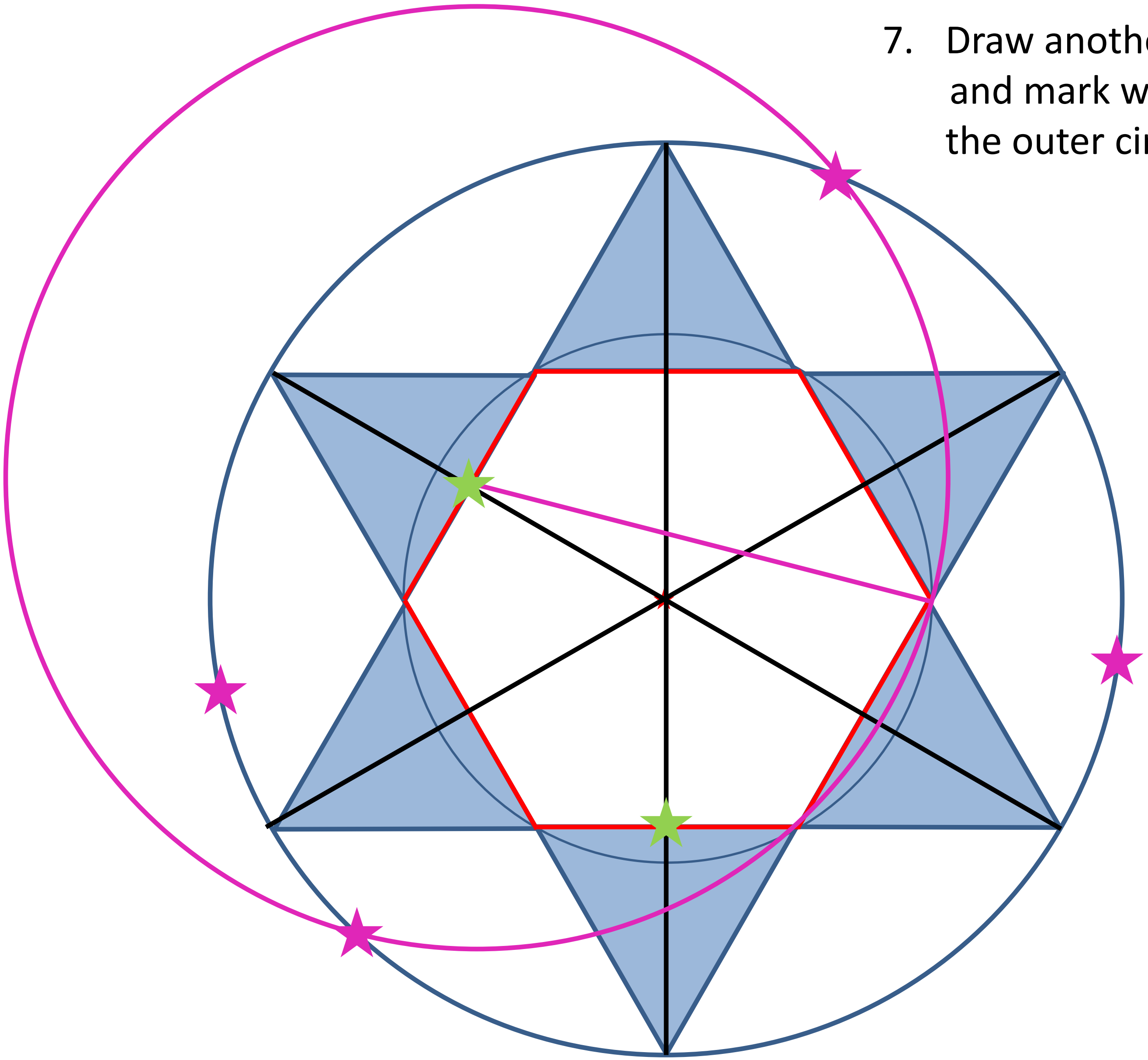


5. Connect outer vertices of hexagram with a straight line

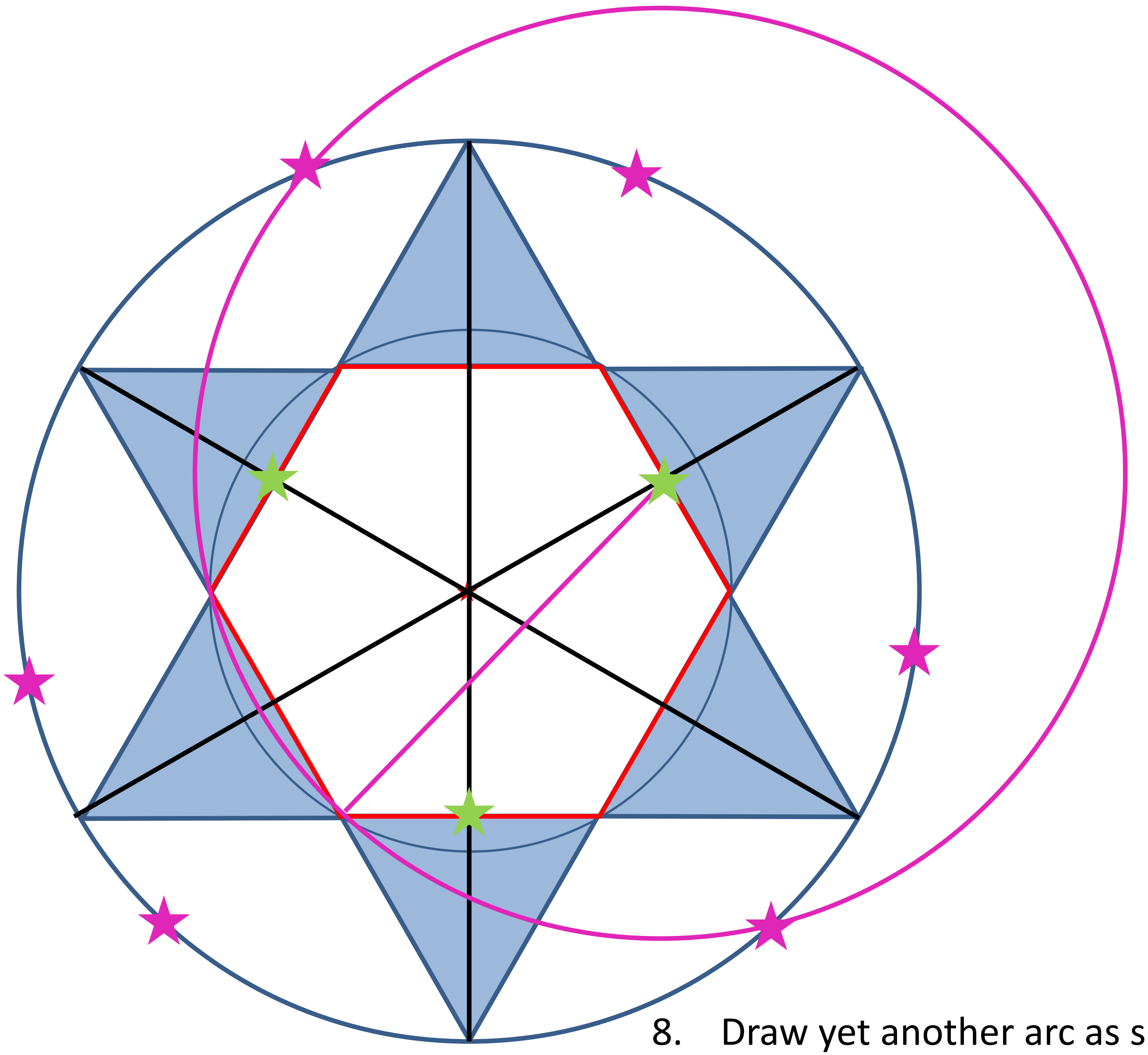


6. Draw an arc from the midpoints of the bottom edges of the hexagon as shown. Mark where this arc crosses the outer circle.





