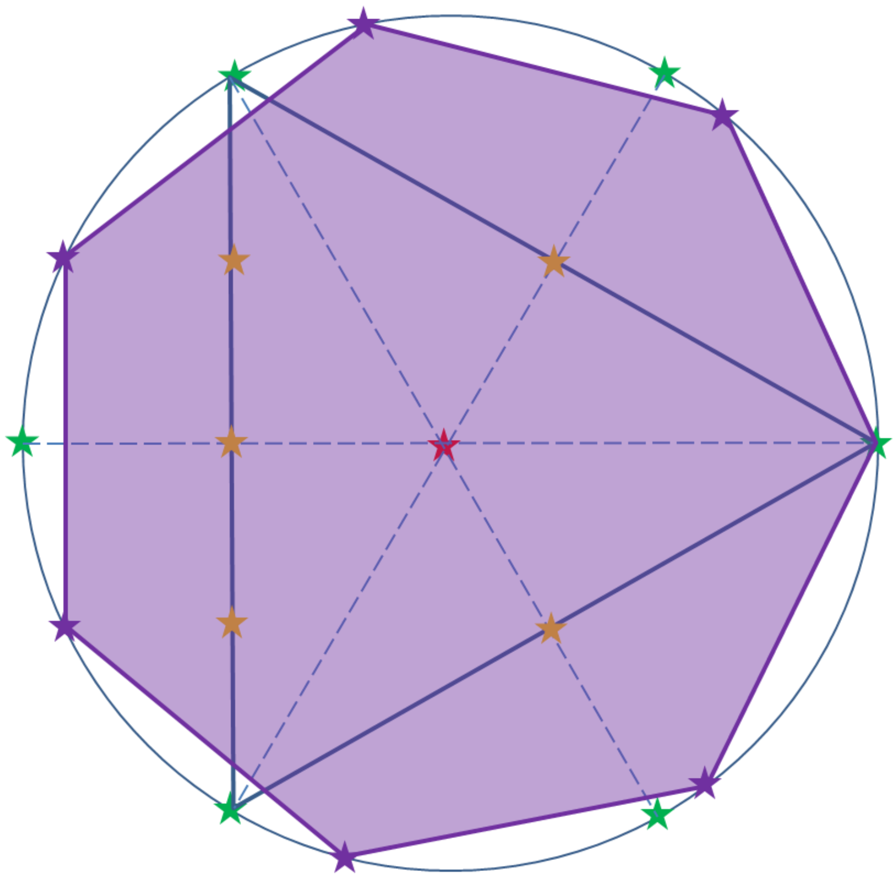
Construction* of a regular heptagon

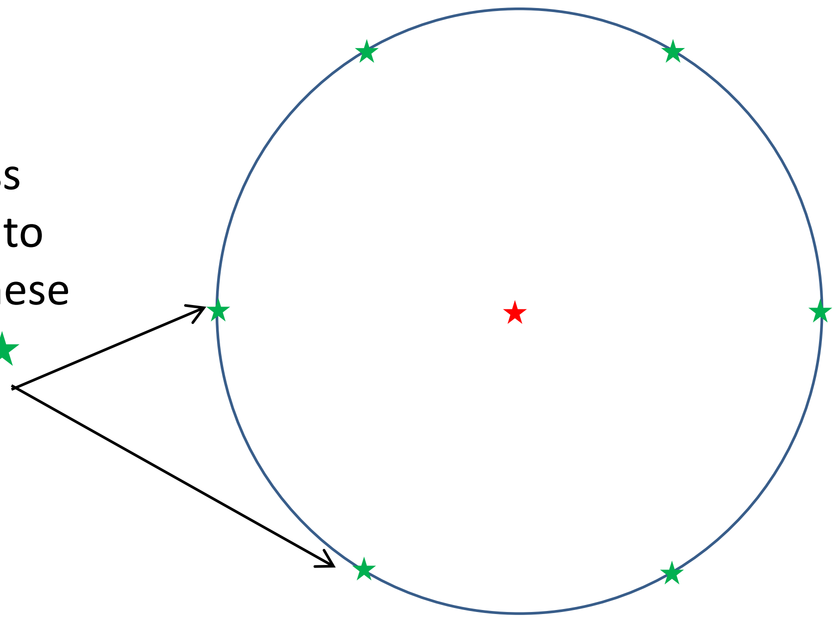
Dr Andrew French



*approximate

1. Draw a circle with a compass. Without changing the opening angle of the compass, ‘walk’ it round the circle and mark points on the perimeter, which will be the vertices of a *regular hexagon.*