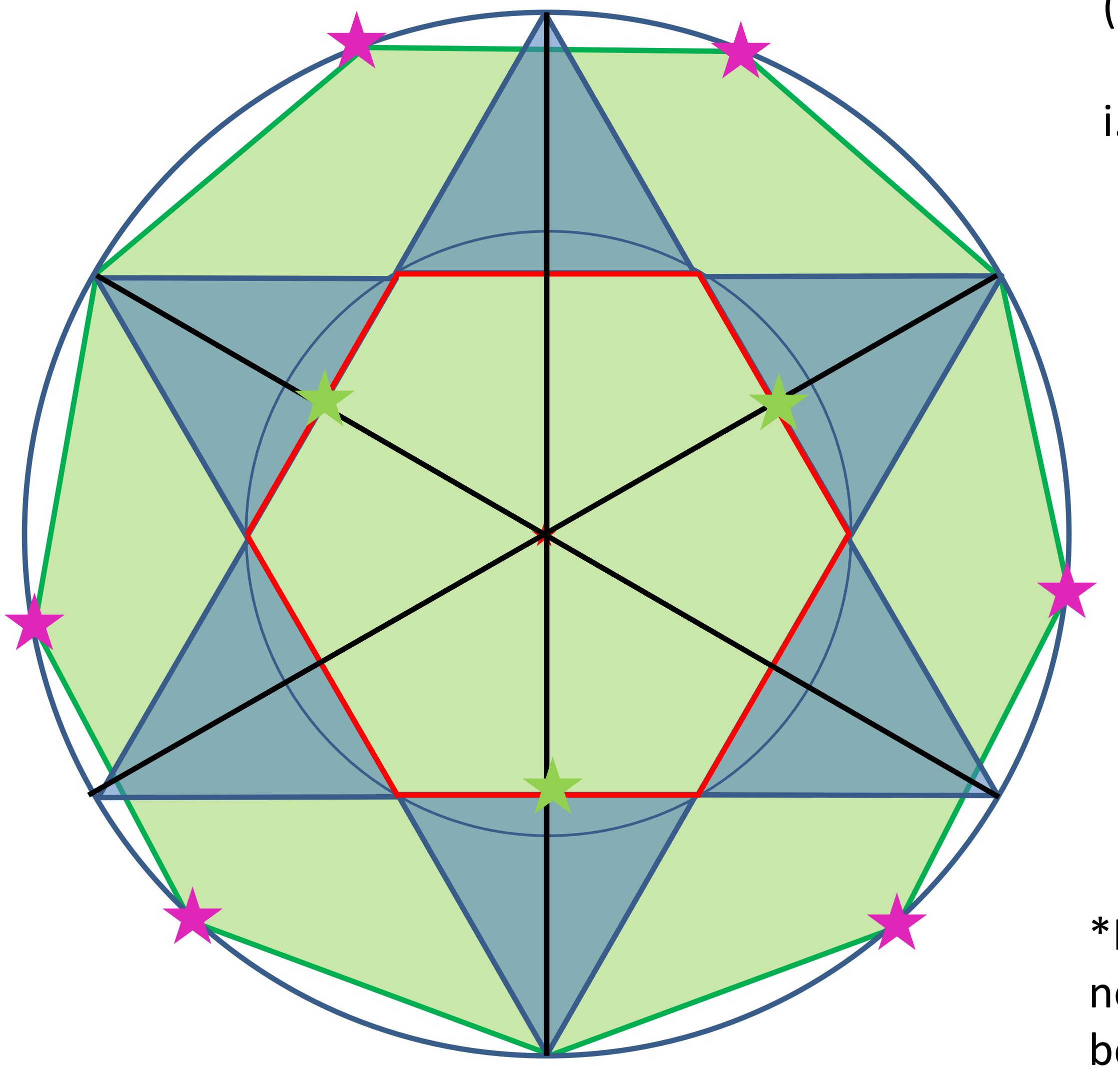
7. Draw another arc as shown and mark where this intersects the outer circle.