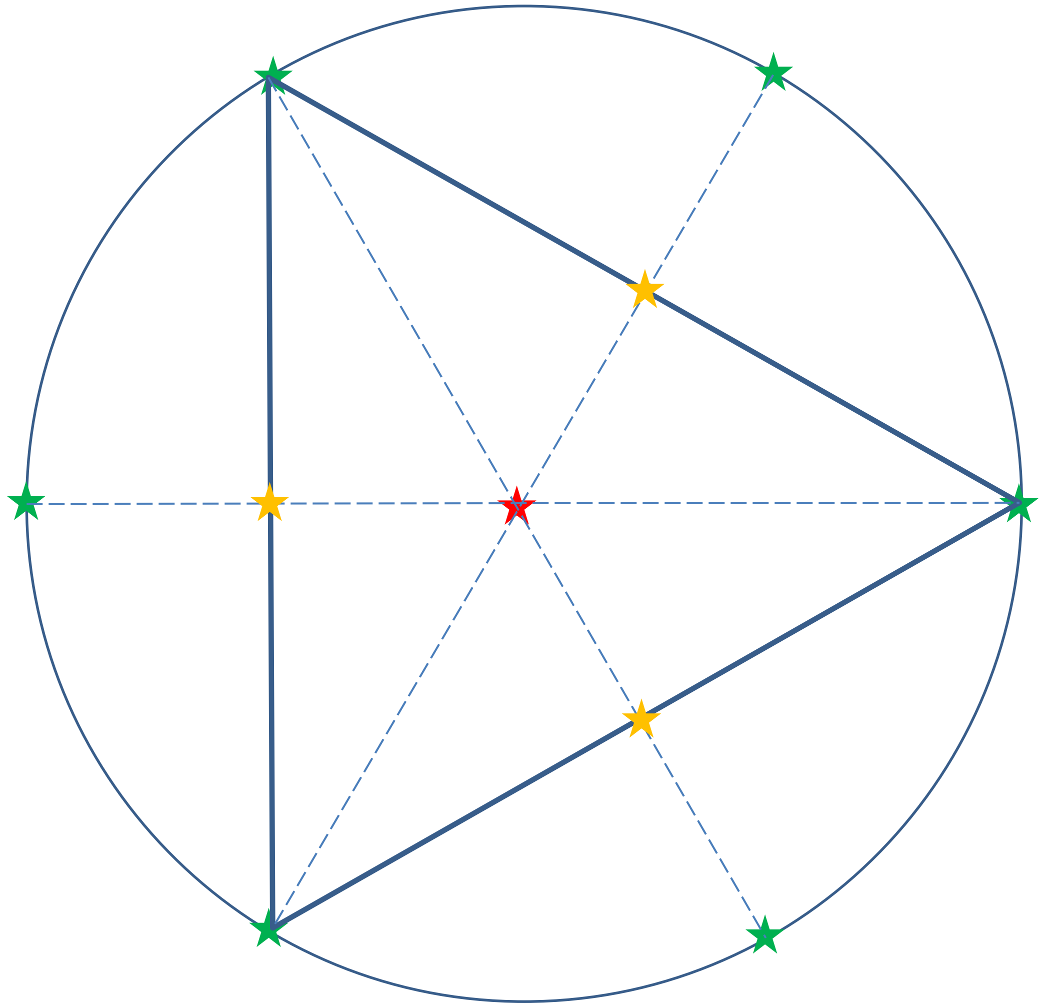
‘Walk’ compass around to mark these points ★



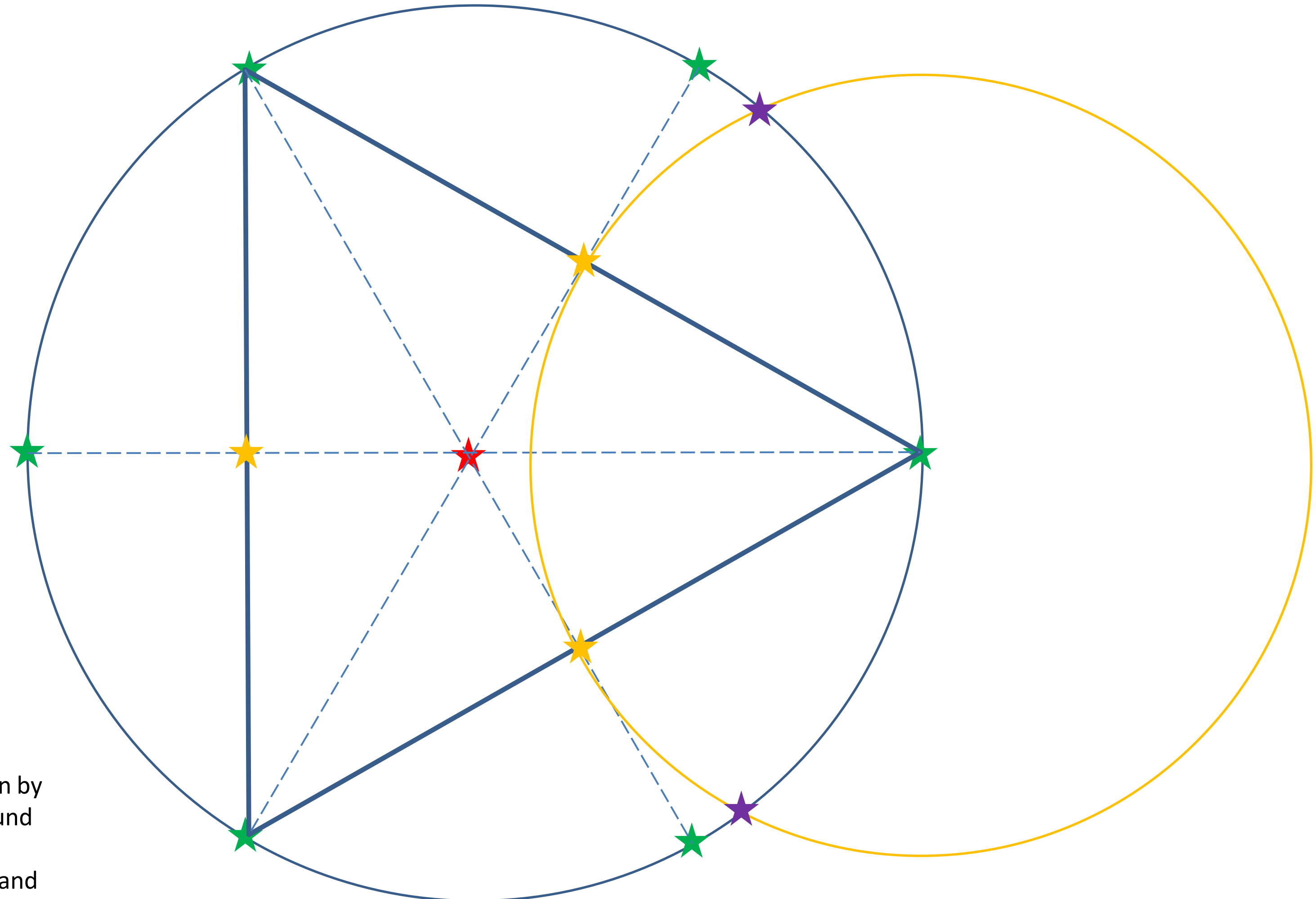
Keep the angle the same as used to draw the original circle