8. Draw yet another arc as shown and mark where this intersects the outer circle

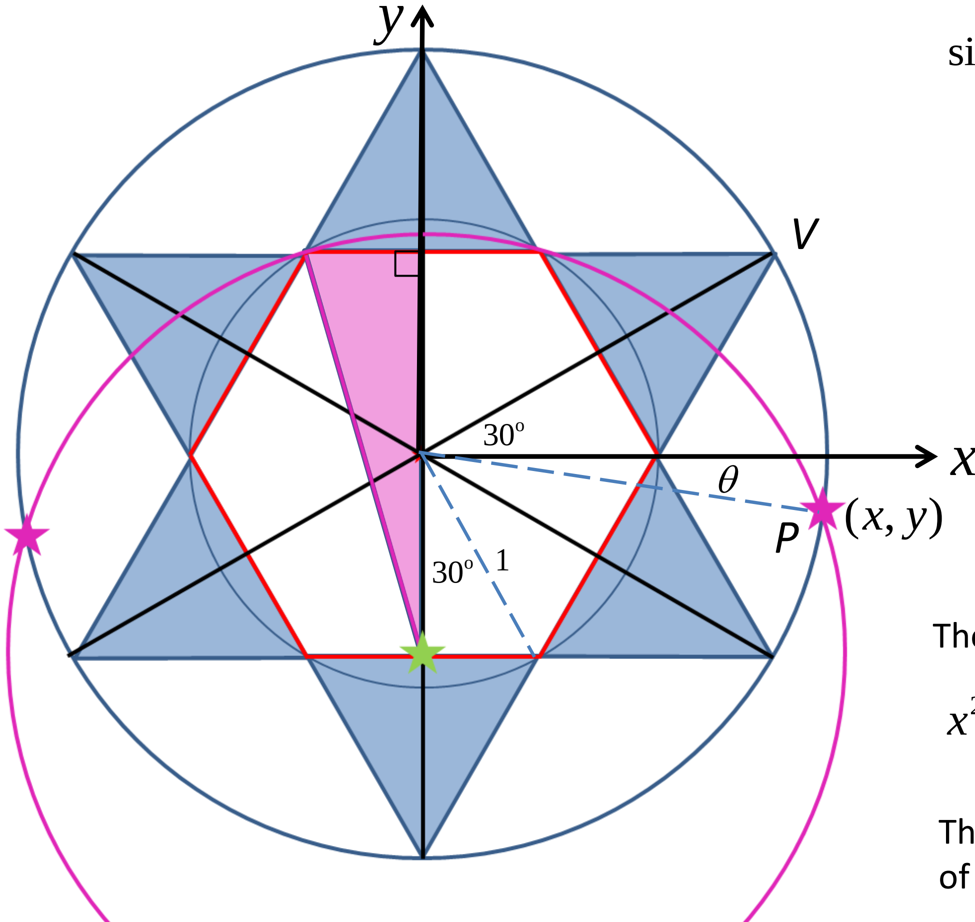
9. Connect up the arc intersections with three of the hexagram vertices to construct an (approximate) nonagon*

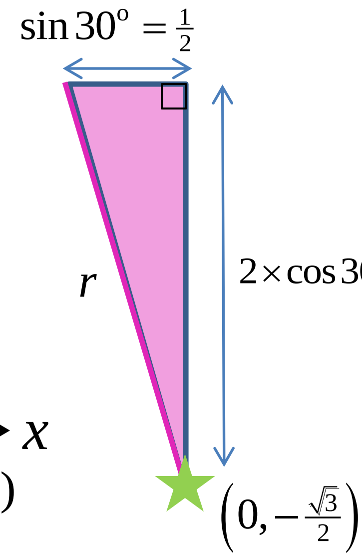
   i.e. a nine-sided regular polygon



*It is not quite a perfect regular nonagon. The edges will not exactly be the same size, but close enough!

Why the construction (nearly) works





$$r^2 = \tfrac{1}{4} + 3 = \tfrac{13}{4}$$

The purple circle has Cartesian equation

$$x^2 + \left(y + \tfrac{\sqrt{3}}{2}\right)^2 = \tfrac{13}{4}$$

The circle which circumscribes the hexagram has radius of $2\times\cos 30^\circ = \sqrt{3}$ and therefore has Cartesian equation

$$x^2 + y^2 = 3 \quad \therefore \; x^2 = 3 - y^2$$

Let P be at angle θ *below* the x axis. $\tan\theta = \frac{-y}{x}$

Hence angle between V and P from the origin is:

$$30^\circ + \tan^{-1}\left(\frac{\frac{1}{2\sqrt{3}}}{\frac{\sqrt{35}}{2\sqrt{3}}}\right) = 30^\circ + \tan^{-1}\left(\tfrac{1}{\sqrt{35}}\right) \approx 39.59^\circ$$

which is only 0.41 degrees out from the correct angle for a regular nonagon. $\frac{360}{9} = 40^0$

Intersecting the blue and purple circles....

$$3 - y^2 + \left(y + \tfrac{\sqrt{3}}{2}\right)^2 = \tfrac{13}{4}$$

$$3 - y^2 + y^2 + y\sqrt{3} + \tfrac{3}{4} - \tfrac{13}{4} = 0$$

$$y\sqrt{3} + \tfrac{1}{2} = 0$$

$$y = -\tfrac{1}{2\sqrt{3}}$$

$$x = \sqrt{3 - y^2}$$

$$x = \sqrt{3 - \tfrac{1}{12}}$$

$$x = \sqrt{\tfrac{35}{12}} = \tfrac{\sqrt{35}}{2\sqrt{3}}$$