2. Connect up *alternate* marks to form an *equilateral triangle* circumscribed by the circle.

3. Connect the circle centre with the other marks to bisect the sides of the triangle.

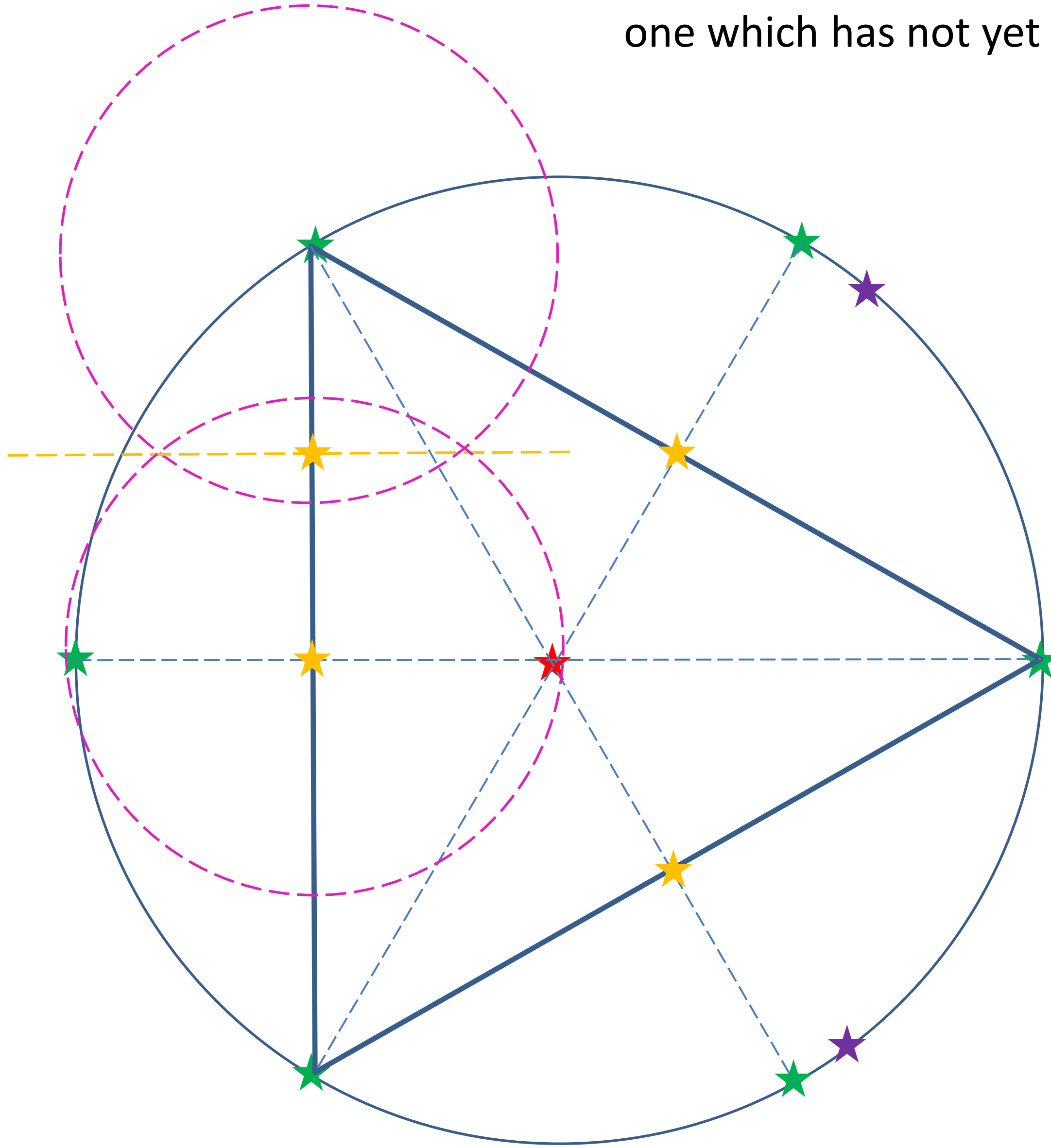


4. Draw an arc which connects one of the triangle vertices with both of the nearest triangle edge mid-points. Mark where this cuts the outer circle. ★

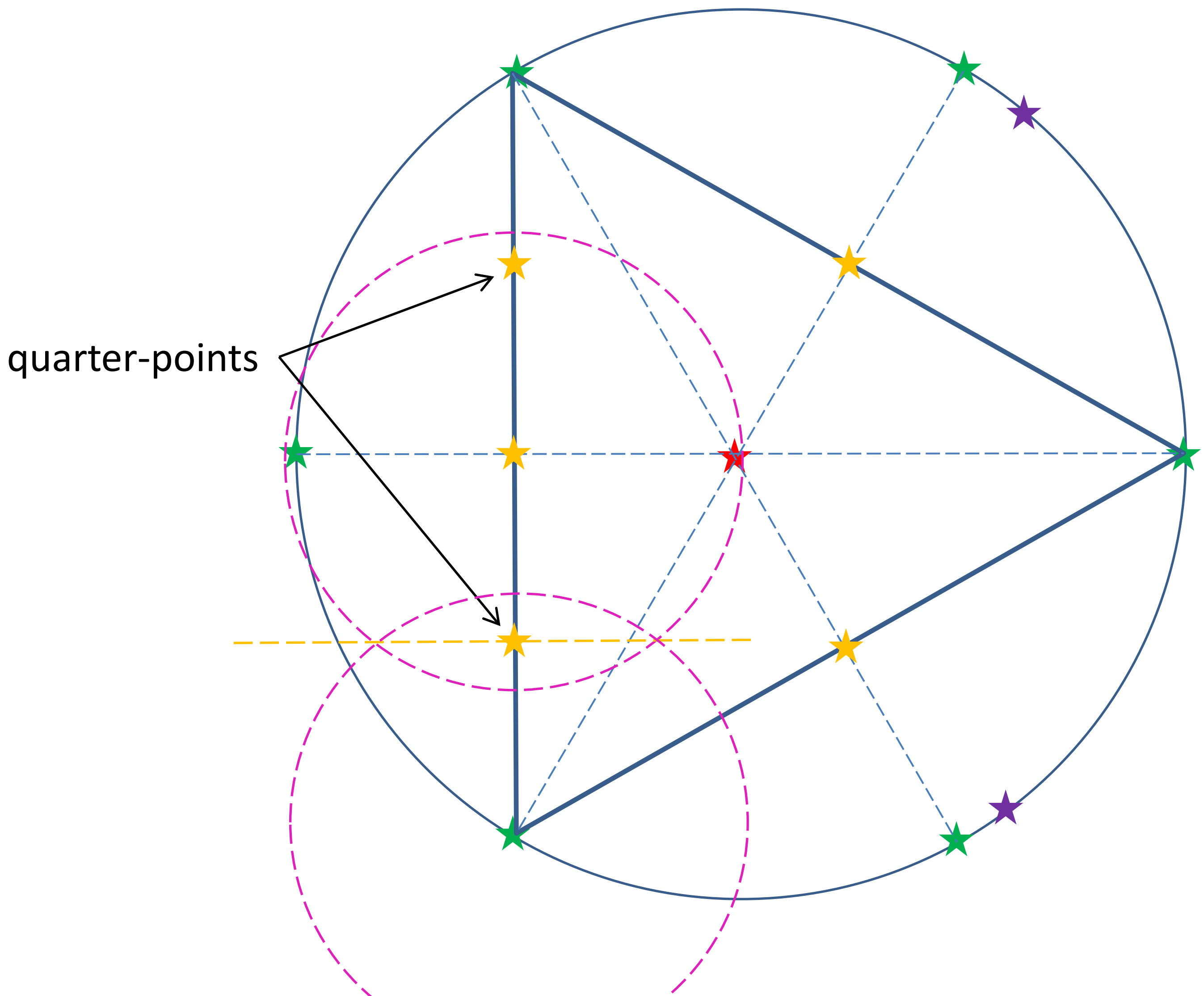


Note we could stop here
and construct the heptagon by
'walking' the compass around
using the green star at the
centre of the yellow circle and
the purple star where the yellow
circle cuts the blue circle.

5. Use perpendicular bisectors to construct the 'quarter-points' of the other edge, i.e. the one which has not yet been intersected by a ★

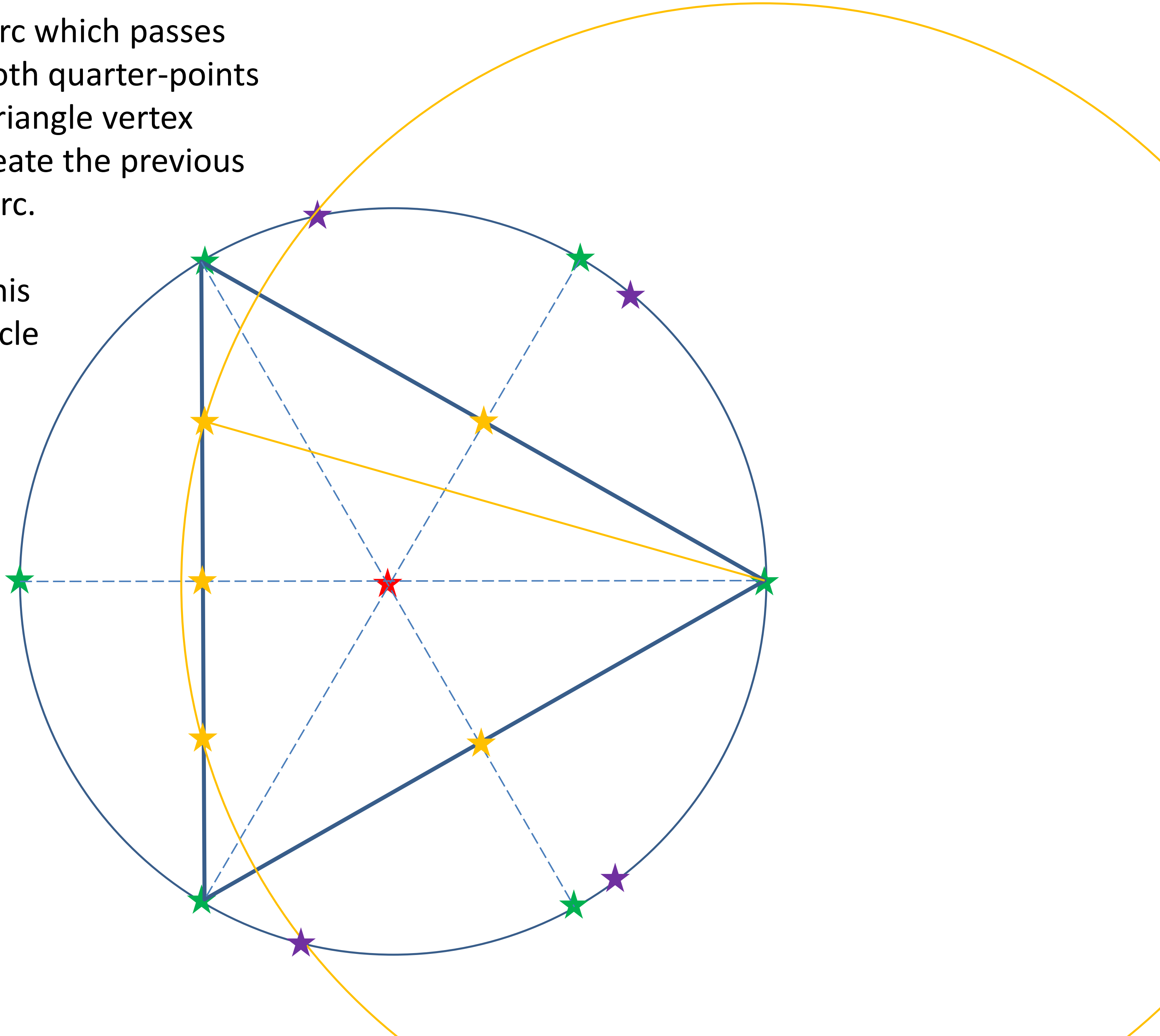


6. Use perpendicular bisectors to construct the quarter-points of the other edge, i.e. the one which has not yet been intersected by a ★

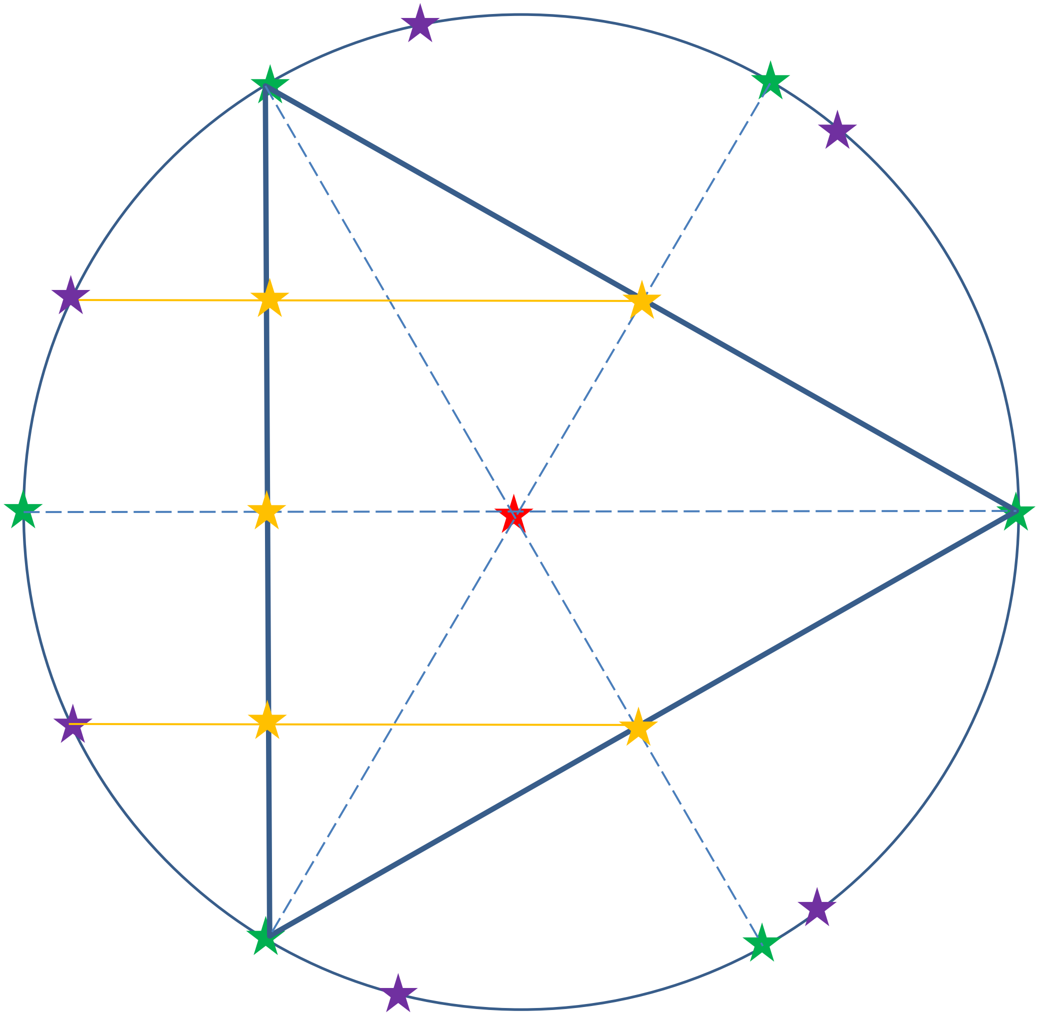


7. Draw an arc which passes through both quarter-points from the triangle vertex used to create the previous (smaller) arc.

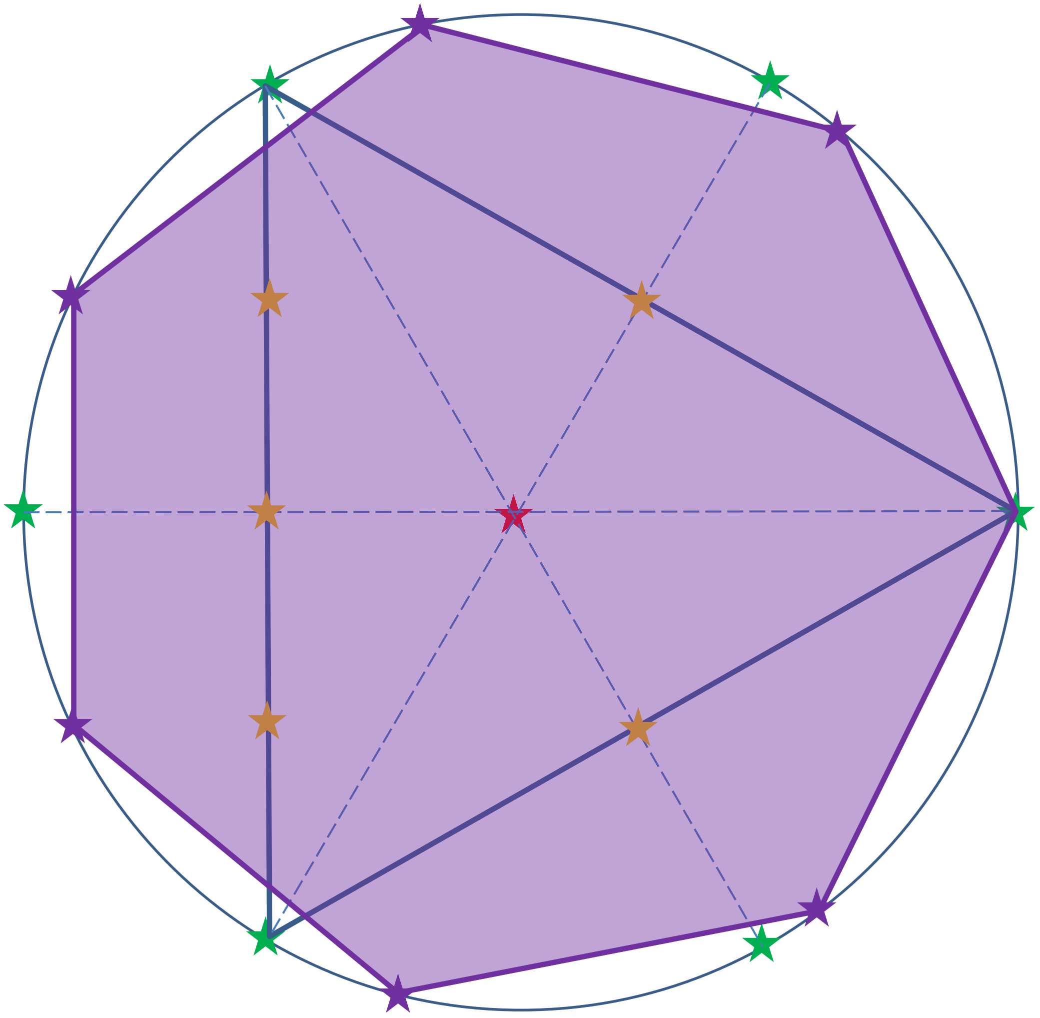
★ Mark where this crosses the circle



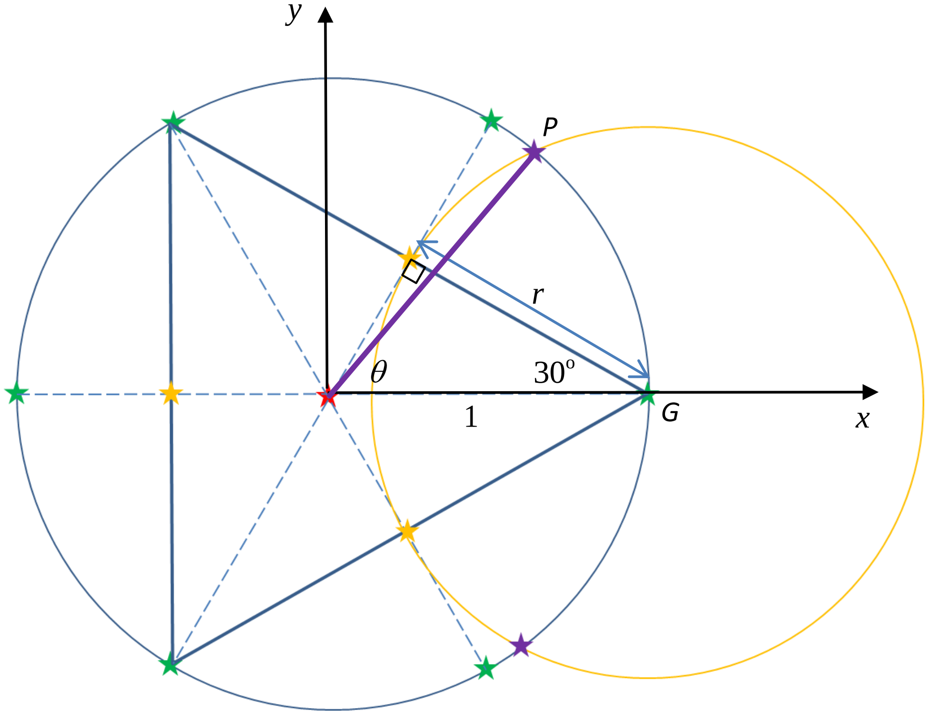
8. Connect the triangle quarter and mid-points with a straight line and mark where this intersects the circle.



9. Connect up with a ruler the marked points on the circle, plus the triangle vertex used for the arcs. You will form an (approximate) *heptagon* i.e. a seven-sided polygon.



Why the construction (nearly) works



For brevity, let us define the radius of the outer circle to be unity and let it be centred on the origin of an x,y grid

The outer blue circle has Cartesian equation

$$x^2 + y^2 = 1$$

The yellow circle has Cartesian equation $(x-1)^2 + y^2 = r^2$

From trigonometry, noting that the interior triangle is *equilateral* and hence perpendicular bisectors of its edges and (opposite) angle bisectors are the *same line*

$$\cos 30^o = r$$

$$\therefore r = \tfrac{1}{2}\sqrt{3}$$

Hence $(x-1)^2 + y^2 = \tfrac{3}{4}$

Let the purple star at *P* have coordinates (x, y)

$$x = \cos\theta$$

$$y = \sin\theta$$

(x, y) are the intersection points of the blue and yellow circles

$$(x-1)^2 + y^2 = \tfrac{3}{4}$$

$$x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$$

$$(x-1)^2 + 1 - x^2 = \tfrac{3}{4}$$

$$x^2 - 2x + 1 + 1 - x^2 - \tfrac{3}{4} = 0$$

$$\tfrac{5}{4} - 2x = 0$$

$$\boxed{x = \tfrac{5}{8}}$$

Hence $\boxed{\theta = \cos^{-1}\tfrac{5}{8} \approx 51.32^o}$

Now if *P* and *G* are vertices of a regular heptagon (as in the construction)

$$\theta = \frac{360^o}{7} \approx 51.43^o$$

Hence the construction is not perfect, but the error is only of the order of a tenth of a degree.